

Design For Quality, Manufacturing And Assembly
Prof. Palaniappan Ramu
Department of Engineering Design
Indian Institute of Technology, Madras

Lecture – 05
Recap and Clarifications Of Basic Concepts

Welcome again to the topic. So, we will continue from the last lecture. There were few outcomes of this particular course that we discussed yesterday right. So, the first one, if you remember if you recall was you will have an understanding on the concept of quality.

The second one was you will demonstrate an understanding on robustness. You will be able to formulate robustness, meaning quantify robustness and use it in a formulation for design ok. While performing design whether conceptual or detailed design, you will be able to design a particular structure or a product or a component for robustness. And the third one that we talked about is the design of experiments.

So, we will understand why your design of experiment is required in and design setup and what is the role of design of experiment in accomplishing a quality based design or a robustness based design. These are the 3 major outcomes that we you can expect out of this course. The couple of things that we discussed yesterday, kind of ticks of these bullets ok the first one is about the quality itself, like how do you define quality. Then we did exchange views on how each one of us looks at quality and then we gave the way in which Taguchi looks at quality.

So, he essentially says that there are 3 components that come into picture. One is the customer one is the manufacturer and the other one is the society ok. So, if a particular product does not live up to the promise that the manufacturer makes or does not live up to the expectation that the user had, it is not only the 2 of them that are affected, but the third the society which could be directly or indirectly related to these people are also affected. So, quality should be measured from that perspective, rather than only a loss for the either the manufacturer or the user.

So, this is something that we saw yesterday right. The another it appears that it is disconnected at this point in time, but today or in the next lecture we will be able to make

the connection, you will be able to relate why we spoke about robustness ok. So, then we kind of discussed what robustness is. Robustness directly put what it means is irrespective of the variation in the inputs, my output variation should be minimal, or it should be within the bounds that a particular are whatever the designer are the user specifies that is what your robustness means.

So, we discussed this example of a furnace, where you make the tiles and there was non uniform temperature was there as a result of it that size of the tile varied. So, that is the variation in the tile size which is what is your output. The input variation was there was non uniform temperature. So, irrespective of the temperature difference the interest was to maintain the output dimension of the tiles.

So, the important point in that robustness is you do not address the cause of the variation itself. So, you will not go and change the non-uniform temperature. That is not what you did, but still you address the effect of the cause of the variation ok. So, that is the point of the important significant point of robustness. So, you do not get rid of the source of variation. The source of variation is still there, but you still design it in such a way that your output variation is minimized.

So, that this becomes important, because in all the settings you will not be able to eliminate the source of variation ok, but still your products performance output cannot vary as much as it wants ok. It should be minimal or it should be within the limits that the designer specifies other user specifies ok. So, if you recall out of the 3 major learnings that we would like to have in this course or that I pointed out, the first one was about the concept of quality itself that is something that we have seen the second thing is an introduction a verbal introduction to robustness.

So, today what we will try to see is how you can quantify this robustness. As we discussed earlier as engineers, whether you want or not you want to quantify things. You cannot say subjectively this student is better than that student I cannot say that, but there should be a metric for me to we tell that this student is better than that student. This product is better than that product from functionality perspective from performance perspective from robustness perspective.

So, you need a metric to measure robustness. So, today we will see what is the metric that was proposed by Taguchi to look into robustness that is one thing ok, but before that

there were a couple of clarification that was required yesterday. One was we discussed quickly about the probability distribution function the distribution function being thin or fat and all that.

