Surrogates and Approximations in Engineering Design
Prof. Palaniappan Ramu
Department of Engineering Design
Indian Institute of Technology, Madras

Lecture – 09
Types of DOE – 2 and some examples

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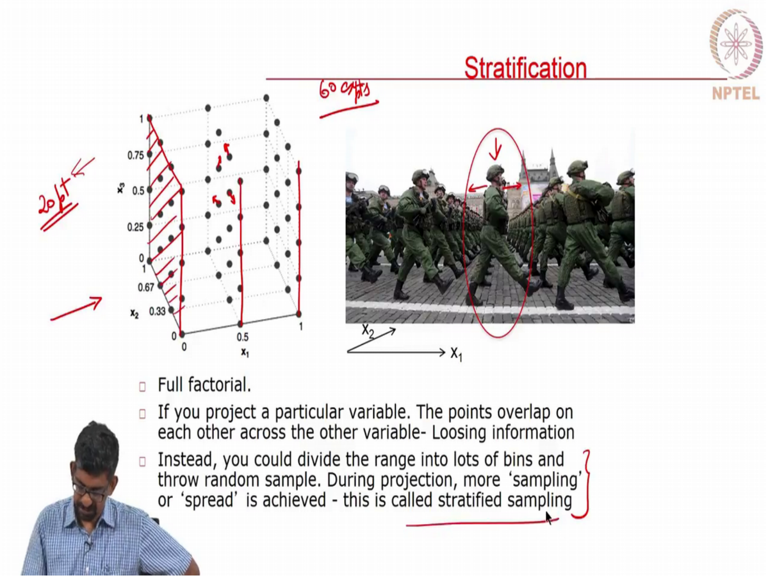

With that idea there is something called a stratification which is what people would like to do. There is a problem with full factorial experiment, ok. I practically want this to be animation, but that is ok. So, the deal is if you take three variable cases, just look at this is very similar to what I just showed you just now that inscribed circle.

So, now take this x 1, and you can see that there are about at this 0.5, there are 1 2 3 4 5, ok. Similarly at this 0, there are 5 dots and at 1, there are 5 dots, ok. So, if you are viewing from this side meaning this particular plane, this plane x 2 x 3, this particular plane if you are looking at it if you are viewing from this side, what information are you getting with respect to x 1; no, the x 1 value is fixed at 0, but you are getting 5 different values.

Student: (Refer Time: 01:28).

Ok, but in terms of x 1 what are you getting?

Student: 1.

We are getting only 1.5. You are not getting the three information you understand? Only the shadow when you are seeing from this side, when you are seeing in this direction, you are only taking the points that are in this plane. You do not have any other information because this is a projection when you take a projection of x 1 and x 2, sorry x 2 and x 3, you are taking only 1 2 3 4 times. So, 20 point information only you are having when you take a projection. Only these 20 points come, but how many experiments you have conducted at each of this design point. You are conducting an experiment, you are conducting 60 experiments. You have conducted in total I have conducted 60 experiments, but when I take a projection, how many information I am getting? I am getting only information corresponding to 20 points. So, there is a problem with the full factorial experiment kind of a situation when you put them, right behind each other. In a projection sense, you are getting only a fraction of the information outside.

So, the uniform, the sampling, the idea on the uniform acceleration is great, but then if you place them right behind each other, you are losing information. So, people wanted better ways to do it ok. That is what is. So, this idea was just to tell you that if you see from the top what we call like a marching thing, it is similar to a design of experiment, ok. So, people are uniformly placed right, but then if you are seeing from this angle, so this is x 1 and this is x 2 perpendicular to that thing. If you are seeing from this side, I may I am seeing only.

Student: One guy.

One guy correct; that does not mean that there is one guy. There are 20 other people in that line. So, only this guys information I will get. When I see from this side from that projection, that is all the ideas, ok. That is why this plot was put here, ok.

So, let us say that I want to see the remaining peoples face. Obviously, I cannot change the camera location. I am only seeing from this side, right. So, if you want to see the other peoples face, what should I do?

Student: View point

Sorry

Student: Change the view point.

That is what I told you to begin with. I cannot change the camera. That is what I said I am seeing from here.

Student: (Refer time 04.00).

So,

Student: (Refer Time: 04:09).

Kind of ok. I asked this guy to come out a little bit this side, the 3rd guy to come out a little bit that side ok, but then the 5th guy again comes behind the 3rd way that I cannot see, ok. So, I want to perturb these points a little bit. So, I cannot go. If I do that and then, I see from this side, I do not guarantee that I will get 60 experiments information, but I might get about 55 58 informations. You get the point, right. So, if I ask these guys to just go perturb themselves a little bit, I may not be able to see all of their faces, but I might be able to see most of their faces. That is the idea about because each experiment costs you and you want to take that much information and you do not want to have any bias also. See that is the advantage of orthogonal array per say. What it does is it does not give you a bias that 1 minus 1 and all it is not giving you a bias with respect to a particular level. It says both levels. I will do the same because I have no information about favoring one particular level.

So, if you project a, this is a full factorial. If you project a particular variable, the point, the overlap on each other across, so you lose information instead if you can do this kind of a thing that is called a stratified sampling ok.

