

Surrogates and Approximations in Engineering Design
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Lecture – 08
Types of DOE - 1

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Identifying important factors



A 10 variable weight function of an aircraft wing

$$W = 0.036 S_w^{0.758} W_{fw}^{0.0035} \left(\frac{A}{\cos^2 A} \right)^{0.6} q^{0.006} \lambda^{0.04} \left(\frac{100tc}{\cos A} \right)^{-0.3} (N_z W_{dg})^{0.48} + S_w W_p$$

FE Simulation
CFD
Cost model

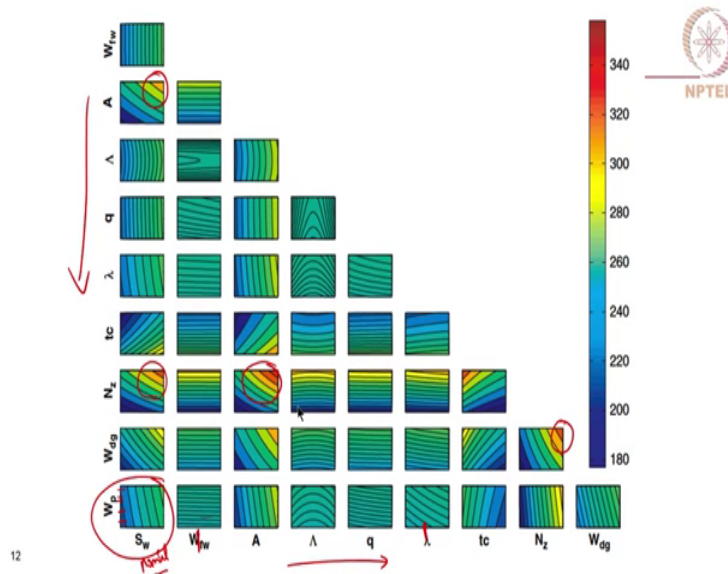
Symbol	Parameter	Baseline	Minimum value	Maximum value
S_w	Wing area (ft ²)	174	150	200
W_{fw}	Weight of fuel in the wing (lb)	252	220	300
A	Aspect ratio	7.52	6	10
A	Quarter-chord sweep (deg)	0	-10	10
q	Dynamic pressure at cruise (lb/ft ²)	34	18	45
λ	Taper ratio	0.672	0.5	1
tc	Aerofoil thickness to chord ratio	0.12	0.08	0.18
N_z	Ultimate load factor	3.8	2.5	6
W_{dg}	Flight design gross weight (lb)	2000	1700	2500
W_p	Paint weight (lb/ft ²)	0.064	0.025	0.08

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So, let us say what they have done here is they have fixed the baseline value which is nothing, but here x 1 x 2 x 3 x 4 that is what is being done here and, then I pick a particular x and then I perturb, it by how much should I perturb it is between minus 10 and plus 10. So, the total range is only 20, but I also should decide how many grids I have, there I had 3 levels or 4 levels, here I would have decided how many levels I have.

So, that eta divided by P minus 1 in this particular case if I had 6 levels, then that eta should be the same for all of these guys then, but then I should divide it by the number of grids, because this particular variable has 1 2 3 4 5 ok. For each of thing will make it 6 so, if the eta was 6 for me in that particular case, then it will be 6 divided by 6 minus 1 or 5 divided by 6 minus 1 ok, if I make this guy to be 5. And then you take this particular guy 0.5 to 1 ok, they are also your eta should be the same, but then my P could vary it can have only 1 2 3 4 5 ok. So, then it will be 5 divided by P minus 1 depending on which P you are looking at anyway.

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If you try to do this what will happen is I kind of get this kind of a plot of course, we will have to run your simulations for each of those cases, for each of the delta that you do for the small perturbation you find out what your value is and this is the graph that I get. Imagine that you do not have this is replaced by a finite element simulation, or some complex plus CFD plus some cost model something.

So, what happens is if I go and change the weight of the fuel in the wing, from 252 to 246 allowed to plug this guy into this setup, I have to run this, I have to run this, I have to run this, then it will give me a response, which I keep these responses. And, now what I am going to do is this I am going plot this, when I am estimating that I am not only estimating the wing, but also meaning not only the weight, but I would also estimate the other stuff, because there could be an interactive effect.

So, if I go and change the weight of the fuel in the wing of course, the wing area the aspect ratio might not change, but the dynamic pressure at the cruise might change ok, ultimate load factor might change ok. The flight design gross weight; obviously, changes ok. So, there could be an interactive effect so, I need to go and estimate each one of these guys as well.

