Surrogates and Approximations in Engineering Design
Prof. Palaniappan Ramu
Department of Engineering Design
Indian Institute of Technology, Madras

Lecture - 02
"Basic Optimization Problem Formulation"

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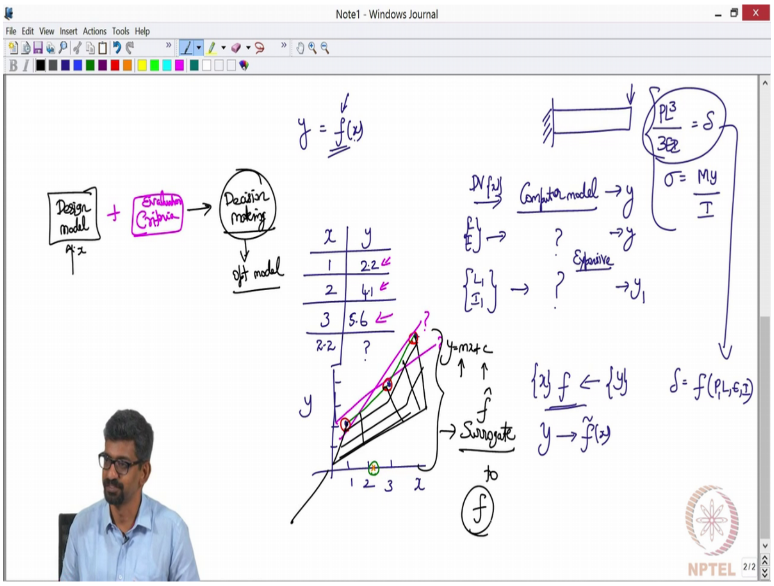


So, what we assumed in the previous cases you have a PL cube by 3 EI S displacement or it could have been some classical stuff that you talk about right. Let us say that you did not have them or this is a complex cantilever beam that you I do not have a closed form solution. But, my problem requirement says that I need to meet some displacement constraints or I need to meet some stress constraints.

And obviously, our goal is to reduce weight in this case or it could be around, I want to reduce stress. You are building a space structure, you are not worried about cost, you are not you are worried about weight though you are not worried about cost. So, material selection could be the highest material that you can build. I want less stress on it, I want less deflection in it, I want less heat emission in a cell phone device that you are doing more usage, but I want less heat dissipation. And the three examples that I just quickly told you like the space structure, the cell phone you do not have a closed form solution like this. But, you need to be able to compute your stresses sorry stresses or a displacement. How can you do that? As we have n number of times we just now, we discussed we can use a.

Student: (Refer Time: 01:57).

Computer model to do this right, but the computer model as we just saw now, what it does it tells you please give me a set of input variables which is nothing, but your design variables and I will give you a response y, correct. But, it does it only for one variable please understand, one combination of your design variable. For the sake of discussion we will say L and I are the design variables.

Student: (Refer Time: 02:30).

Then you feed this guy to the computer model, it will give you a y that is all that is it. If you change this L and I, you will have to redo the computer model it will give you another y correct. So, every time I have a new L and new E, I need to go plug it in and then it will give me a new.

Student: (Refer Time: 03:01).

Delta. So, in this case we just call it y in general ok, but what happens if this computer model is very expensive.

Student: Expensive (Refer Time: 03:22).

Expensive means in terms of time.

Student: Time.

Imagine that it is a very complex cantilever beam ok, cantilever beam cannot be complex beyond a it is a complex structure ok. So, it takes about today, if people want to do some kind of an aerodynamic study which mimics wind tunnel, in a cluster not our clustered in an industrial cluster it takes 4 days for one combination ok. Even some crash studies, in our cluster we run one crash study takes about overnight and you know right that overnight is not just overnight, you are waited for 4 days and then you get an overnight because, there are 599 other faculty who are also competing for the same resource.

So, you will have to look it up from that sense, you cannot just say it is only 12 hours no it is actually 48 hours or maybe more than that not 48. For example, I am saying one simulation waiting time is 32 hours and then 18 hours for running the simulation. So, it is 50 hours actually, that is just to give you an idea. Let us not worry about the waiting period.

