

Design For Quality, Manufacturing And Assembly
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Lecture - 11
Need For DoE And Basic DoE Methods

So, here is an example to motivate the language for the experiments.

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An example


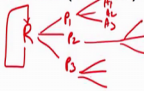
- o Manufacturing process – air bubbles are identified. So rejection. Hoping to determine experimentally some way to fix the problem engineers identified 2 types of resins and 3 amts of prepolymers (100, 200, 300 gms)
 - How would performance be measured
 - QC of the result
 - What are the first 2 factors to be identified and what are their levels

Result can be measured by counting number of bubbles (no units) or size of the bubbles (mm)

QC – smaller the better

F1- resin: two levels → types 1 and 2

F2- prepolymer: 3 levels → 100, 200, 300 gms



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There is a manufacturing process in which air bubbles are identified, I am trying to build a product using some manufacturing process. In the course of the process there are some air bubbles that are created. As a result the product is rejected. Hoping to determine in an experimental sense how to fix this problem, some engineers were recruited and they identified the reasons for the failure. The reason for the failure is a bubble, but they identified the causes. What were the causes? The 2 types of resins and 3 types of prepolymers that were used, the 3 amounts of prepolymers, the it is 100, 200 and 300 grams.

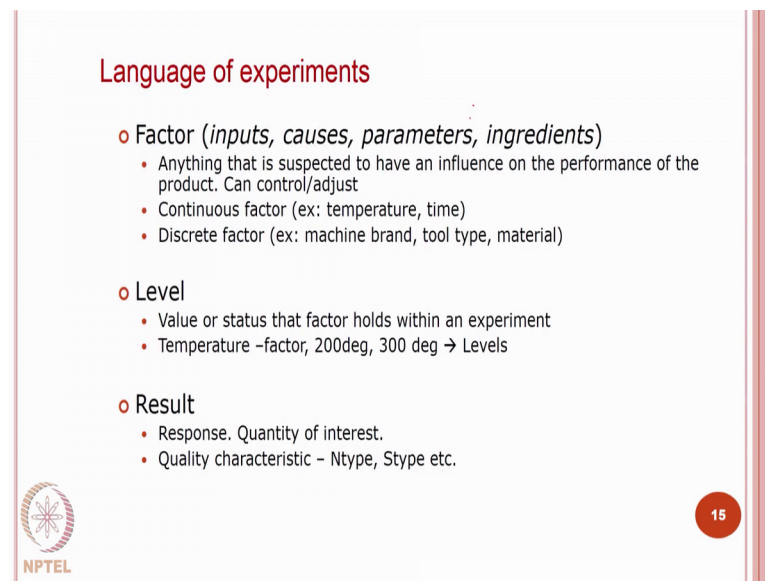
Now, here are a few questions, how would you measure the performance? I think we are losing the thread here ok. This is in continuation to the previous slide where we talked about factors, levels and response. Here is an example I am manufacturing a product, but there are air bubbles inside the product as a result of which the product is being rejected. I want to minimize the rejection, in order to do that I want to understand the reasons

behind the air bubble. I understand the reasons that contribute to the air bubbles are your resins and the amounts of prepolymer. Now, my question is: what are my response? What is my response that is need to be measured? What is my objective in the problem?

Student: (Refer Time: 02:32).

To remove or to reduce the air bubble, to either remove or reduce my air bubble.

(Refer Slide Time: 02:47)



The slide is titled "Language of experiments" in red text. It contains three main bullet points, each with a sub-bullet. The first point is "Factor (inputs, causes, parameters, ingredients)" with sub-bullets: "Anything that is suspected to have an influence on the performance of the product. Can control/adjust", "Continuous factor (ex: temperature, time)", and "Discrete factor (ex: machine brand, tool type, material)". The second point is "Level" with sub-bullets: "Value or status that factor holds within an experiment" and "Temperature -factor, 200deg, 300 deg → Levels". The third point is "Result" with sub-bullets: "Response. Quantity of interest." and "Quality characteristic - Ntype, Stype etc.". In the bottom left corner is the NPTEL logo, and in the bottom right corner is a red circle with the number 15.

