Surrogates and Approximations in Engineering Design

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Lecture – 03
Problem Formulation Example

So, we step back and then we take look at a general optimization problem statement ok.

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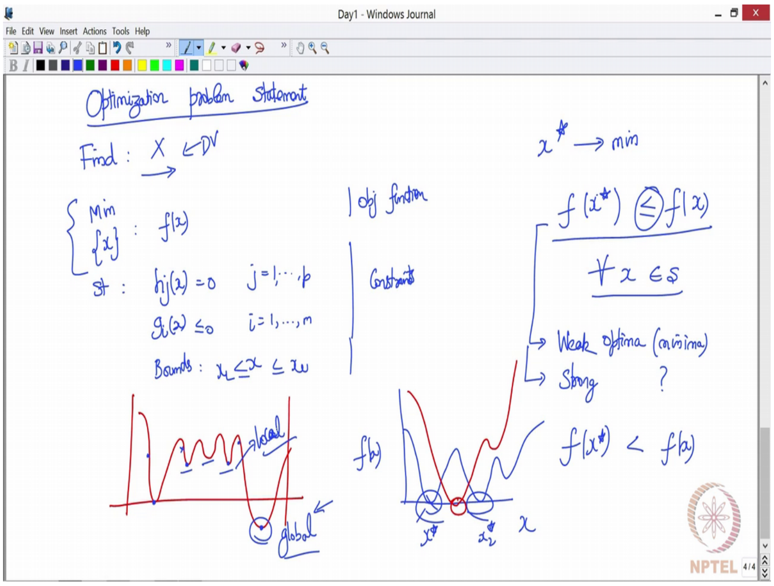

So, whatever we saw in the previous you know a few minutes ago was a subset of this. Any problem statement can be written like this, I want to find for instance X which is nothing, but your design variables that we have been talking about. Then minimize subject to those design variables f of x is what your objective function is and this is what your constraints are. And of course, in this constraint you can also put your bounds if you have x is less than a lower bound and upper bound if you have them ok, these are your constraint generic sense.

But before we go into discussing finding the minimum or minima let us see what the definition of minima means. What is the definition of minima or when do you say a point is minimum or a function is minimum at a particular point? A point cannot be minimum, but a function value at that point is minimum when do you say it is a minimum.

Student: All the other values are higher than.

All the other possible values are higher than that. So, if you say x star is minimum, what does it mean is f of x star the function value evaluated at x star is less than or equal to f of x i. It should be the other way round ok, for all x belonging to s I do not need to put an i. I am looking at it more from a sampling perspective x i means x star is a particular point, x is a collection of points.

So, for all the x's belonging to the space that you are talking about ok. This if this holds good then x star is minimum or f at x star is minimum, this is called a weak optima. And there is a strong optima ok, what it means is this less than or are equal to is there. What it means is there could also be some other point, for sure this is takes a minimum value, but there can also be another point.

Student: (Refer Time: 03:05).

Which takes the same value, that is why it says less than are equal to whereas, what this one says is it is for sure it is.

Student: (Refer Time: 03:13).

Less than.

Student: (Refer Time: 03:14).

So, if you have a function like this they both take the same values should take the same values, difficult to draw. So, let us say this is s and x and this is f of x. So, both of them are this is x 1 star, this is x 2 star they take two values, but in case if I did not have this part let us say that I had a function like this then only this is my minima; the other question that I wanted to ok. So, what is this called the blue curve that I draw what kind of an optimum problem it is you know, something like when you have more than one local optima this is also called local optima's. When I say local there is also a.

Student: Global.

Global ok: so let us say that there is a function like this where is optimum in this function and of course, I bound it here. So, this guy is the optimum, but as you will see next all optimization algorithms work on some starting point. So, what happens is if I start my algorithm here it is likely to go and get locked here, that is why they ask do you do did you do multiple starting points ok. If you started here it will go and end up here ok. So, you could have done an equal sampling and you might end up starting here and it might go and then here ok.

So, what is happening is from what we have discussed here all of these satisfy those conditions. Any valley is a minima so, it is called a local minima, when you have more then it becomes a local and this guy is your.

Student: Global.

Global, heuristic algorithms like genetic algorithm, particle swarm optimization differential evolution all that seemed to work better to give you a global optima, there is no guarantee. But they are more computationally expensive than your gradient based approaches, which does not claim that it gives you a global optima, but one way to deal with that is to have multiple starting points. So, then there is an argument maybe if I am going to use multiple starting point and evaluate the function multiple time, I might as well use evolutionary algorithms true. That is where the designer comes into picture unless you believe that your function is highly non-linear you do not need to worry about it ok.

