Design for Quality, Manufacturing and Assembly
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Lecture- 13
Orthogonal Array – L4 And L8 Example

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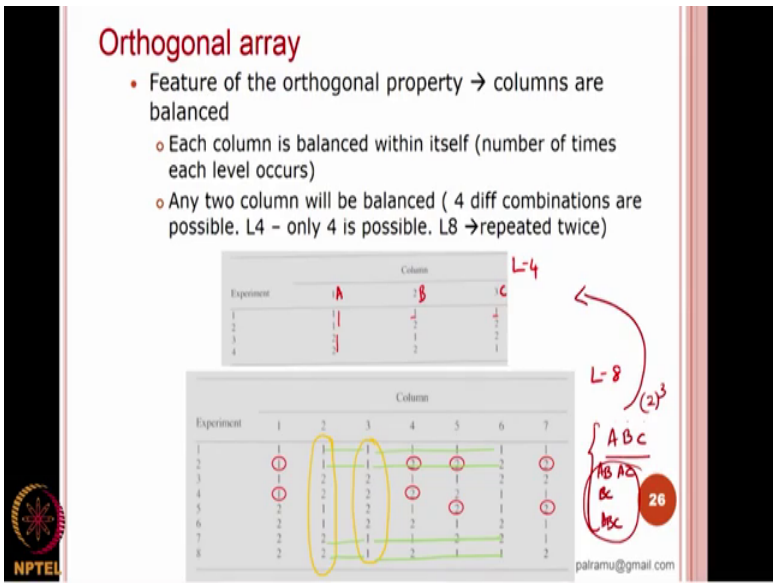

This is a very bleak explanation on how orthogonal array works, but we will go over some properties of an orthogonal array ok. For instance, this is a L4, and this is an L8. What is the meaning there will be 4 experiments; there will be 8 experiments. In an orthogonal array, the elements between any 2 columns are balanced, and within a column they are balanced. We will see what it means.

You take this, this is let us for discussion purpose, we will make it A, B and C. Factors A, B, and C. Level 1 of A, level 1 of A, level 2 of A, level 2 of A. So, is the case level 1 of C, level 2 of C, level 2 of C, level 1 of C. So, if you take, if you take A, 2 level 1s, 2 level 2s, you take any of the column, it will be balanced or if you take any of the 2 columns, if you see 1, 1 together, you cannot see another 1, 1 within this 4. So, any other combination also will come only once, it will not repeat. 2 and 2, you cannot see another 2, 2. 2 and 1, you cannot see another 2 and 1. This holds good between any of these columns A, C; B, C; A, B anything it will hold good.

Now, let us go to L8. Let us see whether it is true, I have helped too. You can take column number 3, take column number 3; 1, 2, 3, 4; 4 1s, 4 2s the column is balanced. Take column number 6; 1, 2, 3, 4; 4 1s, 4 2s. So, the levels are balanced within the column any particular column. Now, you take any 2 columns. Column 3 and column 6 or we will take 2 columns right next to each other it is easy, and then we will go to 3 and 6; 1, 1; 1, 1 happening twice. You can see the rest of the stuff there is no 1, 1. If a combination is happening twice, any other combination will also happen twice, nothing less nothing more, we will see 2, 1. 2, 1 is also happening twice. There is no other 2, 1. 1, 2; 1, 2; 2, 2; 2, 2 you can do this for any other column also.

So, now let us do between 3 and 6. 1, 1; 1, 1 happening twice. 1, 2; 1, 2 no other 1, 2 will be there. So, there is a reason, the columns within a column the levels are balanced between any two columns the levels are balanced. Does this give some information to you? What did we discuss about levels, not all levels are there, they are fixed in some level, they are not fixed in some levels.

Student: Sir, like we did that for the full factorial one there we add there, we did four experiments A 1, B 1, C 1, A 1, B 2, C 1 (Refer Time: 05:02).

That is not a full factorial that is one factor at a time.

Student: One factor.

Multiple factors at a time yeah.

Student: Sir, we were losing information on the second level.

Exactly.

