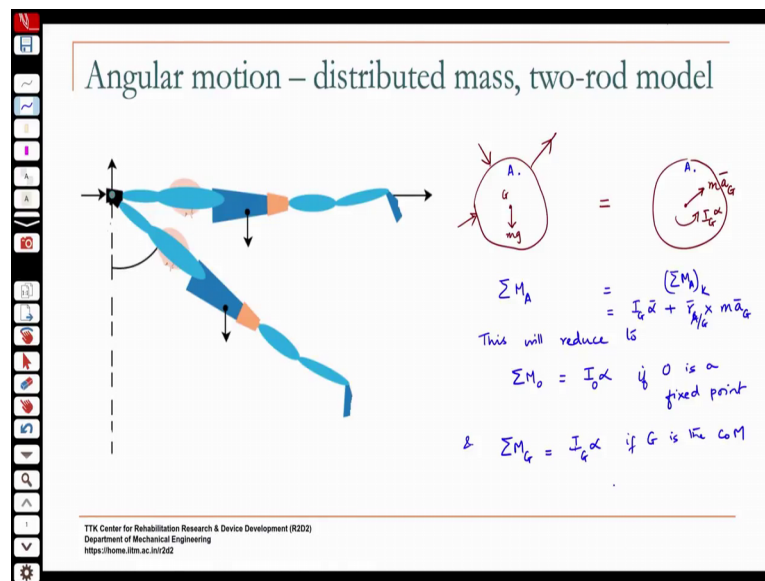


**Mechanics of Human Movement**  
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**Lecture – 32**  
**Kinetics: Angular Motion Part IV**

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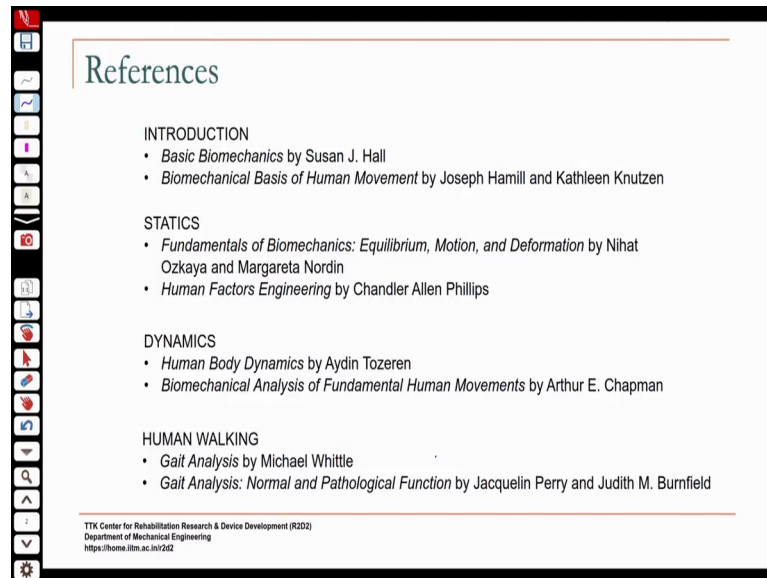
So, last class we looked at some cases of Angular Motion. And as I mentioned to you in solving dynamics problems, it is usually useful to have your kinetic diagram. So, if you have a rigid body subjected to its weight and a bunch of other forces, then you represent it by its equivalent kinetic diagram, which basically shows the acceleration acting at its center of mass and  $I_G \alpha$ .

And this is useful, because if you are computing moments about a general point A, then you can use the kinetic diagram to say that  $\sum M_A$  in this equals  $\sum M_A$  in the kinetic diagram. And this reduces to  $\sum M_o = I_o \alpha$  if  $o$  is a fixed point. And  $\sum M_G = I_G \alpha$  ok, if  $G$  is the center of mass ok, because then the contribution of  $m \vec{a}_G$  goes away.

So, here it will be this would be equal to  $I_G \alpha$  plus  $\vec{r}_{AG} \times m \vec{a}_G$ . So, this is the general form ok and you have to be make sure that you use that ok. Because, in general  $\sum M_A$  is not equal to  $I_A \alpha$  ok, it is that case only when it follows you know  $o$  is a fixed point or you are taking moments about the center of mass.

