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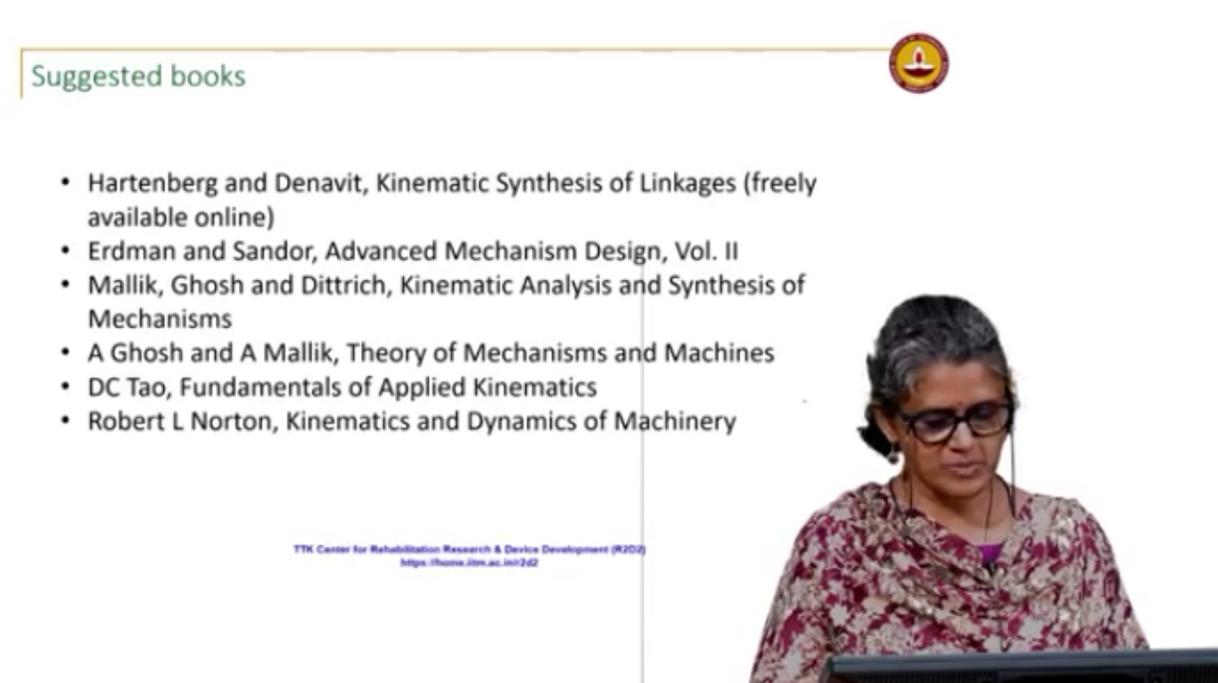
Theory of Mechanisms

Review of Kinematics Fundamentals-1

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So welcome to the course, some of the books you will be familiar with one or two of them from (Refer Slide Time: 00:22)



Suggested books

- Hartenberg and Denavit, Kinematic Synthesis of Linkages (freely available online)
- Erdman and Sandor, Advanced Mechanism Design, Vol. II
- Mallik, Ghosh and Ditttrich, Kinematic Analysis and Synthesis of Mechanisms
- A Ghosh and A Mallik, Theory of Mechanisms and Machines
- DC Tao, Fundamentals of Applied Kinematics
- Robert L Norton, Kinematics and Dynamics of Machinery

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the undergraduate course Norton, Kinematics and Dynamics of Machinery. And also Ghosh and Mallik, Theory of Mechanisms and Machines. We will be using Hartenberg and Denavit which is freely available online, so I have uploaded the PDF on module.

Okay those of you who have done anything in Robotics will be aware of the Denavit Hartenberg convention for designating coordinate frames they are very famous for that. This book is on the kinematics synthesis of linkages and I have uploaded it on module, so most of my figures and all that because this is being recorded I cannot use copyrighted material, so most of my figures will be from that book or figures that we've created.

Then Erdman and Sandor is also a very good book for the analytical synthesis, so the analytical synthesis portions will come from Erdman and Sandor volume 2 which is advanced mechanism design, and then Mallik, Ghosh and Dittrich also has a very good treatment of synthesis of mechanisms and also some advanced analysis that we will be looking at.

