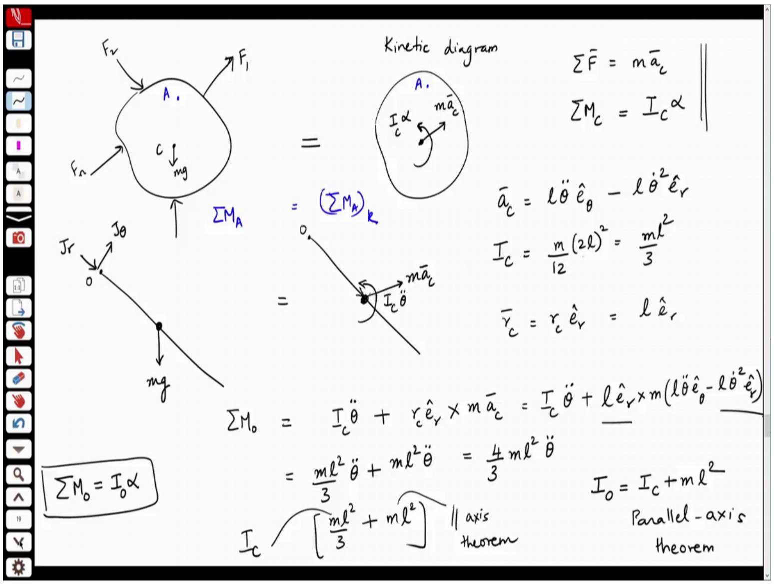
**Mechanics of Human Movement**
**Prof. Sujatha Srinivasan**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Madras**

**Lecture – 31**
**Kinetics: Angular Motion Part III**

In the last class we talked about using the Angular Momentum principle to look at the dynamics of rigid bodies in motion.

(Refer Slide Time: 00:29)

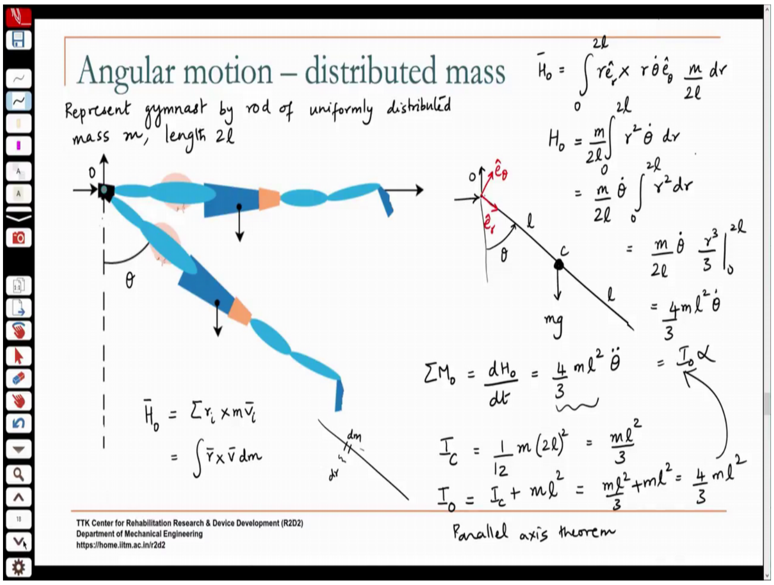



So, if you look at the other way instead of using the angular momentum principle basis behind using the kinetic diagram. So, this diagram essentially if you look at a rigid body that is subject to it is weight and a bunch of other forces acting on it. The kinetic diagram is essentially the resultant it expresses the resultant of these forces and moments.

So, if I have this body subjected to its weight and other forces say F1 F 2 dot F n up to F n. Then the resultant of those is equal to moment about the center of mass a couple A Ic equal to I c alpha and m times a the acceleration of the center of mass of the body. So, this is basically applying these Newton Euler equations to this.

So, this diagram is useful if you want to compute moments about points other than the center of mass. Ok because you will.

