Design For Quality, Manufacturing and Assembly

Prof. Palaniappan Ramu
Department of Engineering Design
Indian Institute of Technology, Madras

Lecture - 14
Setting Up an Orthogonal Array

Welcome back. We have been talking about Design for Quality using Robustness Techniques. Under the umbrella of this topic, we have spoken about design of experiments specifically Taguchis Orthogonal Array. So far we have discussed the idea of robustness, how minimizing variance or standard deviation is equivalent to achieving better robustness which eventually translates into better quality. Very recently we saw how one might want to design experiments to obtain the responses, such that the standard deviation can be quantified or the variance or the spread of the data can be quantified.

One of the widely used techniques more from a manufacturing data perspective or manufacturing examples perspective is the Taguchis Orthogonal Array. We have already looked the history of that. The important take away that you need to remember about the Taguchis Orthogonal Array is the way in which the data is balanced. There is no bias with respect to a particular level, within a column the levels are balanced and between any two column also, the levels are balanced, but that was not the case in one factor at the time or all factor at a time, the kind of design of experiments.

So, unless we have any kind of an information to bias a particular level, we should not be doing it; an orthogonal array, inherently does not build such kind of a bias. Hence, that is one of the major advantages of orthogonal array. For today, what we will try to do for this lecture, what we will try to do over this, we will run through a example that would have used orthogonal array to generate or it would have used one kind of an orthogonal array and responses are generated we will see how to analyze those responses.

N root what we will do is, this we will worry about how to select an orthogonal array which is also one of the important scope for this course, how to select a proper orthogonal array because the scope of this course does not include how to build an orthogonal array. How to build an orthogonal array is more of a computer science or a math department course.

Here, we are only going to use orthogonal array, in order to use we need to know how to select an orthogonal array which is again governed by the budget or the time or the number of experiments that you can do. So, we will start with this experiment and in n root, you will learn those ideas. So, currently we are looking at a plastic molding experiment more like an injection molding let say and this L8 refers to L8 experiment, we have already stated that L stands for Latin square.

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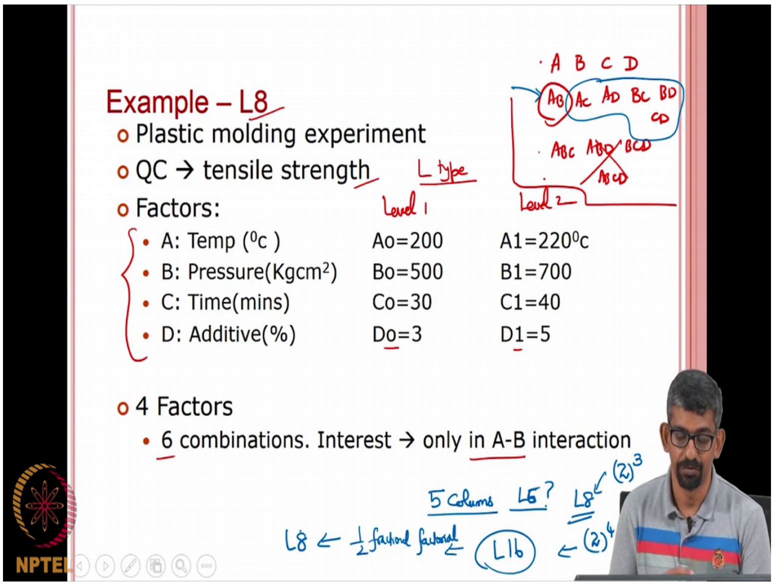

So, when you are talking about tensile strength as a material property, we would like to select a material that will have a better tensile strength, right. So, what do you think is a quality characteristic in this particular case, it will be larger the better L time. But, let us assume that the tensile strength in the factors that affect the tensile strength that has been found out to be temperature, pressure, time and additive, there are 2 levels to it.

For each of the factor, there are 2 levels and some values are given, 200 degree, 220 degree and time 30 minutes and 40 minutes. A 0 represents the initial level and 1 represents level 1. So, in this particular case, what they are talking about is there are 6 combinations in what sense if you have 4 factors A B C and D, what have we seen the single factor, the main factor is A B C and D, the 2 factors are AB AC AD BC BD and CD.

