

Design For Quality, Manufacturing And Assembly
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Lecture - 15
Confounding OA & Resolution Table

So, let us go back to our previous example.

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Factors and Interaction in L8

Experiment number	Column number	Experimental condition	Tensile strength data (kg/cm ²)
	A B AB C AC AD BC D		
1	0 0 0 0 0 0 0	$A_0B_0C_0D_0$	9
2	0 0 0 1 1 1 1	$A_0B_0C_1D_1$	12
3	0 1 1 0 0 1 1	$A_0B_1C_0D_1$	8
4	0 1 1 1 1 0 0	$A_0B_1C_1D_0$	15
5	1 0 1 0 1 0 1	$A_1B_0C_0D_1$	16
6	1 0 1 1 0 1 0	$A_1B_0C_1D_0$	20
7	1 1 0 0 1 1 0	$A_1B_1C_0D_0$	11
8	1 1 0 1 0 0 1	$A_1B_1C_1D_1$	13
Basic mark	a b ab c ac bc abc		$T = 104$
Assignment	A B A×B C e e D		

Handwritten notes on the slide:

- $(2)^3$ (next to Tensile strength data)
- Linear graph (next to Tensile strength data)
- Triangular table (next to Tensile strength data)
- T_{μ} (next to Tensile strength data)
- T_{σ} (next to Tensile strength data)
- M.S. $\frac{M^2}{\sigma^2}$ SNR (bottom right)

So, the first step that you might want to understand is; you want to allocate factors. So, just now we saw that how to allocate factors, but remember in this particular example, we are only interested in a hyphen b, only a cross b interaction we are interested.

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Example – L8

- Plastic molding experiment
- QC → tensile strength
- Factors:

	<u>Level 1</u>	<u>Level 2</u>
• A: Temp ($^{\circ}\text{C}$)	A0=200	A1=220 $^{\circ}\text{C}$
• B: Pressure(Kgcm 2)	B0=500	B1=700
• C: Time(mins)	C0=30	C1=40
• D: Additive(%)	D0=3	D1=5
- 4 Factors
 - 6 combinations. Interest → only in A-B interaction

Handwritten notes and diagrams:

- A diagram showing the L8 fractional factorial design with columns A, B, C, D and their interactions AB, AC, AD, BC, BD, CD. The design is shown as a 2x4 grid of points.
- A diagram showing the L8 design as a 2x4 grid of points, with the first column labeled '5 columns' and the second column labeled 'L5?'. The third column is labeled 'L8' and the fourth column is labeled 'L16'.
- A diagram showing the L8 design as a 2x4 grid of points, with the first column labeled '5 columns' and the second column labeled 'L5?'. The third column is labeled 'L8' and the fourth column is labeled 'L16'.

So, what they are saying I will take a in column 1, b in column 2. So, a cross b will be in column 3, then c, then d. This is as per the Linear graph. So, these two we will call it as dummy. I mean draw line, ok. So, these guys are dummy meaning they will exist, but they do not mean anything.

So, what does this mean? What is the meaning of these guys? So, it says a takes a level of 0, b takes a level of 0, c takes a level of 0 and d takes a level of 0. So, let us take this guy for instance, a takes level of 0, b takes level of 1, c takes level of 1 and d takes level of 0. This is what it is. So, what was a temperature 200 degree, 220 degree. So, 0 means what temperature of 200 degree I am fixing it. B is the pressure, b 1 factor the second level b 1 is 700. So, it is 700 degree. So, I am fixing it at 220 degree centigrade 700 kilograms centimeter square for the pressure.

Then, c is c 1. So, the time is for 40 minutes and d is d naught which is 3 percent additive I am doing and I am getting a response of 15 units kilogram per centimeter square in terms of the tensile strain. Similarly each one of this, each one of the row is a different experiment. So, for each experiment I am getting different results. So, once you get these results, what is it that you can do? There are multiple things that I can do. One is the moment we have already discussed; the moment you have more than one realization which means it is not deterministic or it could be a result of multiple things. We will look for the mean.

So, you can ask μ what the mean value is and then, you can ask for σ . So, this is for across the experiments, ok. This only tells you what the limits bounds are; what is the average of the experiments that you would plan, but what we are usually interested in is, we are interested in the variation across the same experiment. So, if you take this particular experiment where a is fixed at level 0, b is fixed at level 0, c is fixed at level 1 and d is fixed at level 1, if I repeat the same experiment multiple times, the next time I might get 11.5 and the 3rd time I might get 14.