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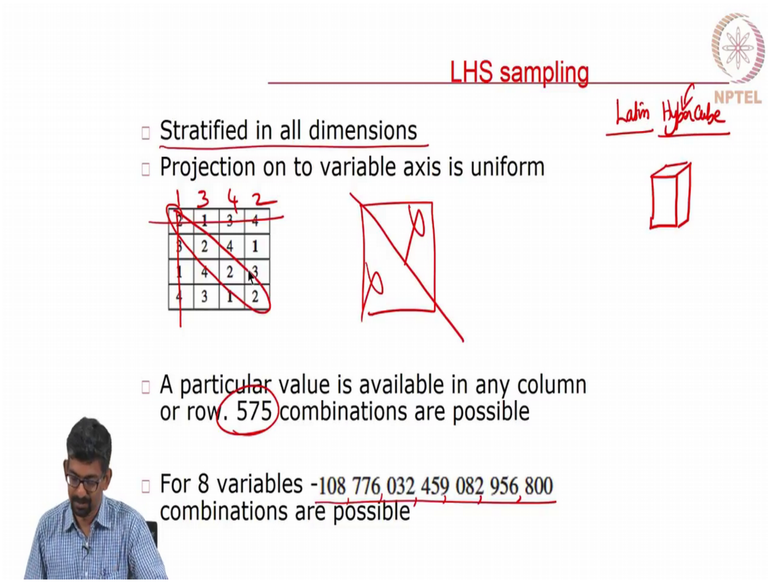

So, one such stratified sampling is LHS. LHS stands for Latin Hyper Cube and S stands for Sample. This Latin is nothing, but the Latin square that we are talking about minus 1 to plus 1, this hypercube is this is square, but then n dimension it can be a cube. Then, fourth dimension I cannot draw, it will become a hypercube.

So, generalization is Latin hypercube sample in two dimension it should just be a Latin square sample ok, but then square is just a simplification simple case of a hypercube,. So, the idea is I would still like to preserve the uniformity of the sampling meaning from an exploration perspective and not from the sampling perspective. That uniformity I might want to keep in terms of all the dimensions. If I am using 5 samples in this particular dimension, I want to use 5 samples in the other dimension and I want to explore each of the grid also. So, in all the dimensions you want to do this stratified and project on to the variable axis is uniform, ok. I do not want to get extra information on any particular variable axis. If I am getting 55 on that axis, I should get 55 in this axis also.

So, the idea is one such combination this is an outcome of a Latin hypercube sample. We will discuss that, ok. What this one says is, I am just going to, it is already bolded there. What I am just showing you ok this is one of a Latin hypercube output. What it says is if you see in this particular row and column, there will be only one 1. So, if I do this, similarly if I do this if I do this I will cover all the entire design space. This is a uniformity. So, you can do the same for any other thing also. This is the one that we are talking about in terms of uniformity and not about the sample placement. If you take 2, I did not expect, but this is an interesting case.

So, what is happening is, this row and this column and get rid of this row and this column and get rid of this row and this column. This one is gone, sorry. Still I am taking care of the entire domain. So, you can do this for any variable you want, but this is only one of the 575 combinations. You have for just a two variable case, ok. I could have like this 1 3 4 2, instead of this. So, if that is the case, then one this guy is gone, this guy is gone. So, I need to choose out of this let us say that I choose, no I am not going to spend time doing that. What I am saying is you can also get other configurations where in whatever I discussed now, we will be satisfied. How many such combinations will be there? 570. Why such combinations will be there and just know as I showed you this is also a case, but does that give me any information on the design space. It is not exploding half of the design space at all. This site is not explored, this side is not half. Almost 100 percent of the design space is not at all explode; it is not giving me any information.

So, though I preserve the projection on the independent variables to be a variable axis to be uniform, I might end up not exploring the design space at all. So, I want some other criteria as well. So, we will see what and for 8 variables, I do not even know what is this called in European sensor; any idea man?

1 followed by 1 2 3 4 5 6 7 8 9, sorry 3 6 9 12 15 18 20 0s 1 followed by 20 0s. I do not know what that number is for 8 variable these many combinations are there and you can do this. So, the point is you want to choose, you want to come up with some other criteria that will give you the best exploration of your design space.

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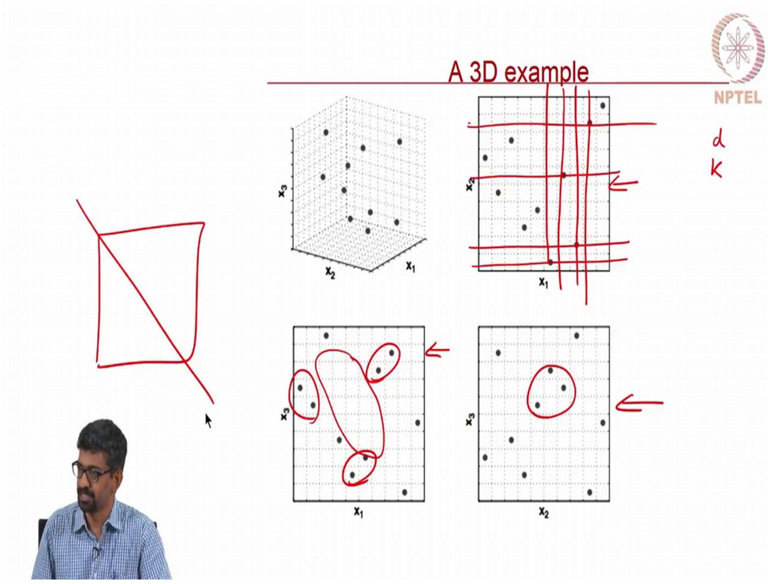

Just to give you 3D example, so what the same idea here let us say that this was x 1 x 2 x 3, ok. I have 10 points I guess 2 3 5 10 points. So, what you do is, you grid each you remember right d dimensions k grid divided. So, 10 grids you make, then what you do is, this is the projection. Please understand this is placed in 3D, but I am just showing your projection. This is the uniformity that I am talking about. Some algorithm I have placed these 10 points in the space, I put three of you in three different directions. You are seeing from outside the room, he is seeing from this side of the room and I am seeing from the other side, ok.

