Design For Quality, Manufacturing and Assembly
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Lecture - 12
Factorial Design

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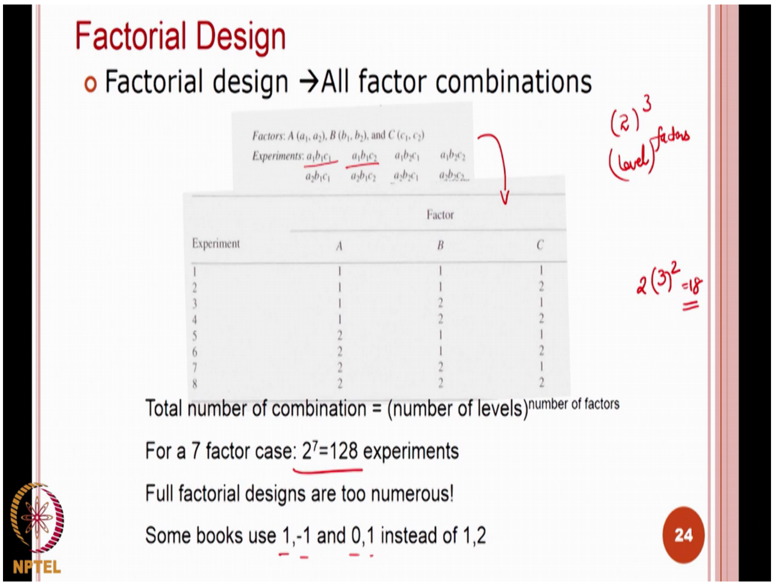

So, we are now going to talk about some factorial design, all factor combination. So, if you after this name; let us say that he asked for three levels, but we will first look at three factors. If there are factors A, B and C, each of them has levels a1 a2 b1 b2 c1 c2. Each of them have 2 levels, then the corresponding experiments if you are doing all the combinations, then it should be a1 b1 c1 a1 b2 c2. Like that all the combinations you need to have. So, it will be 2 raised to 3. Where does this come from? Levels raise to factors. So, I will have 8 different combinations that I will do.

This as you can imagine, the rate will go up very quickly. For instance, if you are trying to do a simple car design, let us not even worry about crash simple car testing. 7 variables is nothing, even 2 levels of those 7 variables you are expected to do 128 experiments. When I say experiments, after that experiment is done which means you will do it until failure, the car is not usable. So, no one is going to waste 128 cars to build a robust. So, this is not the ideal way to go ahead.

This is how do I write. It is given here. Experiment number 1, a1 b1 c1, experiment number 2, a1 b1 c2. That is what we have here. So, now get used to this kind of a table where A1 means level 1 or factor A, level 1 of factor B, level 2 of factor B like that. So, the full factorial designs are too numerous. Some books might use a 1 minus 1 or 0 1 instead of 1 and 2. So, do not get confused for levels 1 and 2 I am saying.

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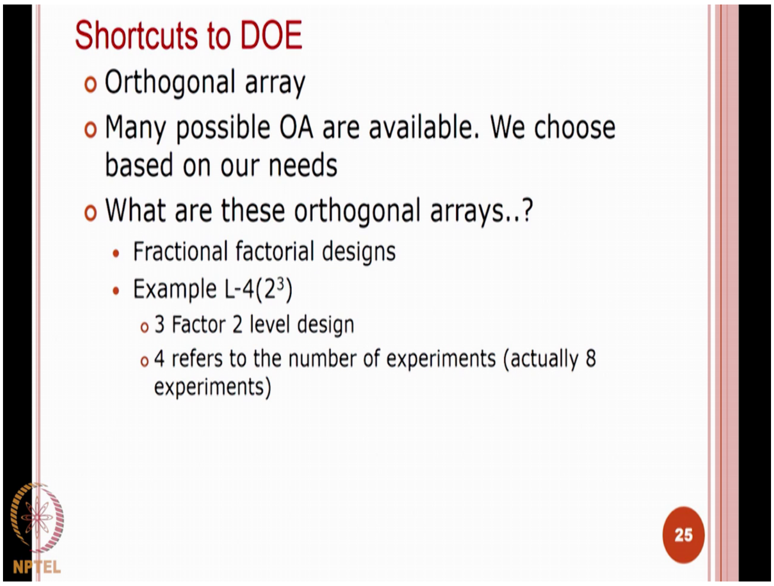

So, shortcut to design for instance, this is a full factorial is a design of experiment. All the possible combinations if there are 3 variables, each of the 2 levels then you need to have 8. So, we saw that right like you saw that 3 factors, but two of them had 3 levels and one of them had 2. So, you had to go for 90, sorry 18 sorry. So, that will be 3 raise to 2 times 2. You remember that resin pre-polymer example there since that pre-polymer and the adhesive that we assumed was 3.