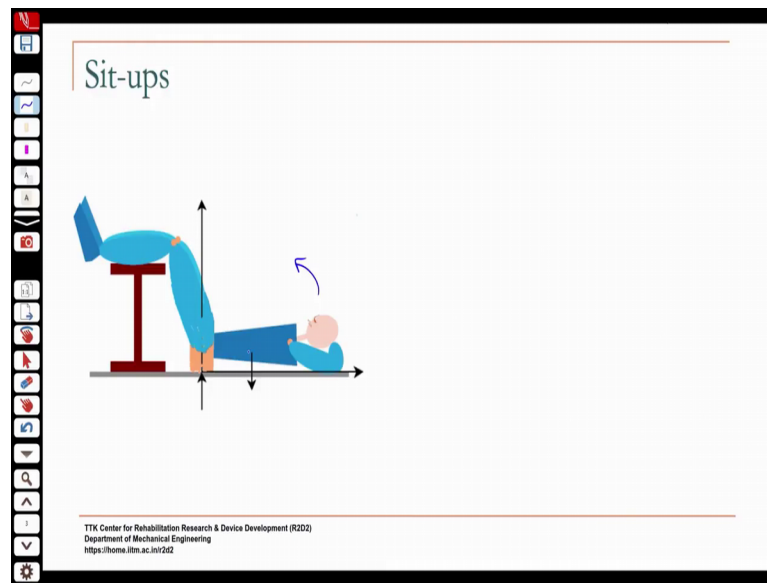
So, this is important to remember, when you are doing dynamics problems involving. So, we saw that in the gymnast case with the two-rod model.

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So, I just wanted to give you some basic you know I have been giving you taking problems from multiple sources in some case modifying them, but these are some of the useful references for this course. So, for the introduction portions, you have these two statics, dynamics and later on human walking, we will be using so, these are some of the references. So, I use this course is sort of brings together a lot of these components, and so there is really no one book that covers what we do in this course, so that is the reason for I am giving you this list.

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So, this is another case of a movement that could be analyzed, I would not go through this, but this is an exercise you could try. So, if you have a person doing a set up right, so, the persons going to move like that, you can compute what kind of moments have to be exerted by the abdominal muscles. We say this exercise is for strengthening the abdominals right abdominal and back muscles also, because the back muscles will be your antagonists.

So, if you look at this exercise, they usually stabilize the pelvis and then you try to if you do it properly, you are supposed to lift your upper body and do this setup, so you can find you can even do a static case. So, if you want to hold it in a particular posture at a particular angle, what would be the moment required to be applied by the abdominal muscles to achieve this. So, this is another example of a movement that you can do.

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### Push-ups

Person modeled as a slender uniform rod of length  $l$  & mass  $m$ . Arms modeled as two weightless rods.  
Find the reaction forces at the hands and feet

$$\begin{aligned} \hat{e}_r &= \cos \theta \hat{i} + \sin \theta \hat{j} \\ \hat{e}_\theta &= -\sin \theta \hat{i} + \cos \theta \hat{j} \end{aligned} \quad \left| \quad \begin{aligned} \dot{\hat{e}}_r &= \dot{\theta} \hat{e}_\theta \\ \dot{\hat{e}}_\theta &= -\dot{\theta} \hat{e}_r \end{aligned} \right.$$

$$\bar{r}_c = \frac{l}{2} \hat{e}_r$$

$$\bar{v}_c = \frac{l}{2} \dot{\theta} \hat{e}_\theta$$

$$\bar{a}_c = \frac{l}{2} \ddot{\theta} \hat{e}_\theta - \frac{l}{2} \dot{\theta}^2 \hat{e}_r$$

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We will do something that is a little bit more not complicated, but you know it involves couple other segments as well and that would be like a pushup. So, in the push up, you have your feet either being held by somebody or you know you try to keep your feet on the ground and then you are going to lower and raise your body by bending your arms. So, the pushups are a good workout for which muscles, it is your triceps we will see that where so, the triceps get a good workout in your when you do a pushup exercise.