So, we can only see projections. We cannot see where it is. In the space you can only see projection. That is what your naked eye can see. So, this is for that persons perspective, this is for one my perspective and this is for that guys perspective, but if you take any of the projections, they will all be the same. I do not know spend time doing 10 by 10, but that is what it will do, ok. When you put a point, you will lock. You lock that particular column and;

Student: Row.

Row and then, you go look for other columns and rows where you want to put the points.

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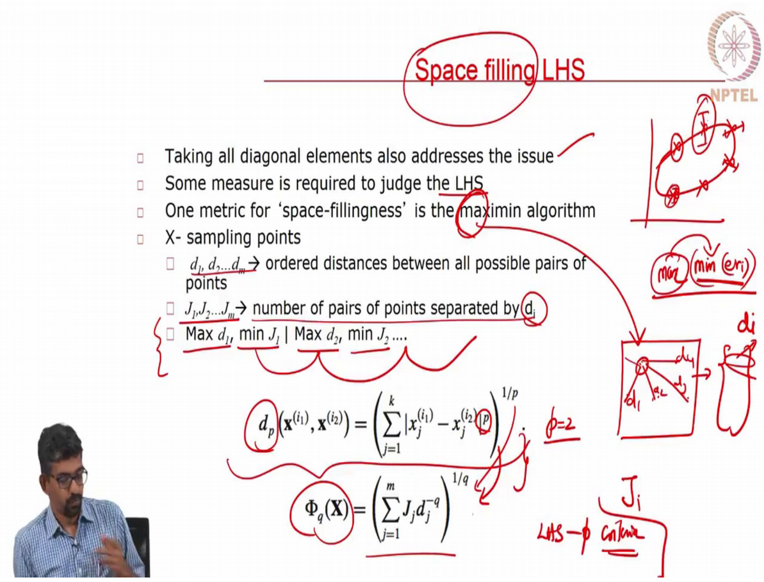

So, because that is taken care of, that is what it does, ok. So, now no I have more points, then you create additional grids. So, you can offer 20 points, then you do 20 by 20 and then, you do that, but you can see, but this is not a great stuff. There is so much of space left out. It has got to do with two things. Maybe this is not I need to come up with some other criteria because these two are very close to replace. These two are very closely placed, ok. Maybe if they are placed somewhere here, these two are very too closely placed. These three are very closely placed given that I am exploding this entire space with only 10 variables.

So, I need to come up with a criteria where I want to send them not only preserve the uniformity property, but also I want to send them across my design space because if you see like that is what there is huge blanks all wrong, ok. How do we do that? So, that is called space fillingness. Until now we are only talking about LHS in general, but the property that I am talking about right now is called space fillingness. So, what they say is this is something that we already spoke, right. You can also do a x 1 by x 2 and then, you can take the diagonal elements. It will still satisfy ok. That is the first one which we have discussed.

So, some measure is required to judge LHS from exploration perspective simple thing called a maximin algorithm; this is available in MATLAB maxmin algorithm is also used in many other stuff. So, what it is saying is you maximize some minimization problem, ok. So, what did we do is, this for instance error, ok. You take a four bar mechanism which is going to give you this kind of a curve. You know what 4 bar mechanism is; does not matter. Let us say a robo it is kind of giving you a graph, but the robot does not go and draw the graph like that. It gives it gets inputs at many points and then, it is good just going to go connect those points, ok.

Now, what happens is that each of this point I could have an error. In such a case, what people will do is, they will say that you want to minimize the errors at each of these points minimize error at each point, then find the maximum of those errors because that is the one that I want to take care of. Let us say this guy had 0 error I know I should not worry about this guy at all, but this guy had maximum error, ok. So, in general you want to minimize the error, but I want to capture the maximum of the minimized errors, ok.

So, the maxmin algorithm for sampling what does it say? Just be with me; we will take a break. So, there are multiple points here. I take one point, I find the distances d 1 d 2 d 3 d 4, ok. It is not done that way, but I am just saying, ok. Then, what I do is, I go back give them into a d i and then, I sort them that is what is this 1 d 1 to d m order distances between all possible pairs of points. So, all possible pairs, this point, this point, this point, this point, this point, this point, point, this point, this point, this point, that is d 1.

Now, what I want to now let let me not tell them. So, the second one is j 1 j 2. So, I took d first the second one that I am taking this j i, these are the number of pairs of points separated by the same distance d i after I do this, ok. In the back I also have an index that says these are all different for the same distance d i. I see what are all the points, how many of these guys you know that pairs I number the pairs and then, I look at it. So, what I want to do is, I want to maximize this d, I want to put these points as far as possible, so that I explore the design space. I want to maximize d 1, and minimize j 1.

Similarly, I want to maximize d 2 minimize j 2. If you go to the book, there will be a theorem that will generally say maximize d i while you minimize j i ok, but there could be a case where you will have to keep doing this in a consequent sense. That is why I wrote it like this maximize, you want to maximize d 1 while you want to minimize j 1 meaning the number of pairs that share that distance you want to minimize that and that is why this is a maxmin and you want to do this simultaneously because as you move this point, all the distances get affected. You understand what I am saying. That is why let us just become a little trickier when you try to use it for higher dimensions. When I say higher dimension even with 10 variables if you do lectures, what happens is you throw 1000 points. It will take over 9 to do because what happens is you will have to, it does the distance every time it goes and recompute your distance,

So, what do I mean by the distance? In this particular case, in a 2D case, it is just a Euclidean distance. So, this p can be 2 in a two dimensional case and or you can have anything, but as it is also pointed out in literature people will say that we will use a Manhattan distance, you can use any distance. Nothing seems to be superior to the other one in this kind of a study.