DC Tao is another old book which has some very nice examples and also fundamentals of kinematics it's a good book to refer to. I think all the other books are available in the library other than Hartenberg and Denavit which you will have access to on module.

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Prerequisites



- Use a compass and ruler to perform basic geometric constructions
- Perform basic vector algebra and manipulate trigonometric quantities
- Define displacement, velocity and acceleration of points and of rigid bodies
- Represent vectors as complex numbers and vice-versa
- Use the chain rule for differentiation
- Draw free-body diagrams
- Use MATLAB, SciLAB or other mathematical programming tools

So on module I have also put together some of the course information, so just some of the prerequisites for the course, you know being able to do basic vector algebra, complex numbers representing vectors as complex numbers, things that you would have done in your KDoM course drawing free-body diagrams, using MATLAB, okay so these are some of the required elements for the course.

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Learning Modules



1. Review of kinematics fundamentals - degrees of freedom, mobility, joints, links
2. Kinematic synthesis of mechanisms for motion, function and path generation
3. Review of complex numbers and its application to mechanism design and analysis
4. Analytical synthesis of mechanisms, Burmester theory
5. Kinematic synthesis of other planar mechanisms: coupler curves and cognates
6. Kinematic analysis of planar mechanisms
7. Force analysis of mechanisms
8. Balance linkages using counter-weights or springs
9. Introduction to the analysis and synthesis of spatial mechanisms

So these are what I envision as the learning modules for the course, we will do a short review of kinematics fundamentals, I don't know how many of you remember from 2 years ago, and then kinematics synthesis we will do both, we will first do the graphical synthesis, then we will review complex numbers and use that for analytical synthesis and the Burmester theory, large portion of that is from the Erdman and Sandor book.

Then we will also look at synthesis of mechanisms with coupler curves and use of cognates which are alternatives for the same coupler curve. And then we will look at kinematics analysis but we will look at some more advanced techniques, so techniques which few techniques beyond what you have learnt in your undergraduate course. A few graphical techniques maybe some analytical techniques as well, force analysis we will do that and then we will look at balancing of linkages using counter-weights and springs, so this is the pad that I'm trying to somehow fit in earlier so that you can use that for your project as well, and then I'll introduce you, so time permitting we will do an introduction to the analysis and synthesis of spatial mechanisms.

Many of you may take a robotics course later on, so at least the serial type of mechanism, manipulators are dealt with the robotics courses, we will introduce some you know close chain mechanisms, spatial mechanisms in this course.

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1. Review of kinematics fundamentals - degrees of freedom, mobility, joints, links
 - Classify joints as higher and lower pairs and determine their connectivity
 - Classify links based on their nodes
 - Draw kinematic diagrams of real-life mechanisms
 - Determine the mobility of mechanisms
 - Construct kinematically equivalent mechanisms
 - Construct and describe the motion of inversions of a kinematic chain
 - Perform number synthesis of linkages

So in the syllabus that I have posted online you will have access to learning outcomes for each of these modules, so it will tell you what are some of the things you should be able to do that I can test you on, okay, things that you should be able to do when you complete a learning module, for instance the first one you have to be able to classify joints when you do the review, classify links, draw kinematic diagrams, construct kinematically equivalent mechanisms, determine mobility etcetera, number synthesis of linkages is also something that we would do, that we have not done before, okay, inversions of kinematic change, so these it gives you an idea of what is expected in each learning module, okay.

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- Kinematics - study of motion without regard to forces
 - Motion described by position and its time derivatives
 - Velocity
 - Acceleration
 - Jerk
- Dynamics - Study of forces on systems in motion
- Kinematics - ensure the functionality of the mechanism
- Dynamics - verify that the parts can withstand the induced forces

Okay, so let's now cover some preliminaries, in this course at least for the majority of the course we will be dealing with the kinematics, so we are going to look at, so what is a mechanism? Mechanism essentially transforms motion from one form to another, and if you look at it, it could be transforming uniform motion to another uniform motion which is for example what gears or chains you know those sort of rope and pulley those sort of things transform uniform motion into uniform motion.

In our case with linkages we are actually looking at non-uniform motion, the transformation of a uniform motion into a non-uniform motion or vice-versa, so we are looking at, we will be mainly focusing on linkage type mechanisms in this course, so when we talk about machines we are also talking about transfer of forces, transfer of energy from, but the majority of the course when we look at design, the design will be based on the kinematics, based on the geometry of the mechanism, how do you transform one type of motion into another, how do you perform tasks based on the kinematics.