And will now model this will model the body, so you have the person modeled as a slender uniform rod of length  $l$  and mass  $m$ . And let us say we model the arms, these are the arms modeled as two weightless rods so, we want to find the reaction forces at the hands and feet. So, let us say at the foot, you have say this is my X Y coordinate system. At the foot, I have  $V_f$  and  $H_f$  the vertical reaction force and the horizontal reaction force and then at the hand, I have  $V_h$  and  $H_h$ .

And I can make an so, these are like four unknowns here. So, a reasonable assumption to make would be to say that this is equal to 0; I can say that I am only supporting the vertical, I am only exerting a vertical force with the hands so, let us say that is an assumption we make. So, similar to the analysis, we have done earlier, I can choose a more convenient coordinate system  $e_r, e_\theta$ , say this is  $\theta$ .

And then I can write my equations so, first I can write  $e_r$  equal to  $\cos \theta \hat{i} + \sin \theta \hat{j}$ ,  $e_\theta$  equal to  $-\sin \theta \hat{i} + \cos \theta \hat{j}$ . And then  $e_r \dot{\phantom{x}}$ , we have done

this before is  $\dot{\theta} \hat{e}_\theta$  and  $\hat{e}_\theta \dot{\theta}$  is  $-\dot{\theta} \hat{e}_r$ . So, if this is the center of mass of the rod at  $l/2$ , so let us say this is located at  $l/2$ , let us say this is  $0.2l$ .

So, then I have  $r_c$  equal to  $l/2 \hat{e}_r$ , I can find the velocity of  $c$  as  $l/2 \dot{\theta} \hat{e}_\theta$ . And the acceleration of this will be if I just differentiate that  $l/2 \ddot{\theta} \hat{e}_\theta$  along  $\hat{e}_\theta$  minus  $l/2 \dot{\theta}^2 \hat{e}_r$ . By now, you should be comfortable enough that you can write this. Now, because my so, once I do this, I can then convert it back to the  $i, j$  coordinates, because that may be easier for me to do to find the horizontal and vertical reaction forces rather than now try so, now I know the relationship.

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$$\begin{aligned} \sum \vec{M}_O &= \vec{I}_c \ddot{\theta} \hat{k} + \vec{r}_c \times m \vec{a}_c = \vec{I}_c \ddot{\theta} \hat{k} + \frac{l}{2} \hat{e}_r \times m \left( \frac{l}{2} \ddot{\theta} \hat{e}_\theta - \frac{l}{2} \dot{\theta}^2 \hat{e}_r \right) = \vec{I}_c \ddot{\theta} \hat{k} + m \frac{l^2}{4} \ddot{\theta} \hat{k} \\ &= \vec{I}_O \ddot{\theta} \hat{k} \\ &= \left[ \frac{ml^2}{12} + m \left( \frac{l}{2} \right)^2 \right] \ddot{\theta} \hat{k} = \frac{ml^2}{3} \ddot{\theta} \hat{k} \\ -mg \left( \frac{l}{2} \cos \theta \right) + V_h (0.8l \cos \theta) &= \frac{ml^2}{3} \ddot{\theta} \end{aligned}$$

Let us first draw the kinetic diagram. I have this ok, I have the vertical force at the hand, I have the horizontal force at the foot and the vertical force at the foot. At  $c$ , I have  $mg$  acting. This is  $\theta$  and this is equal to this is equal to an acceleration and  $I_c \alpha$  that is equal to  $I_c \ddot{\theta}$  that is my kinetic diagram. The resultant of these forces causes the linear acceleration of the center of mass and the angular acceleration of the rigid body.

So, now I have if this is point  $O$ ,  $O$  is a fixed point here, it is a pivot. So, I can use  $\sum M_O = I_c \ddot{\theta}$  along  $\hat{k}$  plus  $r_c \times m a_c$  ok. Here I can just leave things, so  $I_c$  would be over a here I could also use  $I_O$  into it would be easier to do that. This is equal to because  $O$  is a fixed point, it is equal to  $I_O \ddot{\theta}$ .

double dot,  $r \times m a_c$  will only give me  $I \ddot{\theta}$ . So,  $I \ddot{\theta}$  here is it is a uniform rod so, I have  $\frac{1}{12} m l^2$  about the center of mass plus  $m \int_0^l \frac{1}{2} x^2 dx$  by the parallel axis theorem about o into  $\ddot{\theta}$ .