(Refer Slide Time: 01:53)




So, a take for example, in our case where we had the gymnast pivoting about the bar like that. So, if this was the free body diagram you had only you know we were looking at the effect of gravity on the motion of the gymnast ok. So, when you look at in this case for instance o is a convenient point to take moments about because then you can eliminate the two unknowns in the equation.

And. So, if I use say the kinetic diagram for this is this right. So, I have it is subjected to mg and the joint forces J r and J theta. And the kinetic diagram is just this rod subjected to I c alpha and mac ok. Now I know the acceleration of the center of mass I also know the moment of inertia about the center of mass. It is a uniformly distributed rod. So, I use m into the length square by 12 the length here is 12. Now if I want to compute moments about point o.

So, in this diagram I have moments about point o this is equal to on the right hand side I will have I c alpha which is a free couple plus this. So, if I denote the location of the center of mass is a r c; r c is say r c along e r ok. So, I have the moment of this equal to r c e r cross m a c. Here r c is nothing, but r c is l right I took the length as 2 l. So, it is just let the location of the center of mass. So, now, I can say this is equal to I c alpha plus all alpha is theta double dot. So, let me go back to that notation. Theta double dot plus l e r cross m a c is l theta double dot e theta minus l theta dot square e r.

So, the radial component obviously, will not contribute to the moment which you will see because the product of e r cross e r will go to 0 ok. So, same thing as saying the radial con contribution does not contribute to the moment about o. So, I get I c is m l square by 3 theta double dot plus I get m l square theta double dot. From this product the only thing that remains is m l square theta double dot.

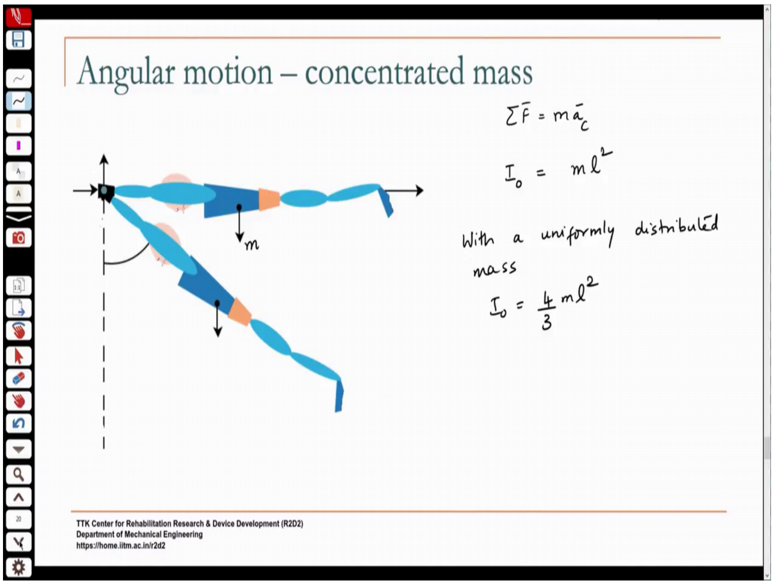
So, this is equal to 4 by 3 m l square theta double dot. Now let me look at this. So, if I look at this term right m l square by 3 plus m l square ok. Let me just look at this term m l square by 3 is what I computed to be I c ok. Now I have this mass m at located at l square by the parallel axis theorem right the moment of inertia about o equals moment of inertia about c plus m l square ok. So, that is what this is giving me.

So, I am getting from this I am getting sigma M naught equal to I naught alpha, but this is valid only when o is a fixed point in the body ok. So, that is why it is always better to use the kinetic diagram because if it was some point which was not fixed then this relation is not likely to hold ok.

So, in this case it holds, but in there are other instances that we will see later where this will not hold. So, it is always better to use the kinetic diagram or start from the angular momentum principle rather than using this formula directly ok.

So, that is something I wanted to alert you to. So, this is the from the parallel axis theorem. So, you can see that they are equivalent ok.

(Refer Slide Time: 07:18)




So, now we looked at the rod as a distributed mass if you simplify it to be a concentrated mass the center of mass will remain the same ok.