So when we talk about kinetics that's when we talk about the forces that are acting, kinetics are the dynamics we talk about the forces that are acting on the systems and motion, and that would be something you have to do, so the kinematic design is the first part of the design, to ensure that you meet your geometrical requirements, then the next step always is to do a dynamic analysis unless your application is such that the forces, the inertial forces that arise out of the motion can be neglected, okay, but in most cases or you know the forces are not significant enough that you're sure that your design will work as this, in most cases though the kinematic analysis is followed by a dynamic analysis to see what kind of forces the links experience, because even if it is a static or quasistatic problem, when you actually the physical links have to be able to withstand forces, so in order to do that you would do a force analysis and that would be followed by a stress analysis you know when you choose, you would look at you know if I use a certain material will it be able to withstand the stresses and deformations that are imposed

on the design and so on, so it's an iterative process, so the kinematic aspect of the design is the first thing that we do, that typically is followed by the dynamic analysis, so kinematics ensures the functionality of the mechanism, dynamic ensures that it can withstand whatever forces are imposed on the mechanism.

When we talk about kinematics we are only looking at the position and its time derivatives, we are looking at the geometrical aspects of the motion. And synthesis is basically coming up with a design, analysis is once you have a design, you want to see whether it performs the way it was intended, so that aspect of it is analysis synthesis is coming up with the design.

So coming up with the design is a very creative process also, okay, and you could perform the same task in a number of different ways, mean I think all of you as mechanical engineers would have been exposed to a number of different designs by now, you would have seen in your various courses what sort of elements are there to put together designs, in this particular course we will look at designs that involve linkages, okay, that involve some kind of mechanism and in the mechanism specifically linkages, I'll explain to you the difference between to shortly because you could do the same thing in a number of different ways, so we are just going to do what is normally called dimensional synthesis, so assuming that a linkage is what we want for our design, we say okay, how do I determine the dimensions of that linkage to accomplish my particular task.

So there are some terms we need to become familiar with,
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Kinematic pairs



- Nature of connections between the rigid bodies determines how they move with respect to one another
 - Different joints or pairs permit different types of relative motions

when we are talking about a creating linkages, so a link is basically just a rigid body, now why do we say a rigid body is an idealization, but it's fairly reasonable, it's a fairly reasonable assumption to make if the deformations in the links, if the deformations in the rigid body are much, much smaller than the motion that it undergoes, okay, so when we are talking about

motion transformation, you are going to be looking at how is, so there will be, in reality there is no such thing as a rigid body, okay, everybody deforms under the action of forces, but we make the assumption that in an ideal case, so that means in a rigid body the distance between any two points does not change, so that is an assumption that we will make.

You can do a kinematic, you can sort of separate the kinematic and dynamic analysis only when you make this rigid body assumption, because if you don't have a rigid body then the forces that you apply to that body are going to deform that body which is going to affect your output, say you try to design flexible mechanisms, you may have heard the term compliant or flexible mechanisms, when you go to do that you cannot do a separate kinematical analysis and then come back and do a dynamic, it doesn't work that way because the dynamics effect the geometry of the mechanism.

So kinematic pairs are basically the types of connections, so you have rigid bodies to make a mechanism we connect these bodies in certain ways to create the mechanism, so the nature of the connection between any two bodies determines how they move with respect to one another, okay, and usually the classification is based on what kind of motions are permitted, so when I join two things, okay, if I join two things and glow it together, then there is no motion permitted, so it essentially becomes one rigid body, okay.

In our case we are looking at moveable mechanisms, if I start attaching everything, glowing things together then you know I'm left with nothing, it's all one rigid body then, then here we are going to look at, we are trying to transmit motion from one link to another, so we are going to look at connections that will enable that kind of a transfer, so that will permit some motion between the links that are joined, so joined does not mean a rigid connection, so connection and we look at the connection based on what kind of relative motion is allowed between the links.

So depending on what degrees of freedom are retained,
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Kinematic pairs



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 - Different joints or pairs permit different types of relative motions
- Kinematic pairs are broadly classified on the basis of the particular dof(s) that are retained



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kinematic pairs can be classified based on that, you know they could be a one degree of freedom, two degree of freedom etcetera, joint or they also have specific names which will give you a clue as to what is the relative motion permitted when you have that kind of a connection between the two rigid bodies, so revolute pair, when somebody talks about a revolute pair it means the two bodies can rotate with respect to one another, okay.