Then, the 3 factors are ABC, ABD, BCD and one 4 factor combination is ABCD. So, if you look at it 4 plus 6 is 10 plus 3 is 13 plus 1 is 14. So, primarily if I have to account for all the interactions, there are about 14 of them. But we are pretending here, in principle if you do full factorial analysis, then you will be able to account for all these things, but in order to account for all these things, we need to do more number of experiments. In order to reduce the number of experiments that is what Taguchis Orthogonal Array proposes; you will get most of the information, but you can save in terms of experiments by at least a percent of 50 or around that.

So, what they are saying is there are 6 combinations in 2 factor interaction. They are not even worried about the higher factor interaction. They are only talking about the 2 factor interactions. They are saying there are about 6 combinations, but our interest is only in AB. This is some additional information that is available; otherwise we cannot make such kind of a decision. What are they saying, these factors are not important. Some expert comes and tells you that only A cross B is important. In such a case, you need to design a orthogonal array, sorry not even design, you just need to select an orthogonal array to run the experiments. So, if you want to capture A B C D individually and AB together, then we are looking at what number experiments or L, ok.

So, primarily we need only 5 columns because, there are only 5 factors that we are interested in terms; combination and main factors, right. So, can we do an L5? If it is available, we can do, but even before that if you need 5 columns, then what should be your L? It should actually be L6 because for L6 there will be 5 columns. Please remember 6 is a number of experiments, then the columns are your factors, so but L6 does not exist that we know. So, the next possible stuff that you can think of is L8.

Please remember this number comes from the fact it is 2 levels raise to 3 factors kind of stuff, ok. That is a full factorial, but in our case if you take 2 level raise to 4 factors because we have 4 factors, so that will lead to 16. So, L16 will be an appropriate experiment, but 16 means you can cover all of these guys, but that is not what we are looking at. We are only looking at half fractional factorial that will be L8 for me. So, we are going to look at L8.

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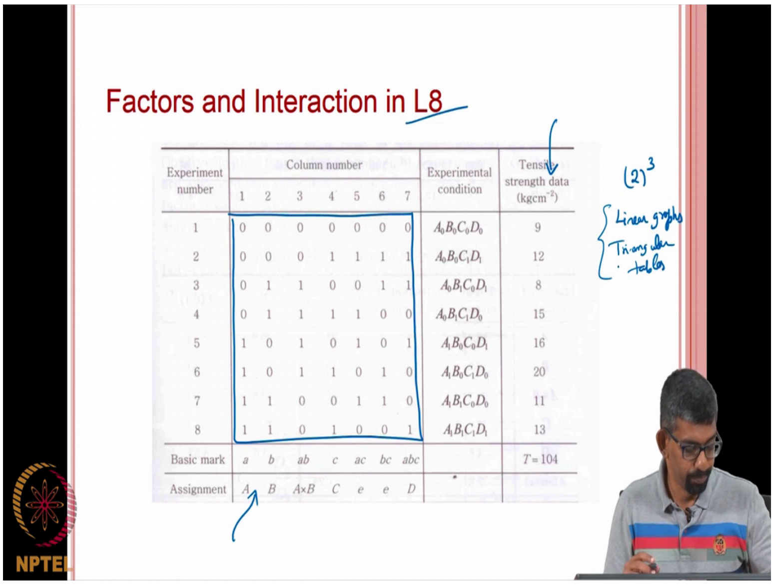

So, this is a generic orthogonal array for L8. It does not matter what example you are looking at, you can use this array; this array is fixed. We do not have to do any research, it is already been done. So, let us say that you are using this orthogonal array in computer science, in mechanical, in aerospace, it does not matter. This combination of the level still remains the same, within a particular discipline whether you are looking at a micro level problem or macro level problem, you are looking at a structures problem, you are looking at does not matter. These numbers remain the same and then, what does change? What is your output response? What are your column assignments? Those are the ones that will change; that is where the designer comes into picture.

These are all already done for you. So, this is L8. So, there will be 8 experiments and there will be 7 columns which are factors. One of the important things that we will see if today is how to assign the factors in this. So, there are two things; one is even if it is 2 raise to 3 meaning 2 levels 3 factors, just ABC. How will you assign ABC to this? Can I just assign ABC AB AC BC and ABC? Is it like that or is there some other way of doing it? Actually there are specific focused of ways of looking at it. Those are called the linear graphs or Triangular tables. So, where I go and get this L8 and this linear graphs and triangular tables, so people have already done these kind of work, but what you just need to do is this.