So, that the when I said expensive it means that computational time, but computational time is related to your cost. There is a cost because, you need computer facility waiting time everything because, in the industry it become very important. I cannot wait for 100 days because, my product release time itself is only 3 months. I cannot wait for 100 days for you to get all the simulations ok anyway.

So, we are slightly digressing, but the question is, if this is expensive what would you like to do? You would like to have something like this, like this ok. So, the question is can I relate if I would call these design variables as x, can I relate this x and y ok; that is what the question is. So, this x when I am writing it as a vector, it means x 1 x 2 x 3, but they could have multiple values of x 1 x 2 x 3. It should actually be written as a matrix in that sense similarly, the y is not a vector it is not only y it could be y 1 y 2 y 3.

So, you usually write it as ok, we write it as y equals f of x; the question is can I approximate this f. So, right now what we are in the regular cantilever beam problem that we are talking about. So, your y is your delta you are saying it is a function of PL comma E comma I that is what it is ok. This equation PL cube by 3 EI is the f, but you do not have this equation then you go to the help of computer model ok, then you want to approximate this f. So, f hat it is not r f tilde ok, it is an approximation it is not the regular f. So, can I approximate it, how do I go about approximating. How can I approximate?

Student: (Refer Time: 06:36) factors.

Sorry.

Student: Using some factors.

Some factors ok, in a simple sense what is happening is let us say 1D problem.

Student: (Refer Time: 06:47).

1D problem I have some x, I have some y. I just give you 3 points and I am asking can you is very difficult ok, this data through a complex process I have gotten this x and y ok. But, I am saying that this takes about 4 days for each of the entry I have spent 12 days to get this data; is there an easier way. So, that I want for 2.2 can you tell me what the values is? It possible for you to do this? This is the question ok.

So, what is the goal I mean what is my goal is can you find a relationship this f can you find. So, what it means is you give me a new x I will give you the new y ok. You cannot say sir you have got this x and y, why do not you go and find yes it will take 4 days, but can you give it to me in few seconds; can you give me a closed form solution and that need not be the.

Student: (Refer Time: 07:49).

That may not be the exact, see please understand when you try to reconstruct this guy you might not end up reconstructing the same equation. You might get something like PL raise to 2.8 divided by 3 I raise to 1.1, you might get something like that because, that is based on data and the data could have errors. Actually, as a matter of fact I should do it like this. This should have been the data because there is always some measurement errors or some other factors that influence. So, if you look at it since, I wrote it the first way the way that I wrote it was y equals.

Student: Two times.

2 x.

Student: (Refer Time: 08:34).

That is what I wrote ok, but I introduce some errors. So, that when you try to construct this f, you will not get it as 2 times x you might get something else. You might get 2.1 x or 1.9 x whatever, it is you will get. Student: (Refer Time: 08:50).

One simple way is just try looking at it is it is not necessary. One way when you have data is to plot them ok. The moment I say this most of the people answer the question ok. So, 2.2 means it was somewhere here, then 4.1 somewhere here, 5.6 will be somewhere here. This is the type of data that you have, the red circles are the data ok. Now, I am asking can you tell me at 2.2 what will be the value of y. How can you find it?

Student: Mind interpolation.

Now, I change my mind and then I am asking for, can you tell me for 3.6 what the values is?

Student: Sir, making a graph and then at that point were the graph intersects.

So, how will you go about graphing, but 1 second let me go back to the question that I asked him. So, 2.2 you said that you will interpolate between these two values and you will find it, but then I changed my mind because we are professors right. So, what we will do is this I will suddenly ask 3.6. What will you do? You will again go and interpolate and you know that I am going to give you 100 such questions. So, every time you will go and interpolate; it is interpolation, but you will have to do a wise interpolation that kind of takes a form that he says. What is that you will do?

Student: So, we will make a graph joining the points.

You will make the graph joining the point, but there I can join the multiple ways.

Student: (Refer Time: 10:40).

So, one is like this now, you decide how I want to connect the other one; just like this there is some mathematical issues when I do this.

Student: You should find the relation between a y and x.

And that is the overall goal correct.

Student: (Refer Time: 10:55).