So, these are all called local optima's ok. So, often times in structural engineering we are happy with local optima unless you believe that it is a non-linear problem and you really need to find your global optima. With local optima is still we are happy enough ok. Let me ask you a now, let us just solve a simple example so, that you understand what we are looking at.

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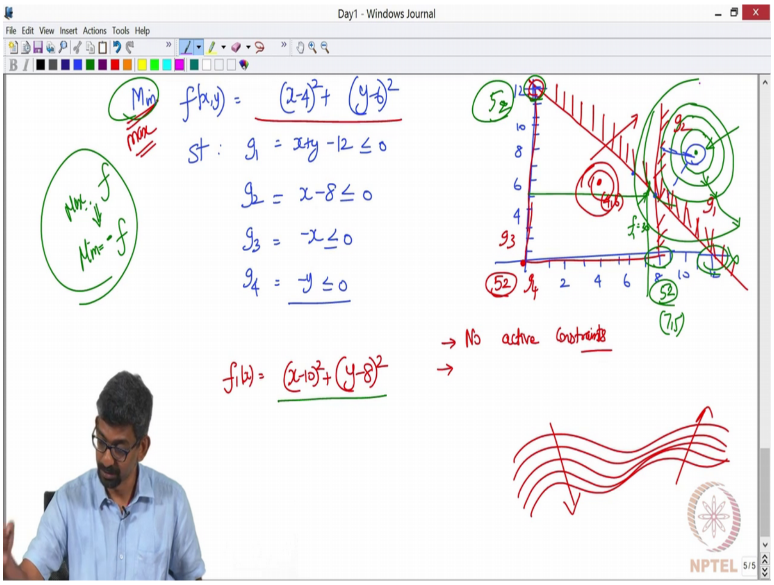

Minimize f of x y very simple problem, after this we will switch to slides, this I want you to try on your book ok. So, just if you observe here I wrote everything in the form of less than or equal to ok. This is the generic way of writing a optimization problem, if you have inequality constraints you always write it as less than or equal to. See for instance this; what does this equation mean it means.

Student: Y is greater than.

Y is.

Student: Greater than.

Y is greater than or equal to 0, but since I want to stick to a format of less than or equal to I am writing it as minus y is less than or equal to 0 ok; that is a way in which it is usually written. Now, what is this objective function mean x minus 4 whole squared plus y minus 6 whole square it is a.

Student: (Refer Time: 08:31).

It is a circle with.

Student: Center 4.

With center 4 and 6.

Student: (Refer Time: 08:41).

So, 4 and 6 that is the center ok. Now, let us also draw the constraints g 1 equals x plus y minus 12. What does it mean when my x takes 0 my y should be?

Student: (Refer Time: 09:25).

Similarly, when my x is.

Student: Y is 0.

Y is 0.

Student: (Refer Time: 09:35).

It is not to scale then some other intermediate value because, it is a linear equation just two points is good enough. But if you want you just connect some 6 plus 6 ok, this 6 and this 6. So, that is a line and then the way we write this is it says less than or equal to. So, this is your infeasible domain ok.

So, this is your feasible domain right now, just before that this entire rectangle was your domain. But now, when I put your constraints you are saying that this half this side is not feasible, only this is your feasible domain ok. Do not get confused with this tick mark and this is to mark in feasible domain ok. Then what other constraints you have?

Student: X minus 8.

X minus 8, what does that mean? X is.

Student: Vertical (Refer Time: 10:50) vertical (Refer Time: 10:51).

X minus 8 right. So, x should be greater than 8.

Student: Less that (Refer Time: 10:58).

Sorry less than or equal to 8 sorry. So, you derive sorry you draw a.

Student: Particular.

Line here and then it should be less than. So, only this is your good area ok. This is your g 2, this is your g 1 then what is your g 3 it says x should be.

Student: Greater.

Greater than.

Student: 1 0.

Sorry this is the other one.

Student: (Refer Time: 11:25).

This is your third constraint which is.

Student: (Refer Time: 11:30).

G 3 and this is your y should be greater than 0, which is your g 4 ok. So now, this region here is what your feasible domains ok. Now, tell me out of this in this feasible domain which means, the constraints are satisfied where is your optima.

Student: Center, center.

Where is your optima in the sense, when does this function take the minimum value.

Student: (Refer Time: 12:05).

Can I take different values? Yes, it can take different values depending on what your x and y, you plug in different x and y it will give you different f, but when will it be the minimum.