Language of experiments

- Factor (*inputs, causes, parameters, ingredients*)
 - Anything that is suspected to have an influence on the performance of the product. Can control/adjust
 - Continuous factor (ex: temperature, time)
 - Discrete factor (ex: machine brand, tool type, material)
- Level
 - Value or status that factor holds within an experiment
 - Temperature -factor, 200deg, 300 deg → Levels
- Result
 - Response. Quantity of interest.
 - Quality characteristic - Ntype, Stype etc.

NPTEL 15

So, that objective is what your response is, response is the quantity of interest. If you see here response is what the quantity of interest is. So, what is your quantity of interest? For the first question, what is your quantity of interest?

Student: (Refer Time: 03:03).

Is it the amount of air bubble?

Student: The presence of an air bubble.

So, if is 1 air bubble better than 10 air bubbles.

Student: No.

Why?

Student: (Refer Time: 03:18).

So, it is not only the number, but it could be the size. There could be 10 tiny air bubbles and there could be 1 big air bubble. So, it is it does not mean that 10 is bad than 1, sorry 1 is better than 10. So, we can use a word density. So, that is my quantity of interest. So, the performance is measured by measuring the number of air bubbles. What is the quality characteristic?

Student: (Refer Time: 03:54).

Smaller the better, do not say l type lower the better, l stands for larger s type smaller the better. What are the factors to be identify and what are their levels?

Student: The 2 (Refer Time: 04:12) 3 and (Refer Time: 04:14).

What is your factor 1?

Student: (Refer Time: 04:16) resin 1.

Is the factor, first factor. What are the levels?

Student: Resin 1, resin 2.

There are 2 resins resin a resin b resin 1 resin 2 whatever. What is the second factor?

Student: Prepolymers.

What are its levels?

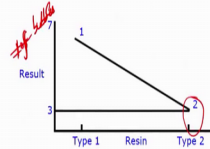
Student: 100, 200, 300.

We are not going to solve this problem, this is only to get you introduced to the language of the experiments. The result can be measured by counting the number of bubbles, no units or size of the bubbles together. Quality characteristic is smaller the better, factor 1 is the resin: 2 levels types 1 and 2. F 2 is a prepolymer: 3 levels that is 100, 200 and 300 grams.

(Refer Slide Time: 05:10)

Investigating one factor at a time

- Quick, cost effective, no complex formulas
- For 1 factor, levels are changed while rest all factors are frozen.
- At least 2 experiments at 2 different levels (not at each level) is necessary
- Previous example – to study the effect of resin:
 - Sample 1: resin type 1, result = 7
 - Sample 2: resin type 2, result = 3



So, you can actually take a look at this problem itself. You want to understand how this resin and the prepolymer interact, that is what is going to give you the quantity of interest which is the air bubble. So, how can you go about doing it?

Student: (Refer Time: 05:46).

So, what is doing is what he is saying is I will fix the resin, resin 1 and I will change the prepolymers 100; I will check out how many number of bubbles 200, how many number of bubbles? 300, how many number of bubbles? How many experiments you have done?

Student: 3.

3 experiments you have done, then what you will do you will change.

Student: The resin.

The resin from resin 1 to resin 2 and you will do 3 different experiments, 6 experiments you will do and you will know which resin and what polymer have this combination correct; this works nice for two-dimensions. Now, let us bring the third dimension into this, some kind of adhesive material I am I am just saying. So resin, prepolymer, adhesive you fix resin, prepolymer 3 times and let us say the adhesive also can have 3 levels, you understand what I am saying.

So, you will have you will have a resin, you will have 3 prepolymer combinations. Then, for each of this prepolymer I will have an adhesive like this I will have for other 3, I will have for this guy correct.

Student: Yes sir.

And then I will also repeat this R 1 to 2 is that all, but you can look at the number of experiments what has happened.

Student: (Refer Time: 07:58).

Initially in this case it is only 6, but now it is.

Student: 8.

1 2 3 4 5 6 7 8 9.

Student: (Refer Time: 08:06).

9 times 2 is 18. Let us imagine that this was a product design combination, you have to build 18 proof of concepts to make a decision let sorry.

Student: We can do a finite number of (Refer Time: 08:25) with different combinations and rundle them (Refer Time: 08:27).