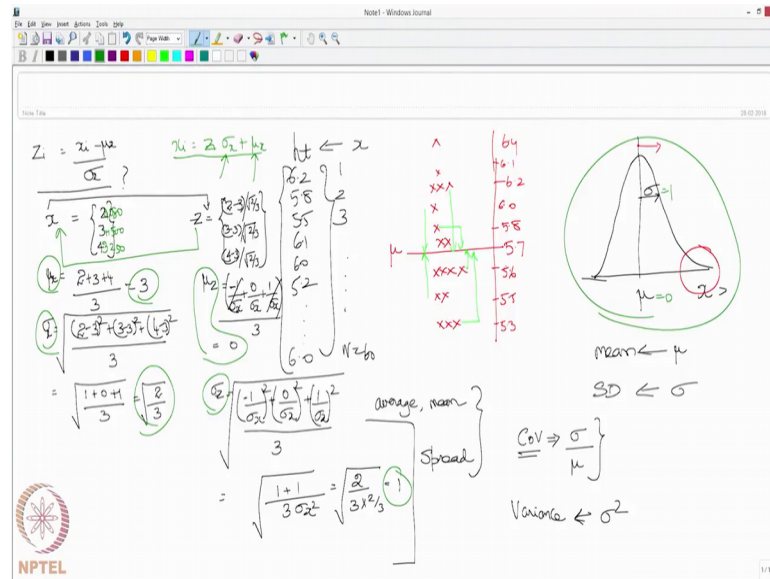
So, what we discussed yesterday was predominantly in this course we will talk about one particular distribution that is the normal distribution or the Gaussian distribution, which is the bell shaped now if you see the church bells you know it is that shape. So, it is called the bell shaped curve right. So, this is this has some very nice properties because you can you just need to know 2 moments to describe a normal distribution, which is your mean and your standard deviation. Mean is nothing but your average statistics statistician call it expected value because it is not the real right.

So, there are 2 terms in statistics one is called the population and the other one is called the sample ok. The population is something that you would like to have ok which is the ideal case, but oftentimes you might not be able to get the population. So, what you will do is you will take a sample a significant representation of that population is what called the sample. So, oftentimes when you want to estimate the average what you do is you go and estimate the average on the sample.

So now this average you hope this average to be the average of your population. As long as your sampling was right, this will be equal to the or it will be close to the populations average that is why it is called the expected value ok. It is usually written as $E[x]$ means the variable that you were talking about right. So, it is called the expected value ok. The statisticians if you see the word that they use is called the expected value, but the regular engineers and leman you can use the word average or mean.

So, for a normal distribution, you need only 2 quantities to describe it the one is a mean and the other one is the standard deviation right.

(Refer Slide Time: 07:08)



So, in order to describe this guy, I just need the mu and I need the standard deviation. So, now what does this standard deviation actually mean? Ok that is something that we wanted to discuss ok.

Now, let us say that I want to take the height of the students in this class ok. There are about a 60 students and I want to have the height of the students in this class room. So, it is an array of the heights of the students right. So, 6.2, 5.8, 5.5, 6.6, 5.2 I do not maybe 6 there are some height of the students right. So, there are multiple ways that you can look at this information.

If all of you were of the same height, then I do not need to worry. If someone asks me what was the height of the students in your class 5.8 the job is done, but right now if I step out of this room and someone asked me what is the height of the students in your class. I will say I am likely to say the average height is this the moment, the moment you are going to have more than one output value the usual metric that you start to use is the average.

So, I guess I do not have to explain you what an average is right. So, you know what an average is, it is going to be summation of in this case if I will call height as x. So, the summation of this x i over the total number that is your average it is also called as average mean the other one is a spread this is called a ok. So, this is more of a position. It

says where your distribution is going to be sitting it will it be here, will it be here, will it be there.

So that is described by your mean and whether this distribution is going to be whether this distribution is going to be fat, please understand it is that the same mean. The mean is the same, but my standard deviation is different in this case. My standard deviation is different in this case ok. So, if I have a larger standard deviation then it means that it is a fatter distribution. So, you will see what it means whether it is a thinner distribution or fatter distribution how does it affect this is important from a robustness perspective. So, we will we will discuss that one, but what exactly is this standard deviation? What does the word standard deviation means?