So, when I did that I am able to plot, this graph it is just called a tie plot, what it says is the way that you look at this is this is your S w is your wing area weight of the fuel in the wing. And then aspect ratio quarter quad sweep. Aspect ratio quarter dynamic pressure

and then it keeps going like this. The same ones are listed on the other side also the y axis also of course, S_w will be just 1 so, I am just taking of the off diagonal terms. So, I am starting with in the reverse fashion I am doing $W_f A$ by q it goes in this direction.

So, what this one says this particular graph, if you take S_w is fixed at its nominal value and I am perturbing W_p I perturbed W_p different values and I am taking a contour of S_w ok, I am just looking at this guy ok. So, with respect to W the fuel weight ok, why this guy is at its nominal value if I am changing W_p what is the effect that I am getting. You can also look it up the other way around ok, I am fixing this W_f and I am changing $W_d g$, what is the effect that I am getting.

I am fixing this λ and then I am fixing the λ the baseline value and, then I am checking what my W_p is by varying it ok. Similarly $W_d g$, $N_z t c$ so now, what this one says the legend is the same for all of them, this one is pretty much here, this particular graph it does not say anything S_w is what your wing area sorry and, what is this guy W_p is the paint weight the paint weight; obviously, will not have an effect on your wing area. So, that is what is captured here whereas, let us look at this particular stuff, because that is where the red is large right. So, that is between a and N_z , what is A the aspect ratio, what is N_z the ultimate load factor.

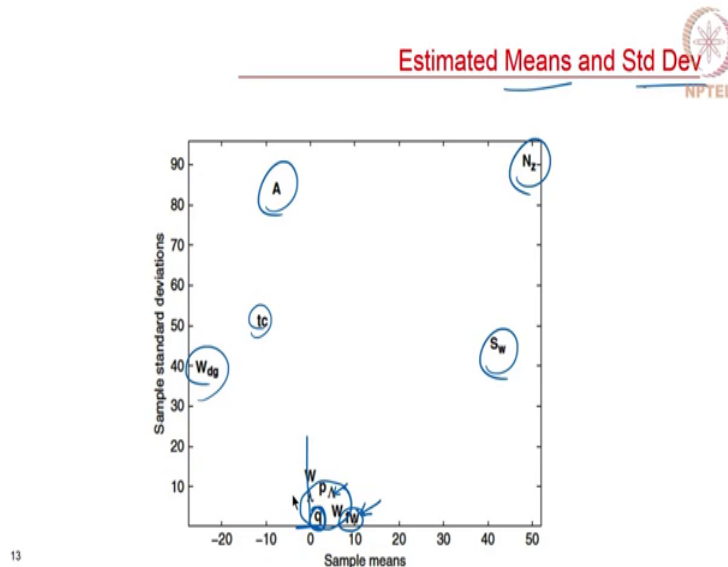
So depending on the shape, the aspect ratio also tells you what is going to be the shape, or at least the size if not the shape ok. That affects your load factor and what it says is if you go right on your a ok, on your aspect ratio if you keep increasing that guy, what is happening is you are also increasing your N_z ok. So, you do not want to keep increasing that that is one way. The other thing is if you look at it this N_z is with S_w which is nothing, but your wing area and you can see that the wing area is also related to your aspect ratio ok.

So, the point is these kind of graphs can give you some idea ok. So, let us say that see this S_w was also related to my aspect ratio ok, there was a relationship and this one is $W_d g$ and $N_z W_d g$ his flight design gross weight has an effect on the ultimate load factor; obviously, because these aircrafts are structural fluid interaction right. So, it will have a direct or an indirect effect on the weight per say ok.

So, you can get a basic idea and then you can say that ok, these are the interactions that I am interested in or this guy does not have an effect direct effect. So, if this was not

important I will get rid of this for instance this q does not seem to play a major role, I do not know what was q dynamic pressure at cruise, maybe only at the stall speed and all that it plays a role in this case it does not play a role. So, you can kind of leave out those variables, or you can create sub variables so it now we look at how those sub variables are obtained.

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So, this Morris what he said, he said that I want to have the mean and the standard deviations right. So, from this particular plot I am just plotting here, but I can also get my mean and standard deviation from these plots, if I do that if I am plotting the mean, sample mean versus the sample standard deviations. This is how the variables are getting distributed. So, what it says is this is the origin and, around the origin there is a bunch of people who are sitting there ok.

So, what but then; obviously, this q seems to be the most ok, it is not kind of play a role ok, because there is no variability there is no mean value which is what we saw in the figure also right q I arbitrarily told q ok, but then it turns out that that is so, is the case there is a little bit of variability of the order of 10 for this guy and whereas, the mean is much lesser than that.