Student: (Refer Time: 12:16).

When.

Student: (Refer Time: 12:17).

You plug 4 here and when you plug 6 here.

Student: (Refer Time: 12:19).

It will give, I mean this is geometrically you can see visualize you can see and this is your optima 4 and 6. So, what is this called? It is said as an no active constraints, I removed all the constraints let us say. What is your optima?

Student: Same.

It is same.

Student: Same.

So, the constraints are not playing a role in forcing your optimum ok. Let us change the same problem meaning retaining the constraints, but I am converting the objective function ok. What I am doing is, I am giving you a new objective function f 1 of x is x minus 10 the whole squared plus y minus 8 the whole squared. I should have drawn it in the center, but it is ok. Now, I will use a different colour for this guy as usual we will plot this is also an equation of a circle. So, we will say x is 10 and y is 8. So, this guy is somewhere here center the center of the circle is not the optima because, he is in the.

Student: (Refer Time: 14:00).

Infeasible domain; now, it is an optimization problem correct because, you want to minimize. When will it be the minima? When you plug in 10 and 8 here it will be 0, you put any other value this guy is going to take positive values now. It will become 1 2 3 20, it will take different for different values it will take different, it will have different f 1 values right. So, what is what does it mean is it is going to keep.

Student: Increasing.

Increasing, I am just drawing I do not know how it is going to increase. It is likely to increase this way. And these are all iso contours ok, it is not like what I have drawn they are all iso contours. They are [FL] circles on above with the center being the same meaning the radius is changing that is all the center is the same my radius is changing. There is just iso contours imagine, that you are inflating a circle it is becoming a bigger and bigger balloon ok. Now, tell me guessed by commonsense by visually looking at it you can tell me where the optima will lie.

Student: (Refer Time: 15:14).

The way especially the way I have withdrawn one more circle I draw it will touch the optima ok, where will it I mean I did; I did not mean to do that, but then when I draw I realized and then I put a slightly lesser radius.

Student: (Refer Time: 15:29).

It could be a I do not know it could also be a, what he called.

Student: (Refer Time: 15:36).

Yeah.

Student: Center.

I mean you do not need to give me the point, tell me when it is a one feature that you observe.

Student: (Refer Time: 15:48).

For the optima.

Student: It one (Refer Time: 15:50).

Should be is different you do not know should be right now.

Student: Yes, sir.

Should be is derived out of this observation ok, let me ask you something ok. So, I keep doing this right. So, then suddenly there will be a circle let us, not worry about completing it is a circle that is completed right. But then can this be an optimal circle because, it is inside the feasible domain hm. So, I do not know let me give a value of f equal to 30 for this guy, I am just giving a number it is not 30 I do not know I am just giving a number f 1 equal to 30 ok. So, if I show you this figure and I am saying that f 1 equal to 30 is the minimum function value that you can get.

There is more resource to reduce, there was a person who ran faster than Usain Bolt which meaning what, there is more resource in terms of time. If you are going to put the best person ok, let us there let us say that in spaces going to go run take something and come back you need the best person to run ok. You think you will take Usain Bolt or Gatlin.

Student: (Refer Time: 17:09).

Based on the data you should take not based on history ok, you will only take Gatlin because he is likely to reduce more. So, this is this is Usain Bolt because, he still can improve in this direction ok. So, you should get a better candidate. So, this is not optimize you still have more. So, this should give you some information on any problem, where the optima is likely to fall. Where will it fall? Not only in this problem, in any problem based on this discussion that just now we had it should be in the.

Student: (Refer Time: 17:46).

See please understand that your objective function is increasing in this direction or it is decreasing in the opposite direction. In our problem, what we are saying is just keep your focus on me ok. Your objective function is increasing in one direction which is this direction ok. And what is your actual goal?

Student: Minimize.

You want to minimize, what it means you are going in the opposite direction of how the function is increasing this is when optimization comes into picture ok. So, I am running like I do not know like 6 minutes per kilometer ok. Now, you are saying the best performer is 7 minutes, there is no optimization in my case I do not need to worry at all ok. If you tell me if you want to go and run this half marathon you like to run like 5 minutes per minute sorry 5 minutes per kilometer.

Then I need to worry about what I need to do so, that I am going in the opposite direction ok. Because, it is easier for me it should be difficult for me that is when it is optimization ok. So, it is conflicting it is easier for you to relax it is difficult for you t on the other side to performance ok. So, that is the idea that is something that you need to keep in mind. If you are trying to optimize and you are getting, you are relaxing then it is not optimization please understand you should.

Student: Constraint.