So, what you are actually doing there is you have a bias on B and C's first level. So, you are fixing it, that that does not happen here. The beauty of this one is this L4 can be derived from L8. There is another stuff right, how can you do that, because this seems that is the reason that they have 1, 2, 3, 4 instead of alphabets for what A, B, C, you can write it like that. But, what will be the column headings for this L8, you understand the question, and there is also a relationship, number of experiment versus the number of columns, it is n minus 1. If you have four experiments, you will have three columns. If you have eight experiments, you will have 7 columns ok.

Now, what is the difference between L4 and L8 you think, of course the number of experiments, but in terms of the factors and can you imagine ok. Here is a question for you ok, 8 experiments means what 2 raise to 3. This could be a full factorial experiment. Now, if it is 2 raise to 3, I have 3 factors A, B, and C. Each of them at 2 levels 1, 2; 1, 2; and 1, 2. Now, tell me what are these columns?

Student: (Refer Time: 07:10).

Sorry.

Student: Those are the factors.

Tell me, assign the factors to the columns? You are having only three colou 3 factors right, but you have 7 columns to assign, you get the question?

Student: Yes, sir.

The standalone factors are three, but how many are there 1, 2, 3, 4, 5, 6, 7 this is a full factor. This is another way of looking at it. If I am reducing this from L8 to L4, what am I going to compromise, I am going to, I am happy with the fact that, I do not have to run 8 experiments, I have to run only four experiments, but where is my information lost.

Student: (Refer Time: 08:27).

I cannot account for these (Refer Time: 08:31). I can only account for A, B, and C, you get it. In a similar fashion, if you have 2 raise 4, 4 factors, you have A, B, C, and D.

Student: 16.

16, full factorial is 16. You can bring it down to L8, you will lose the three factors ok, because there it will be A, B, C; A, B, D; B, C, D you lose those, can you bring it down to L4?

Student: (Refer Time: 09:12).

Sorry.

Student: We have to remove A, B; A, D; A, C.

Yeah, see from L8 to L4 itself your interactions are lost. So, from L16 to L4 for sure your interactions will be lost. But, my question is 2 raise to 4 means, there are 4 factors. So, can you reduce L16 to L4?

Student: No sir, A, B; B, C; and C, D.

A, B; B, C; and C, D what about A, D? What about B, D?

Student: (Refer Time: 09:52).

So, then you are biasing that is the whole point, you cannot bias ok. So, there is an order in which you will knock things down ok. You cannot just knock things down just like that ok.

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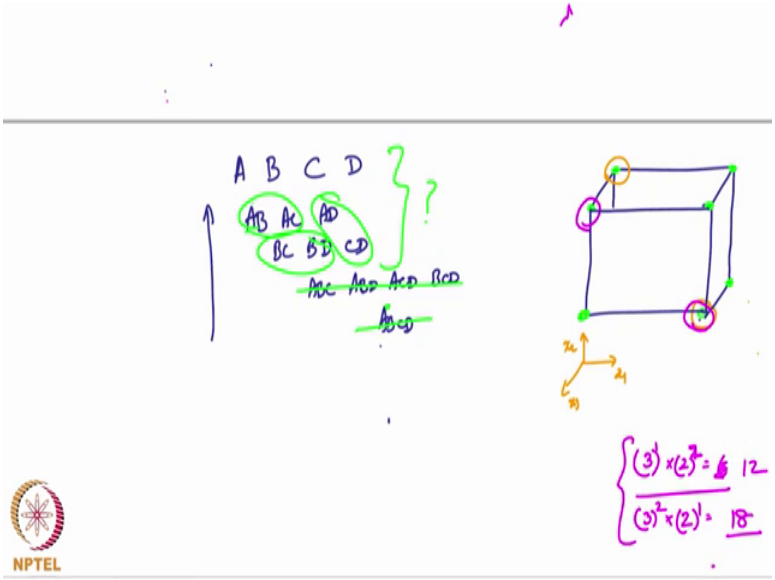

So, for an for example, you have A, B, C, and D. So, A, B; A, C; A, D. B, C; B, D; C, D. A, B, C; A, B, D; A, C, D; B, C, D; A, B, C, D, so that will give you a 16, 4, 7, 10, 14, 16, 15 correct 16 experiments, 15 factors ok. So, when I am going from this is L16, when I am going from L16 to L8 the way in which I will proceed to knock down is this. I will first knock down this guy. Then I will knock down three factor guys, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 still it is not ok, correct because if it is L8, I have how many columns?