That is the overall goal, I just I am just trying to help you I am just trying to help you to go towards that you are absolutely right that is a overall idea. Because, the moment you have y equals 2 times x, this simple right I give you the x you can find what the y is. But how do you know it is 2, it is not 3, it is not 1.8, it is not 1.4. So, this is I please understand this is not that necessarily the way to go about it, I am just I just plotted I just plotted and then I am asked. I thought that it might give you some indicators to do that ok, I am I am not saying that this is the way to do it ok.

So, if you do it like this there is some issues with the function itself about continuity and all that ok. So, what might be the way to do it?

Student: (Refer Time: 11:43).

So, one way to look at it very simply is you look for.

Student: (Refer Time: 11:49).

As you pointed out I want to approximate the function y equal to x. So, one simple question is how does this data look like? I know it does not fall in a straight line, but it is pretty close to a straight line that I would draw; straight line means not a flat line, but the line like this. Just trying to make sure that I am not putting a light colour, there could be other questions why did you choose this, can't you choose this? Yes, I will choose there is an alternative, this is what I told you there is an alternative ok.

The question is how will I choose, which curve I will take. I can write again draw like million curves here with these 3 points, million is just a number to tell you that there are lot of curves that again wrong ok.

Student: (Refer Time: 12:34).

Sorry.

Student: Least square method.

So, there is a method called the least square ok, least square fit which we will discuss when you go to something called polynomial response surface so, you choose that. So, I just wanted to expose you to a problem ok; you will use some kind of a method that says that need not go through all the points because, as I pointed out there is always error in these numbers themselves. So, you want to do something called a best fit, that best is a criteria that you would choose. For him the best was battery backup, for him the best was.

Student: Aesthetics.

Aesthetics, for someone the best was the selfie camera. So, the best differs, but for him that was the best fit for him that was the other one was the best fit ok. So, the best is what your criteria is right. So, the criteria allows you to take you towards the goal or let us call the evaluation criteria. I do not know whether how good is this, I will say the design model kind of takes you to your decision making. In whichever field you are looking at this is what we are looking at, you are interested in making a decision. If you are a metallurgist, you want to know what material model I want to use that is a decision making.

For which you need to have a design model which is a finite element or an experimental model that will give you some values, for different conditions of stress or strain. And appropriately, you need to have an evaluation criteria. What is it that I am looking at? I am not interested in the elastic zone, I am interested only in the plastic zone and I am looking for a second order representation, I am looking for a bilinear model, I am looking for a Cook's model kind of a representation. Is it an exponential model? Is it a generalized Pareto model? So, I would use something like that. So, whatever is the evaluation criteria then I make my decision based on that ok. So, this is the overall idea.

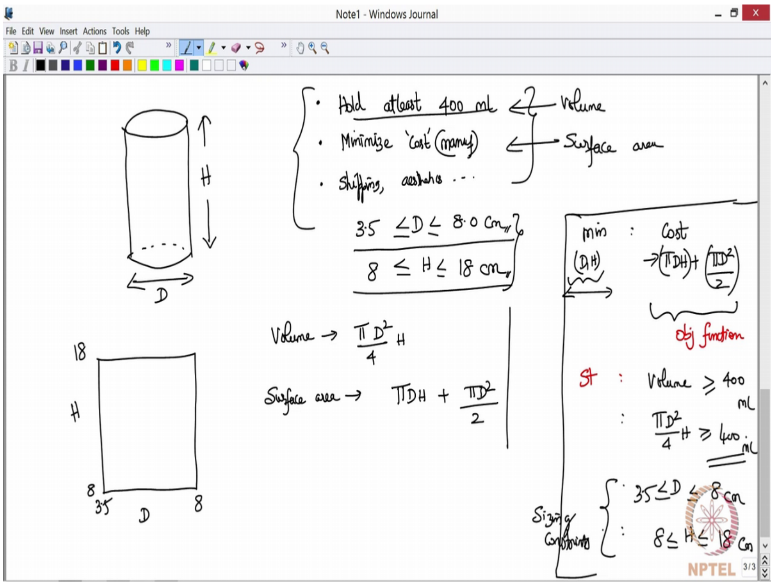
But in this sometimes your design model is what we are talking about; the design also comprises your analysis right. I would not have if you give me a new set of x, it should generate a y new set of y so, that I can evaluate that particular criteria. So, I need to have an f, I might not have an f explicitly all the time then I need to construct that f; this is what we just discussed just now ok. A simple you can just call this line as y equal to mx plus c then you just need to find your m and c because, x and y is something that you already have that that is the simplest way of looking at it. The moment I make this into a n dimensional problem, this is not a line anymore it becomes a surface.