So, you can do as many experiments as many repetitions as you want for this particular experimental setup. Then, for each row I might want to find the μ and standard deviation. Then, I might want to like that I will find for each of the rows $\mu_1 \sigma_1$ $\mu_2 \sigma_2$ $\mu_3 \sigma_3$. Then, I will have to find out which of them gives you the maximum μ and minimum σ . So, there is something called SN ratio, alright. So, what does it tell you, Signal to noise. So, it is μ^2 by σ^2 .

So, if σ is large which means a noise is large, then this value is less. You actually want a better value of SNR meaning the mean is also max more, the σ is less. When the σ is less, then your SNR is going be large. So, μ^2 by σ^2 is what you are looking at in this particular case, but there are also other types of SNR depending on what your quality characteristic is, ok.

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Anova Table..

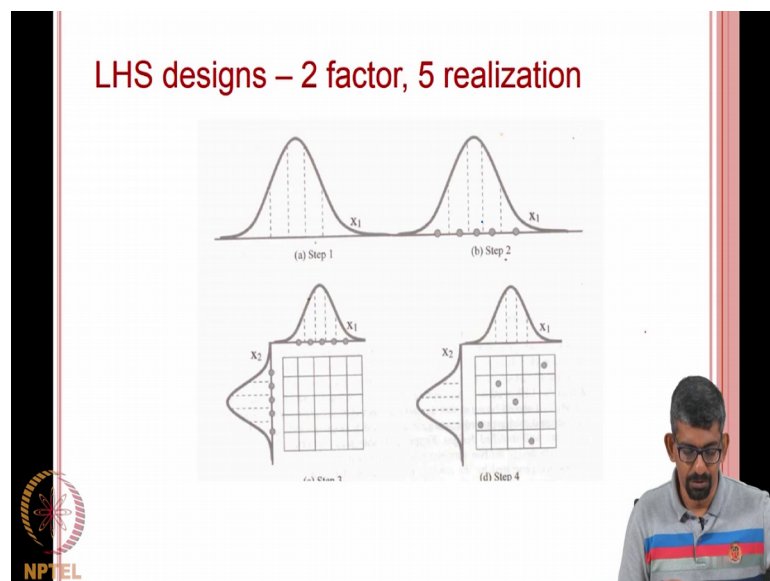
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Source of variation	Sum of squares	Degrees of freedom	Mean square	F_0	$F(0.1)$
$\rightarrow A$	32	1	32	25.6	8.53
$\rightarrow B$	12.5	1	12.5	10	8.53
$\rightarrow A \times B$	24.5	1	24.5	19.6	8.53
$\rightarrow C$	32	1	32	25.6	8.53
$\rightarrow D$	4.5	1	4.5	3.6	8.53
e (error)	2.5	2	1.25		
T	98	7			




There is something called Anova which is out of scope for this particular course. anova stands for Analysis of Variance. So, this anova has its roots from the statistics and interestingly what it says is like it takes your responses and as the name suggest, it decomposes the variance into the different factors that are involved. What are the factors that are involved? A b a cross b c d in this particular example. Then, there are ways in which you will be able to find out there is something called an F statistic. It will tell you which one is more important and stuff like that, ok.

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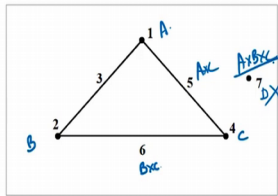


So, this is just to give you an idea.

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Confounding (mixing) of main effects and interactions

Consider a L8 with three factors



$A, B, C \rightarrow 1, 2, 4$
 $AB \rightarrow 3$
 $BC \rightarrow 6$
 $AC \rightarrow 5$
 $ABC \rightarrow 7$

$A \times (B \times C) = A \times B \times C$
 $1 \quad 6 \quad 7$

$B \times (A \times C) = A \times B \times C$
 $2 \quad 5 \quad 7$

$C \times (A \times B) = A \times B \times C$
 $4 \quad 3 \quad 7$

All main factors and interactions are estimated. High resolution experiment

Handwritten notes on the right side of the slide include: $(2)^4 \rightarrow 16$, L_8 , $A B C D$, $AB AC AD BC BD CD ABCD$, $AB AC BC AD BD CD ABCD$, $AB AC BC AD BD CD ABCD$, $AB AC BC AD BD CD ABCD$.