Student: 7.

So, I can accommodate only seven, but I have A, B, C, D; A, B; A, C; A, D; B, C; B, D; C, D. Now, what will I do?

Student: B, C; B, D sir.

Student: B, C; B, D; C, D.

That is some order that I wrote, you have no idea, which one is more sensitive. So, there is something called confounding. So, what you do is you couple two guys, there is a reason, there is a way in which you can couple, I am just drawing something here ok.

Now, what happens is you assign one particular column to A, B and A, C. Later we will see, when we analyse and all that you will you will identify that column number 5 was has is very sensitive, but in that case column number five could be a could be A, B or A, C, you do not know. If column number 5 turns out to be effective, then you will have to do additional experiments to understand, whether it is A, B or A, C ok. So, this is called confounding anyway, so I guess, you got the idea of the property of an orthogonal array. So, tomorrow someone shows you an orthogonal array, you know whether it is an orthogonal array or not, because you just need to look at whether they are balanced ok.

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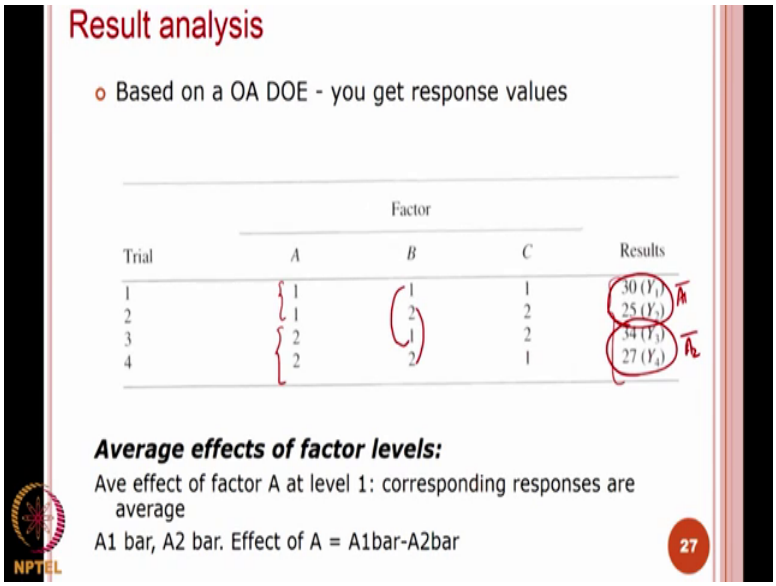

So, we are just trying to do a result analysis here. We take the same kind of an experiment. And then, we say A, factor A, B, C there are two levels. It is all balanced, it is an orthogonal array and these are my results. Now, there is an interesting question. In

the previous one, in the previous case that we discussed, for identifying the what you called the effect of A there are only two rows. In one row it was A1, and the other row it was A2 so, I went ahead and subtracted. But, in this case how will you do that, because there are 2 rows with level 1, and there are 2 rows with level 2, which one will you choose, you get the question with this data I want to know the effect of A, how will I do that.

So, you have B1 and C1, then you have B2 and C2, but for both of them it is a level is 1. Then for some other random change A level is 2. The difference between the previous one, and this one has there you had only 1 row, where A had level 2, but in this you have 2 rows in which A had level 2, and you have 2 rows in which A have level 1. And your B and C are not held the same, they are changing. If you take these two rows or any two row, 1 1 never repeats so, you cannot bring that argument. B and C is the same, so I will compare those two and actually that is exactly what we wanted. What have we said? So, one way to look at this is the moment you have more than 1 what you do?

Student: Sir, averaging.

We have talked about averaging and standard deviation. So, what you do is you average the value for these guys, so that becomes my A1. And I average these 2 guys, they become A2. Now, you do not worry about what your values of B and C are, because that was our limitation in the previous one. How can I not worry about it, boss you do not need to worry, because you are looking at an orthogonal array, and it is already balanced.