So, then what people said is this becomes really trickier because you will have to keep going and doing that and finally, they came up with a slightly different criteria. There is a little bit of algebra coming from this to here which is slightly out of scope for this course. So, what we will do is, we will take it for granted that you do this as a sum for all the points. Basically j is the number of points, right. I mean j is this guy, j runs from 1 to m combinations j. Each pair that I take d j raise to minus q and 1 over this is the same 1 over q that you take, ok. The q is usually defined by us. We usually keep it as 0.5, but you can also change it accordingly, ok.

So, this is the criteria that you want to maxmin that is what you will try to do. This is the criteria that you want to maximize. I mean you use the maxmin algorithm to deal with this particular criteria, ok. This is your LHS phi criteria that they say.

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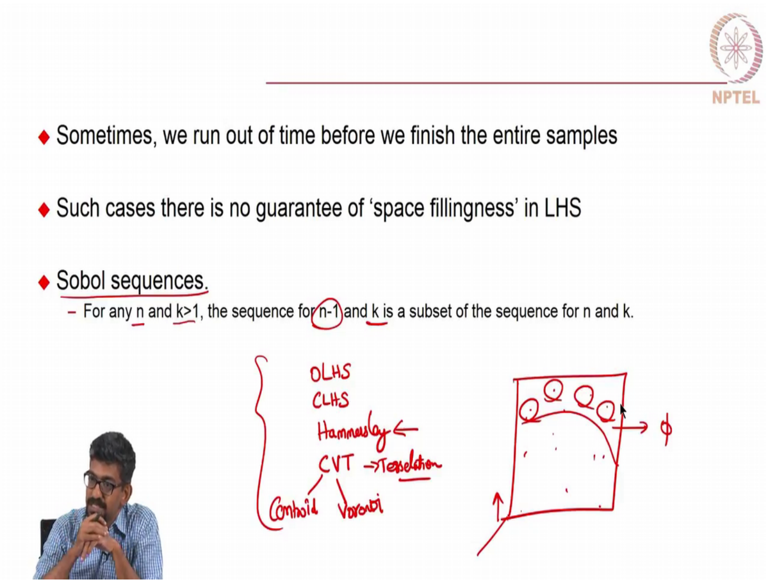

So, there is one limitation though to do this what happens is I told you I give you a cluster and I told you the cluster is in your hands for one day, 24 hours. You give me a sampling response, you sample and you give me a response ok, but I just told you and then, I told you that your time starts now. You do not know how much time each simulation takes. So, you assume that each simulation will take about 1 hour and then, you took 24 points in this space and then, in some order you started going in this direction faster, you went like this, ok. So, you covered these many points. 24 hours is over and you do not have information at these points. There is a problem with the LHS because the criteria of phi was generated based on all these guys.

Now, with this response you cannot make a wise decision because they have actually taken these points into account, but you do not have the responses there. So, the space filling has properties lost immediately. There is a problem with LHS. Instead in such cases what you can do is, there is something called sobol sequence, there are multiple other stuff. LHS is widely used. So, we are talking about if you go to MATLAB there is also an optimal Latin hypercube sampling. This is a better way of doing this max min algorithm, ok. There is also a constrained Latin hypercube sampling people do, Hammersley. There is something called a Hammersley sequence which you can use and T stands for triangle, V stands for varmy and C is central.

Student: Centroid.

Centroid sorry not central; centroid.

Student: Voronoi Tessellation.

Voronoi Tessellation, not triangulation sorry. So, there are multiple other sampling schemes that you can use, but they also have this kind of space fillingness property that is not a problem. Actually this Hammersley and all has a very nice property, ok. More than space fillingness what happens is, they also have a memory because when you do Latin hypercube even the optimal Latin hypercube sometimes you could get such big voids ok, but Hammersley sequence has a memory property where it says is in that 2nd iteration, I put a point here. So, in the 8th iteration and not up put a point right next to it while, so much of space is unexplored. So, it will go and put this point elsewhere, ok. So, that property is not there in LHS. It does not worry about your previous points. It is more like I do not want to use the word, but it is more like a stationary process, ok.

So, the interesting point of sobol sequences for any n and k being greater than 1, n is the dimension and k is the number of grids that we are talking about, the sequence for n minus 1. So, you can take any n minus 1, the dimension and k is a subset of the sequence for n and k. Understood what I am saying? You can reduce and it will still be a subset of it, ok. What I am saying is these four did not run, ok. So, you can take this because in this two dimensions that we are talking about ok, so even any other projection that you take will be a subset of the original one. So, the uniformity in the sampling part are in the sampling will be preserved.

So, in that sense it is a good idea to use sequences like sobol, ok. I just wanted to pull a couple of example from our research study to show you such things can be used. Basically I am going to show you something on design of experiments right now and the next example I will also show you design of Metamodel. Of course, the Metamodel part we will discuss next, ok. So, we are interested in designing.