Similarly there is no variability for this f_w , but he is at least mean wise. So, he has an effect f_w has an effect by itself it might not play a role in interaction f_w ok. So, f_w if you can see mostly it is straight lines, it is just linearly increasing if you see here, other

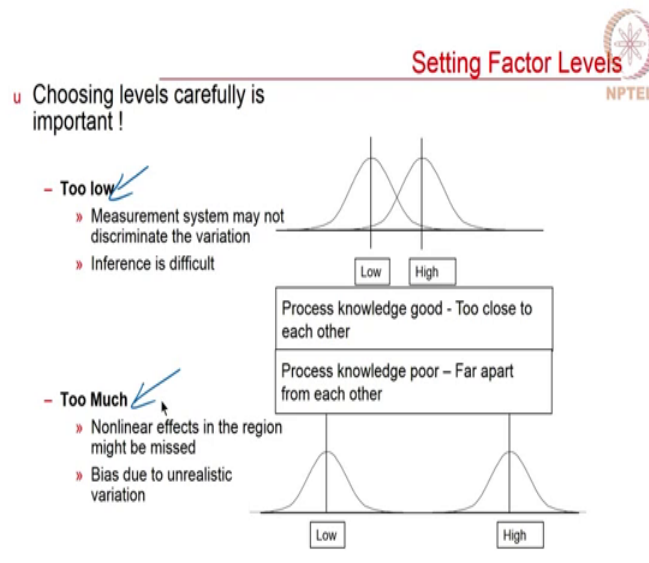
than this particular graph all of them are linear in sense for this particular guy, if I change this how much is it varying same, if I change this how much it is varying same, if I change this how much it is varying it is varying by an order of 10 or 20 that is all it is varying meaning like if I fix this guy.

If you look it up from the other sense, if I do S w then this is what is for f w ok. So, you can keep doing that and this graph gives you the same amount of information and then, but it also tells you these important guys N z a S w; N z is your ultimate load S w was your wing area, but what it says is this guy has by himself he is important and in interactions also he is important N z was a dynamic cruiser, or whatever ultimate load factor sorry S w is your area.

So, the area in one sense is also the same story 40 and thing both mean as well as standard deviation is important ok, these guys have negative means. So, maybe the correlations are negative in the sense ok, if it is increasing they will decrease it is like that t c and W d g, flight design gross weight how is a negative correlation anyway, but the mean is in a negative sense, but it is important because the value is more it is not around your 0 0. So, it kind of gives you an information this is what Mitchell proposed in terms of the sampling ok, you should put it in such a way that if I get these details it gives me information about the interactions.

So from the plots, it also says that we can we can kind of appreciate what is happening in this particular mean versus standard deviation ok. The idea of that Morris method is important for the sampling stuff that, we are going to discuss next that is why I just out of the loop, I discuss Morris method and then I am showing this for you ok.

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Now, then there is also people you know talking about how do you want to set your factor levels, if you are taking these two or your bounds minus 10 to 10, should you have just 0 in between or do you also want to have minus 5 and plus 5, or should you have minus 2.5 minus 7.5 2.5 and 7.5 ok. So, the deal is if you can have low you can have many of course, budget wise the number of experiments that you need to do is more or less accordingly ok.

So, you need to make a wise decision ok; however, let us say that you have some information that says that this is a linearly varying quantity, then what would be your wise choice how many levels would you choose


Student: Two.

Good enough, because it is only linearly varying for a line linearly varying line on the surface, I just need 2 or 3 points for the surface 3 points is good enough ok, because it can vary linearly on both sides, if it is a surface that is all you need to know; however, someone came and told you this aerofoil thickness to chord ratio is not linear, but it kind of has a quadratic can be anywhere, I am just drawing some quadratic here how many points do you think you need in one dimension. In two dimensions then N dimensions you need to use a Pascal triangle equivalent to understand, how many coefficients you need to find ok. So, that is what this particular thing talks about ok. So, you have issues with too low you have issues with too much also. So, you want to choose, but there is no

way I mean like you will have to use these kind of engineering experiences and all that to decide upon your factor levels that is all.

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Experimentation Strategy



- Be aware of the experiment's inference space.
- Use two-level designs early stages.
- Utilize bold level settings initially.
- Subsequent experiments will describe complex relationships.
- Spend less than 25% of budget on first experiment.

"The purpose of analysis is insight. The best analysis is the simplest analysis which provides and communicates the insight which can be obtained from the data. Complexity, for sake of sophistication, is obfuscation. Unfortunately, there are people who are afraid of clarity because they fear that it may not seem profound."

D.J.Wheeler

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So, these are some generic strategies that you know the other the co author of these slides have given ok, which he feels is very important. So, you have to be very clear about what is the inference space, you cannot just take raw data and then say that ok, I am going do this ok. This is really required if this is what it is what does that output mean, for that output data wise this input is required, but physically does it make sense that is what the inference paces, use two level designs at the early stage higher level designs at a later stage. So, two level designs kind of give you the basic idea.