You understand, you are not biasing on one side, you are not biasing on one side. Can I do the same average in that previous one, yes you can, but you are biasing. You understand what I am saying, there were three A1s and only one A2, there were three B1s and only one B2, there were three C1s and only one C2. So, you the way in which you connected your experiment, you have already biased it. You had a favourism for A1, B1, and C1, but that is not the case here.

So, number 1, the balance that we spoke about in terms of the columns and between the columns has a meaning, you are not biasing on particular level, and you are not fixing your, the other two factors also, they are taking the different combinations. So, what you need to do is we will call this A1 bar and A2 bar and then, if you take the difference between them, it will be the effect of A and B, sorry A. Similarly, if you want to do for B,

what will you do? You will take the responses corresponding to this in this, you will take an average, you will take these two responses, you will take an average and subtract it ok. Now, in terms of using the information what do you see the difference between that previous approach, and this approach. There how many experiments, did you conduct?

Student: 4.

4 here.

Student: 4

In computing the effect of A, how many experiments value are you taking here?

Student: 4.

There.

Student: 2 2.

There for effect of B, how many experiments are you taking?

Student: 2 2.

So, you are getting or you are using more information to find the effect of A. Otherwise, you are doing four experiments there, and you are using only two experiments to get the effect. Here, you are using four experiments, and you are doing four experiments, and you are using all the four experiments to get the effect. That is an advantage. There, if you plan to do that, it will be wrong, because you will bias, you can say sir, if you are averaging here, there also I will average. If you average, it is not correct, because it is B1 is appearing three times, and B2 is appearing only once. And necessarily your A and C, they are not changing also in a uniform fashion, they are not staying the same also. So, it is it is not the correct way to do that then, this is what that discussion is.

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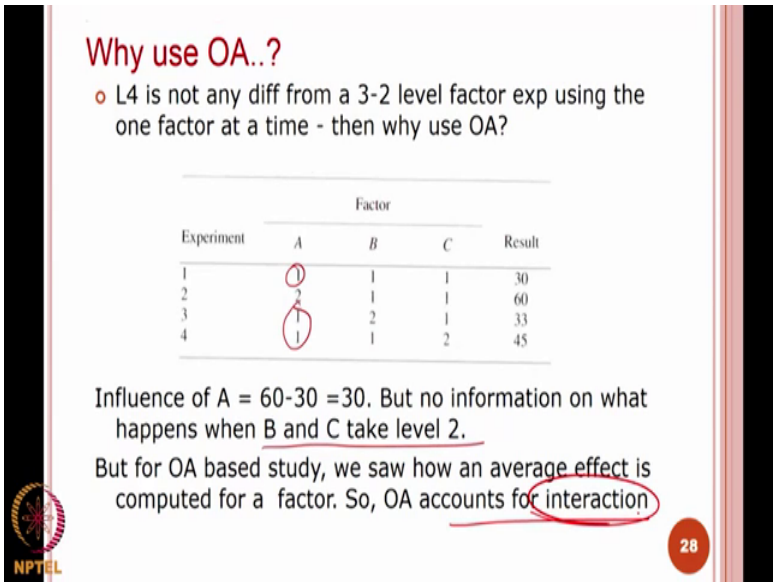

L4 is not any different from the 3-2 level factor experiment using the one factor at a time, then why use orthogonal array? So, this is that experiment. So, if you see here, we have three 1s ok. Influence of A is, but no information on what happens when B and C take level 2. But for an orthogonal based array based study, we saw how an average effect is computed for a factor. So, we accounts for the interaction that is important. And we saw how that interaction is taken into account.

