Design For Quality, Manufacturing And Assembly
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Lecture – 04
Introduction To 6 sigma Concept

Now, you might feel that the concepts are disconnected, right now. But actually there is continuity. So, just hold on so, this is only the first lecture. So, just hold on, but as we go in you will feel you will understand the connectivity between these stuff. So, so far we have discussed two things. One is a simple a verbal discussion on what quality is about.

So, we introduced Taguchi and then we said how Taguchi said there are three things that come into picture, the user the customer, sorry the customer the manufacturer and the society. And then we talked the second concept that we talked about was robustness. So, in robustness what it says is irrespective of the variations in the input my output variation should be minimal or it should be within the specifications that I state. And then we used a simple example to derive that concept home.

So, before we wrap up for today we will discuss one more concept called the 6 Sigma. So, off light you can you know see these some courses called 6 sigma black belt certification courses. And there are many companies you can see there are this 6 sigma trainers, 6 sigma leadership things, and then they even wear badges on that. So, this is a you know heavily followed and it is what we call like a very important topic for the companies which manufactured. And today it is not only in manufacturing it is there in everything, it has become process product engineering in everything it has come.

So, it is important and it is directly related to what we are talking about. So, in the beginning of the class I asked what sigma is. And someone replied so, can I ask someone else can does someone else remember what that sigma is? I asked what sigma is and someone answer.

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Standard deviation right.

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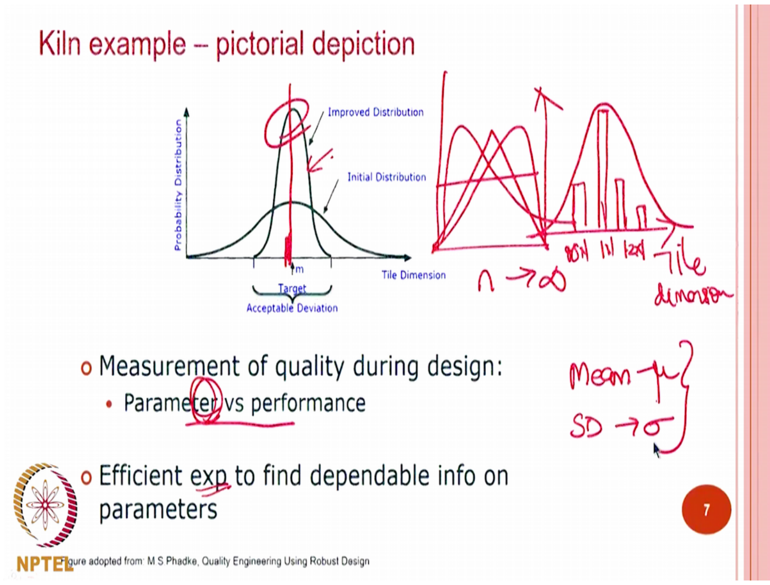

So, if we go back to this problem here, did I use the word fat and thin distributions? That is described by your standard deviation. Predominantly these hold good for distributions that look like this. What are these types of distributions called? These are probability distributions PDF Probability Density Functions. What type of distributions these are? These are called the normal distribution or the Gaussian distributions, hat shaped distribution it is like a hat that you wear like a Mexican hat that you wear. Hat shaped distribution, there are some interesting properties of these guys this distribution per say. It is a symmetric distribution.

You just need to, I am unable to draw a straight line, it should have been on the right side a little bit. It is a symmetric distribution. It is very interesting to have this distribution. There are also other types of distributions; for instance, you know left skewed distributions, you can have right skewed distributions, you can have uniform distribution, you can have triangular distribution. And I can keep on doing this for another 60 minutes, there are so many distributions are there.

.But if you see in the manufacturing process industry or most of the product design and all that, we always talk about this shape only that is the beauty of the Gaussian distribution, or the normal distribution. If we plot the height of the people in this class and n tending to infinity, it need not go to infinity with 60 about 150 people. We will more or less get a distribution like this that is a beauty of normal distribution.