I can easily show this, this is equal to  $I_c \ddot{\theta}$  plus  $r \times m \int_0^l \frac{1}{2} x^2 dx$  into  $\ddot{\theta}$  minus  $\frac{1}{2} m l^2 \ddot{\theta}$  along  $e_r$ . So,  $e_r \times e_r$  that will go away, I will be left with  $I_c \ddot{\theta}$  plus  $\frac{1}{4} m l^2 \ddot{\theta}$ . Which is nothing but  $I_c$  is  $\frac{1}{12} m l^2$  plus  $\frac{1}{4} m l^2$  into  $\ddot{\theta}$ , which is nothing but  $I \ddot{\theta}$ , does not give you anything new. So, this is or I can do it in one step as this so, I get this is  $\frac{1}{3} m l^2 \ddot{\theta}$ .

Now, from this diagram from the left hand side, the moments the reason I am taking moments about point o is I can I will be left with only one unknown, I can eliminate  $H$  and  $V$ , so I get  $m g$  into.

Student: (Refer Time: 15:04).

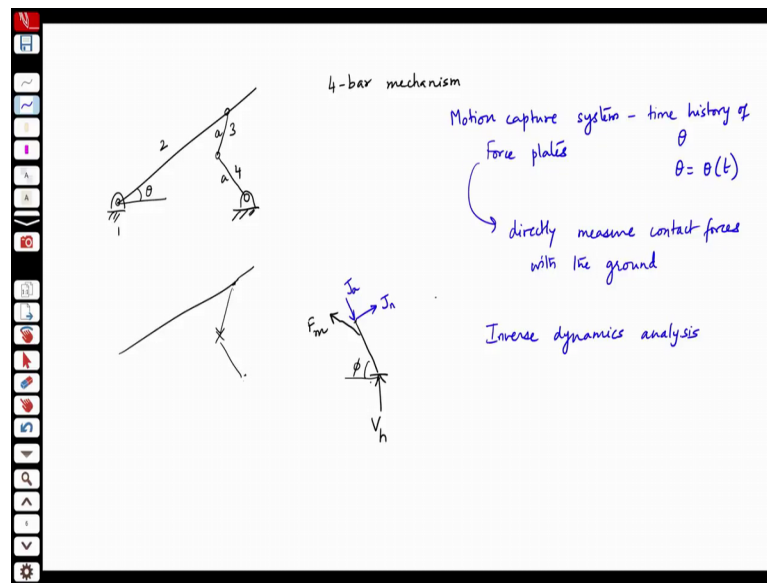
$\frac{1}{2} l \cos \theta$ , so this is minus along  $I$ , sorry.

Student: (Refer Time: 15:22).

It is a I can just I because both of that is a planar problem, I can remove the  $k$ . So,  $m g$  the clockwise moment is  $m g$  plus  $V h$  into this distance is  $0.8 l$ . Because, this in the previous diagram, I showed you is  $0.2 l$ , this is  $0.2 l$ . So, this is right under, you know you keep your hands right under your shoulder, when you do the push up. And you have  $V h$  into  $0.8 l \cos \theta$  is the momentum, so that is this is equal to  $\frac{1}{3} m l^2 \ddot{\theta}$ .

So, if I know the time variance of  $\theta$ , how is  $\theta$  varying  $\theta$ ,  $\dot{\theta}$ ,  $\ddot{\theta}$ , which can be determined experimentally, then I can solve for the reaction force at the hands. Now, how will that, now I am really interested in seeing what is going to happen at the triceps, what kind of a workout am I getting for the triceps muscle.

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So, if you look at this model of the person doing the pushup, then I can equate it to do you recognize this mechanism? It is essentially a 4-bar mechanism ok, my 4 links are the fixed link, 2, 3 and 4. So, it is like four links pivoted together so, what is that tell me, so this is basically a 1 degree of freedom system then. So, if I know theta, I know the lengths of these two arms, so I know basically the orientation of these two.