That is the point that is a point ok, but sometimes you might not be able to do that; how can you do in a statistical sense is what the questions. I just want you to appreciate and these numbers will go in an exponential fashion we will see. So, what he just pointed out and we just discussed right now, is investigating one factor at a time. We fixed one factor and then we are seeing the effects ok. It is quick, there are no complex formulas you just need to conduct the experiments. For one factor the levels are changed while the other factors are frozen, correct.

You see here in this particular example I fixed R, I fixed P 1 to understand what is the effect of A and then I figured out A 2 is a good combination. My experiment cannot stop there because, I could get a better combination with P 2 and R 1, R with P 3 and R 1 and some other combinations of A. R this entire thing with respect to R 2 also could give me

a better result. So, that is where the problem is for one factor the levels are changed while, rest of the factors are frozen.

You have to do at least 2 experiment at 2 different levels, correct. If you have 2 levels you have to do 2 experiments: level 1, level 2 you have to do that. You are not conducting please understand there is a difference, you are not conducting 2 experiments at each level; you are conducting 1 experiment at each level and there will be 2 levels at least. So, you need to do 2 experiments, I hope you get it right R 1 and R 2. Let us say that I fix something, I am coming in the opposite direction A 1 P 1 I fix, I want to know the effect of R, then I have to do at least 2 experiments R 1 and R 2. For the problem that we are talking about the x axis here is the resin type. So, resin type 1 type 2 and this is the result. What is the result in our case?

Student: The number of bubbles (Refer Time: 11:10).

Yes, it is the number of bubbles or the density of bubbles. So, it is 1 ok, this is basically for the level 1 and the level 2 that we are talking about ok. Meaning for type 1, I have got 7 bubbles and for type 2 I have got 3 bubbles. So, which one is better?

Student: (Refer Time: 11:42).

There is no 3 ok 3, 3 bubbles which means this type 2 is a better option. Sample 1 the resin type 1 the result is 7 air bubbles and then for the resin type 2 the result is 3. What if you need to know the result for another type of resin, I changed my mind someone came and told me actually there is one more resin in the market. But, I want to know what will be the corresponding number of bubbles for that resin. Do I need to repeat an experiment or can I is there a easier way for me to understand what will be the number of bubbles?

Student: Sir, resins characteristics are somewhat similar to the both of these.

Student: We can (Refer Time: 12:40).

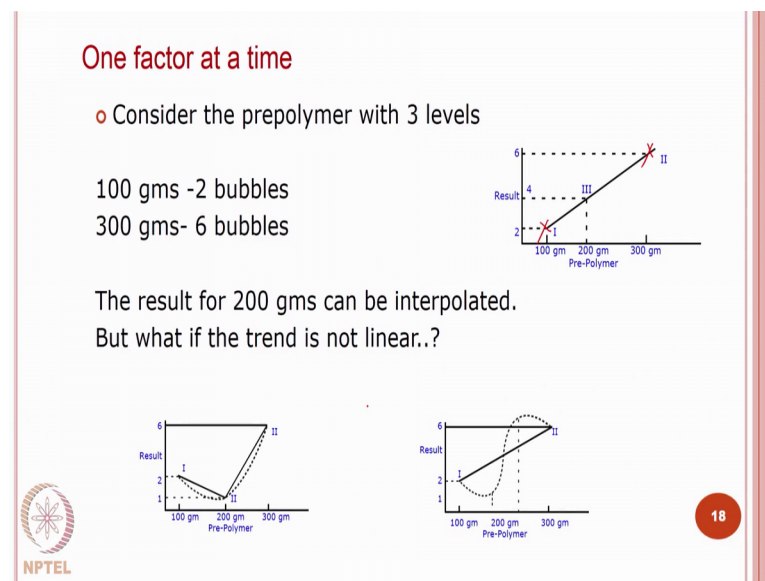
So, if the resin characteristics are somewhat closer to the existing resins; basically I need to know the, I need to know something about the resins in terms of their composition, their ingredients, their performance ok. Then I can make that kind of a conclusion, if I am able to correlate the resins themselves then I can do an interpolation here, otherwise I cannot that is all to answer.

Student: (Refer Time: 13:12).

The primary answer that he gave is correct you will have to repeat the experiments unless you have some correlation information between the lessons. Since, discrete because this is considered as discrete right. However, within the discrete if I know the properties then it is continuous. You can come up with a arbitrary property, you might not have a material you can call it as a fictitious material.