So, you need to find a surface that will fit that will run through the points of course, I have other points as well it is just not a projection ok. So, this f that you are going to construct this guy that we did just now is what we call the surrogate. Because, it is not the actual function it is only a f hat it is a surrogate to f, but f is ideal what I call in my courses this is god given that is all. If god comes and asks you what do you want you asked for?

Student: (Refer Time: 16:22).

Should ask for f ok, then later you will know why you should not ask for an f ok. If god comes and asks you are trying to do an optimization, he comes in says I will give you one wish what do you want. You say god please give me f job is solved ok, you do not need to spend 4 hours 4 days to run simulations at different points; f is there your job is solved from whatever we have seen up as of now correct, agree fine.

(Refer Slide Time: 16:53)



(Refer Time: 16:52) So, that is a H and this is the D. So, the reason that we are going to discuss this problem is I will introduce the formal way of casting an optimization problem ok. So, what I am interested in this one is I want to design such a cylindrical structure such that it will hold at least 400 mL right yeah, 400 mL content water any soft drink that you want. I want to minimize obviously, the cost the manufacturing cost yeah.

So, some shipping and aesthetics etcetera impose some constraints which are on the design variables. They cannot vary between they cannot vary beyond 3.5 or 8 centimeters. Similarly, H also cannot vary beyond 8 and 18. This is your problem, I would not say statement this is what you are looking at. Now, how will you cast this as a math in mathematically how will you do that, how will you convert this stuff that I have told you or I have given you in a mathematical sense.

Student: Hold.

When you say hold at least 400 mL, what does that mean?

Student: Volume.

Volume of this guy; so, let us first write what that is. So, volume and I want to minimize the cost; let me say manufacturing and it is in this particular case directly related to material let us say, the amount of material that you are using. Do not do not think too much on the machining or the laser and all that, when I say cost it is only the material cost that we are talking about. How will you go about evaluating the material that is required for making this?

Student: (Refer Time: 19:31) thickness.

Thickness is there, but let us say its constant thickness.

Student: surface area (Refer Time: 19:40).

So, that is not a variable. So, it is.

Student: Surface.

Surface area correct thickness is one thing, but if I am going to use a 3 mm or a 5 mm, I will use the same across. So, what will matter is what is the.

Student: (Refer Time: 19:57).

Length and width or what is the diameter of that thing that I did. So, what is the surface area of this guy? Sorry, what is the volume? Pi.

Student: (Refer Time: 20:06).

Let us do it in D.

Student: D (Refer Time: 20:09).

So, that because B is a variable here we use pi D square.

Student: (Refer Time: 20:13).

Pi D squared by 4 that is all.

Student: Into H, into H.

Into H ok, then what about your surface area?

Student: Pi D H plus the (Refer Time: 20:25) 2 pi 2 2.

100 time, what is it?

Student: (Refer Time: 20:29).

Pi.

Student: 2 pi (Refer Time: 20:31).

Pi D H.

Student: Plus 2 2 (Refer Time: 20:38) pi D square by 2 (Refer Time: 20:39).

Pi D squared by.

Student: 2.

Do you know how this came?

Student: (Refer Time: 20:47).

Because, the area is pi D square by 4.

Student: 100 only.

But I have a top surface and I have a bottom surface ok. Hence, 2 times pi D squared by 4 become pi D squared by 2 and this pi D H is D times H is your rectangle, but of course, in this cylinder I just take the pi into account. So, it becomes pi D H. So, it is a surface area of the cylinder that we are talking about ok great. Now, you are in this case you know all this ok, but again let us say that it was a complex tank you do not know that. Then you will have to build a CAD model and then CAD will give you the volume, which is essentially it is breaking it into known volumes that is all ok; anyway let us not worry about that part.