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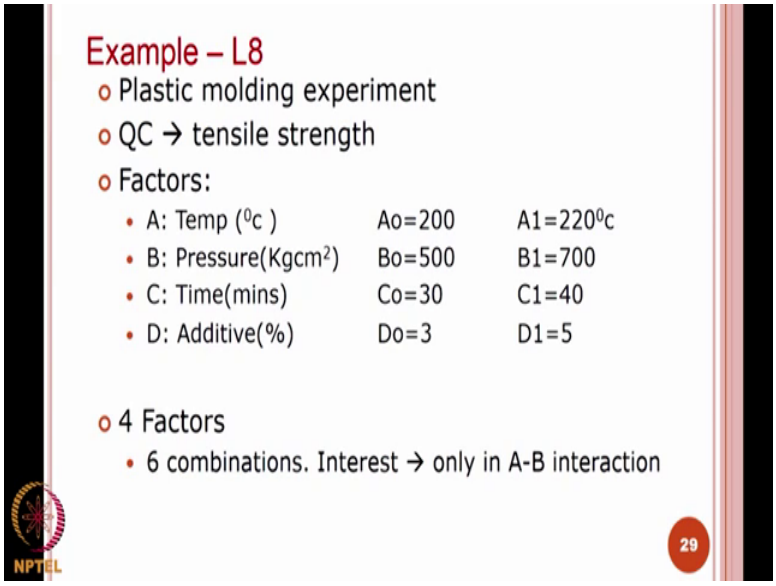

So, here is a plastic molding experiment. So, what is the quality characteristic is the tensile strength, but what kind of a quality characteristic is this what type.

Student: Continuous.

No, no what type quality characteristic?

Student: Larger the better.

Larger the better there is no more, larger the better ok. So, I want to find a combination in which I get maximum strength, what are the factors the temperature, the pressure, the time, and the additive. And there are some levels that are given here, A naught A1, B naught, B1. So, here is the deal, 2 levels, 4 factors. So, totally how many combination for a full factorial?

Student: 16.

2 raise to 4 is 16. In this case, whoever is doing this experiment, my interest is only in the A-B interaction ok, meaning A, B, C, D and A, B that should be only 5. So, I am only interested in the A-B interaction, not in other interaction. So, you need to have prior information to make such a conclusion otherwise, you are biasing. In terms of the columns I have A, B, C, D, A cross B, that is all I have only 5. For an L4, which is the minimal how many do I have columns. So, I cannot use, I need at least 5. The next possible is L8.

Student: L8.

There is no L6 or anything ok. So, L8, because it has to be reduced, I will show you, how the reduction also happens ok, from L8 to L sorry from L4 to L8 we go, but in L8, how many columns you have?

Student: 7 sir.

7 columns, but I have only 5. I need only 5. So, there will be two columns will be designated as dummy, do not worry.

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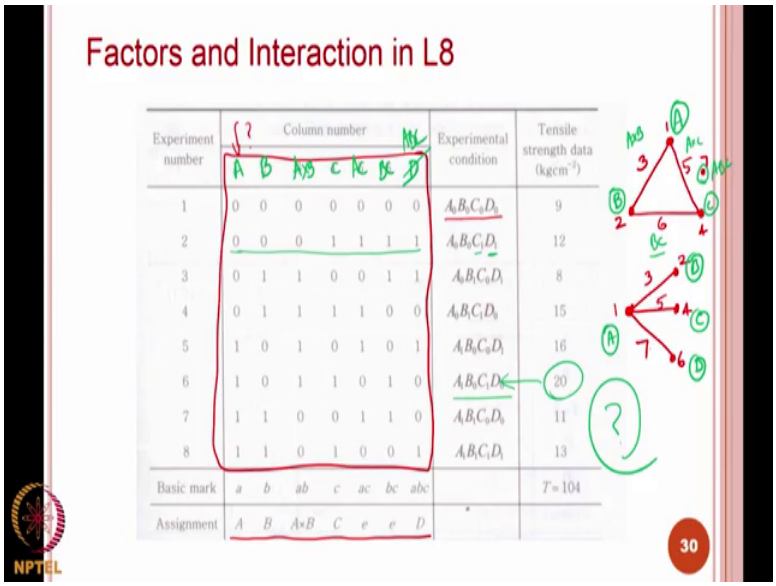

We will discuss in the next class on how this is assigned. For right now do not worry about that, this is an L8. Whoever you are, whatever work you are doing, this is L8. I am a computer scientist, I am planning to do some kind of an optimal scheduling problem, I am using orthogonal array, for you also L8 is this. I am a manufacturing engineer, I am trying to understand the effect of manufacturing parameters on the surface finish, for you also this is an L8. I am an aerospace engineer, I am looking at different parameters on my wing and tail and fuselage to understand the flight time, for you also this is an L8.