It should it should in it should be in the opposite side, it should pull you in the opposite side, it should pull you in the opposite side ok. If this is what is your optimization side and then your function is also decreasing in the side; meaning your cost is also decreasing in this side it does not make any sense ok. If you want better features in your cell phone, what is going to be the cost it is going to be.

Student: High cost.

It is going to be a higher cost. It will not be a lower cost that is for you, for a cell phone manufacturer it is the other way around. If I put more features it is increasing, but then I want to put it in the band where my people will get it. So, I want to cost I want to bring down the cost in each of them where can I do it, but I still want to give the same features that is exactly this problem.

So, the function is increasing in one side and you want to minimize that. So, ideally speaking I would have liked to have this that particular point is what I would have liked to have because, it is 0 that is the best optima that I can get because you cannot shrink beyond a.

Student: (Refer Time: 20:33).

Dot correct, you cannot shrink beyond a dot. So, that is the best optima that I could have got, but I cannot now let us see how far you will have to relax. So, I am going I am going this side I am keeping ongoing. When do you think I know that I have reached my optima? You do not need to give me the function value. I myself do not know what the function value is. In this when do you think that I will are you or going in this direction whichever, direction it is. When do you think that you will you know that you have achieved or you are you are near or you are at the optima.

Student: Like at the constraint.

When you hit the.

Student: Constraint.

Constraint.

Student: (Refer Time: 21:09).

That is why I put this specifically.

Student: (Refer Time: 21:12).

I am come in this direction and I have hit the constrained.

Student: (Refer Time: 21:16).

But is that still an optima: no, because with respect to this constraint with respect to this constraint I am not.

Student: (Refer Time: 21:22).

I am not. So, it should touch it should be towards the feasible domain ok. It need not be this point, please understand it need not be this point. There is no need that all your constraint need to be active ok, that is what the first problem we saw. None of the constraint need to be active, it is still optima.

Student: (Refer Time: 21:46).

It is just an inactive constraint that is all. So, in this particular case I guess if you if I had properly drawn the circle whose a actually hit somewhere here ok. It will hit somewhere here, it is not on that and that point is likely to be f equals 18, if I am not wrong ok. It should be 7 comma 5 that you can do for yourself, it should be it will be somewhere here. Maybe somewhere here it should be 7 comma 5, that will be your thing.

There are other observations that you can make out of it is now, this is the direction in which my cost function is increasing correct dot circle. So, it is increasing in this direction. In the same view which is the direction in which my constraint is increasing. If you have observed me I actually showed.

Student: Yeah.

In which direction is my constraint, it is actually going in the.

Student: Opposite direction.

So, my function minimization I want to keep it smaller and smaller whereas, my constraint is going in the opposite direction. So, it is that is when optimization comes into picture ok. You have only 24 hours ok, you want to spend time with your friend, you want to do research, you want to attain classes. Sometimes people call you ruthlessly on a Saturday to attend an old course, you need to come for that.

So, beyond all these things you want to do sports, you want to do you want to go to a movie everything, but then the time is only 24 hours ok. And then again we are going to put constraint that you need to sleep for 6 hour, you need to sleep for 8 hour, certain things you cannot do it in the night, certain things you cannot do it in the day. So, with all these constraints you are getting only lesser and lesser a time, but you want to do more and more things ok.

So, it is coming in the opposite direction, you want to go in this direction, but the time is pulling you this way. So, that is when optimization comes into picture that is exactly what this is, the constraint is it going in the opposite direction and you your objective is in I mean the constraint and the objectives are in opposite direction. So, they should be conflicting this is the basic idea of a problem formulation. Sometimes we get papers where the objective function and the constraints are really not conflicting, but they would not show that ok. Maybe, if they would have figured out if they would have written the proper formulation, instead they will just say we want to minimize something subject to this thing ok. And then they present some results also, but then it is very evident in the result that there is they are not conflicting.

So, they will always be in the minimum bound whatever, minimum bound you take they will always be there. Irrespective of whatever, you do they will go into the minimum bound, because when you extend the design space beyond the minimum that will again become the minimum ok. So, it unless it is conflicting you will not get that kind of a result. Sorry unless, unless it is conflicting then it is not optimization that is what it is ok. Now, let us go back to this example, the first example that we discussed and I have a question for you, but small difference instead of minimization it is.

Student: (Refer Time: 25:10).