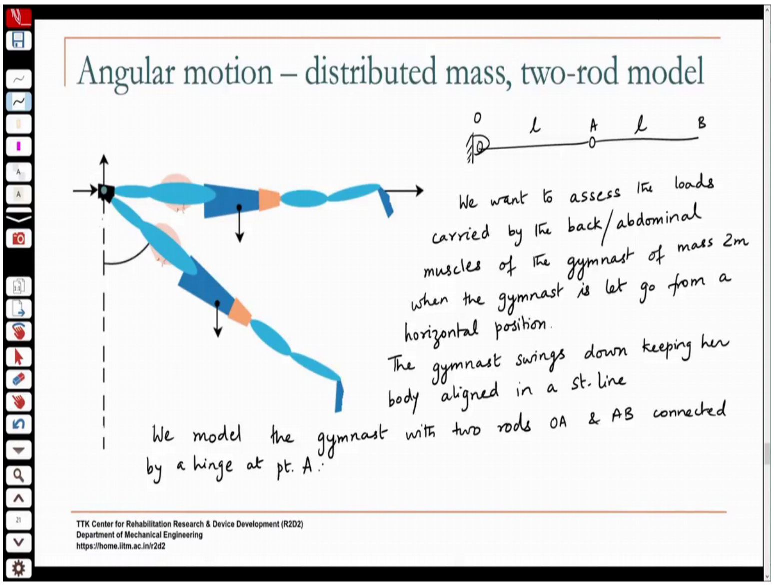
So, your sigma F equal to ma. The kinematics remain the same they are not influenced by the. So, the kinematics stay the same because the location of the center of mass remains the same. What will change is your moment equation because now your I naught in this case will be. So, this is m at a distance of l square.

So, your I naught will be m l square. In the previous case with a distributed with a uniformly distributed mass I naught was 4 by 3 m l square we just saw that ok. So, if you use the concentrated mass approach here actually in this case under estimating the moment of inertia actual moment of inertia is higher than using when using a model with a concentrated mass in this case ok. So, that is the tradeoff here.

So, you should be aware of you know when you use a particular model what may be the limitations of that model. So, otherwise the analysis of this is the same and I am not going to go through it, but just to show you that if you make this kind of an assumption because a point mass assumption is simpler than doing an integration right. So, of course, for a uniform you know rod we all already know the formula. So, it is does not really make a big difference.

But in other cases you would have this is the compromise you make the estimation of your moment of inertia would be different ok.

(Refer Slide Time: 09:36)




Now, we look at another interesting case ok. So, this was we assume that the athlete is applying whatever internal moments are necessary to keep the body straight ok. Suppose you want to figure out what exactly you know. So, an obvious thing would be you have an upper body and you have a lower body pivoted at the pelvis right.

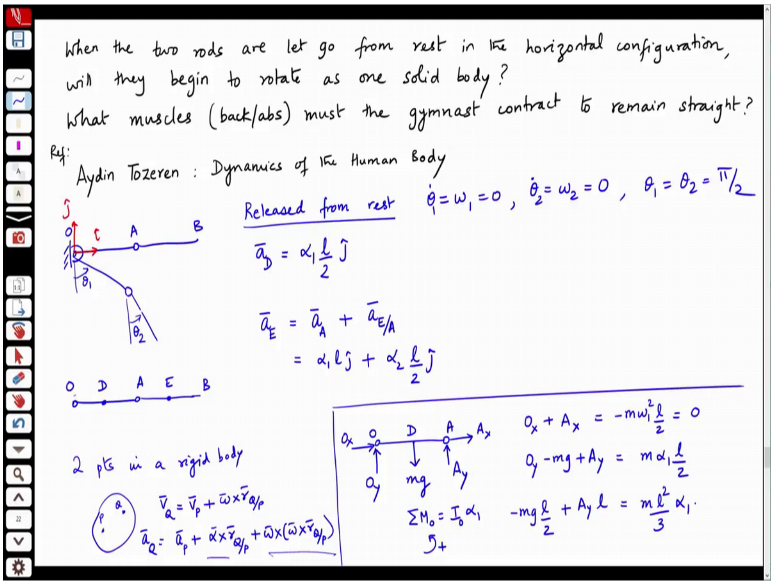
So, the obvious extension would be to look at if I want to compute what kind of a moment the back or abdominal muscles have to exert in order to keep the body straight how do I do that ok. So, we can start off with a simple two rod model. So, where I say that this is modeled as two rods with a hinge joint ok. And I want to first know if I just if it just acts suppose the person is let go from this horizontal position under the action of gravity what is going to happen to these two rods will they tend to rotate the with the same angular accelerations or you know what is going to happen ok.