So, please understand, the L8 that is prepared is available across the board, for everyone that is, so it is fixed, but what as a computer science engineer, as a mechanical engineer, as an aerospace engineer, that you need to decide is, what are the facts that are assigned to these columns ok. There is something called triangular tables and linear graphs that let you assign that, I will I will discuss that, that is not a problem ok. So, now, but we will just discuss quickly. Experiment number 1, this is the experimental condition what does this mean A ok, that is because ok, let me just quickly tell you what a triangular, sorry linear graph is.

So, if you go to the back of any orthogonal array book, you will have these orthogonal arrays. And there will be these linear graphs that are provided along with them. For L8, this is one linear graph. So, it will be said 1, 2, 3, 4, 5, 6, 7 this is how it would be given. There might be another thing, I am not sure whether the numbers are right, but this is

how it works ok. What this means is, these tables the linear graphs are given with the table.

You need to now assign, what factor goes to what column, how you do is. You say A goes to this particular. I assign A to 1, you assign B to 2, you assign C to 3, then your 4, 5, 6, are frozen. What is my 5, A cross B; what is my 6, A cross C; what is my 4, B cross C. I am not sure whether that is correct meaning, this is how you analyse the table the numbers are correct or not I do not know that, we will figure out. A 1, B 2 yeah, so this should be 3 actually. So, sorry, so this is 4, this is probably A, C means, this is 5.

So, if you look at this A B, the third column is A cross B ok, the third column is A cross B that is what is written here also A cross B. The forth we have assigned for C. In case, in this case we are not worried, in case we have to complete this C, then 5 is A cross C; then 6 is B cross C; 7, I am going to assign D. Please understand you cannot sacrifice A, B, C, and D without sacrificing AB, AC, BC correct.

So, this can be for 4 variables or this can also this can also be ABC. This could have been, if I am not considering D, this could have been ABC get the point, but you go with the linear table. But, this enough work has been done to come up with the linear tables. What could be the other way, it could be this one A B, then I think, this will be 3 yeah ut cannot be like this, I am sorry. This should be 3, this will become 4, 5, 6, 7, this is only for discussion purposes ok. This is for all said and done this might not be correct also, but the process is correct. The graph itself might be wrong, but the process is correct.

So, if I provide, if I give A for this guy, B for this guy, then AB is in 3, C, D. Now, I have two factor interactions 4 of them AB, AC, AD anything with respect to A I have. BC, BD I will not have. And you need not assign only A to 1, you could have assigned B to 1, you could have assigned C to 1 that is not a problem. But, this is what the linear table is ok, there is something called a triangular table, which we will see ok. Anyway this is the idea to tell you like what is the factor.

We will see what this means, so all of them are 0. If you take the first row ok, which means it is level 0, so A is 0; B is 0, C is 0, D is 0. If you take this particular column up with this particular row, it says C 1 and D 1. There is no physical meaning for AC and BC having 1, because there is only A, this is just an interaction, you can only fix the factors, not the interactions. The factors are only for the levels are only for the factors.

So, if the level of my A is 0, and the level of my B is 0, level of AB is automatically 0. If level of A was 0, and level of 1 was level of B was 1, then A cross B level is 1, that is what you will see here ok. So, there is no physical meaning for A cross B 1 or A cross B 0, it comes from what levels A and B take there is no physical meaning for that. Whereas, there is a physical meaning for A taking level 0 or level 1 ok. So, you conduct all these experiments, there are eight different setups, eight different values you get. Now, there are two things that you can do, what can you do, you can find the statistics. What was the objective our problem in this problem, what was the objective in our problem in this problem, what was our objective in this problem, you forgot what the problem, what was the problem?

Student: We have to model the model combination of A and B.

Student: Maximise tensile.

That and all was not the objective. The objective was to maximize your tensile strength that is all ok. So, the point is simple, I have given you a bunch of data. So, what you do is you look for the maximum strength, which is the maximum strength in this?

Student: 20.

So, you can say ok. You will get the maximum strength at this combination, if you run this experiment is that right, yes or no? Student: Yes.

It is just statistics in the given condition yes ok, but you should also remember that this is not a full factorial experiment. So, there could be some combination that you did not try could lead to a better result. So, how do we analyse that, we will discuss in the next class ok, fine.

Thank you.