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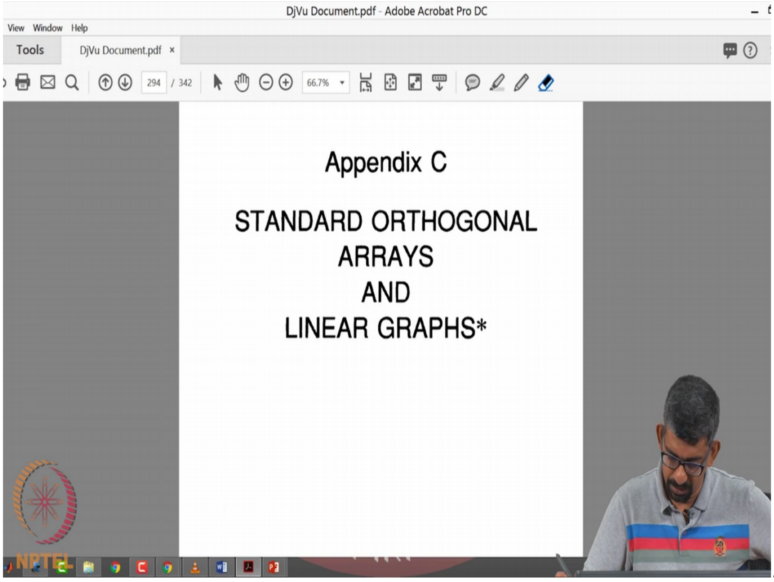

If you go to the back of any book that we have prescribed in, so this is for case book on Robots design.

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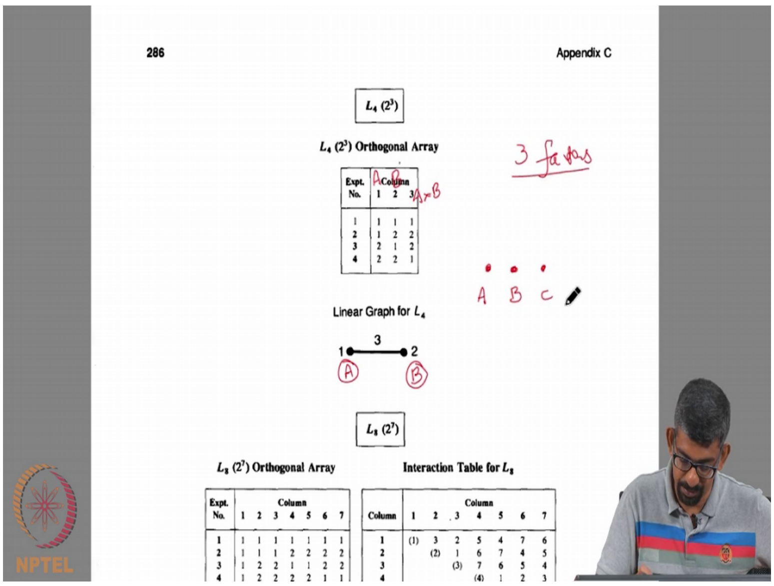

So, if you go to that, you can very clearly see that there is a orthogonal array. For instance, L4 in the bracket it says for what conditions 2 raise to 3 meaning 2 levels 3 factors, but L4 means I cannot account for the interactions. So, you can see there are 3 columns and this is our, how do I assign the factors to the column. So, one way to look at it is something called the linear graph. So, what it says is 1, 2 and 3, this is standard.

What you just need to do is in this case there are 3 factors, right. So, I could say AB. If I say a goes to column 1, B goes to column 2, then that is what I have written here.

Then, column 3 should be A cross B oh, but we have been looking at 3 factor. In that case for any kind of an orthogonal array, there is a combination that you can write it like this. It is understood ABC. You are only looking at the main factors. So, you can also have A B and C or you can choose to have ABA cross B, ok. So, this is 2 factors 2 levels and it is a full factorial experiment. If I am just going to assign A B and C independently, then it is a half factorial experiment of 3 factor experiment, 3 factor with 2 levels. So, originally you need to do 8 experiments, but L4 only half of those experiment half of it.