But if you consider it as discrete, let us say that you are an end user you do not know any characteristic you are just a user of the resin; then it is the resin a b and c or resin 1 2 and 3, you do not know anything about it. It is a discrete stuff so, you need to conduct a additional experiment, you cannot interpolate that is with the discrete factor. The next one that we are discussing is the prepolymer with 3 levels.

(Refer Slide Time: 14:17)



The prepolymer was 100 grams, 200 grams, 100, 200 and 300 grams. For 100 grams the result was 2 air bubbles and for 300 grams the result was 6 air bubbles. And, for 200 grams the result was 4 air bubbles. Now, I have a question should I do or should I have done all the 3 experiments or I could have gotten away with 1 or 2 experiments.

Student: Since, we do not know what properties of prepolymer drag with the (Refer Time: 14:48) resin. So, we cannot actually identify if it is going to be what kind of

relationship it is going to be between the resin and the prepolymer and what is going to cause bubble. So, we need to be we need to conduct the experiment.

Yeah, what he is saying this since we do not know the relationship between the resin and the prepolymer, we might not be able to reduce the number of experiments. But let us say that you know the relationship, what can you do about it or how will you go about it.

Student: Then, inter interpolating it is the better option.

You will basically try to interpolate, but from an interpolation perspective you need to know whether it is a linear, it is non-linear that is what he said. We might not know the relationship between your prepolymer and the resin, if we know that we at least have a rough idea that it is linear, it is non-linear then we can say. Let us say that it was linear right now, I have constructed after the 3 tests we have kind of show that it is a linear relationship. But, let us say for whatever reason you know a priori that it was a linear relationship then you could have gotten away with just these 2 numbers. For the third-one I know you can interpolate from these two ways. So, for 100 grams it was 2 bubbles and for 300 grams it was 6 bubbles and you can so, you can interpolate for 200 grams it will be 4 bubbles.

The result for the 200 grams can be interpolated, but what if the trend is non-linear. If the trend is not linear or you were to believe that the trend is not linear then you need at least 3 points because, at least quadratic right n minus 1 quadratic means second; so, you need at least 3 points. If you believe that it is highly non-linear then you would expect at least you should have at least 4 points, that is what these 2 graphs are talking about ok. So, that is where the designer comes into picture. The designer should have an idea on whether it is a linear function or it is likely to be quadratic or non-linear highly non-linear. What do we know until now, with respect to design of experiments. We have not gone into design of experiments just the experiments ok.

(Refer Slide Time: 17:16)

What do we know until now

- Experiments on at least 2 levels are necessary to learn about a factor's behavior
- For cont factors, at least 3 levels when nlin behavior is suspected
- Extreme nlin \rightarrow 4 levels is desirable
- No idea on the physics, time constraint \rightarrow 2 level experiments
- Nlin applies only when continuous factors are considered



Experiments on at least 2 levels are necessary that is what we discussed right, level 1 level 2 at 2 levels are necessary to learn about a factor's behavior. For continuous factors at least 3 levels, when non-linear behavior is suspected. If it is highly non-linear you need about 4 levels or 4 points. If you have no idea about the physics and time is a constraint then you can go ahead with this 2 level experiments, you just need to know the bounds. And, non-linearity applies only when you have continuous variables, with discrete variables anyway you need to go and calculate; does not matter how it varies, you are going to go and calculate at each of the levels. But there are slightly smarter ways of doing this stuff, instead of fixing one factor at a time you can also try to do several factors at a time. So, we will see how we go about that.

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Several factors at a time

- One factor at a time – minimum two experiments
- When multiple factors are present, it is possible to run $N+1$ experiments and get the same information of $2 \times N$ experiments effect

Factor	Level →	
	1	2
A: time (minutes)	4 (a_1)	8 (a_2)
B: temperature (F)	176 (b_1)	225 (b_2)
C: pressure (lb/in ²)	1500 (c_1)	1750 (c_2)

Technically, I need to run two experiments for each factor ($2 \times 3=6$ experiments)



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Right now, the way that we have looked at it is you need to run at least N times 2 that is what we discussed here right number of experiments to understand the effect. But, there is a slightly smarter way you do not need to do $2N$ times, but you can get away with it with N plus 1 times. If you see what I am talking about is, I am talking about this one, I am talking about this one. You at least require 2 levels to understand R 1 R 2 P 1 P 2 A 1 A 2 at least 2 experiments you should have done.