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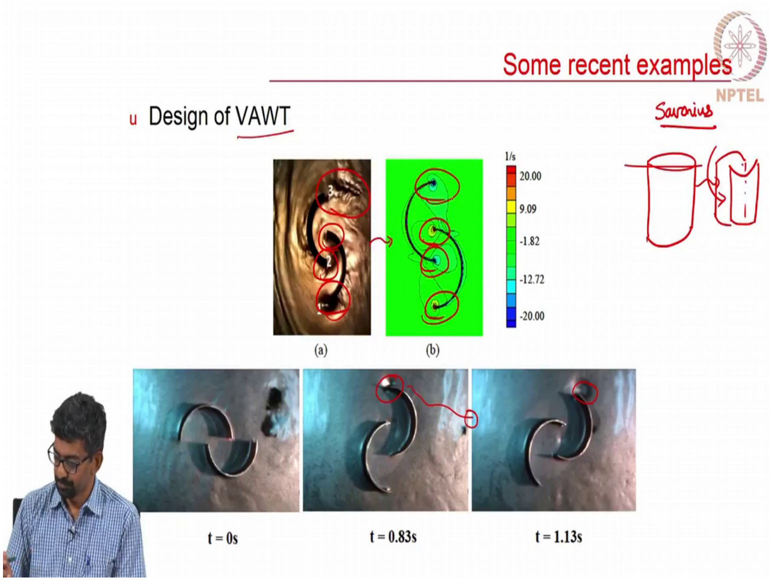

What means a Vertical Axis Wind Turbine, he type of wind turbine that we have designed is called savonius, ok.

So, savonius is a very simple idea. So, you can see that you can take a cylinder, I mean just cut it into two halves and it will become you just move the center and this is how it looks like. Of course, it is mounted on a shaft and as wind comes, it starts rotating in a particular direction, ok. So, what we wanted to do is this first benchmark because we are interested in a computer model. So, this was done in a water tunnel in the applied mechanics department and we created an equivalent computer model and we wanted to evaluate this qualitatively, ok. So, you can see the vortices formed in these regions are all captured. Well, that is what we wanted to do because those will tell you what is the pressure and the vortices those are the plots that you want to have and there is something called the strouhal number.

So, we are seeing that when the vortices are created, you can see the vortices is created and then, it is leaving. So, the vortices will travel like this and then, we will go and capture. So, within 1 second or over 4 seconds, how many vortices are being shed is that the same in the computer model that is something that we wanted to check.

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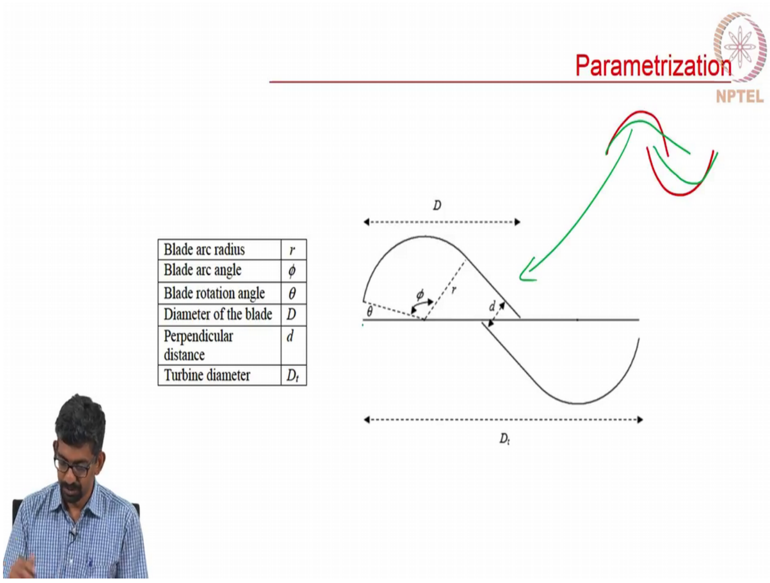

This was the initial benchmarking. Then, we wanted to study which factors are important stuff like that. So, the classical savonius model has a blade like this, but then in literature people have suggested that a change in the design will help for obvious reasons something like this. This is called a batch type model is what we have here. So, we wanted to parameterize. We want to see how much should be the arc angle. The blade arc angle government should be the blade arc radius. How much should be the rotation angle, how much this is called the overlap ratio. How much should be the perpendicular distance, should it be close to each other or should it be far away? So, these are all parametric study as you might know.

So, first we wanted to parameterize them and we also wanted to understand what is the effects on each one of them. These were from our experience as well as from the literature. So, we do a basic representation, parametric representation.

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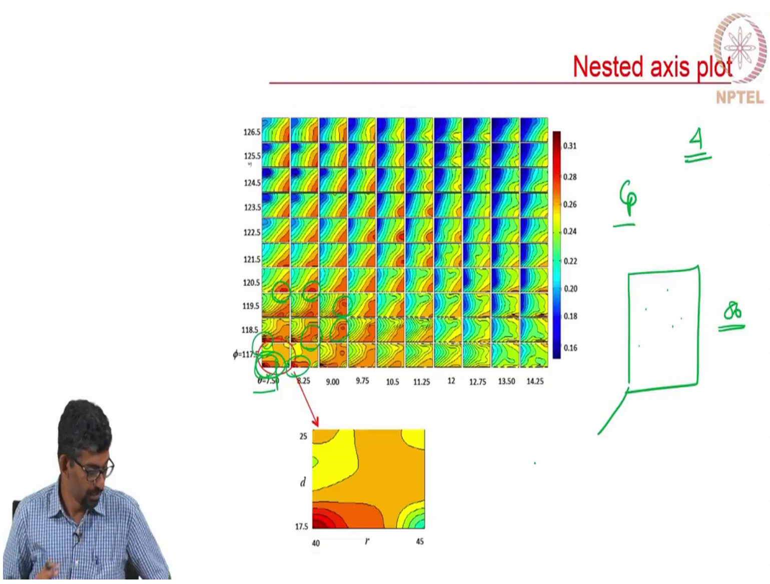

Then, what we have done is similar to the nested axis plot that I showed you, ok. I did not show you a nested axis plot; I just showed you a tight ok, but the same idea is here, but what is happening is currently we are investigating 4 variables in this r, d, phi, theta and phi. So, the way this happens is I am going to fix theta at 7.5. For this particular type I am going to fix phi to be 117.5. For this particular tile within the tile r and d varies, ok.