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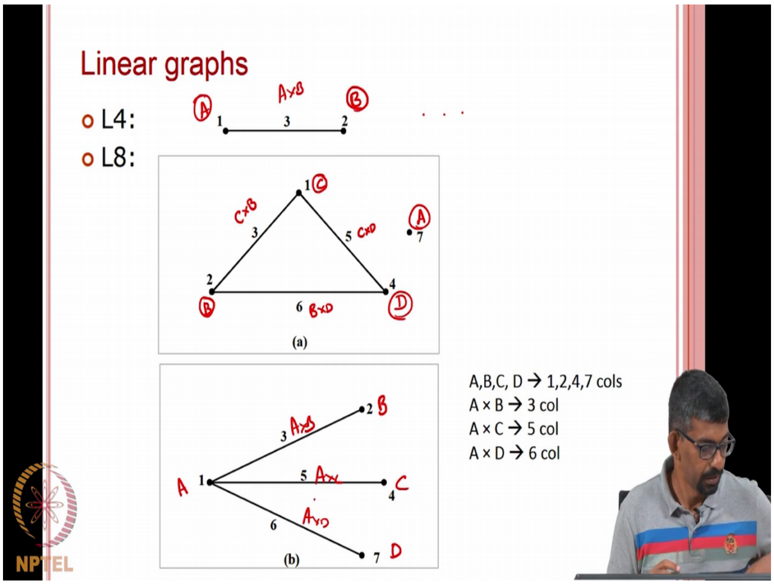

So, we just saw for a general L4 if you do this A B, then 3 could be A cross B or you can have a B and C. In a similar fashion, let us see for L8 how you can do this. So, you do not have to take A to B in 1 you know I can say C is in column 1. Then, B is in column 2, then what happens automatically column 3 will be C cross B. Then, let us say that you take D in column 4, then C cross D is fixed, B cross D is fixed. So, 2 factor interactions with 3 factors gives you 6, then this one could be my A.

So, this is one possibility for L8 meaning 1 2 3, sorry 1 2 3 4 5 6 7. So, L8 means 7 columns. So, 7 factors is something that I can account for. This is one way. There is another way which is this linear graph. So, in this it is slightly different. Let us say for whatever reason someone came and told you that interactions of C are very important, then I will say A for this guy B and C. So, each node I am assigning a single factor and the link will give me my interactions A cross C, sorry C cross B. This should have been D C cross D, but you should also appreciate the fact that it is biased towards this C interactions. That information you need to have, otherwise you cannot use this one. So, the way that we have written here is if you give A B C and D, then A cross B, then A cross C, then A cross D, these are the combination, then the single factor that will come into picture.

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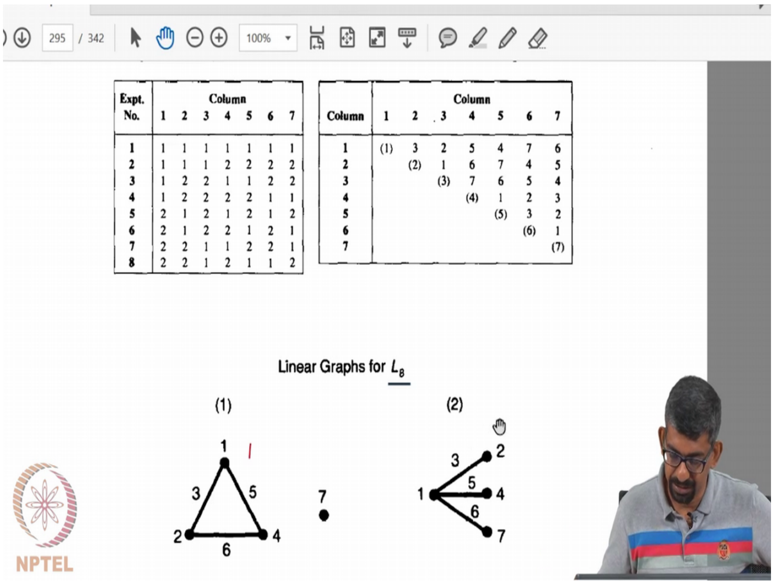

So, now if you go back to the book basically for any of the array that you are talking about, whether it is L8 or L4 or L16 or L32, whatever you are talking about, you will have such linear graphs, ok.