And very specifically if you look at the degree of freedom of a kinematic pair,
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Kinematic pairs



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- Kinematic pairs are broadly classified on the basis of the particular dof(s) that are retained
- DOF of a kinematic pair is the number of independent coordinates required to completely specify the relative movement. Also called connectivity



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then it is the number of independent coordinates required to completely specify the relative movement, okay, it's also called in the case of joints it is also called the connectivity of a joint, you say a joint of connectivity 1 or connectivity 3 that means those are the number of degrees of freedom that are allowed by the joint, and it's very important that you remember that it is the number of independent coordinates to specify that the relative motion.

And the coordinates, the independent coordinates are called the pair variables, (Refer Slide Time:15:37)

Kinematic pairs



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 - Different joints or pairs permit different types of relative motions
- Kinematic pairs are broadly classified on the basis of the particular dof(s) that are retained
- DOF of a kinematic pair is the number of independent coordinates required to completely specify the relative movement. Also called connectivity
- Coordinates are the pair variables

Lower pairs

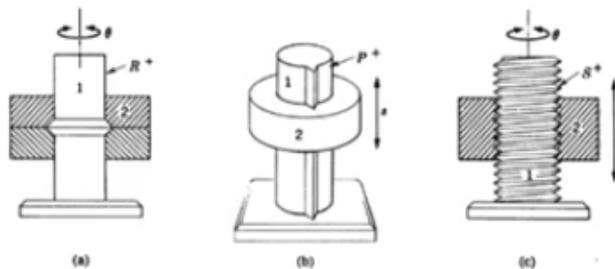


Figure courtesy Hartenberg and Denavit, Kinematic Synthesis of Linkages

I'll explain, so if you have two bodies 1 and 2, and 1 can only rotate with respect to 2, okay, you can see here that this projection ensures that one cannot translate with respect to 2, so since 1 can only rotate with respect to 2 this is the pair variable, theta is the pair variable for this kinematic pair which is also called the revolute pair, okay.

Then if you look at this one, you should have all seen this before this is just, what is known as a prismatic pair, so you can see one cannot rotate with respect to 2, it's prevented from rotating with respect to 2, but there is sliding that happens, the relative motion between 1 and 2 is that of sliding, this is called a screw pair, and the revolute in prismatic pairs are actually special forms of the screw pair. So in the screw pair as this nut rotates, (Refer Slide Time: 17:08)

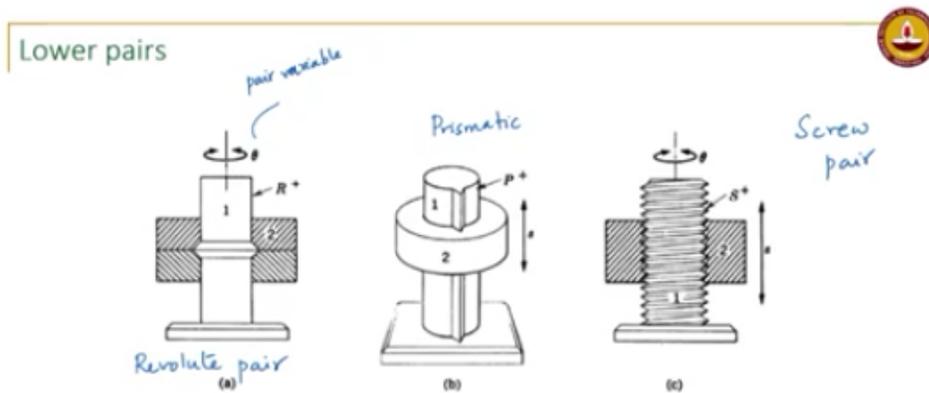
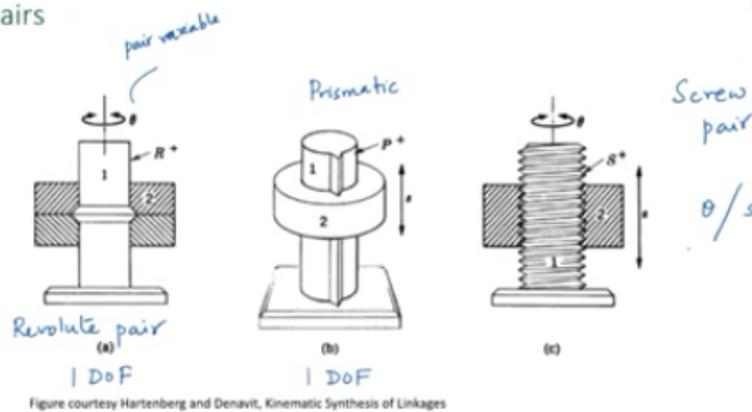