So, what it means is it is to assume linear relationship to begin with, but you will; obviously, identify the nonlinearities over a period of time. This is all the management language so, subsequent the experiments will describe the complex relationships that is one thing yes. So, this is a interesting point because, your budget is usually given up front they say I give you about 1 lakh rupees now decide your experiments.

Let us say that your experiments are going to cost you 10 thousand rupees, each you do not up front design 10 experiments ok, because you want to reserve multiple levels of experiments. So, you would spend about 30000 to begin with then another 30000, then your 40000. So, roughly you budget about 25 percent of your total budget for the initial level of sampling that is your initial design ok that is what it.

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- Randomized design
- Blocked designs
- Latin Hypercube designs
- Full Factorial Design
- Screening designs
 - Fractional Factorial Design
 - Plackett Burman Design
 - Orthogonal arrays
- Response surface designs
 - Central Composite
 - Box Behken Design



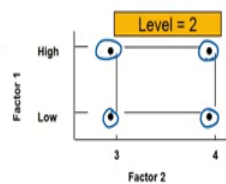
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So, I guess we touched upon this we came to this slide yesterday that know ok. So, there are different designs, what we are going to talk about to begin with will be full factorial design and fractional factorial design, which is what this kind of an orthogonal array is a type of full factorial; is a fractional factorial design.

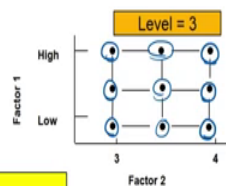
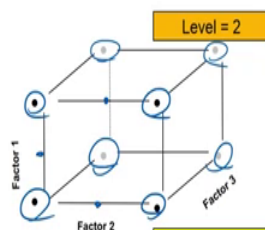
Will just quickly we are not going to get detailed into it and, then we will talk about statistical experiments like Latin hypercube samples.

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- Full Factorial Design
 - Used for full design exploration
 - Smaller # of factors
 - Experiments grow in size



$n = 2$
 $k = 2$
 $\# = (k)^n$
 $= (2)^2$
 $= (2)^4$



$(3)^2 \rightarrow 9$

L^k runs (L : # of levels)



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So, what does a full factorial design mean ok, you have you remember discussing in chemistry the crystal structures FCC, BCC ok. So, you just remember that body centred cube, face centred cube that is all the idea is ok. So, if you take about two levels what it means, there is factor 1 there is factor 2 2 levels means.

Student: (Refer Time: 16:24).

Factor 1 has a low and high, factor 2 has numbers 3 and 4 what I am saying is each combination I will run it. So, this is this guy is one this guy is one this guy is one and this guy is one so, there are 4. So, if you can draw a rectangle all the 4 corners this is the maximum combination I can have you understand that right. So, it is a full factorial design. There is a small catch to this you think that this is so, simple why do we need to worry about it ok.

Let us assume in this case your n which is your dimensions number of variables was 2 and your level, let us call it K was 2 as well. So, this is usually given as number of experiments is usually given as K raised to n . So, let us say that you retain your level to be 2, but you increase your number of variables to 7, not even 7 5 let us say 5 variables this is acceptable right 5 is not too much, how many experiments you need to do you, make it 7 how many experiments you need to do 128 experiments you need to do.

Imagine that you are talking about what you call at your rotor helicopter, you know like your helicopter failure you are trying to understand our under oscillatory landing condition, what happens to your helicopter of course, you want to understand failures. So, the helicopter will fail can you are crash car crash can you crash or fail 128 helicopters 7 factors is nothing, if you talk about a car crashing or an helicopter crashing 7 variables is nothing people will laugh if you say that I am taking 7 variables to evaluate it that is very less.

So if you take 7 variables and do itself you get 128. So, full factorial experiment might be a possible way, but it is not a feasible option because you have to keep on doing it So, if I do so, just to give you an idea if I do a 3 factor sorry 3 level what does it mean in the same two factor 1 factor 2, 3 levels means what this is factor 2 at this particular level I have 3 levels, for this factor level ok.


Similarly for this factor at this particular level I will have 3 realizations. So, in total 9 which is clear from this right so, the level is 3 raised to number of factors was 2 so, it is 9 get the point ok. So, now, if there were 3 factors just to give you an idea what will happen; obviously, I will go to factor 1 factor 2 and factor 3, I am going perpendicular to the screen, but did the factor 3.

This is how the increment happens, you see immediately you increased it only by a factor, but the number of points increased by 4 in that particular case, it was 4 initially for just factor 1 and factor 2, when I brought factor 3 it immediately went by another 4 points. Similarly if I had done 3 levels in this I just did two levels, if I did 3 levels, then I would have got 9 points just for two variables just similar to this.