So, we want to evaluate that, because that will then help me determine what kind of internal muscle moments I will have to apply in order to keep the body straight. So, let us look at this case. So, we want to assess the loads carried by the just for simplicity this time I will take the mass as 2 m because it is easier to analyze I have two rods muscles of mass 2 m when let go from a horizontal position.

So, the gymnast swings down keeping her body aligned, but first we want to see because we want to evaluate the loads exerted by the back or abdominal muscles we model it as a as two rods connected by a hinge we just want to see what is going to happen if there if you could not apply a moment in that region how is this system going to move two rods let us say O A and B O A and A B connected to. So, let us say each has a length l and a mass m by a hinge at point A.

So, we want to first answer the question.

(Refer Slide Time: 14:29)




When the two rods are let go from rest in the horizontal configuration, will they begin to rotate as one solid body. Intuitively you can say no ok, but if they did rotate as one solid body then how would you know that they rotate as one solid body their angular accelerations would be the same ok. If we get different angular accelerations for the two rods that means, they are not rotating together as one solid body.

And then if you want to make it rotate together as a one solid body give it a single angular acceleration we have to apply moments ok. So, what muscles? So, these are the questions what muscles probably a combination of the back and abdominal muscles must the gymnast contract to remain straight. So, I will tell you some of these problems are adapted from this I will give you a list of references that I use for this many of these problems are adapted from the dynamics problems are adapted from Aydin Tozeren this is the author. I think it is “Dynamics of the Human Body” I will give you the correct I will give you a list of references or if you look at your syllabus actually you have these books listed in the.

In this book for this problem they have it where the gymnast has their feet in the rings and then moves down with their arms stretched forward ok. So, let us compute the angular accelerations of O A and O B. So, it is released from rest. If this is theta 1 this is theta 2, but I am going to for simplicity we now know how things are going to move we you know we know what the accelerations etcetera are. So, I will not go through the whole formulation with R e R and e theta and all that.

So, I will directly write it for this horizontal case ok. Since we know that we are we are looking at this particular configuration. At as soon as they are let go how is it going to move. So, at that instant we are interested. So, I will just go ahead and formulate it accordingly. So, since the rods are released from rest theta 1 dot equal to omega 1 equal to 0 and I will designate theta 2 dot as just for simplicity as 0 both are released from rest.

So, omega 1 equal to omega 2 equal to 0. Now let me call the center of mass. So, if I look at this rod O A I will call this center of mass as D ok. So, I have the acceleration of D I can write it directly what would be the acceleration of d? In terms of alpha it is just alpha 1 into this distance is l by 2 along J in the horizontal configuration. So, we are analyzing it released when it is released from rest at theta 1 equal to theta 2 equal to pi by 2 at that particular configuration. My alpha 1 and alpha 2 are what I am interested in finding ok.

Now, I have when you look at interconnected bodies, I can if I take E as the center of mass of the second rod. Rod A B that is another acceleration I am interested in. I want to find the acceleration of E from. So, for a system for in a rigid body if I have 2 points in a rigid body say P and Q. If I know the velocity of P I can write the velocity of Q as the velocity of P plus plus omega cross r of Q relative to P right.

Similarly, I can write the acceleration of Q as the acceleration of P plus alpha cross this plus omega cross omega cross the relative velocity vector. So, in planar motion this all simplifies right this will be alpha times r Q P in the tangential direction this will be in the radial direction.