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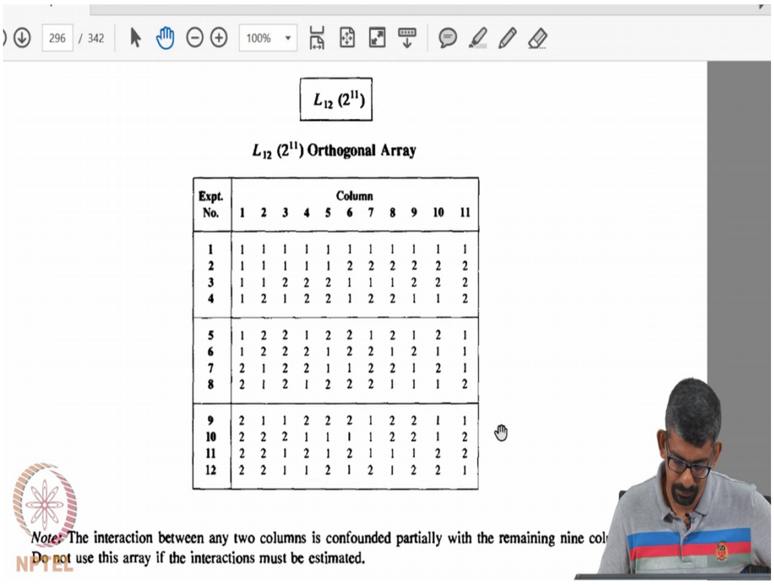

L_{12} (2^{11})

L_{12} (2^{11}) Orthogonal Array

Expt. No.	Column 1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1

Note: The interaction between any two columns is confounded partially with the remaining nine col
Do not use this array if the interactions must be estimated.

So, they are even talking about L16 array for 15 factors, ok. That is the maximum that you can use, but you use less.

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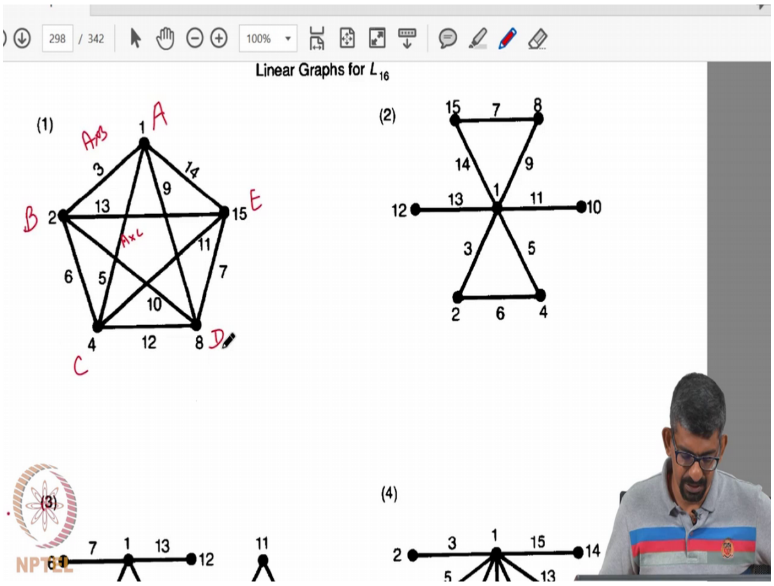

So, the point is what happens is this your linear graphs, becomes slightly complicated as your number of factors and their interactions increases. So, you can see here just for 15, it becomes slightly confusing, ok. We will quickly see in this, we will quickly see in this. So, let us say that you do A here, B here, C D E, so A cross will be in this one as expected, then A cross C will be in 5, then A cross D, then across E, then B cross E. So, you can keep doing, right. So, this becomes slightly complex as the number of dimensions increases.