So, I am going back to our triangular tables and sometimes what happens is you take an example of 2 raised to 4. As we saw in the previous example, it was said that there were 3 factor. There are 3 factors and 2 levels, sorry there were 4 factors and 2 levels, but only one 2 factor interaction was important. So, it was ok. You can just still survive with L8, ok. So, in this particular case if you had 4 factors and 2 levels, there are 16 experiments that you need to do so. Obviously, I will do L16, but I cannot afford L16. I can run only L8. So, now you are coming to the real problem if I have.

So, if you have the different combinations that we are talking about, you want to know to select which of the factor because you cannot have the privilege of having all the factors. So, the idea in selecting is very simple. What is it that you are going to knock down? You have to compromise. What is it that you are going to compromise? So, instead of 16 which is a case in this, you want only 8. So, select 8 out of this 16 that is a basic question, but it is not that difficult. What you knock down is, you knock down from the more number of interactions, ok.

So, for instance you knock this guide. Obviously, these three guys also will be knocked down. Now, the other one is 1 2. This you cannot do because main factor you need for sure, then 2 factor interaction, then what is of importance 3 factor, then 4 factor interactions, ok. So, in this sense if you see this will account for 4, then here are 6. So, in

total 10, but I can have only 8. So, this 4, I cannot compromise this 4. I need to have, otherwise there is no point.

So, 4 plus 4, out of these six, I can take only 4 which 4 can I take and how do I, how am I going to take if you say which 4, so to answer that question we need to have some additional information just like the previous example and says anything that has got to do with an interaction is important, ok. 3 even then you will be left with 1. What you do, you can treat it as a dummy one, but let us say that you do not have any information on which one factor is important, ok. You do not know whether a is important, b is important or c is important and their corresponding interactions are important. Please understand one factor by itself might not be important, but that factor in an interaction with another factor could be important.

So, you will have to be careful about that part. So, in this case let us pretend that you have no information on the interaction. In such a case, what will you do? How will you deal or how will you choose the rows? Are the number of experiments appropriate? What you could actually do is, something called confounding. What we are trying to do is, you can confound the main effects with the interaction effects. What does that mean?

So, let us say I have a and I have c, then I have if I put c here, if I put c here, what will happen is I will get only, sorry a b ok. I will only get a b a c b c right, but a b b c you will not get anything with respect to b d because d will be here. So, is there a way to deal with this? Yes, there is a way where you can combine this stuff. So, we will see what that combining is, but before that this is the one that I spoke about. For instance, a was here, b was here, c was here. So, this is b cross c, now I am going to combine one main factor and one interaction to give me a cross b cross c in 7.

Look at the point or I can do b cross b cross a cross c. That is also equivalent to a cross b, but if you look at it 2, 5 will give you 7, 1 6 will give you 7. So, here instead of saying d, I can have a cross b cross c provided I do not have the 4th quantity. That will give me the full factorial experiment. Now, what I am doing? I am actually removing this a cross b cross c and I am putting my d there.

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Confounding contd..

- What happens if need to add another factor?

Res 4:

Column number						
1	2	3	4	5	6	7
A	B	A×B	C	A×C	B×C	A×B×C
		C×D	A×B×D	B×D	A×D	D

Res 2:

Column number						
1	2	3	4	5	6	7
A	B	A×B	C	A×C	B×C	A×B×C
B×C×D	A×C×D	C×D	A×B×D	B×D	A×D	D

The second row just shows the confounding effect. One will use first row with factor D in column 7

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That is what happens if you need to add another factor. So, this is your full factorial experiment, this is called resolution 4. That part we will talk next, ok. A B A cross B C A cross C B cross C A B C.

Now, there is also another way of looking at this. One is instead of this A cross B, I am actually putting D. So, let us put D. I use a red colour. It is much better, ok. I put D. So, what if I have that, then column 1 has A and column D has 7. So, column 6 should have A cross D, then I have C and A cross C, or I can also have B cross D, right. So, B is in 2 5 D is in 7. So, B cross D will be in 5, then I can do A cross B cross D. So, it is 3. So, A cross B is in 3 and then, D is in 7. So, 4 will have A cross B cross D.

So, you can keep building and that is how this 2nd part of this resolution 2 is built. So, what is the meaning of having two layers ? So, what you are going to do is, you are going to confound, you are going to combine these guys, ok. You are going to combine these guys. So, what it happens is, what it means is, you do not know whether it is A cross B or C cross D or A cross C or B cross D. You will run it and let say your analysis says this column number 5 is important. If it says, then you do not know whether it is A A C combination or A B D combination.