So, it is 2 times N , but you do not need to do that you can still get away with N plus 1 that is what we are saying here. Of course, there are some so, there is a factor the factor is you have to read it this way; this is your factor. This is another example the factors are A: time B: temperature C: pressure, the respective units are here. Each factor has 2 levels: level 1, level 2 and here are the values and I am using this notation; a 1 means level 1 of factor A, a 2 means level 2 of factor A and I am giving you the values 4 8 whatever it is. Now, how many experiments do I need to run? 2 experiments for each factor, how many factors I have?

Student: 3 factors.

3 factors. So, I will have to do 6 experiments do you agree, you have time 2 temperatures and then 2 pressures so, 2. Then 4 for time it will be another 2 so, it is 6 experiments you will have to do.

(Refer Slide Time: 20:40)

A smarter way to experiment

Experiment	Factor			Result
	A	B	C	
1	a_1	b_1	c_1	45 (R_1)
2	a_2	b_1	c_1	60 (R_2)
3	a_1	b_2	c_1	20 (R_3)
4	a_1	b_1	c_2	35 (R_4)

- Only 4 experiments ($N+1$) in the place of 6 experiments ($N \times 2$)
- Can get the same result – that is effect of a is: subtract row 2 from row 1
- Though all factors are considered, its still one factor at a time. The way you conduct the experiment is different



So, the deal is we said that there might be a smarter way to do it right, you do not need to do 2 times N, you do not need to do 6 experiments you might still be able to do it with about 4 experiments. So, here is the way; this is your experiment number of experiments 1 2 and 3 and 4 and these are your factors A B and C. This is your result for each combination, for each experiment what is my result and I call my result as response. So, response 1 2 and 3 4, 4 experiments, 4 results.

Now, look at the way that the experiments are conducted level 1 of a, level 1 of b, level 1 of c. Please note fixing b 2 b 1 c 1 and changing only a 2 right, I am fixing b 1 and b sorry b and c 2 b 1 and c 1 and I am changing only a. So, I kind of get the effect of a; look at the third experiment I revert back to a 1 while c is still fixed at c 1 and I change b from b 1 to b 2. So, now, I can get the effect of b. So, is the case the far with the respect to the final row, where I do, I fix a and b to 1 and then I am getting I am changing c to c 2.

So, in the place of 6 experiments I am doing only 4 experiments, but still I will be able to get the effect of a comma b comma c. How do I get the effect of a? For instance, if I am interested in the effect of a I have to take these two guys; 45 minus 60 or 60 minus 45 that will be the effect. If I am worried about b then I have to do this guy and this guy, that difference 45 minus 20 will give me the effect of b. Similarly, for c I have to do 35 minus 45 that will give me the effect of c. Imran is it clear?

Student: Ok.

Remaining all factors is the same. I am changing a particular factor so, that I will give the effect of a factor. There is some limitation though in this, what is that?

Student: We are only considering the two levels of the (Refer Time: 23:15). The effect is linear.

No no one at a time, what is it you are considering only.

Student: We are only considering 2 levels, what are the considering 3 levels in each of the experiment that could not be able to.

And that is ok, that will come back that that is ok. In this particular course, we will look predominantly at 2 levels. It does not mean that you cannot do 3 levels, you can do 3 levels, but our discussions will be restricted to 2 levels. But, even in 2 levels there is a limitation in this, what is that?

Student: One factor.

One factor.

Student: At a time.

That is fine as long as I get the information how does it matter one factor at a time.

Student: The linearity you can find.

What is it?

Student: Sir, the factors are non-linear.

What is the problem?

Student: We like in the graph that we show.

Student: If the curve is linear we can 2 minimum experiment.

That is required only if you want to interpolate, we are not trying to do any interpolation we are only trying to understand the effect of some factors.

Student: (Refer Time: 24:32).

I have still not got all the information in this.

Student: We do not know if the factors have an implicit relationship with the output or it is (Refer Time: 24:40) with implicit relationship.


That is not entirely true; if you have this data input and output you can construct that relationship.