So, in one shot I am able to visualize 4 variables. So, this is one way of doing it, but this is a maximum you can do. You cannot visualize 6 variables. For instance, you can only visualize 4 variables. So, today this visualization of data is a big stuff ok. Now, with all your sensors and all that you get a lot of information, but how do you visualize this data in higher dimensions is a big problem. So, like this we see and then, we figured out that you know there are spots, this is coefficient of performance is something that we are plotting, ok. We had to perturb a little bit, we have to run computer simulation and that is why we build a computer simulation to begin with. It is expensive, it is still a CGD model and it takes a few hours to run, but we did about let us not worry.

So, we did few simulations and then, we have you know we have some zones of interest, ok. This is just based on your nominal value and a little bit of perturbation. It is not the actual fitting. Then, what we did is we took the n dimensional space which is the four dimensional space, we put a large neighbor cube sampling. I think there are about 86 samples 70 plus 16 samples at the corners we did and then, we fitted a surface to get the optimal result. Finally, the optimal results was also manufactured and if you want you can see the stuff in the workshop.

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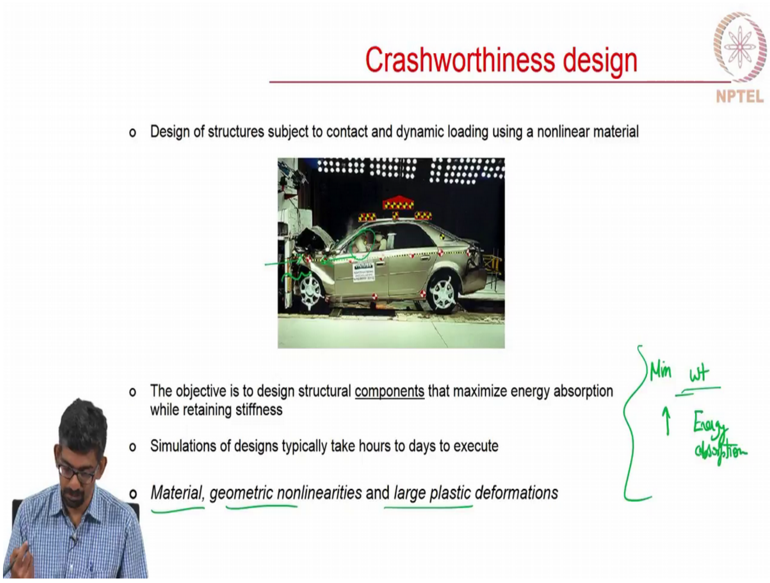

In our workshop, it is there. There is another interesting problem that we work on as a crash worthiness design.

So, these are some pictures that is lifted out of the internet. This is typically the car crash experiment, right. So, after this the car is lost meaning you cannot use the car. So, you cannot use as many simulations as I mean as many experiments as you want. So, computer simulations is a better way to do. This is a very interesting problem because material geometric and large plastic deformations, ok. So, material non-linearity means large plastic deformation material and that is also boundary condition. Non-linearity is there meaning you start with a boundary condition, but during the crash, the boundary condition changes. So, it is a boundary condition and non-linearity is also there.

So, this is an interesting problem because you want to minimize weight and at the same time you want to increase the energy absorption, ok. You want to increase the energy absorption and even in energy absorption only in the frontal part, you want to increase energy absorption because if you want to do energy absorption, I can put a concrete. Of course, my weight will get affected, but I can put a concrete in the front. It will absorb all the energy, it will not send it to the dummy or the pace engine, but there is a problem because crash propagates this way and our inertia propagates this way, ok. So, you will go and bang on the concrete, ok. So, there is some head injury, neck injury criteria and all that that people try to do.

So, this is a conflicting objective, right here ok. So, you wanted to minimize, but also you wanted to increase your energy absorption. So, you will have to come up with innovative materials and designs to do that, but that is not what we are.

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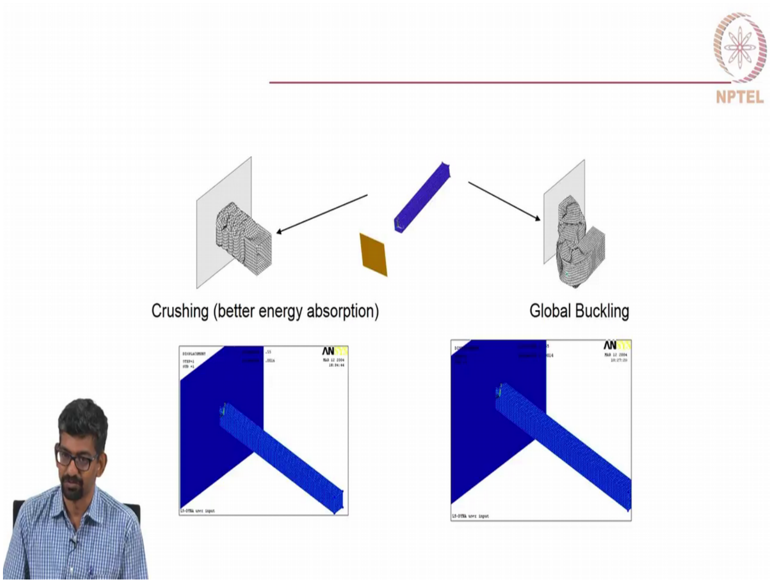

Focusing on this is a simple crushing and buckling kind of an experiment with that we try to do.