So, the idea is I want to understand the spread of the data ok. So, if someone asked now your teaching is designed for quality course. After you guys take the exam if someone asked me what was the performance in your class. I would say well the average was 70 percent, but this only tells you part of the story. What could be the other part of the story? Yes that is a specific information. So, he is asking how many people scored greater than 70 percent and how many people scored less than 70. That is that is one another information what else.

Sorry.

So, how close to 70 did they.

Ok.

One is the distribution the frequency of this course that is one thing name one easy thing on trivial stuff ok. So, when a cricket player comes into the ground to play some statistics come up ok. Then here when he or she enters on the screen you see some statistics which says that x y z, it is his name. And then there is some statistics what is that.

One is a strike rate the other one is the average score that he has done. So, that is the first metric that comes to your mind ok. So, strike rate is how much he or she is going to hit in 100 balls ok. That the rate at which that is why it is called the rate right, what else.

What was his highest score ok? One thing which will look very negative probably that is the reason that they do not telecast that one is minimum, what is the minimum that person has got ok. So, why this is required is you need to have an average score.

So, I expect you to score that what is, what does that average means if a particular batsman is coming into the field and then I project this number and I tell you that this is the average what does that mean is? I expect you to score close to that average this way or that way, but that is a meaning of average on. An average this person is going to do that, which means on any random game that this person is going to enter this batsman is going to enter he is likely to score that lightly around that ok.

But there is also a situation that this person has scored a maximum value ok. There is also a case where this person has scored a minimum value. So, oftentimes you need to know the range ok. There are this one classical book called how to lie with statistics ok. So many times when you look at performance of a company or performance of a share you might not want to see what is the average performance ok. You want to know what was the maximum, that the share went and what was a minimum that the share went. That will tell you what are their chances of your profit or what are the chances of your loss.

The company's performance you need to look at those statistics ok. For instance, your gross domestic product GDP is an average metric. It is good you know it is a good indicator there is no point about that ok, but it talks only in an average sense ok. If your average is going to move to the right if your average is going to move to the right that can happened by just putting more points on the sorry, more points on the right side ok I can have more richer people earn more and the GDP will move to the right ok.

So, you know it does not talk anything about the people who are below the poverty line ok. So, if you go only by the GDP it tells you only one part of the story. The GDP are usually presented with the minima or the maxima are with the not the minima or the maximum, but the minimum value on the maximal value and the bounds that govern it ok. So, that only gives you the entire story. So now, the question is it is not enough, if you just take the mean, but you also need to know what is the spread of the data.

So, how can I measure the spread of the data ok? So now, imagine that this is the axis ok, of these values of height and I let me draw an average at 5.7 ok. So, I am just imagining

that 5.7 is the average. It could be now I am just going to say I am going to say there are 2 people at 5.8 5.9 6 6.1 6.2, there is some guy at 6.4.

Some numbers 6.4 6.2 5.8. This is not to scale. Similarly, I might have someone at you know 5.5 5.6 5.3. So, I might have about like some 4 people. I might have about 2 people I might have about 3 people and if I have about something of that sort.

Now, this average only gives me see this can also directly translate it into a histogram. If you look at it, this is just tells you the frequency if I have about n people ok. How many of that n is at 5.7 or 5.8 how many of that is at 6. How many of them are at 6.2 how many of them are at 6.1? So, if you make these into a rectangles that will represent a histogram directly it is just 90 degrees off this way that is all.

Now, the point is i in a relative sense is, what I can come up with a spread the other one would be the absolute value, but the one way of looking at how do I measure the spread is in a relative sense. So, the natural the first moment that you have is your mean. So now, what I am going to say is this on an average, on an average I expect the next person to be on of this height, but if this person is taller then I am going to see how taller because it is a relative metric ok.

So, what I am going to do is I might want this information. I just want to know, I just want to know this information. All the green lines are the distances of each of the point each of the realization each of the sample, to it is own mean. Meaning this height there is a mean height. So, how far is this realization, from this mean will give me a spread data.