(Refer Slide Time: 25:02)

A smarter way to experiment

Experiment	Factor →			Result
	A	B	C	
1	a_1	b_1	c_1	45 (R_1)
2	a_2	b_1	c_1	60 (R_2)
3	a_1	b_2	c_1	20 (R_3)
4	a_1	b_1	c_2	35 (R_4)

- Only 4 experiments ($N+1$) in the place of 6 experiments ($N \times 2$)
- Can get the same result – that is effect of a is: subtract row 2 from row 1
- Though all factors are considered, its still one factor at a time. The way you conduct the experiment is different

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What I want you to observe a changed from a 1 to a 2, while you fixed it at b 1 c 1 b 1 c 1 b changed from b 1 to b 2, while it was fixed at a 1 c 1. Can something else happen? For the effect purpose yes, you need to change b 1 to b 2 c 1 to c 2 a 1 to a 2. What about the fixing?

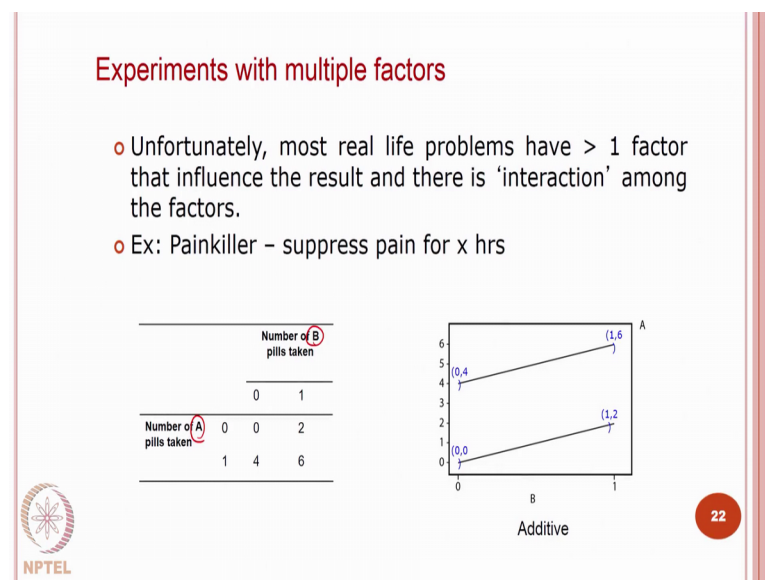
Student: We can fix at the experiment 7.

You are fixing; I could fix, I could have started with the a 2 b 2 c 2. This experiment also will give me the effect of a, but this effect of a need not be the same effect of a because, these responses are might are likely to change you understand. So, I am not factoring what happens if you go to the second level of the frozen variables, it might be entirely different that is one limitation. The second is this could also be an interaction I could have a b 1 c 2 b 2 c 2 or b 2 c 1, whatever that is also possible why should I always freeze them correct.

Student: Yes.

Do you get the point? You can study that provided you have more experiments ok. So, this is certainly better than the previous one, but it still does not give you all the information. So, though all factors are considered it is still one factor at a time. You are considering all the 3 factors together, but you are changing only one factor at a time while, you are fixing the other factors that is important. What if I fix the other 2 levels, your results might change which I cannot predict. There what you mention might play a role if I know the explicit relationship, but that is the whole point if you know the explicit relationship you do not need to do the DoE, you can directly solve the problem.

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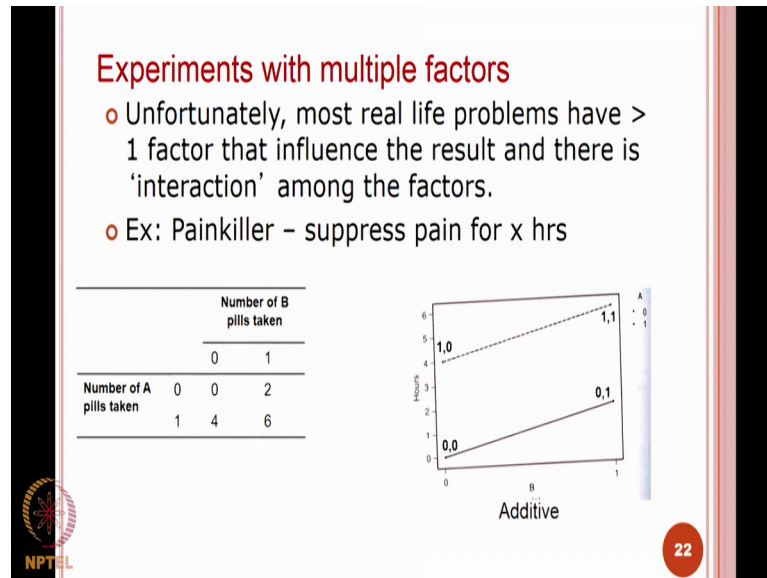