Now, how will you because you have just got the quantities. So, you give me a new D new H I will get the volume, you give me a new D new H I will get the surface area. So, what are these called? This just satisfies the analysis part you understood right. These equations are nothing, but the analysis part you give me a new D and H I will give you the new volume on the surface area ok, which means you give me another new D and H I will give you another. So, there is an alternative, I can alter and you see one is it could be fat and it could be short for the same volume or it could be slender.

Student: (Refer Time: 22:10).

It is slender and long for the same volume or you can get a trade-off between the two. So, basically by increasing decreasing D and H for the same value I can get multiple combinations as well. So, now which one will I choose that is called your criteria. What is your criteria in this particular case? It says you want to minimize your cost which is your goal primarily and it should hold at least.

Student: 400.

400 mL ok, see there could be multiple solutions also just now as we pointed out correct, but it is also governed by this requirement. I want to maximize the data storage capability on my cell phone ok, go get what is the latest phone my iPhone, what you know what is it OnePlus.

Student: OnePlus.

OnePlus or yesterday I saw some Nokia some 50000 rupees 59000 rupees they might have, but today storage has become cloud. But, let us say some something else you know speed or battery backup something like that I want to maximize, but then I cannot get a like a 60000 worth phone. So, either if time is there you work you wait for a sale, otherwise what you do? So, you say there is a constraint. So, the cost should be less than 30 k in that tell me, what I will get ok? So, you will have to compromise a little bit. So, that maximization is based on the constraint that you have.

So, now whether it is a slender or fat is governed by these guys. Now, we want to cast this into a proper scientific or mathematical not scientific mathematical way of doing it. So, how would we write it in a general optimization census, I want to minimize you want to minimize your cost. So, I am just verbally writing at first and that is nothing, but let us do it parallel that is nothing, but your surface area which is pi D H plus.

Student: Pi D square.

Pi D squared by 2. What are my design variables? D and H, this is how it is written minimize, underneath that your design variables and what is your objective function. This guy is entirely called your. So, if you go to departments like management and all that it is usually called a cost function. We can also call it as a cost function because, eventually it will relate only to cost such that.

Student: (Refer Time: 25:05).

What is my constraint here? Volume should hold at least 400 mL. How will you write that?

Student: Volume is greater than or equal to (Refer Time: 25:14).

At least means.

Student: (Refer Time: 25:17).

400 is ok, if you give more I am happy. It should hold at least 400 ok. What is the cutoff? This is basically the cutoff. What is the cutoff? The minimal you need to have as 80, you get 85 yes, you will be there, but you need to have a minimum of.

Student: (Refer Time: 25:37).

80 to get in. So, such that my volume is greater than or equal this sign is very important 400 fine. So, you just need to plug in your equation here for this particular case.

Student: (Refer Time: 25:56).

Do not get into complex conversions. So, if they either centimeter or meter ok. So, I need to just ok, to make it generical put the units here. So, you want to minimize the cost such that this is done. So, this is the basic problem formulation. But now, how are you going to solve this? It is all good, you have written it in a standard optimization format and all that.

Student: (Refer Time: 26:27).

Sorry design variables are here D and H.

Student: (Refer Time: 26:32) the constraints on bound some.

Correct there is some bounds also are there correct sorry. Those are also constraints correct, such that the D cannot vary this is your problem formulation. Now, how will you go about solving this formulation? We came from the verbal statement to some what I call the mathematical ok. There are more numbers in this formulation compared to this statement ok. So, this is all this is how you write this is the global way of writing an optimization problem.

You write this problem statement, any engineer who practices optimization will understand ok. They might not understand your cost or volume, but for sure they will understand the equations that you have written, the way that you have done this. This is how it is usually done. To be more precise, you might want to bring 400 to the other side and then say less than or equal to because, always those constraints are written that way. What are those constraints called? Any idea these are called.

Student: (Refer Time: 27:53).

Inequality constraints. If I said please come up with the design that gives you exactly 400, then I will replace this inequality by an equal to sign.

Student: (Refer Time: 28:07).