And if I introduced the third factor I would have got another 9 points which is 18. So, you understand right. So, here imagine what will happen 3 levels raised to 5 how much is that 9 9 81 81 times 3 is 243 243 experiments with just 3 levels 3 levels is nothing, if you look at it with moderate even quadratic you saw you need 3 points.

For just for 5 variables so, it kind of exponentially builds up, so, there is a problem with full factorial experiment you cannot do a full factorial as you wish there is a problem.

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Full Factorial 
NPTEL

Trial #	A	B	AB
1	-1	-1	1
2	-1	1	-1
3	1	-1	-1
4	1	1	1

2 Factorial

Coded Factors

Trial #	A	B	C	AB	BC	CA	ABC
1	-1	-1	-1	1	1	1	-1
2	-1	-1	1	1	-1	-1	1
3	-1	1	-1	-1	-1	1	1
4	-1	1	1	-1	1	-1	-1
5	1	-1	-1	-1	1	-1	1
6	1	-1	1	-1	-1	1	-1
7	1	1	-1	1	-1	-1	-1
8	1	1	1	1	1	1	1

3 Factorial

$(2)^3$


$$\begin{matrix} A & B & C \\ \hline AB & AC & BC \\ \hline ABC \end{matrix}$$

And this is the way people try to do, for instance if it is two raise to 3 ok, this is 2 raise to 2 this is 2 raise to 3 3 factors and two levels there are about 8 experiments and, these are

all the coded factors meaning what values which you it will take meaning coded factors means minus 1 is 1 level and 1 is another level that is all ok. So, if A is minus 1 means it is a low level A is 1 means it is the high level. So, B so this basically tells you what are all the different combinations see, imagine simple you imagine this to B minus 1 minus 1 minus 1 that is what is this.

So, like that there should be 8 points I do not want to spend time writing the combinations ok, but just for the sake of this thing. So, in this particular case only factor 1 changes 1 so, 1 minus 1 minus 1 like that for 8 points there will be the other things and this AB BC CA is governed by that lets not these are all the interactions these are all the interactions ok.

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Types of Design
NPTEL

◆ Fractional Factorial Designs

- Full factorial is too expensive when there are too many factors
- Used to find "vital few" or quick assessment of all factors


Full Factorial Design

→

Factors	Levels	Symbol	Experiments
2	2	2^2	4
3	2	2^3	8
7	2	2^7	128
15	2	2^{15}	32768
2	3	3^2	9
3	3	3^3	27
7	3	3^7	2187

◆ Sparsity-of-Effects Principle

- Most statistically significant effects comes from main effects and lower order interactions

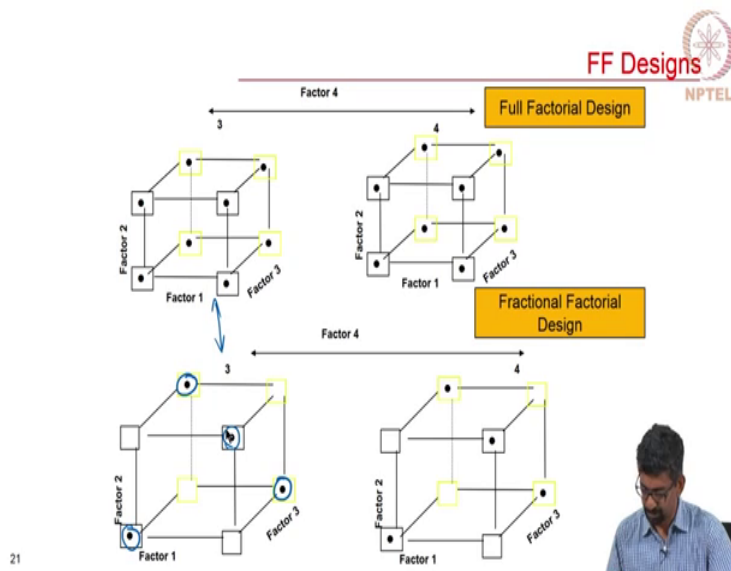


So, now, what people said is or rather Taguchi was the first guy to propose that was you actually do not need to run all the experiments ok. So, for instance if you take 3 factors, you have single factor A B C you have 2 factor interactions. And then you have 3 factor interactions. The assumptions and the observations are assumptions and observations are this effect is going to be smaller than these effects and these effects are going to be smaller this ok. The second thing is you cannot estimate these without estimating this you cannot estimate A 3 factor interaction without estimating the 2 factor interactions. So, that is the hierarchy. So, first you will have to do this then this then this.