Figure courtesy Hartenberg and Denavit, Kinematic Synthesis of Linkages

it's also translating with respect to 1, so if you take 1 as the bolt and 2 as the nut you have motion that is both, there is theta as well as S involved here, right, so there is, so with the revolute pair how many degrees of freedom do you have? You have one pair variable you have a one degree of freedom system, similarly with the prismatic pair, your pair variable is the translation with respect to some reference you say that it has translated so much, this is again one degree of freedom joint.

What about the screw pair? Again even though the pair variable could be theta or S, because the two are related by the lead of the screw, okay which is inbuilt, (Refer Slide Time:18:10)

Lower pairs



so it is not that two are not independent, so either of these can be the pair variable, but the two are not independent, so if S is 0 that is the case of the revolute joint, $\theta = 0$ is the case of the prismatic joint which means you have a lead of either 0 or infinity which denotes the revolute or the prismatic joint, okay, the lead is 0 then that is the revolute joint, there is no translation happening, if the lead is infinity that means you have a prismatic pair. So in screw theory they use this kind of a notation, so the screw pair is a common notation that can be used to designate all three types of joints, so this is also a one degree of freedom joint.

If you notice all these have surface contact and those are typically called lower pairs, so joints which have surface contact are called lower pairs because you have lower contact stresses that come into play as the two links move with respect to one another,
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Lower pairs

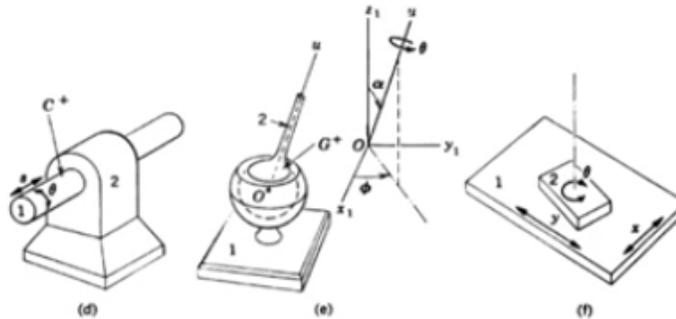


Figure courtesy Hartenberg and Denavit, Kinematic Synthesis of Linkages

more lower pairs you have in this case unlike the screw joint you have what is known as a cylindrical pair, and here the two motions are independent, you can have rotation which is independent of the translation, okay, so that in the case of the screw that was not the case, the rotation was dependent on the translation or vice-versa.

In the case of this spherical which is referred to as the globular pair, you can see you have like a ball inside a socket and that is, that pretty much are the translation, no translation is possible with a spherical, with a cylindrical pair it's a two degree of freedom joint, (Refer Slide Time:20:50)

Lower pairs

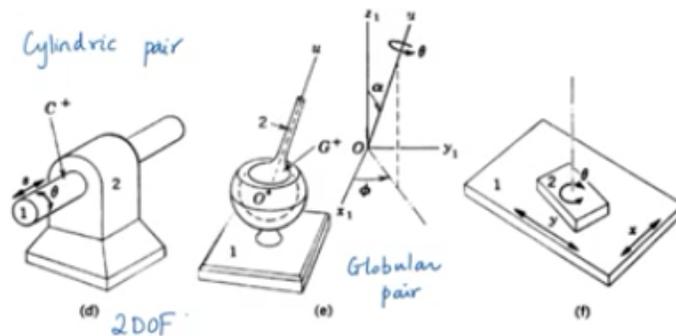


Figure courtesy Hartenberg and Denavit, Kinematic Synthesis of Linkages

the globular pair is a three degree of freedom joint which are all the three degrees of freedom are all rotational degrees of freedom.