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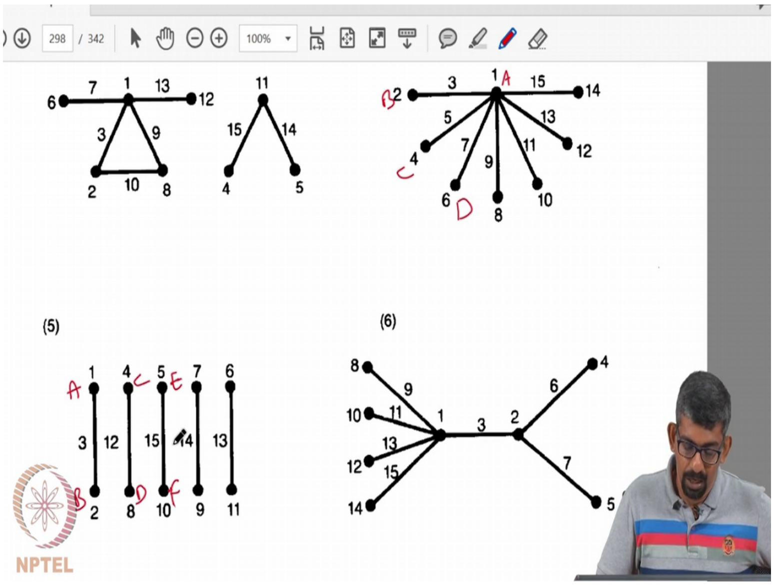

So, this is like similar biased testaments. So, if you take a here, you will get all the interactions with respect to A, but you will not get an interaction with respect to let us say, sorry C and D, you will not get B and D, you will not get, but with respect to A AB AC AC AD and whatever factors you have, until then you can or there could be an interaction like this. You know there is only interaction between AB CD EF.

So, in such a case, you will have to have this information that these are the factors that are important and these are the interactions that I am interested, if you have that. So, there are multiple combinations of linear graphs.

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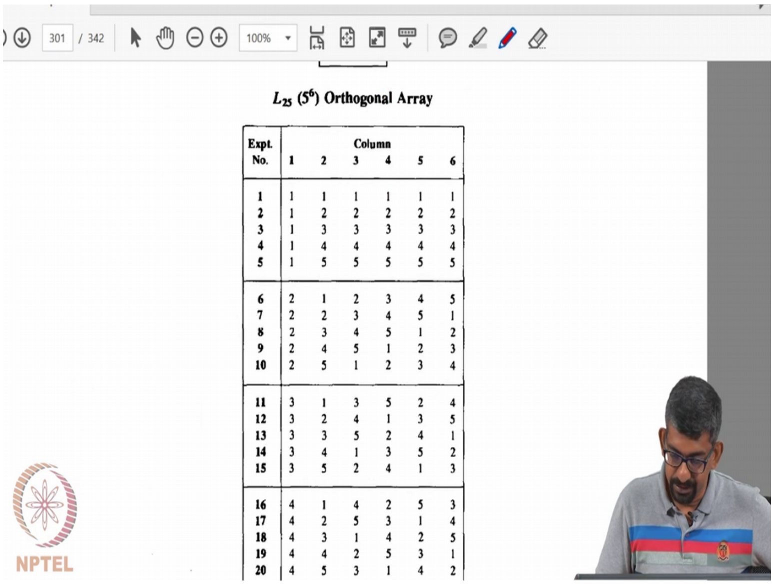

L_{25} (5^6) Orthogonal Array

Expt. No.	Column 1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	3	3	3	3	3
4	1	4	4	4	4	4
5	1	5	5	5	5	5
6	2	1	2	3	4	5
7	2	2	3	4	5	1
8	2	3	4	5	1	2
9	2	4	5	1	2	3
10	2	5	1	2	3	4
11	3	1	3	5	2	4
12	3	2	4	1	3	5
13	3	3	5	2	4	1
14	3	4	1	3	5	2
15	3	5	2	4	1	3
16	4	1	4	2	5	3
17	4	2	5	3	1	4
18	4	3	1	4	2	5
19	4	4	2	5	3	1
20	4	5	3	1	4	2

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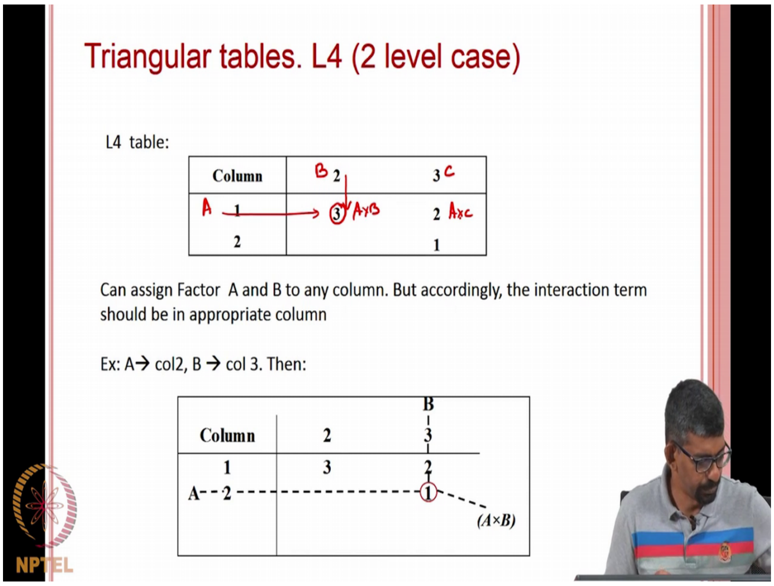