Then it becomes a equality constraint. So, usually your constraints are equality constraint, inequality constraints sometimes you call these bounds, which are usually the geometric bounds or it could be other bounds also; cost some bounds it could be ok. You could because you can also recast this problem if you want. You can say I want to maximize volume, such said the cost is less than some value you can recast that problem that way also ok. So, it depends what your constraint.

So, you also have bounds we usually call functional constraints these are called functional constraints. These are just geometry curves static constraints equality, inequality constraint are the most widely used criteria, I mean terminologies. And this kind of equations, where you have quadratic or anything more than that or anything greater than linear is called a non-linear formulation; the formulation the equations are non-linear.

Instead, if you had a just a linear equation D plus H such that 2 D plus 3 H should be less than something, 4 D plus 1 H should be greater than something that is called linear programming ok, LP problems. If you go to management operations research most other problem that this always linear programming, which means your objective function and your constraints are linear in nature. So, please understand there is a distinction between linear regression and linear because, we are going to extensively use regression. Here the equation itself should be linear, then it is called linear programming.

Student: Linear programming.

This is a very simple case non-linear so, NLP non-linear programming. So, non-linear functions that is what we are using, LP problem and non-linear problems and then equality constraint, inequality constraints, then what?

Student: (Refer Time: 30:09).

There is a concept of design domain. So, in this case what are your design variables?

Student: D and H.

So, it need not be a rectangle or a square, but in this case it is D is one variable and H is one variable. It is going to vary between D varies between 3.5 and 8 and H varies between 8 and 18. Now, in this design space you can get something called a feasible domain, that is governed by what?

Student: Constraint, constraint.

Constraint. So, if you can because now right now with just these two constraint this is the feasible domain. What I have here is what the feasible domain is just with these two constraints. If I draw the contour of the volume on top of this guy, I know wherever it is greater than 400 my optima will lie only in that. It is likely to lie on the curve, but it is lie it will lie only on the other side of it. So, that is called the feasible domain and what else. So, in terms of your design variables usually, we try to have them independent and most of the time they are independent. Even when you write y equals f of x, they say y is the dependent variable of course, because as you change x it will change.

However, sensitive it is for x 1 it might be very sensitive for x 5 it may not be that sensitive, but still it is sensitive. So, y is the dependent variable and x's are independent variables and your design variables are your independent variables. So, usually people are very careful with the design variables because, they define the dimension of your problem; the degrees of freedom of your problem. So, when say they say the optimization problem is very complex they ask, you do not know the function. No, yes I have ways to approximate the function. They ask what is the dimension of the problem? Today it again varies between departments you go and say in computer science I am solving a.

Student: (Refer Time: 32:40).

20 degree or 20 design variable problem, it is very difficult they will laugh because their design variables are all in 100's and 1000's. There they solve these networking problems ok, but anyway the quality of the problem and not the quality the type of the problem that you solve are different, but usually we define it this way. In the context of design problems even 15 dimensions is large, but today people are trying to solve 35-40 dimension problems. So, this is difficult usually done if you are single digit it is ok it is a it is a good number.

So, those are the things about a design variables, but actually when you plug in the design variables you should be able to get an f. So, preferably numerical in nature it could be integer, it could be continuous variables or it can also be categorical variables sometimes. You cannot have for instance I instead of giving you I, I can say can you tell me what is the type of I. So, what happens is 1 is triangular, 2 circular, 3 is.

Student: (Refer Time: 33:43).

Hallow beam ok. So, it is going to be categorical. So, if you put 1 it means that it or you can call it a b c, 1 means it is still numerical it a b c you can call; a is this b is that c is that. So, you will have to make a decision based on that, meaning if you put a it means that it is a circular one and appropriately you will use your I. So, that is the that is a point and accordingly your problem will be casted; it will change you cannot just use I directly. You will have to use how do you compare this I with that I.

So, there should be something like a bounding variable and then what is the best one that you will take out of it because, still your objective function might be to minimize your cost. So, this is how we write the simple optimization problem stuff. I am just trying to make sure yeah, these are also called Sizing constraints. We call it as bounds or sizing constraints fine. So, this is just to get you started on the this stuff.