So, some little bit more examples with multiple factors. Most real life problems have greater than 1 factor that influence the result and often there is an interaction that is what I told you. So, you will see rather than just hand waving as there is interaction there are some examples here. So, someone has a headache, in order to take care of the headache painkillers is given. The painkiller is supposed to suppress the headache for certain hours, that is what this problem is about. There are different types of painkillers: painkiller A, painkiller B. So, I can take either 0 painkiller of A which means I am not taking A or I can take 1 painkiller of A.

Similarly I can take no painkiller of B or 1 painkiller of B. If I am taking no painkiller the pain is suppressed for 0 hours, if I am taking painkiller B the pain is suppressed for 2

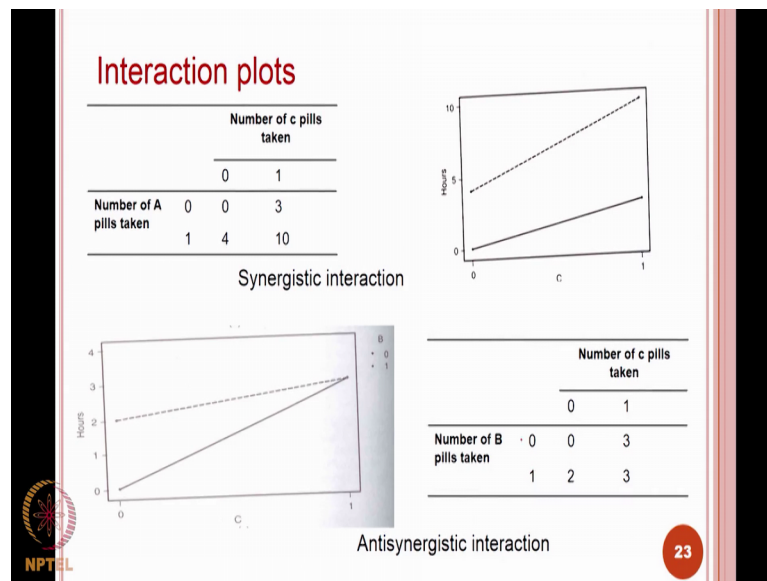
hours. If I am taking painkiller A alone, the pain is suppressed for 4 hours, if I am taking both of them this A and this B then the pain is suppressed for 6 hours, which is an addition of this and this; 4 plus 2 is 6. This is additive in nature, it is not necessarily always additive. So, this is 1 interaction, if I had them together then they might actually add up.

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So, 0 of B 0 of A 0 over suppressed 1 of B 0 of A suppressed similarly you can do. So, this is 1 comma 1 means they are both and the number of hours it is suppressed is 6, in this case A is 0 B is 1, suppression is 2 hours. So, in this case B is 1 A is 0 so, the suppression is 4 hours. So, the same data is presented in a graphical sense.

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Now, there can also be interaction plots, I mean there is a synergetic first is additive just 2 plus 4 was 6. In this case if you see 0 0 of course, 0 hours; if 1 pill of A was taken 4 hours suppression c this is another pill so, c suppression was 3 hours. If I took both of them together it was 10 hours, it is not just 4 plus 3 it is much more than that they are working really well. So that means it is something like a composite ok. There is some positive relationship between them and hence they are suppressing.

You can also have an anti-synergetic stuff, if you see here B, if I take B alone it is suppressing for 3 hour 2 hours and if I sup if I am taking c alone it is suppressing for 3 hours. But, if I take both of them it is still suppressing only for 3 hours which means the effect of c overtakes the effect of B. So, that is the anti-synergetic; meaning there is no use of having B, B is insensitive.