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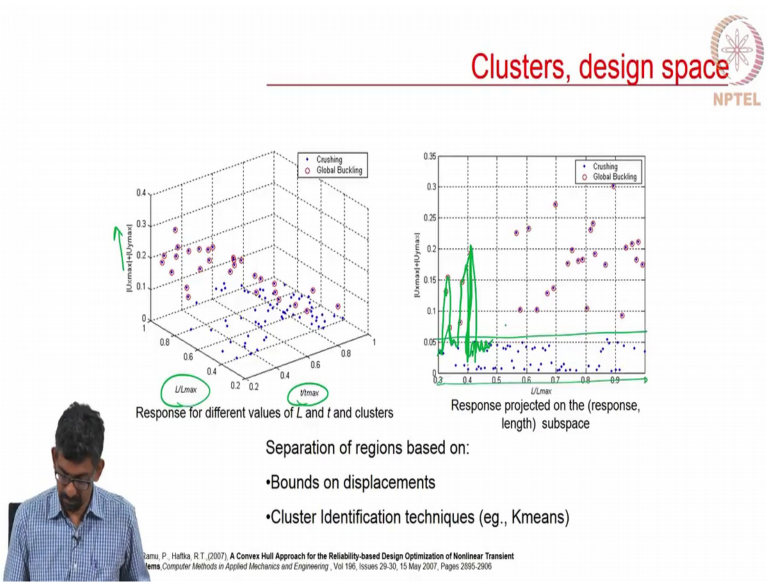

So, the point is we took some two design variables at t and L and we have some displacement constraints, ok. This says whether it failed or no it is, ok. Wherever you see the red circle, it is failed and wherever you see the blue circles, it is I mean the blue dots, they are fine. Interesting point that I want you to look at is this is out of LHS design do not worry about all that; l u you look at the function. It is very clear that beyond this, it is failing because that is what we have defined. If your displacement, the summation of the displacement absolute values if it is greater than 0.05 or 0.06, it fails that is what we are saying. So, that part is clear, but for small changes in L max ok, from here to here it was like this for this value it goes up here, then it comes down and in between it went up here, it went here, it came down, then it is here, then it was going up, just going up z ok. If I connect it like that, it is a highly non-linear function, ok.

I have so many realizations here and for each realization, I have points. I am not necessarily connecting it properly, ok. Probably it is somewhere here and then, it goes up and then, it comes. So, it is a highly non-linear function. Then, there is a problem do not worry about this.

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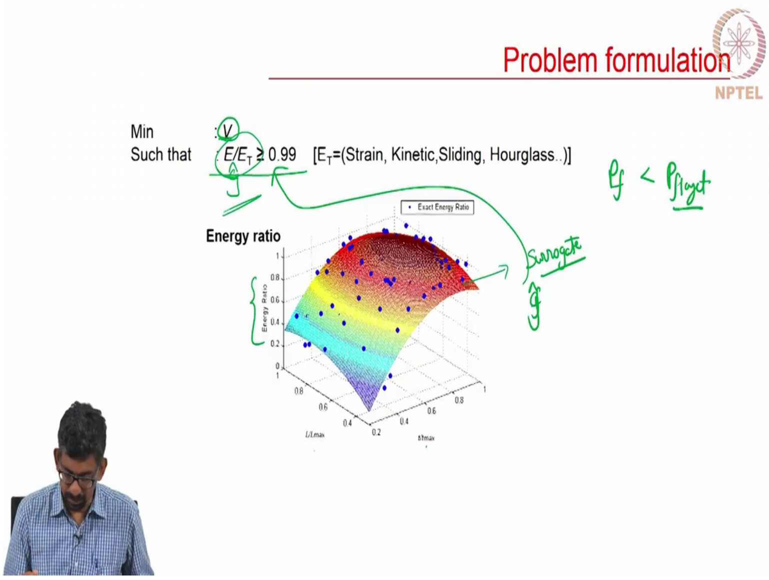

So, what we are trying to solve in that problem is, I will go back to that problem. What we are trying to solve is we wanted to minimize the value, sorry the volume such that some energy ratio what we are doing in this particular case is for t and L which are my design variables, there are different points, right. Through LHS design, we are estimating what the energy ratios are and we are fitting a surface. This is what we are going to talk about. This is a surrogate that I have fitted, ok.

Then, this surrogate let us call it g hat or whatever it is, it will be used here. To do the optimization this is something that we saw yesterday. If you have this function g hat you know how to do the optimization. That is what we saw yesterday, but right now I only have a criteria. I did not have this equation I do not have an equation of E over E t in terms of L t. So, what I am doing, I am creating all these blue dots that are in space which comes out of LHS sampling and then I run the simulations at each of this point and then, I am fitting a surface. We will see how I fit the surface in the next module, ok. Once I do that, I will be able to use it here ok, but it does not stop here. We will also say in the problem that the probability of failure should be less than some target variety of failure, ok.