Many of the real life process follow a Gaussian distribution. Yes, there are there are always exceptions; as a matter of fact we work on under such areas where, there they do not follow a normal distribution Gaussian distribution. So, the Gaussian distribution, this particular course we will discuss only the Gaussian distribution. And you can take it from me about most of the real life problems that we will discuss we will follow a Gaussian distribution.

So, yeah the reason that I went back to that is, given a Gaussian distribution there are 2 parameters that I can use to describe a Gaussian distribution, right. So, if I ask someone comes to my office, and you know, like I come at like 11 O'clock and then my student next or says is; sir, there was a student who came to meet you, then I say do you know who it is, he says I do not know. So, then I asked can you describe, so the person goes ahead and says. So, so tall person was wearing specs, he was a stout student, he was looking athletic. It was a male, it was a female, he had a beard. So, these are all representations, right.

So, characterizing; so, this student is trying to characterize the person who came to meet me so that I can identify. Now let us say that I have scheduled meetings that morning with 2 people are I have asked 2 people to come, then I know whom he is talking about, because he has given me the characteristic, I know, it is not the second guy it is the first guy that I asked to come, right. So, in a similar fashion a normal distribution is a symmetric distribution, it is around it is mean is worth, there are 2 parameters that is described that is used to describe. One is your mean and the other one is it is standard deviation that is all you need. For other distribution for instance, this is predominantly called a log normal distribution. There are other quantities called Skewness and kurtosis. Skewness says how much it is skewed in one particular way and kurtosis is a 4th order moment.

So, whereas, for a Gaussian distribution you need only a average which is called the expected value or mean. These are the different words that are used. And then there is a standard deviation which is represented by the notation called sigma. So, there are 2 terms that I would require one is the mean which is usually represented with mu and then there is a standard deviation which is usually represented with sigma. And this is the notation that we will use that we will use for the rest of the course. Now as I pointed out this sigma that we are talking about is the standard deviation, right.

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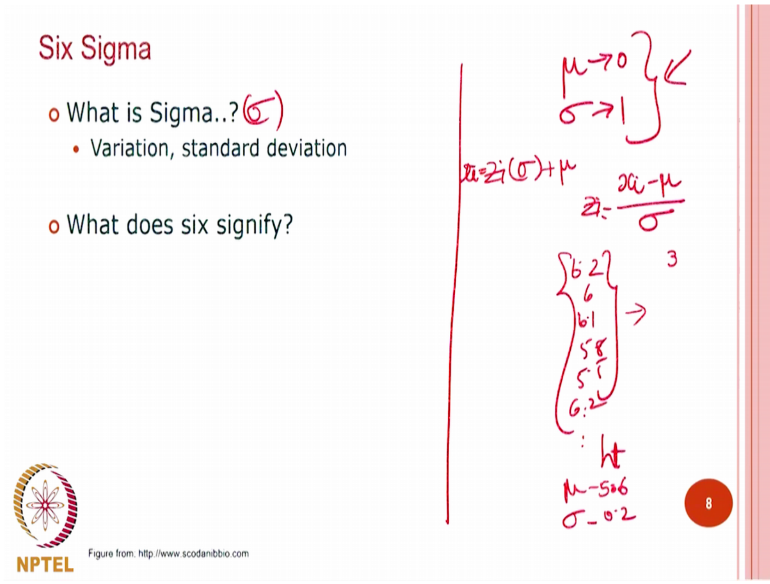

So, does anyone know what 6 sigma stands for? So, there is a small digression we will have with this I will wrap up. This is a small digression, but what do you think is the use of this kind of plots? What do you think is the use of these kind of plots, why do you need a PDF or probability function? What can you find out of this? Variation you just need data you do not need these plots. That is a very generic statement, you can model the uncertainty is one thing yeah. So, there is a very interesting perspective on what these plots will enable you to do. I am sure that you have done this in your 10th 11th or 12th grade.