In that case, you will have to do additional experiments to identify. So, let us say that out of all the experiment, it will tell you only column 5 was important. So, you say experiments in column 3, you say experiments in column 6.

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A confounded L8 Table

Confounded two factor interaction are analytically impossible to separate the interaction effects.

		Factor and interactions						
		A	B	<u>A×B</u>	C	B×D	A×C	A×D
		Column number						
Trail no.		1	2	3	4	5	6	7
1		1	1	1	1	1	1	1
2		1	1	1	2	2	2	2
3		1	2	2	1	1	2	2
4		1	2	2	2	2	1	1
5		2	1	2	1	2	1	2
6		2	1	2	2	1	2	1
7		2	2	1	1	2	2	1
8		2	2	1	2	1	1	2

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So, this is what it is a rewritten stuff, ok. Confounded 2 factor interactions and analytically, this is impossible to select two separate interaction effects, ok.

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Resolution Table

Res 2: Column assignments

Factor	L8	L16	L32
A	1	1	1
B	2	2	2
C	4	4	4
D	7	7	7
E		8	8
F		11	11
G		13	13
H		14	14
I		17	17
J		18	18
K		20	20
L		23	23
M		24	24
N		27	27
O		29	29
P		30	30

Number of trials	Number of levels	Number of factors	Resolution
4	2	1	4
4	2	2	4
4	2	3	1
8	2	1-3	4
8	2	4	2
8	2	5-7	1
9	3	1	4
9	3	2	4
9	3	3	1
9	3	4	1
16	2	1-4	4
16	2	5	3
16	2	6-8	2
16	2	9-15	1
18	3	1-8	1
27	3	1-3	4
27	3	4-13	1
32	2	1-5	4
32	2	6	3
32	2	7-16	2
32	2	17-31	1

Resolution Code:

- 1 = Only main factors can be estimated (main effects are all confounded with any order interaction). Low resolution.
- 2 = All main factors and groups of two-factor interactions (all main effects are unconfounded with two-factor interactions; two-factor interactions are confounded with each other).
- 3 = All main factors and two-factor interactions can be estimated (all main effects and two-factor interactions are unconfounded).
- 4 = All main effects and all interactions can be estimated. High resolution.

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So, this is one way in which you can include one more factor into an existing orthogonal array. So, this is just resolution table. I used the word resolution, right. So, the top level resolution is called resolution 4 which is the full factorial experiment, right. So, what it says is if the factor and the column assignments, so A will be in column 1; if it is L8 and D will be in column 7, if is L8, ok. So, it keeps going like that.

So, take this particular case. So, all main effects and all interactions can be estimated that is called your highest resolution experiment. For instance, if you take L16, if its resolution number is 4, then what happens is factor 1 to 4, number of levels is 2. You remember right? 2^4 , that is what is this, but then my resolution is 3. What does it mean? All the main factors and 2 factor interactions can be estimated. All the main effects and 2 factors interactions are unconfounded. That is important, ok.

So, 2^5 is how much 32. Half of that is 16. So, L16 I am taking. So, what it says is if you are taking A B C D and E, so these are single factors, main factors. So, 5, then AB AC AD AE 4, then 3 7, then 2 9 10, so 15; so you will get all the factors, the 2 factor interactions like A B AD AE BC BD BE CD CE DE, right. So, 2 4 6 8 10, 10 plus 5 is 15. So, 15 columns and then, 15 factors you are taking this into account. So, it is done.

The next one is if however if you wanted to have AB CA DB DA BE and all that, then you cannot use this particular stuff. So, if all the main factors and 2 factor interactions unconfounded, that is important. Then, it is called level 3. What is level 2? All main factors and groups of 2 factor interactions, when you say groups of 2 factors, 2 factor all main effects are unconfounded with 2 factor interactions. 2 factor interactions could be confounded. So, this is what we saw. The 2 factor interactions could be confounded whereas, the individual factors are not confounded, ok. This is what we saw.

Resolution code 1 means only main factors are there, only main factors can be estimated. Main effects are all confounded with any other interactions low resolution, ok. I mean you can just say I mean there is no meaning of interact factors being interactions being confounded with. For instance, if you take this particular case, we can say in 7 this is confounded with this, this is confounded true, but then you get the same factor of combination ok, multiple times, fine.

With this, I will drop up today's lecture, ok.

Thank you.