But I will not have one value. I am going to have if this we call it as the distance are the difference. Ok for this point I have a difference for this point I have a difference for this point I have a difference. So, for 60 points I will have 60 differences. Now I need to make this into a one metric and these are all distributed these are all not are the same; obviously, they are not at the same place.

So, how do you do this under such situation? Have you heard about the word rms the phrase rms root mean square ok? So, whenever you are faced with multiple outputs, and you want to get a like a qualitative one quantitative representation for that you usually do the root means square.

But why do you need a square? The distance can be positive or negative ok. So, that is the reason. So, what we do is this we say x_i minus μ ok. So, this is the distance, this is the green line that I have presented here. Squared because this x_i minus μ could be positive or negative; why 5.8 is an x_i the mean was 5.7 if I difference it is one or sorry point 1 whereas if I did 5.6 minus 5.7 then it becomes minus 0.1 ok.

So, if you sum them up they will even out. So, you do not want to do that. So, you penalize it. The more you are away from the μ the more is a weightage for you. So, I square it and then it is going to be the sum of i running from 1 to n .

So, this is squaring and I need to do a mean when I divided by n , I add them up and then I divided by n . Then it becomes square mean then I have to take a route. So, that it goes back to the base unit that we spoke about ok.

So, this is just a distance unit. So, it will go back to that distance unit. So, this is what is the standard deviation. So, it is basically a spread of the data. So, oftentimes these 2 guys need to be discussed together. If I just tell you the spread of the data that I have is 10, is it good or bad? I have a data; I tell you the spread of the data is 10 do you think it is good or bad it depends on the average ok.

So, usually we use the word called coefficient of variation CoV. So, what it says is it is σ over μ ok. So, that is what he nicely said he told this variation has to be compared with respect to the mean ok. So, let us take the case of σ equal to 10. You cannot comment whether it is good or bad. You need to know what their mean is. If the mean was 100 and the standard deviation was 10, it is only one tenth of variation; however, if my mean was 20, and my standard deviation was 10 it is a bad deviation it is about 50 percent deviation ok.

So, lesser the deviation compared to the mean in terms of the ratio compared to the mean the better is your coefficient of variation. If it is a manufacturing process your manufacturing process is better ok. If it is or not some performance it is a good controlled performance. So, usually the word the coefficient of variation is used and it makes sense ok, to do it that way and the other term that is usually used is variance, which is standard deviation squared ok it is the square of the standard deviation ok. So, there were some questions on this yesterday and that is why I am going through this in detail ok.

So, the next one that we so, is this clear so far the mean the standard deviation and these are the 2 basic stuff that we will discuss in this course. That is all there is something called signal to noise ratio that you need to discuss for that you need to know these information. The other one that I wanted to discuss is yesterday, I pointed out that you can normalize all the data ok.

So, this is just a data analysis technique it is not necessarily applied only in statistics. Tomorrow when you are trying to plot something there could be there could be 2 variables x_1 and x_2 ok. x_1 can be measured in meters and x_2 might be measured in microns ok.

So, in that case what happens is if you try to plot them in their actual values, you know what happens if you are going to map meters with respect to microns is one knee power minus 6. The deal is you might want to normalize the data to plot it hence what people usually do is this they plotted between 0 and 1 in each axis ok or you can normalize one axis to log of that particular data. In a similar fashion what we are trying to do in this case is, we are only trying to make the mean of any data equal to 0 and the standard deviation to 1. So, what we discussed yesterday is we said z_i will become x_i minus μ over σ . This z is the usual notation that is used, is a usual notation that is used ok.

So, the deal is what is this going to do for you? That was a question that was asked ok will take a simple numerical example a simple example. So, let us say your x is 2 3 4. What does it mean? What does it mean of this?