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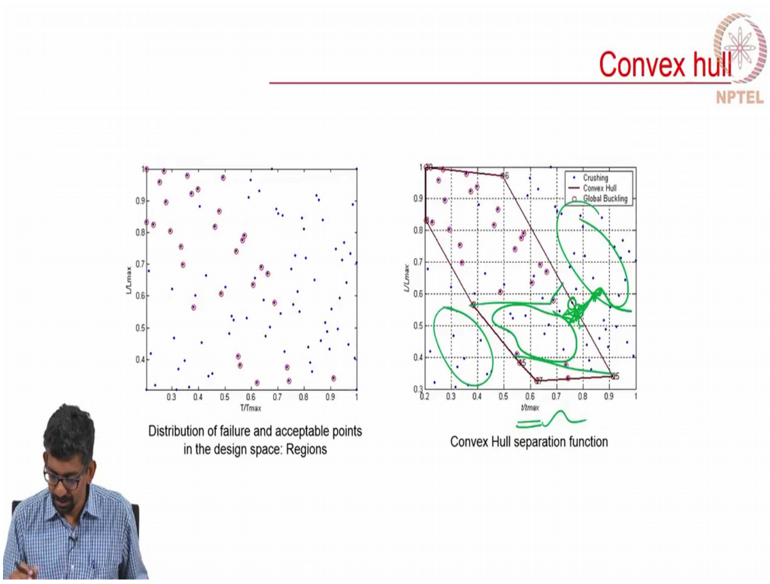

That is slightly out of scope I will also tell you, so what happens is when this function was so non-linear, what we try to do is, we try to take a slightly different look at it, ok.

We took it in terms of the design space and not in the output space, but only in the design space and this is what we call the Island Failure Zone meaning because this is also good space, this is also good space is called this is a failure zone, ok. We use some computational geometry technique called convex hull to do that, but unfortunately it also includes some good point, but we were able to do some work with that, but later we came up with some technique called the alpha shape which will do something like this, ok. That is not the point of the discussion.

So, what we do in this is after all this you will not take this point to be, this point is outside. You do not take that to be an optima because what we are saying is there is some randomness in t and L for a small randomness this will actually fail ok. So, how usually we take that into account in our deterministic design is we use is a 1.5 safety factor, so that this point gets pushed here which means it is a heavier design ok, but still it is safe from these perturbations, small perturbation or small randomness. This could be a manufacturing error. You ask him to generate like 5 mm, but he did 4.8 mm. You put it in the car your car will immediately fail, ok. So, you want to push it that side.

So, let us say that this was 6 mm. So, even if he gives you 5.8, you are ok. So, it means that it is a over design, but we wanted to minimize this over design, then what people say is we know that this follows some kind of a distribution and then, they estimate what is a failure. They do not work for 0 failures they try to minimize the failures as much as possible. That is a paradigm, ok. So, we have used what I am trying to tell you is this kind of gives me an idea on the design space. I do not have to do an optimization with this per se, ok. Right now that is what some of our students are trying to look at is how does that design space vary. Does it even because you will see in the next surrogate model, you need to know at least you need to have some idea of your function, then only you can fit a function to it. You cannot say that I have 0 idea about how the function varies. Then meaning like that is not a wise way of approaching the problem, ok. So, you need to have some understanding of how the function is going to vary over your design space, so that you can do that, ok.

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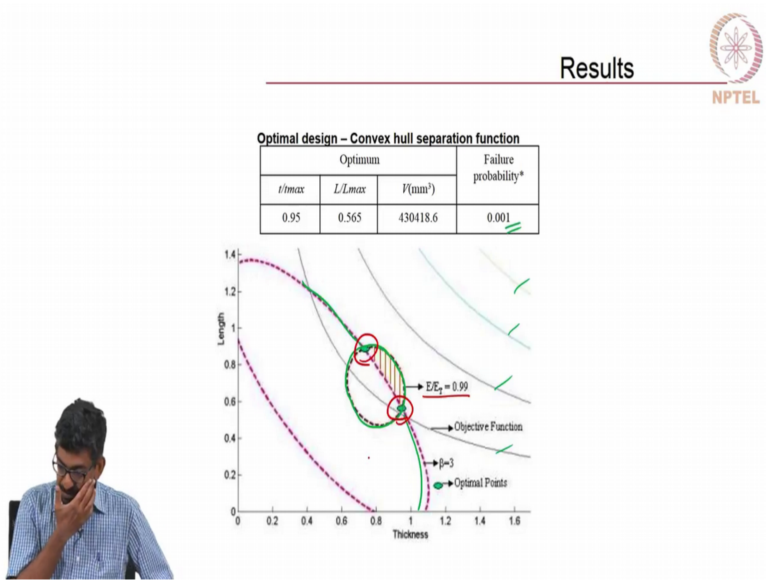

Then, finally I mean in this particular stuff we also did what we call an optimization study and then, as you can see that we had some failure probability, but interestingly in this point there is a local optimum was there, ok. So, this was the let me see this is that failure probability constraint that we have this is our w times t. I guess the objective function these are the objective functions. This is the failure curve for the reliability, and this is the energy curve. So, this is the energy curve that we are talking about. So, if you did the starting point from here, it will reach here. If you did a starting point from here, it will reach you. So, this was a classical case of a multiple local optimum case.

So, we first did and accidentally while running the algorithm again I gave a different I mean I did not intentionally give a different starting man. Then, we went and plotted and saw, then we understood that it was like this. So, there is always has double when you use a gradient based algorithm which was not of his use is to use different starting points fine. These results were just shown to you to motivate where all you can use design of experiments, you can for understanding the design space, to understand this to do two factor analysis. We can do and of course, you can also use it for optimization obviously.