Student: 3

Yes, 3 levels. So, how many factors? 2 factor resin pre-polymer and the adhesive. So, that is 9 times 2. So, that will be 18 experiments and that is what we got. So, that is a full factorial experiment there yeah, but there are also other types of full factorial experiments that you might already know in a different sense. We will just briefly discuss that.

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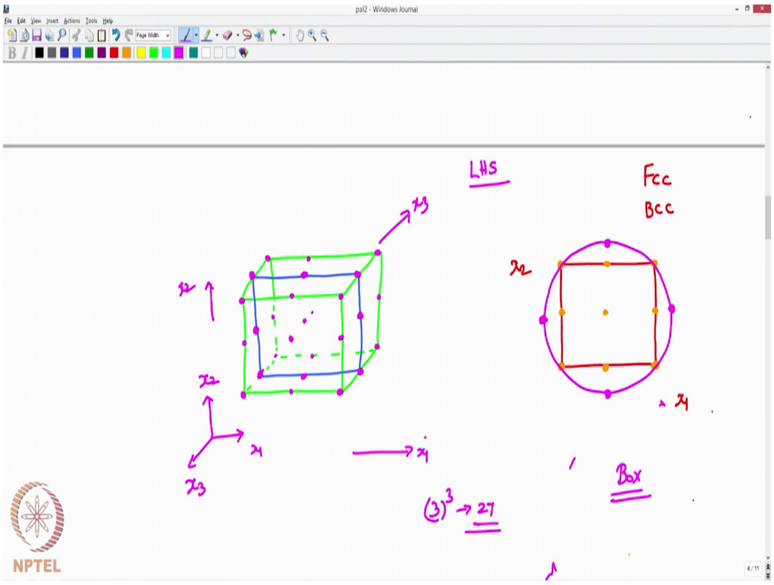

Have you come across this a face centred cube, body centred cube. In one sense that is what we are talking about. Imagine that this is x 1 and this is x 2 variables. What does it mean? It means 2 levels of x 1 that, is all you are talking about 4 experiments, the 4 corners. In case, this is what 2 raise to 2 factors 2 levels. If I had 3 levels of x 1, I had 3 levels of x 2, then what happens is, I need to have these combinations also. How many?

Student: 9.

9 times 2 raise to, sorry.

Student: 3 raise to.

3 raise to 2, simple. What do you imagine? I am going to do what are the next acceleration easily, Just 2, let us say 1 2. This is for x 1 x 2. What about x 3? I am having stuff like this that is what is x3? The depth is what the x 3 is. So, where will be the other point p? It is along x 3 direction. So, it is just a projection you got it. So, if it is a cuboid like this, then there are 8 points. Does that tally to what we have discussed? It is 2 levels. How many factors? 2 raise to 3 is, ok.

Now, if this was 3 something at the centre, so it becomes difficult. Now, you have to project that, ok. So, then it is here, then yeah it will be here, then it will be here, then it will be here, and this side that side yeah you might also want to have one at the centre here. So, it will be 3 times 3, sorry. Not 3 times. It is 3 raise to 3, that is 27 points, right.

So, the way that I have drawn is only 9 plus, not even 9 plus 9. So, it is only 18, but then you will get points here, this, this and this. So, if you do all that because here you will have here yeah. So, I have added 9 points now.

So, that will give you 27. Did you get what I did? I just did this. I just sliced in between and I put points there, yeah. So, that will tally up to and there will be a point in this stuff all put together, you will get 27 points, ok. So, this is one way of looking at things, then what people suggested is you can in order to get some more information because if you look at projection wise, you are only replicating the designs in terms of x 1.