So, essentially it is because if you look at 2 points in the rigid body. How can Q move relative to P see if when I say this is a rigid body it means the distance between points P and Q does not change as the body moves.

So, there can be no velocity component of Q in the line joining P and Q. The only way Q can move relative to P is along the circle centered at P. So, that is where this comes this is your the more the circular motion that is happening about the point P. And similarly the acceleration is also due to that you have the centripetal component and then you have the tangential component ok. So, that is the basis for this equation because Q can only move along the circular path because the distance between P and Q do not change because of the assumption we have made that this is a rigid body.

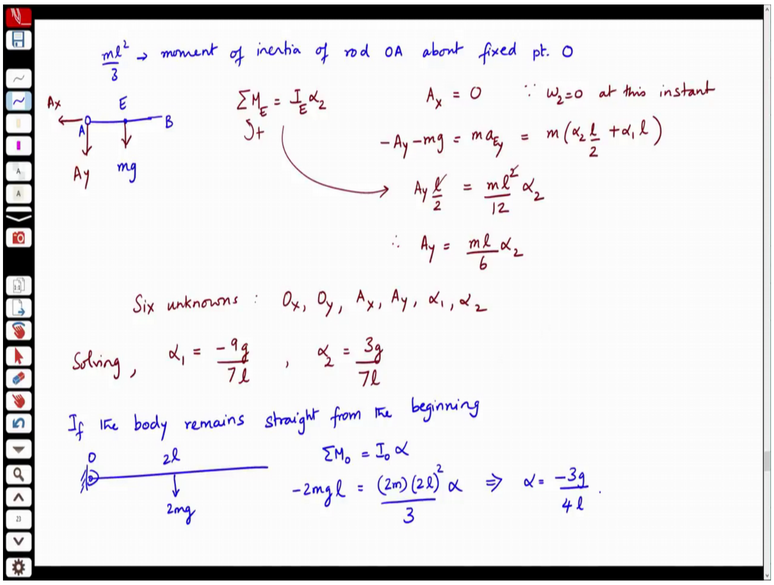
Why is the assumption of a rigid body valid in most cases because if the deformations are small relative to the motions that the body undergoes then the assumption that this is a rigid body is a valid assumption ok. So, in this case now if I look at this point E. Point E lies on A B; A also lies on O A ok. So, I can write the velocity of sorry acceleration of E as the acceleration of A plus the relative acceleration of E with respect to A. And this is equal to acceleration of A is nothing, but alpha 1 into l along j because it is moving about this fixed pivot o. And then acceleration of E relative to a because omega I am assuming is 0 at that particular instant.

So, there is no radial component. Again this component is also the it is alpha 2 into l by 2 j. So, these are the 2 accelerations of the centers of mass of the two lengths ok. Now let me look at the free body diagram. So, I have O A. And I have point D. So, I have mg acting at point D. If I draw the free body diagram I have the reactions we call it o x oy at the pin joint it cannot support a moment it can only prevent horizontal and vertical displacements and then A is also a pin joint ok.

So, I can say the reactions at A are A X and ay. So, this becomes my free body diagram for O A and. So, I can apply sigma f x equal to max sigma f y equal to may and because O is a fixed point I can also apply sigma M naught equals I naught into alpha 1 ok. So, I have O X plus A X please make sure you are also working it out with me. So, that if I make a mistake you can fix that. This is equal to minus m omega 1 square into l by 2, but omega 1 is 0 right it is the radial component O X there is no radial component here because I have assumed it is starting from rest. that is equal to 0 then I have O Y minus mg plus Ay equal to m into alpha 1 l by 2 and sigma m naught equal to i naught alpha 1 which gives me if I take moments about o because then I eliminate these two A X will also not have a moment about O.

So, my only unknown will be A y ok. So, then I can I get minus mg into l by 2 I am assuming counterclockwise positive. So, minus m g into l by 2 plus A y into l equal to what would be the moment of inertia of this rod about O it will be m l square by 3 into alpha 1 ok.