2 plus 3 plus 4 divided by 3 summation of x_i divided by n in this case n equal to 3. So, it is 4 plus 3 is 7 plus 2 is 9, 9 divided by 3 is 3. So, the μ now is 3 what is your σ ? σ is going to be 2 minus 3 the whole squared plus 3 minus 3 the whole squared plus 4 minus 3 the whole square ok. And then you have to take the n in the denominator which is 3 the square root ok.

So, this is going to be 1 plus 0 plus 1 divided by 3 which is 2 over 3, will you keep it like this it is actually square root of 0.66 we will keep it like this ok. Now what I am going to do is this I am going to try doing this is a die. Ok I am going to convert this x_2 e z is it. What I need to do I need to do 2 minus what is the μ here, what is the μ for that data. See if you want to do you can even make it like this μ of x and σ of x . So, what is

the μ of x here it is 3 divided by the sigma how much is it $\sqrt{2}$ over 3. Similarly, you are going to do 3 minus 3 divided by $\sqrt{2}$ over 3. 4 minus 3 divided by $\sqrt{2}$ over 3 ok.

So, if I want to find the mean of z , I just have to sum these guys up. What will it be 1 divided by the sigma of x plus 0 3 minus 3 is 0 divided by the sigma of x plus minus 1. Correct? What is 2 minus 3, sorry 2 minus 3 is minus 1 divided by sigma of x that is nothing but $\sqrt{2}$ of 3 plus 0 of sigma x plus 1 of sigma x . Ok divided by 3. What will you get? This guy will cancel out with this guy this is 0. So, you will get 0 as your μz sorry. In a similar fashion if I were to do sigma z .

So, what do you do? You have to take this and find out what the standard deviation is. So, 1 divided by sigma x minus your standard deviation sorry minus your mean which is 0 ok. So, I need to take a square of that right. So, I will say 1 over sigma x , I am not saying minus 0, because this is the same value squared plus 0 over sigma x . Squared plus this is actually minus 1 over sigma x i. So, plus 1 over sigma x the whole square ok this is being retained the same because my means is 0 the mean of z is 0.

Now, I need to make a mean then I need to make a square root. So, this will be what if I do this sigma x 1 plus 0 our sigma x squared divided by 3. So, this is 2 by 3 ok. And times my sigma x squared. What will that be this is one square root of 1 will be 1. So, what did I get? A mean of where did I start from? Ok I started with the mean of 3 and the standard deviation of $\sqrt{2}$ over 3. And they went to a mean of 0 and a standard deviation of 1. Ok this is the standardization.

But I can always get back these data I can always go back from here to here. I just need to say x_i is equal to $e z_i$ times sigma x plus μx . I will always know sigma x and μx . I can reconstruct my x_i that is not at all a problem. Now what is the benefit of going from the x domain to the $e z$ domain because now I can replace this 2 by 20080.

I can make this guy 34500. I can make this 43250. This is some numbers. You do the same process you will get μx you will get a sigma x . You go and then you subtract that mean divided by the standard deviation you do again you will get the μz to be 0 and your standard deviation to be 1. Irrespective of the set of data that you have you can always convert it to 0 and 1 mean and standard deviation ok.

So now in order to find the probability, I will deal only in the z domain I will not deal in the x domain. Once I get the probability it is easy for me again to go back to my x domain. Because I just need to know only this information. So, in terms of the probability content preparation I only need this guy for whom the μ is equal to 0 and the standard deviation is equal to 1.

So, I just will have only one chart where the μ equal to 0 and the standard deviation is equal to 1. Any data I will convert it into a data where the mean equal to μ the μ equal to 0 and the standard deviation equal to 1. And I will be able to compute my probability then again I will go back to my x domain. So, this is the whole idea of that normalization is it clear? This was one thing I just told quickly yesterday, but there was questions. So, I am explaining it today is that clear ye fine ok. So, this is something that I wanted to tell do you have any questions? No.