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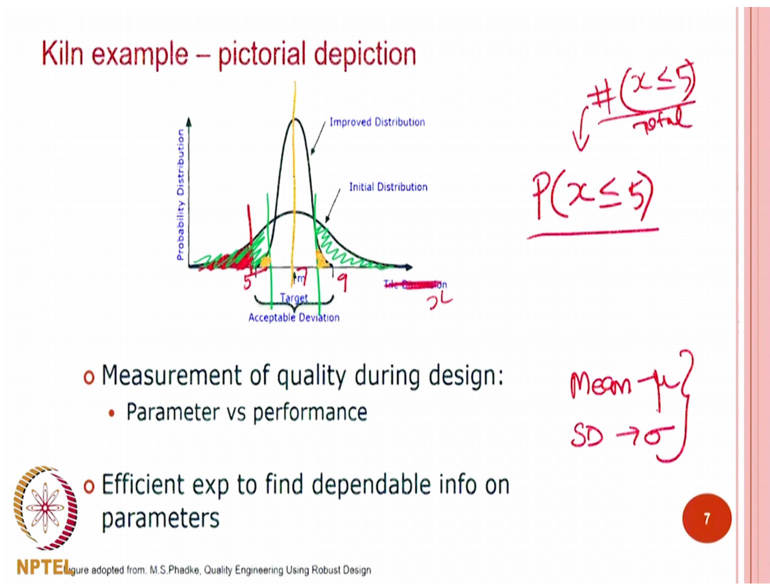

I will write something and tell me whether you recognize that. Have you done something like this? What this one reads this, probability of x in this case the tile dimension would be x let us say is going to be less than or equal to 5. So, you are trying to predict this plot gives you the capability to predict some stuff. So, this becomes very important. If I have this plot for the tile example, if I am going to randomly draw the next tile, I can tell what is the probability that that random tile will be less than or equal to 1 square foot are greater than or equal to 1 square foot. That is very important; this probability is nothing but the ratio, what is this x a number of times.

This hash means number of times; number of times x is less than or equal to 5 divided by the total. That is what is this probability it is just a ratio. So, this plot let us you do that. So, let me just modify this a little bit. So, let us say that this was 5 some other example do not worry, this is not a tile dimension, this is not a tile dimension, this is some variable x, this is 7, this is 9. So, this one tells me if I am drawing in a random fashion, what is the probability that this x is going to be less than 5. I need to understand the area under the curve. And if you remember we would have given you a chart, that would have used this drawing at the top, and then it would have given you a table in the x and y, and then you will have to go and find that one.

Now, the question here is this was for a tile dimension example; you can go and do something at the strain gauge which is at the level of microns. I can measure the height of you people which are on meters. I can measure the weight of you which are in kilograms or pounds. I can do some force measurement which is in Newton. And these are all could be in different units and different numbers but I still need only one chart. The chart that we gave you in the 10th or the 12th grade to find out what the probability is. How is that possible? You understand the question? I can start from one e power minus 6, and I can go to one e power 6, at entire bandwidth I have given you examples for all. And I for each of them I have a different problem, I am asking probability of x less than phi e power minus phi probability of x less than 100 probability of x being greater than 250,000.

So, my requirements are different for each of these problems, but still I can answer all of them using just that one chart. So, it is one size fits all kind of a problem; one chart will answer all the problems provided it is a normal distribution. So, that is called a standardized normal distribution, which means the mean is 0 and the standard deviation is 1. So, any problem meaning any data that you gather can be brought to this level, how? I can use a simple stuff called x i minus mu over sigma. So now, imagine that I took the is an array of height, I to the height of the people in this room, 6.2, 6, 6.1, 5.8, 5.5, 6.2. I just keep on going or I can replace this with some other stuff.

1 e power minus 3, 2.5 e power minus 3, 0.9 e power minus 2, it can be any other data. But I can bring this data in such a way that, I find the mean of this guy, and I also find the standard deviation of this guy, this data and then I subtract that mean value. So, let us say that roughly for this the mean value is about 5.7, I am just assuming, 5.7 here. And the standard deviation is about 0.2, then what I need to do is, is I need to say 6.2 minus 5.7 which is 0.5 divided by divided by 0.2.