So, what he said is if you are able to compromise certain things like these interactions, a little bit you do not need to run those many experiments, with only limited number of information with the only limited number of experiments, we can get as much as much information as we can much more than your what you call like, half it is not half meaning a half fractional factorial is not equal to half full factorial ok. You get much more information than a half full factorial for a fractional half fractional factorial of strength half.

So, full factorial as we pointed out is too expensive, sometimes it is used to you find the vital few ok. So, this is just to give you an idea with about 15 factors and two levels this is the amount of experiments that you need to do ok.

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Let me just do not worry about the top one, these are your full factor experiments, if you are talking about factor 1, factor 2, factor 3, then I need to have 8 experiments here right. Instead if I am looking at a fractional factorial, what it says is you do not need to run 8 just compare these two images. You do not need to come you did not need to run 8 of them, you need to run only 4 so, but then the 4 should have participation from all 3 of them.

So, what it says is if you take this guy, you take this guy, you do not take the other guys, you take this guy there are ways in which you can do this in this particular thing you can just say that I will take the diagonal quantity. And then I will not take the projections I

will take only the diagonal quantity, but that is not how it is done. So, do not worry about that point we will discuss how that is done ok.

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Design Characteristics
NPTEL

- ◆ **Confounding**
 - How effects are combined with each other
 - For example, main effects confounded with 2-way interactions or 2-way with 2-way

$\rightarrow AB$
 $\rightarrow BC$

- ◆ **Resolution**
 - Ability of design to represent independent effects without them getting combined
 - » Separated effects
 - » Resolution 3 means – Main Effect confounded with 2-way
 - ◆ (or A = BC)

- ◆ Higher resolution better, but more runs



There are also ways in which you can confound meaning confounding means you can combine ok, you take particular columns of the experiment and then you say it belongs to AB as well as BC, I do not know which one, but if that particular column becomes important, then I know that it is either AB, it is either AB, or BC, then I will do additional experiment to find out whether it was AB or BC meaning interaction wise. So, we will talk of the talk about that a little bit in detail.

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$L^{(k-p)}$ runs

L : # of levels

p : Reduction factor

Example

3-Factor $\frac{1}{2}$ Fractional Factorial

NPTEL

$L_8 \rightarrow L_4$

Trial #	A	B	C	AB	BC	CA	ABC
1	-1	-1	-1	1	1	1	-1
2	-1	-1	1	1	-1	-1	1
3	-1	1	-1	-1	1	1	-1
4	-1	1	1	-1	-1	-1	1
5	1	-1	-1	1	1	-1	-1
6	1	-1	1	1	-1	1	-1
7	1	1	-1	-1	-1	-1	1
8	1	1	1	-1	1	1	-1

- Resolution III Design - Main effects confounded with 2-way interactions

- ♦ Goal : From full factorial design for 3-factors, choose 4 runs
- ♦ The only way is to combine effects !!
- ♦ One way - Combine C with AB (or B with AC or A with BC)
 - Choose rows where C=AB (or B=AC or A=BC)
- ♦ That's half-fractional factorial

Handwritten notes:

- Column is balanced within
- Any 2 Cols are balanced between

A B C
AB AC BC
ABC

So, now, what does this fractional factorial experiment means ok. So, in this particular case if you see we are doing 2^3 ok, 2^3 meaning 2 levels 3 factors. The factors are A B and C what it says there are 8 experiments to begin with, but you want to reduce it to 4 experiment that is what we saw here there were 8 experiments actually, but I told you that you just need to run only 4 experiment. So, what does that mean how will you reduce it.

ok There is a property of this orthogonal array ok, we will just go back to an array and will discuss that, there is a property of this orthogonal array, what it says is over I am sorry this is not a orthogonal it is a full factorial will. So, I have to go here to explain yeah. The column is balanced within itself, any two columns are balanced between them, what is the meaning you take any column you can see the number of entries right minus 1 once, twice, thrice 4 times 1 once twice 3 times 4 times.

So, minus 1 and plus 1 will always be balanced if that is 3 this also will be 3 if that is n this will also be n, you take any two columns for the sake of discussion will take AC and BC you can see 1 1 happening once 1 1 happening twice. Any other combination also will happen only twice, what is that minus 1 minus 1 happened once minus 1 minus 1 happening twice that is it will not happen less or more.

One senses I should not have used this, because this is again a full factorial experiment the one the actual experiment was a full factorial experiment, but the idea still remains

the same the column is balanced with in and any two columns are balanced between them. So, you want to take this L 8 L stands for the Latin square and that is what this L stands for. And then the L 8; 8 stands for the number of experiments so, in this there is actually an L 8.