So, if I find theta for the body, then I basically know the orientation of the arms as well. So, then I can isolate the arm alone or even geometrically. This is you do not even need to do the analysis for a 4-bar mechanism in this case, this is just I said the arms are of equal length  $a$  and  $a$ . So, if I know this then from here I just take an arc equal to  $a$ , I know right below that another arc and that is my location of the arms. So, I know the angles of these as well so, I know the complete configuration, once I know theta.

Now, I can go back and look at I know once I compute  $V_f$  using the entire body, I can go back and just look at the arm alone. So, for instance, if I look at the forearm, I have  $V_f$  sorry  $V_h$  acting on that ok, this is my elbow and this would be the force in my triceps muscle, so at a particular so, even assuming it is quasi static.

So, even for static analysis, I could use the same thing. If I know the inclination of the whole body, then I can basically find this inclination call it  $\phi$  ok, then I know how  $F_m$  is acting from anthropometric data and I can compute the joint reaction forces at the

elbow ok. So, this is how you would go from the whole body analysis to an individual segment.

You can look at what is happening at a particular joint on a particular segment and what kind of so obviously, here again we are making the assumption that only one muscle is acting, but that is not going to be the case ok, it is going to be the net moment actually. The moment created by  $V_h$  is resisted by a net moment at the elbow joint ok, so that would be the sum of the moments of the triceps as well as the biceps or the elbow flexors.

So, this is how you would kind of carry through the analysis like we did for the previous case, where we computed the for the gymnasts, we computed the net moment at the abdominal in the in that region, the net arganest moment would be the moment created by the abdominal muscles there. Here you can find out what would be the force that would have be in the triceps muscle? You will find that the force can be pretty high, so it is about one-third the body weight or so. So, it gives you a real good workout for the triceps muscle the pushups can give a good workout for your because, the larger force, you make the muscle apply, then you are activating it you making it stronger ok, so that is the purpose of this kind of an analysis.

The other way, you could measure these forces in a lab, you can actually you can measure the kinematics and then compute the forces. In the lab, you also have some things called force plates. So, typically you have a motion capture system using multiple cameras to give you the time history of theta. So, you can find theta as a function of time, which means you can find theta, theta dot, theta double dot. And then you would have force plates, which directly measure the forces of contact ok, so that is another way contact forces with the ground.

And in biomechanical analysis, typically both are measured. Although you know that if I just know the kinematics, I should be able to get the dynamics ok. But, in a lot of biomechanical analysis and you will see that later, when we look at walking for instance. There are a lot of errors associated with motion capture systems.

Motion capture systems typically have some kind of a marker that is attached to the body. And then they track the motion of that marker, the cameras locate those markers at different instants of time. And what happens with that is one you know the skin the



markers move with the skin ok, sometimes they the cameras may not capture deposition of the marker.

So, there are a lot of errors that are associated with determining the kinematics and then you there will be noise in the data, so you have to apply some kind of filtering to get the time profile of that variable. So, because of that, the kinematics alone may not give you the right force picture. And therefore, in many lab settings, you measure both the ground reaction forces and the kinematics why do you need both, why do I why not just measure the forces?

Student: (Refer Time: 24:27).

If I want individual segment, if I want to start doing you know look at individual segments right and try to compute internal forces, then I need the motion of those segments ok, so that is why I need the kinematics as well. The ground reaction forces alone will not be enough; I mean they will give me the overall force on the body. But, if I want to compute at individual joints, you know in or estimate forces in individual muscles, well individual muscles is not going to be possible, but at least the moments that are created about those joints, then I need both the kinematics as well as the kinetics kinetic measurements.

So, this kind of analysis, where you use both this data is called inverse dynamic analysis, and we will talk about that shortly. Before that, I would like to do, so where you know the kinematics, you may either compute the ground reaction forces by using a model like this or you may actually measure the ground reaction forces directly using force plates. And then compute internal joint moments and joint reactions, so that kind of an analysis inverse dynamics analysis. And before we get to that, I would like to do one more example, one more interesting example of looking at arm swinging during walk.