For instance, in this case if you are taking the projection of x 1 and x 2, in this particular case that we are talking about if you are taking a projection of x 1 and x 2, any depth information is lost. Though I have 27 points, my valuable information in x 1 x 2 plane is only 9 because the remaining all are in x 3. They are not any different in terms of x 1 and x 2. They are same x 1 x 2 for me. You understand what I am saying? The depth all the depth information is lost. So, the case here also if you are taking just x 2, what are the information? Only these three are my information. The remaining they do not change. X 2 does not change in this direction. So, there is only three different values of x 2 for me.

Instead what people suggested is can we push this a little bit away here? How do you want to do that? They said let us do something like this. Instead of keeping this guy there, can I have it here? I might not get any information with respect to this point, but certainly with this point I get more information you see for the same 1 2 3 4 5 6 7 8 and 9 points. Now, I will get 1 2 3 4 5 information in x 2 instead of 3 information in x 2. You get what I am saying from a projection perspective.

Student: Yes sir.

Then, they said no you should not have it like this. You should actually have the circle inside whichever way it is, you can do it and you can get, ok. Then, there were something called box designs. Box is a name of the guy who came up with this Box and Bencam designs, ok. They also grew famous, but at some point in time people suggested that this is not the way to go. We will have to use statistics as a basis to come up with these designs and that is how there is something called LHS, Latin Hypercube Samples. We will discuss them that came into picture. So, with this background we wanted to see the full factorial experiment that we discussed just now. Has that background you understand, right that cuboid cube whatever we looked at.

Now, orthogonal array is a fractional factorial experiment. What it says is you do not have to go and factor in all that information. We will give you some spots. It will not give you the entire information, but you can get most of the information with which you can make a decision. So, what are those points? How are we going to go about it? What are those points is something that we will discuss. I will go back to that cuboid example and then tell you, but how orthogonal array itself is constructed is not the scope for this particular course, but we will discuss how to choose an orthogonal array, how to use it, how to analyse the results based on a design of experiment orthogonal array. Those are the discussions we will have.

So, where do you get the orthogonal arrays? There are some websites which offer orthogonal arrays for as many variables as you want in the field of computer science. People construct orthogonal arrays for about 1024 variables and even more than that, but this is of not great interest to people like us who are in product design predominantly mechanical product design. In our case, if you go to about 20 to 30 design variable that itself is too much. So, why I am telling you that is, orthogonal array required for our purposes are already available. You just need to choose the type of orthogonal array. We will see what you mean by choose.

Student: Sir, so far we assumed that all the factors have the same number of levels, but if they are.

Oh no, no, no. You can have different, no that is not true. The one that we discussed the pre-polymer resin had different types of.

Student: Sir how did we get to their formula levels, to their power of factors? So, if all factors are different level, what is level we choose? What are the factors?

We just discussed that, right. For instance, if all of them were the same, you just do level raise to factors. The other example that we had, we had 2 factors 3 levels. So, you did 3 for that particular thing, you did 3 times 3 raise to 2 and then, you multiplied it with the 2 factor stuff. Let us say that in that same example you had 4 factors in which two of them had 2 levels and two of them had 3 levels, then you would do 2 raise to 3, sorry 3 raise to

2 multiplied by 2 raise to 2. That is straightforward that comes from the full factorial. This thing that I showed you now 9 projected on the other side will give you 80. That is what we are looking at.

It is just like an extrusion when you are extruding only the point at the corner on the other end also, you will get only points at the corner. So, 4 plus 4 will be 8, but when you have a point at the centre, when you are extruding, you will extrude that point also. So, along that it will also have a point at the centre. So, that is why you will get an additional 9 points. So, it will be 27, but you can try doing that one of the variables. You can make it 2, that x 1 x 2 x 3 that we did just now. You fix x 1 to be 2, x 2 to be 3. So, when you project, how much should you get?

Student: 2 power 2 plus.

One factor has 3 levels, one factor has 2 levels what will you get?

Student: 1.

Sorry.

Student: 9.

You will get 9. No man if you have 2 factors with 3 levels, you will get 9. If 1 factor has 3 levels, one factor has 2 levels, how many points will you get? You can either think pictorially or you can do it not mathematically, numerically also. So, one factor 3, 3 raise to 1 is 3.