So, we are interested in going from L 8 to L 4 so, how will you reduce it from L 8 to L 4 so, that these two properties are preserved, but you do not go about doing it is very simple you can see, what we are trying to do is we are trying to combine this C to AB what we are saying is C wherever meaning see C, where factor C and AB take the same values where are they taking 1 and 1 minus 1 and minus 1. So, I am going to choose those rows and say bar I am sorry I am going to only take those rows and this will just become 1.

So the only way so, if I want to run if I want to reduce L 8 to L 4 ok, then I have to get rid of certain rows otherwise I cannot do that, because the 8 experiment should become 4 experiments, but which one will I use, or which one will I leave out is what I need to choose and we are using this idea to do that ok. So, now, what happens is in this particular case I have kind of assigned A B C A B C and all that ok, but that is a different discussion on how to there is something called a linear graph, there is something called a triangular table use in which you allot those stuff ok.

So, one way that you can do is combine C with AB or B with AC you can do anything, because you remember what we spoke about right like A B C AB AC BC and then A B C. So, first thing is I might want to topple this A B C. So, how can I do I can combine C and AB that is A B C, or I can do A and B C so, you can you can look at that for instance minus 1 minus 1 ok, they will be the same rows minus 1 minus 1 minus 1 minus 1, I am sorry are you able to follow typically I should have used another colour and, then you can see this one again 1 and 1, it will be the same set of rows.

So, the choose rows where C equal to AB or B equal to AC, or A equal to BC and then you use them that is your half fractional. So, wherever I have those cases I will use them ok, but one thing that you need to remember and, this is when you are using to factor what is happening is the number of columns, if you measure these number of column there will be 7 for an L 8 experiment you will have 7.

(Refer Slide Time: 32:09)

Example
4-Factor 1/2 Fraction Factorial

Trial #	A	B	C	D	AB	BC	CD	AC	AD	BD	ABC	ACD	ABD	BCD	ABCD
1	1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1
2	-1	-1	-1	1	1	1	-1	1	-1	-1	-1	1	1	1	-1
3	-1	-1	1	-1	1	-1	-1	-1	1	1	1	1	-1	1	-1
4	-1	-1	1	1	1	-1	1	-1	-1	-1	-1	1	-1	1	1
5	-1	1	-1	-1	-1	-1	1	1	1	1	1	-1	1	1	-1
6	-1	1	-1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	1
7	-1	1	1	-1	-1	1	-1	-1	1	-1	-1	-1	1	1	-1
8	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	-1	1	-1
9	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
10	1	-1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	1	1
11	1	-1	1	-1	-1	-1	-1	1	-1	1	-1	-1	-1	1	1
12	1	-1	1	1	-1	-1	1	1	1	-1	1	-1	-1	-1	-1
13	1	1	-1	-1	1	-1	1	-1	-1	-1	-1	1	-1	1	1
14	1	1	-1	1	1	-1	-1	-1	1	1	-1	-1	1	-1	-1
15	1	1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1


- Resolution IV Design - Main Effects are captured, 1 to 3-way is confounded and 2-way interactions are confounded with other 2-way interactions

NPTEL
(2)⁴
= 16
↓
x → L8
x → L4

ABC
|
|
|

ABC
AB BC
CD AC

AC




For an L 4 experiment how many how much you will have, you will have 3 columns so, that is why I want to know which one will I combine I will only take the first 3. So, I will take minus 1 minus 1 1 minus 1 1 1 minus 1 1 minus 1 1 ok. Similarly if you want to do for a 4 factor experiment 4 factor half fractional factorial experiment, this is out of this thing like a software or something so, A B C D A B A B C D up to A B C D ok.

So, basically this 2 raise to 4 which is 16 if you look at it there will be 16 experiments in this. So, I can reduce this to L 8 which is half fractional factorial or I can this is half fractional factorial, if we do to one fourth then it will be L 4 ok. So, the point is can I do L 4 practically no mathematical yes, because how many columns will L 4 have 3 so, it can have only A B C, but you have basic variable itself A B C D and as we have discussed you need to have at least the first layer done, you can compromise the other things AB BC BD CD AC AD, then A B C you can keep writing all the combinations.