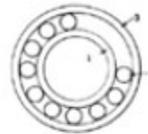
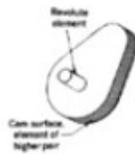
And then you have what is known as a planar joint, that's also a lower pair where if you have a block, moving on a surface so that can translate as well as rotate with respect to that plane, can you think of an example that you may have come across that involves a planar pair? Duster on the board yes, so duster on the board that's an example of a planar pair, something else in a mechanism that you have all used, first year drafter, you drafter the scale that is an example of a planar pair, this is a planar pair and this has 3 degrees of freedom.

You can have what are known as higher pairs,
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Higher pairs



- Cylindrical roller
- Cam pair
- Rolling ball



where you may have say a cylindrical roller, okay assume this is a cylinder, if I'm moving this on a, rolling this on a plane, okay, if it is pure rolling then again the rotation is related to the translation, so again it's a single degree of freedom, but it's called a higher pair because you only have line contact, okay, so when you don't have surface contact those are called higher pairs typically they have line or point contact.

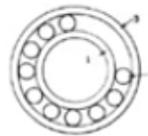
You may have a cam pair, so cam surface like this which is say in contact with either a rod or a roller you know, and again you have line contact and that is also called a higher pair, so if I represent it in 2D I could have something like this, I could have rod that's in contact with the cam, so this here, so this connection here,
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Higher pairs



- Cylindrical roller
- Cam pair
- Rolling ball

line contact



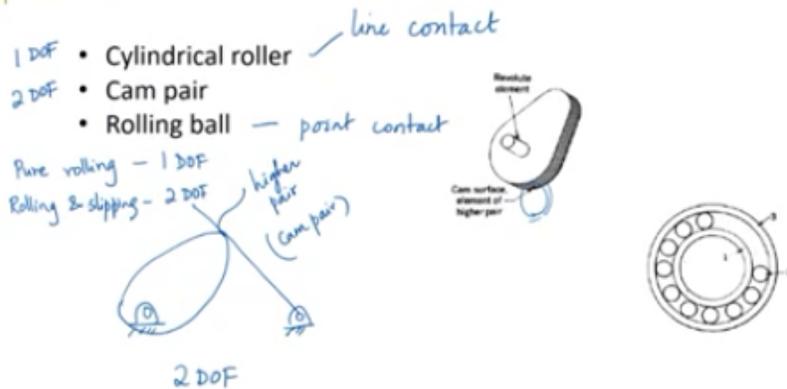
so you see that this is different from the typical connection that we think about, this connection here is a higher pair, this is a cam pair.

And how many degrees of freedom does it have? A cam pair has, it can have independent sliding as well as rotation, so I can rotate this which causes, so this is again a two degree of freedom kinematic pair, cam pair is to cylindrical roller if it is pure rolling, then I only have it's a one degree of freedom, cam pair is 2 degrees of freedom.

Rolling ball again if it is pure rolling you could have one degree of freedom and it's a point contact, rolling ball is a case of point contact and pure rolling implies one degree of freedom, otherwise if there is rolling and slipping it is 2 degrees of freedom.

Here is an interesting case where you have this link 1,
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Higher pairs



and this is a ball bearing, okay, you have a bunch of balls here moving in that race and then connected to another element 3 which is, so if you look at the connection between 1 and 2 that's a higher pair, okay, if you look at the connection between 2 and 3 again rolling, you have a higher pair but overall this joint typically if you look at the joint between 1 and 3 you're only looking at a revolute pair, okay, you're only looking at that, so the actual physical nature of the joint may not matter, it maybe be, you have to look at what is the motion allowed between the links of interest, okay. Sometimes you will substitute higher pairs by lower pairs, (Refer Slide Time: 26:06)

Substitution of Higher pair by Lower pairs

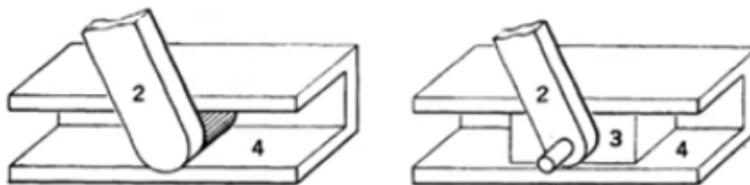


FIGURE 2-6 Substitution of two lower pairs for a higher pair; note the additional link.

Figure courtesy Hartenberg and Denavit, Kinematic Synthesis of Linkages

so in this case you have a link, this link to has a pin attached to it which kind of rolls and slides in this slot, okay.