What is that? You take it is the same, what you call the constraints are all the same. Imagine that the green is not here that is all, the green circles iso contours are not there. Iso contours is something that you need to get used to, if you are doing metamodels and optimization ok. So, if you had a function like this let us say the iso contour of that function is this it is increasing in this direction; it is decreasing in this direction ok. Now, tell me what is the maxima of this thing x minus 4 y minus 6, you cannot say 4 comma 6 because that is the minima.

Student: (Refer Time: 25:58).

What is the maximum value that you can have? So, what is happening the circle is increasing in this direction. So, I can keep going enlarging in this side right, but.

Student: (Refer Time: 26:08).

Only in my feasible domain; so the constraint is your 24 hours I am capable of going out with friends, I can spend time on dinner, I can do research, I can do 5 papers per month. I have the capability to make 50 papers, one of the major things that prevents me from doing that is the time part of it. So, this is a constraint I can keep on increasing, but then it will be limited by the.

Student: Constraint.

Constraint. So, can you tell me any idea?

Student: (Refer Time: 26:40) 0 0.

Sorry.

Student: 0 0.

0 0 ok. So, 0 0 what will be your function value?

Student: 52 (Refer Time: 26:51) 64.

Some 52, 16 plus 36.

Student: 52.

I will give you another point.

Student: (Refer Time: 26:58).

This guy.

Student: Yeah.

So, he is 12 minus 6 is 6 36 0.

Student: 16.

Sorry 16 plus 36 is also 52 is that all.

Student: Intersection of (Refer Time: 27:25).

That point.

Student: Right (Refer Time: 27:30).

Sorry, sorry not that point sorry this point sorry feasible domain sorry correct. Accidentally I mark then you are alert, this is not the point this point 8 minus 4 is.

Student: (Refer Time: 27:44).

Still 4.

Student: (Refer Time: 27:45).

6. So, that is also 52.

Student: That is y equal to 0.

Sorry.

Student: That is 16 (Refer Time: 27:56) y is equal to 0.

Y equal to 0.

Student: (Refer Time: 27:59).

So, 6 squared is 36 8 minus 4 is 16 so, it is 52. The function values right. So, what does that mean?

Student: (Refer Time: 28:06).

So, you have.

Student: Multiple optima.

Multiple.

Student: (Refer Time: 28:12).

Optima's. Do you have global optima?

Student: All of them could be global.

It is you lost the meaning of global then, global means is one guy.

Student: Say they all have the same function value.

Now, there are multiple optima's that is all each one is a local optima, there is no global optima. So, it is less than or equal to or sorry less than or equal in the maximization greater than or equal to; it is not like greater than none of this is greater than all other points. So, you can call a weak global optima if you want, I do not know whether that phrase exists you can call it a weak global optima because, there are also other optima solutions ok. So, that is an example ok, but in our course we will only talk about minima. Why is that I am not interested in profit huh?

Student: Standard (Refer Time: 29:07).

So, because.

Student: (Refer Time: 29:09).

There is only a small difference between a minimization and maximization. What is that?

Student: Negative side.

So, minimizing f or rather maximizing f is equal to minimizing.

Student: Minus (Refer Time: 29:28) minus.

That is all, you are just going to flip the function and whatever where the peaks there will become the valleys. So, it is easier to develop algorithms for one understanding and extend it for the other one, nothing will change it is the same thing ok. Instead of looking for positive you need to look for negative. So, instead of going and changing the analysis every time what you do is in the form of casting you do it because, the analysis remains the same after that.

So, that is just a simple way of doing it. So, we will always talk I mean I might give questions in the assignments or the exams this is not for you with the maximization ok. Because, the usual mistake that people do is this they will formulate it everything, but finally, they will have to multiply it by the negative to get the maximum value. They will say the optimal value is minus f and they will finish it ok. So, we usually give that problem as a assignment problem or a homework problem ok.

So, the one of the reasons this particular example was introduced was to achieve three different things ok. One is to get you introduced to the contour kind of a stuff, where which is what we will be using it ok. So, you need to if you ask me you told too many things in this slide; what is it that we need to do we need to take home very importantly. So, you need to know that let us call this objective function I need to be q j ok. Let us just call it constraint, this is not a constraint this is the objective function right f.

So, f increasing in what direction, decreasing in what direction, iso contours that is one stuff. Of course equality, inequality constraint there was no equality constraint, but inequality constraint, active constraints. And in active constraints, how constraint need not play a role. Even in this one only one of the constraints will play a role, the other one will be an inactive constraint.

And of course, these guys are also will be inactive. Only one constraint would have played a role in the other example as well, that was one thing. And of course, the third-one was the maxima minimization. In maxima you also saw multiple local optima's that is ok. So, this example was introduced for that particular case.