On the other hand, there is also something called a triangular table which provides a similar information, but it is easier to use for a larger factor case. So, this is our table, a linear table looks sorry, a triangular table looks like.

So, what it says is, it gives you a column numbers, and what it says is now you assign factors on one of these columns. So, if you say a for column 1, then let us say you are saying b for column 2, then what it says is on column 3, it should be A cross B. That is all. For instance, if you took a in column 1 and you took C, it cannot be in the same problem. It is a different problem, ok. C in column 3, then your column 2 should be A cross C, ok. So, yeah this is the same thing that is explained in this part.

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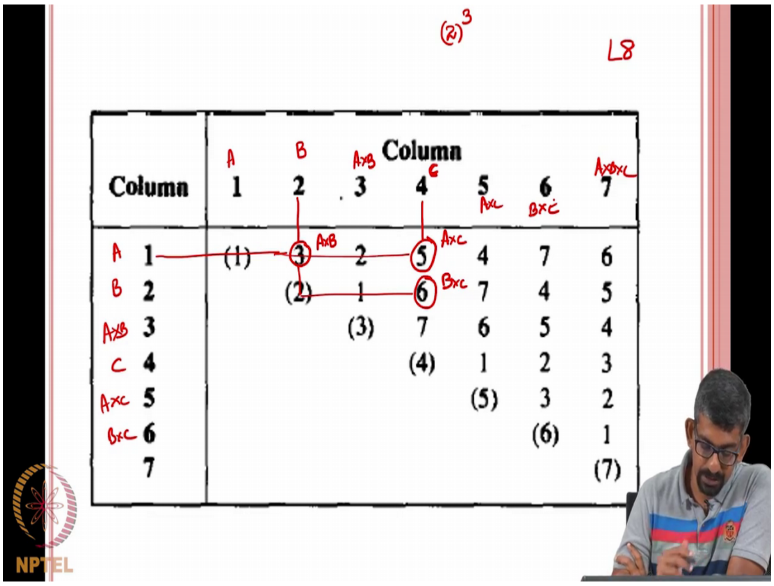

Now, this is for L8 condition. So, this might sound a little confusing to begin with, but we will start. So, as I pointed out, it is 2 raise to 4 case it should be ok, then only we can take a half-factorial or if you even take let us start with 2 raise to 3. So, it is a full factorial, but we will see how the columns are assigned. Let me take A for column 1. So, obviously column 1 column 1 is A. Now, if I assign B for column 2, then what this one says if you come from here to here, then A cross B will be in column 3. Since column 3 is already decided, here we should just say A cross B. In my next column would be my next factor assignment would be C. So, if I take C here, A is in column 1, and C is in column 4. So, column 5 will be A cross C.

In a similar fashion if I take B column 2 and I come here, C is in column 4. So, it will be B cross C. There is also another way of looking at it, but this is not the correct way though, but it will work out if you look at it, A is in column 1; B is in column 2. So, kind of A cross B is 1 plus 2. So, A is in column 1, C is in column 4. So, 1 plus 4 is 5. So, A cross C will be in 5. Similarly, B is in 2, C is in 4, 2 plus 4 is 6, so B cross C will be in 6th that is one way of looking in it.

So, like this if you do, so this will be A cross B, this will be C. Now, we know our these guys are A across C and B cross C, then you do not have a choice, but to say your final D will be in, so you can see in this. So, A cross B, A cross C, B cross C, all the 2 factor interactions you have got. So, this sorry, this is only if you have 4th parameter, otherwise this will be A cross B cross C. We saw this will be A cross C and this will be B cross C, correct yeah. So, this is triangular table also gives you the same idea, but it is easier when large number of factors are involved.