So what kind of a kinematic pair is that between link 2 and 4? Higher pair, which specific higher pair you would call it? Cam pair, so anytime there is rolling and sliding you'd just call it a cam pair, that's a cam pair and this is 2 degrees of freedom, okay, so if you look at the motion of 2 relative to 4 it has 2 degrees of freedom, now you can replace, so this is sort of the low cost way of doing it, I'll tell you why, so you can replace this by separating out the rolling and the sliding, okay, so you can have a revolute joint with a block, so you put a block in between which can only rotate, so the 2 can only rotate about 3, and 3 can only slide with respect to 4.

The reason the one on the left is use, so again now by adding this extra link between 2 and 4 you still have 2 degrees of freedom, but you have added an extra link and between, what happens is making these sort of, this joint, achieving this joint between 3 and 4, a prismatic joint, achieving a good prismatic joint is difficult, it's manufacturing wise it's a difficult task to get the tolerance right for smooth motion, so a low cost version of something that requires that kind of an arrangement would use the pin and this, so but you can substitute a higher pair with lower pairs, okay, and the remaining different ways of constructing the kinematic pair, the actual physical construction will be different, so if you look here this is, you may have one link that shape like this,
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Kinematic pairs



- Many different ways of physically constructing a kinematic pair
- However, only the allowed relative movement decides the type of kinematic pair

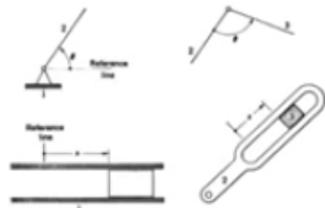
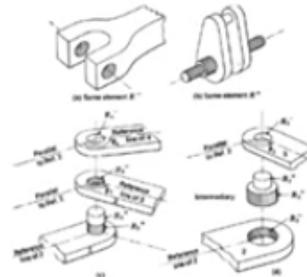


FIGURE 2-2 Conventions for showing pair variables.
Figure courtesy Hartenberg and Denavit, Kinematic Synthesis of Linkages



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another link with a pin going through that is your hinge joint, okay, that would be the pair that.

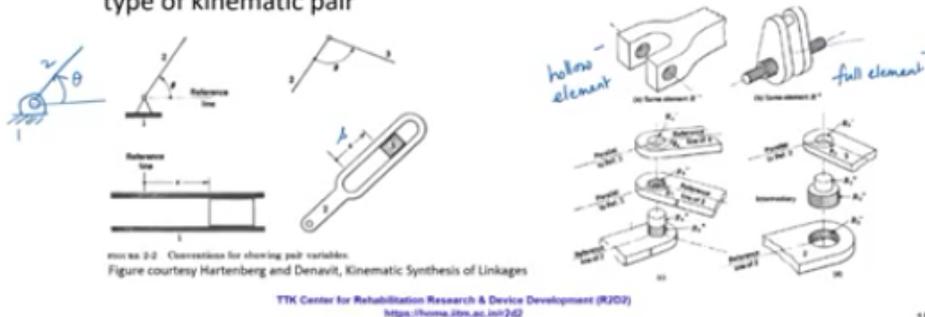
Similarly you may have one link, so this is called the projecting element is called the full element, so if you see here it uses a lot of pluses and minuses here, you see R^+ , R^+ this element is the full element of this revolute joint, okay, full element is denoted with that super script plus and the hallow element is denoted by minus, so the hole into which the pin goes that is denoted as R^- , for the revolute joint it's R^- this is R^+ .

And if you have more than one link, so you could have the same, in the physical construction you could have one pin onto which you attach two different links, so they're actually 2 revolute joints, so here you would have, so if you have R3 and R4, you would have an R3+ and R4+ physically they are on the same link, but you would count them as two different joints, because to specify the position of link 3 with respect to link 2 you need one variable, to specify the position of link 4 with respect to link 2 you would need another variable, so for revolute joints are typically denoted by a circle like this, and if it is attached to a fixed link something like this or this triangle this shows that, that link 1 is fixed and with respect to some reference the pair variable will be specified. Or if you have some other you know, it need not always be with reference to a fixed link, it's pair variable with respect to the other link so theta is the pair variable for 3 with respect to 2 or vice-versa. Similarly for prismatic joint you have S as your pair variable for this.
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Kinematic pairs



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- However, only the allowed relative movement decides the type of kinematic pair



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