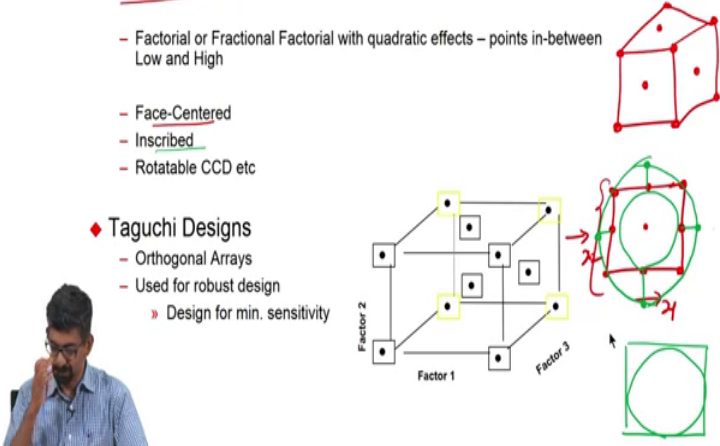
So, you can leave out these, but at least this should be preserved. So, your minimal thing if you are talking about 4 factors is at least L 8. So, if I want to take this L 16 and reduce it to L 8 what I need to do I need to kind of knock down this A B C D s to begin with so, A B C D means I can do A B C D separately, or A C D and B. So, if you look at it I am taking D and A B C in this particular case. So, wherever the pink colour is mentioned, it means they are taking the same value minus 1 minus 1 1 1. So, I can just take only those

values and then the blue arrows tell you what are the rows, that we will retain. So, we can retain we can bring it down to L 8 in that sense ok.

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Other (Screening) Designs 

- ◆ CCD (Central Composite Design)
 - Factorial or Fractional Factorial with quadratic effects – points in-between Low and High
 - Face-Centered
 - Inscribed
 - Rotatable CCD etc
- ◆ Taguchi Designs
 - Orthogonal Arrays
 - Used for robust design
 - ▶ Design for min. sensitivity



So, there are also other types of designs like your central composite design, there is a face centred ok. So, central composite design as the name suggests, under that there are different thing one is the face centred rotatable CCD and all that, what is the face centred says it says take a particular face, put a point at the centre also. So, that is what we have here yeah so, 1 2 3 and 4. So, there is a point in the centre there will be a point in the centre goes like that ok.

So, if I build another guy then I will have if I have another face here, I will have it like that and then you can keep on building it ok. So, that is a face centred, but then what people also suggested is if you are going to put additional values ok. So, can I put it here, there is a problem of putting it there, because if you see from this direction it was still only giving you 3 variables I mean sorry 3 values non variable sorry.

If you take $x_1 \times x_2$ what is happening is in this particular line my x_2 values are the same, for these 3 points my x_2 values are the same. So, they are not though I am doing 3 experiments, though I am doing 3 experiments I am getting only with respect to x_2 I am getting only one information value ok. So, is the case with anything so, what people suggested is instead of doing that you can kind of to a circle.

And this point will be pushed here, this point will be pushed here, this point will be pushed here and this point will be pushed here ok. You might still not you still what you call like lose information with respect to this and, this particular dimension, but at least from this dimension you get additional information.

Student: But that that I will go out (Refer Time: 37:37) what does it means (Refer Time: 37:38).

So, if that is your problem you can do it this way also, it need not be a I mean that is what is an inscribed circle is ok. So, the idea is the idea is like this actually you can put a square outside what I meant is; so, you can just push it accordingly whichever way you want ok. So, that is your inscribed circle and also people talked about rotatable CCD is using matrix notations and all that ok.

(Refer Slide Time: 38:26)

Characteristics of good design

- ◆ **Randomization**
 - Order of experimentation is random
 - Removes bias

- ◆ **Replication of experiments**
 - Captures variability in measurement
 - Helps to decide statistical significance of effects

- ◆ **Orthogonal**
 - Independence of factors

Independent Factors

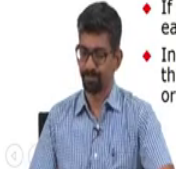

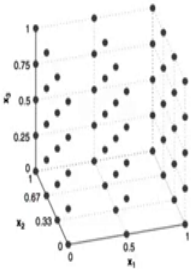

Trial #	A	B	AB
1	-1	-1	1
2	-1	1	-1
3	1	-1	-1
4	1	1	1

$A^*B = A^*AB = B^*AB = 0$

So, usually the Taguchi design is used in robust design principles, which is slightly out of scope, but we spoke yesterday write robust design means, if you have a particular response that is of interest you want to have the variability very less ok, that is called robust design you do not want a very flat variability. So, people also suggested that you need to do random experiments, you will have to replicate the experiments and orthogonality provides you the independence of factors ok.

(Refer Slide Time: 38:57)

Stratification



- ◆ Full factorial.
- ◆ If you project a particular variable. The points overlap on each other across the other variable- Loosing information
- ◆ Instead, you could divide the range into lots of bins and throw random sample. During projection, more 'sampling' or 'spread' is achieved - this is called stratified sampling

So, from here what we wanted to do is this one thing this Moriss Mitchell method and all these full factorial ideas, what they brought is this kind of a uniform sampling, uniform sampling is one thing, uniformity is one thing. The idea is I do not have any information about this design space. So, I do not want to focus my samples at any particular slot. So, I want to explore them uniformly that is the idea ok.