So, let us make it 0.5 point let me make it 5.6 so, that it is easy for the calculation. 6.2 minus 5.6 is 0.6 divided by 0.2 is 3, correct? So, like that you can keep doing it for the rest of those guys. So now, this will come into a new stuff, and this data will be between meaning like it is normalized on mean of 0 and variance of 1; which means this x i, x i x 1, x 2, x 3, x 4, x 5, x n right. So, at some point it will have the mean value so, it will be mu minus mu. So, it is 0, similarly x i will be the max and then mean divided by the standard deviation which is what is the range so, you will have a range of 1 so, it is a normalization. So, the data is with the mean of 0 and a standard deviation of 1.

So, any data can be made into a standard distribution. Then you can use that chart to find out, then what you just need to do this is go back and whatever value you get you just need to multiply this is usually called Z i. So, what you have to do is after you find the Z i you just need to do Z i times sigma plus mu. This will take you to the original this will this will take you to the this will take you to the original space that we are talking about this will be your xi. So, we are talking about that sigma is what we are talking about, and when I said sigma is 1 what 6 sigma says is you can go 6 standard deviation, meaning 6 units in the distribution, that is what it talks about.

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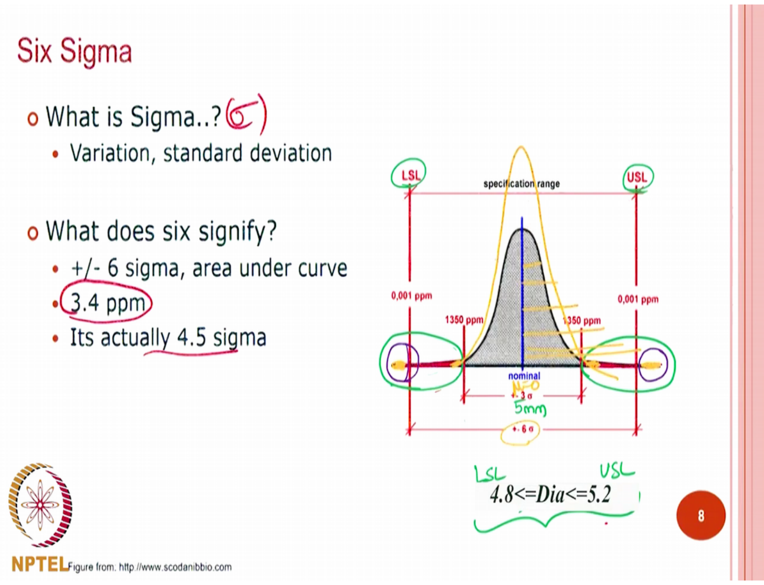

So, it basically says plus or minus 6 sigma. So, this is one clarification that you might want to make sure that you take away from this class. This it is not plus or minus 3 sigma, 6 sigma means it is plus or minus 6 sigma on each side it is 6 sigma. In total it 12 sigma, it is not 3 sigma this side 3 sigma this side so, it is 6 sigma no. And as I pointed out in the beginning of the class, what does it actually mean? It means 3.4 parts per million. What does it mean? If you are going to manufacture 1 million washers or any product, since washer is a standard example that we are using, about 3.4 of them can fail or it is allowed to fail. That is the level of quality.

So, we will come back to this what why what is this 4.5 sigma in the next lecture probably we will talk about it, because we are already running out of time. So, I will just finish with what how do we look at this. So, what does this 3.4 parts per million mean? Meaning what is the meaning not the mean standard deviation mean, what is the meaning of that. So, this is the normal distribution that we are talking about. This is a nominal which means the mean the expected value the average value that we are talking about, and it is going to vary. You see this plus or minus 3 sigma it means 1350 parts per million can fail.

Where is the failure? The failure is here, you see this area, this is a failure, the area under this region this is what the failure is. So, if I counted this area if there is a way in which I can count the number of parts, they will tally up to 1350 parts per million both sides.