Student: 6

Times 2 raise to 1, so 3 times 2 is 6. So, what will happen is 2 factors in x 1, 3 factors in x 2 you will sweep it along x 1 and 3 will go from here to there. 3 plus 3 is 6. That is all. If I had a point in between in x, then you will have you understand what I am saying, right. So, that is not a big deal that I mean actually that was the reason that I actually discussed that.

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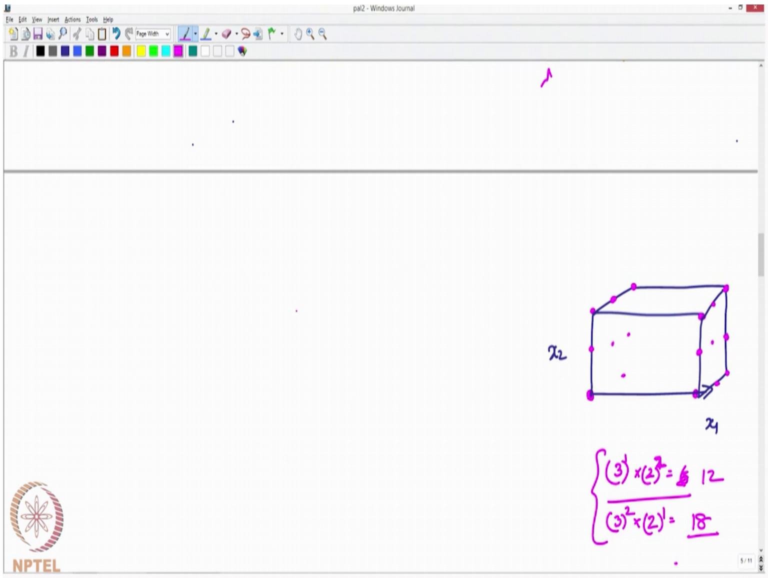

What I am trying to tell is to answer your question. I have 3 factors in this guy. If I have to propagate, then I have only at the next level I will have three levels for this x 2. So, it is 6. How can I without drawing this, can I say. Yes, you can say because it is 3 levels for x 2. Only 1factor times that is 6.In a similar fashion if I want to extend this guy, so depending on if you have 2 variables in not 2 variables, 2 levels in x 3, it is just a projection.

So, how many you will have now 2 times 2 is 4. So, you should have this is 4 times 3 is 12. So, 6 times 2 will be 12 for you. So, I will have one here and then, I will have something here. Now, if I made this 3 instead which means when I am projecting, I will also have stuff here in between. So, this guy, this guy, this guy is it, right. In between I will have this guy in this guy. So, now instead of that 3 times 3 raised to 2, 2 raised to 1, so 9 times 2 is 18. So, 6 in between 6 after that 6 is 18 points. So, this is all I mean this is just to give you a pictorial depiction.

So, what are these orthogonal arrays? They are fractional factorial experiments. It is not the full factorial. It is only the fractional factorial. For example, this is the way which we usually use, it is called L4, the way in which an orthogonal array is called is L x. X is the number of experiments when I say L4, it means?

Student: 4 experiments.

4 experiments are there, but you also need to look for this guy 2 raise to 3. What it is saying is actually 8 experiments can be reduced to 4 experiments. So, it is a half fractional factorial, ok. In some cases, you can also do 2 raise to 4 which means 16 experiments can be reduced to 4 experiments which is one-quarter, ok. Instead of 16 experiments, you will do 4 experiments; one quick stuff as in how you can do that.

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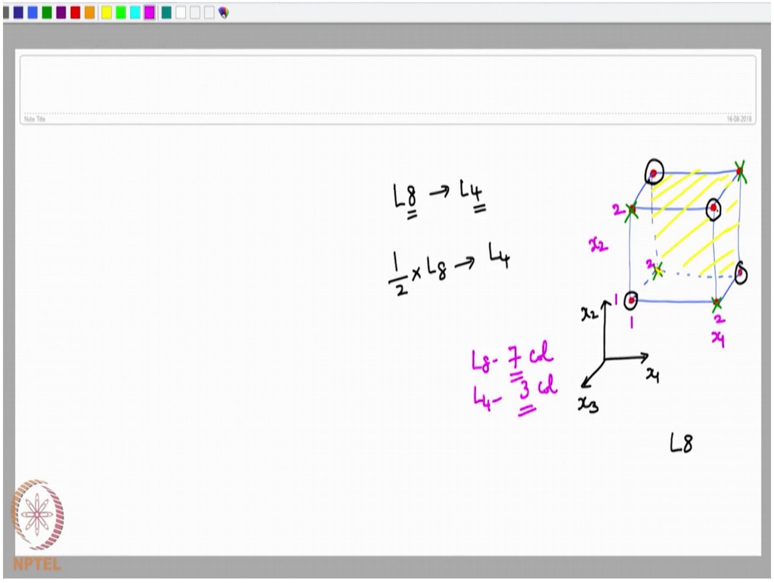

We are looking at L8 experiment. This can be represented using a cuboid. Let us say there are 3 axis; x 1 x 2 and x 3 which represent the factors primarily, but let us say I do not want to do 8 experiments. Is there a possibility that I can reduce the number of experiments, but still get most of the information? That is the whole idea of orthogonal array.

Here we are going to see in a pictorial representation how that can be done for L8. All these corner points which are represented by the red dots are nothing, but your experimental conditions. For instance, if you would say this point as 0 0 0, this is 0 1 0 and this guy would be 1 0 0. So, there are different combinations, right. All these would refer to the edges of the corners, the corner points and not the edges; the corner points of the cuboid.

Now, I want to reduce these 4, sorry 8 red dots into 4 red dots. It should also be logical. So, I cannot just knock off the 4 at the bottom or I cannot knock off the 4 at the front. If I do that, then I am losing one axis information. I am losing x axis information if I am taking off all these four. So, there should be a logic behind which we knock off these, right. Now, what I am going to do is the red dots that I am going to retain, I will denote them by putting a black colour circle around it, ok. So, let us say that I start with this guy and I am retaining this. So, since I retain this, anything that is directly connected with it, I am not going to retain. So, I am not going to retain this guy. I need to choose a colour, so that we know that we will not need it. I will take a green colour. So, this will go off and this also will go off.

However, this guy will be retained because he is not directly connected to that point. So, interestingly if you see if you take only this plane, the plane of x 1 x 2, this is x 2, this is x 1. Actually there are 4 points to represent the corners. What I am trying to do is, I am only taking the two diagonally opposite points. We could have started with this point and then, we would have ended up retaining this point. So, right now I just started with this point, hence I am retaining this point. The other two which are connected to this directly are knocked on.

Similarly, on x 3 I will have to knock this out because this is directly connected to it. So, I am going to knock this guy off. Now, we will have to go to the next plane which is represented by this plane. It is kind of the back end plane. So, this is knocked out already. What I want to do next is, I have 3 points; 1 2 and 3, but as we know this guy is connected to this point, I would not have this. So, the only two other remaining guys are naturally these two guys, ok. So, if you see 4 points that we have chosen, they are not directly connected to each other, ok. There is at least one more point that is intermediate and we are able to get these points.

Pictorially this is the idea behind which the reduction works. Basically we are trying to go from L8 to L4. What does it mean? 8 experiments to 4 experiments I have to run half times. L8 to go to L4, 8 number of experiments need to be reduced to 4 number of experiments which obviously means you need to knock down few experiments. The ones that we are marked with the green crosses will indicate those experiments that you want to get rid of meaning indicate the experimental setups that you want to get rid of.

For instance, let us take this case that x 1 has 2 levels, it is trying to, so this is x 1, right. So, this guy is x 1. He has levels 1 and 2 here and this is x 2 that has levels 1 and 2 here and this perpendicular to the plane of the monitor is our x 3 and that also has 1 and 2 here. What we are trying to see is this in L8. All combinations would have been possible, but right now that might not be the case. This will be reflected when you look at L4 array compared to L8 array. So, this is the basic idea by which the rows are knocked down.

Accordingly as a result of it, the columns also will reduce because if you take 2 level cases, L8 will have how many columns? It will have 7 columns. Similarly L4 will have 3 columns only. So, the number of factors that can be accounted also goes down. When I say factors, it is not only the main factors, but it also means the interaction factors. It could be 1 1, 2 factor interactions or it could be higher order interactions as well.