What 6 sigma says is in a similar fashion if you are going to go to plus or minus 6 sigma, the number of parts that we will fail for you is going to be 3.4 parts that is all can fail, you understand? So, you see the region right.

So, if I were to you see this region. That small region here it is not even that much, yeah, the one that I have circled is the region that corresponds to 6 sigma. Whereas, this region the green region that I have pan, I mean within the green you should see the red one that corresponds to the failure.

So, what this means is for the washer example, this nominal is 5 mm. And let us say the allowable is 4.8, it should be less than that dia this is your lobe. So, what it means is if it is 3 sigma then I would put the 4.8 here and the 5.2 here. Then it will about 1350 parts can fail this side plus this side, both the sides together. Now tell me if I fix this, because this figure is not like that, this USL allow LSL means Lower Specification Limit and Upper Specification Limit.

So, this is what is the lower specification limit; and this is the upper specification limit. Now, if I take this to be the lower and the upper specification limit, how can I make this 6 sigma. You understand the question? How can I make this 6 sigma, meaning I should make this region smaller, how can I make that? You get the question, since I have drawn everything we will wrap up with that question. You get the question or you want me to repeat it, you get the question? How can I do that, just from the figure only you can tell how can I get it? It is the same as you are the tile example, the kiln example. I want to reduce this area; how can I do that?

In this one if I put my failure to be here, you can see the amount of failure with the initial distribution correct, and you can see the amount of failure with respect to the new distribution, correct? How would I achieve that by, what did I do in terms of the graph, in terms of the plot? Sorry, you know in terms of the graph what I have done? What have I done? Fine, you improve the distribution, but what has happened? In terms you have shrunk this particular distribution, what does shrunk mean here? What is the meaning of shrunk in this case, it the meaningless because it is a normal distribution there are only 2 parameters, one is the mean and the other one is the, but the mean is the same. The mean is the same the mean has not changed between this distribution and this distribution, what has changed?

Student: (Refer Time: 23:01).

The standard deviation has changed, which means I have come from a fatter distribution to a thinner distribution. Hence I have been able to reduce the number of failures from the green region to the yellow region. Do you understand that point? In the similar fashion if I want to reduce this green region to this region, meaning do not worry about the location, only worry about the quantity. So, if I want to bring this region you know this much, I can only fail this much.

Not the red region, then what I can need to do is I need to change the standard deviation, I need to minimize the standard deviation such that this will become like, it is already Gaussian it is still a Gaussian, but I just need to maintain the same mean, but minimize my standard deviation so that my failure is very less. That is the underlying numerical idea behind robustness. So, see this plot it is an interesting question.

So, this is the mean of equal to 0, and then a standard deviation of 1, a standard deviation of 2, a standard deviation of 3, standard deviation of 4, 4 units, standard deviation of 5, standard deviation of 6, 6 units. So, this is easier with a unit standard deviation; otherwise, if I take the height of you, we figured out that standard deviation was 0.2. Then I have to go 0.2, 0.4, 0.6, 0.8 for some other thing it will be a unit of 10 meters. 10 meters, 20 meters, 30 meters, 40 meters, you do not have to worry, you do not have to go and do different, you normalize the data, you use the same chart, it is simple.

So, 6 sigma only tells you how far you can go from the mean. And it should be at the lower and the higher specification limit, because anything beyond that is your failure. Anything beyond this LSL and USL is failure for me.

So, this is the spread that I can have beyond that it is failed. So, if that failure the number of failures is not, to me, then I need to my distribution thinner so that the failure gets reduced, fine anything else? No, it is simple just imagine you are spreading your legs. And then there is an immediate shadow that is there. The amount of shadow is what the failure is.

But if the shadow is deeper, the failure is more I do not want to have that. How can you reduce your shadow by coming thinner, and thinner no by contracting your legs even further. So, finally, when you stand straight like this, you will not have any shadow on your side simple like that, any other question? So, we will wrap up for today.