(Refer Slide Time: 27:28)




So, m l square by 3 is the moment of inertia of rod O A about fixed point O. So, if I had the rod mass lumped at the center my moment of inertia would just be m into l by 2 whole square m l square by 4 which again is an underestimation it is actually ml square by 3.

So, now I draw the free body of A B and. Let us say it is center of mass is at E. So, I have mg now my Ax and Ay will be in the direction opposite of what I took earlier right. So, I have ax. So, I have mg I have Ax. Now in this direction and Ay in this direction and B is a free end. So, there are no reactions there. In this case I know A y now from the previous equation right. So, if I go back here I can now I know A y in terms of alpha 1 I do not know alpha 1 it. So, I could express both in terms of that. I can apply about the center of mass that always applies the moment equation.

So, I could say sigma M e equal to I e alpha 2. This is perfectly valid and I can then a from my sigma f x sigma f y I get Ax equal to 0 because there is no radial acceleration because omega 2 is 0 at this instant. Then I can write minus A y minus mg sorry yeah minus mg equal to ma e along the y direction which is again the acceleration of E is m into alpha 2 l by 2 plus alpha 1 l. I found that in the previous alpha 1 l plus alpha 2 l by 2 that is the acceleration of A E.

If I apply this for the third equation I get A y into l by 2 mg no moment due to mg no moment due to A x is equal to Now it is about the center of mass. So, the moment of inertia is ml square by 12 times alpha 2. So, I can say therefore, A y equal to I can cancel this out this out I get m l by 6 alpha 2 this is A y. So, essentially in these equations I have. So, for each free body I I have three equations no A y causes a moment about.

Student: (Refer Time: 31:31) mg

Mg about E in a counterclockwise manner which is positive ok. I am taking counter clockwise positive. So, I basically my unknowns are.

Student: 6.

I have 6 unknowns that I can solve for each rigid body has three equations the six unknowns I solve for are O x O y.

Student: A x.

A x.

Student: A y.

A y.

Student: Alpha 1.

Alpha 1 and alpha 2. So, if you solve for these you get please do that I get alpha 1 equal to minus 9 g by 7 l alpha 2 equal to 3 g by 7 l an easy way to solve for alpha 2 would be to take the moment about to that do it anyway you can solve it is these are not very hard to solve they are fairly simple algebraic equations, but this is what you get what this tells you is that.

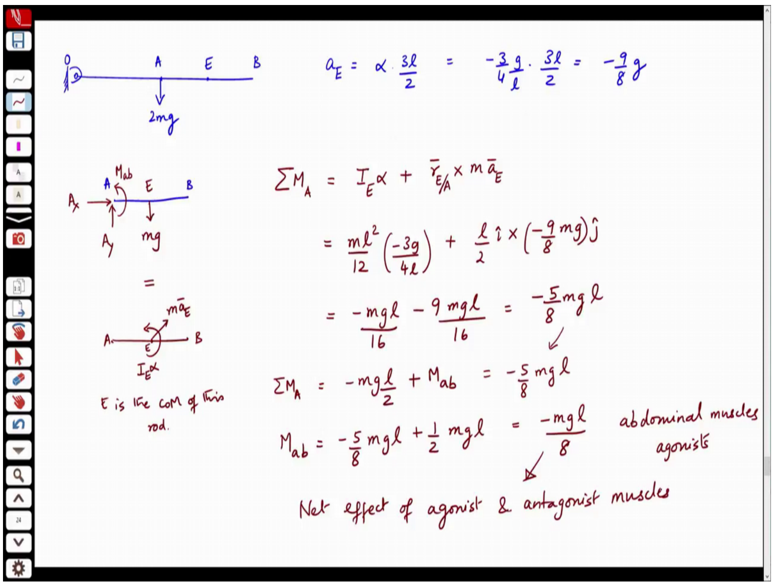
Student: (Refer Time: 32:42).

So, they try to rotate in a opposite directions with unequal accelerations. So, that means, you cannot just if you stay completely relaxed your upper body and your lower body will tend to move in different directions. If you are let go from rest in that position you are not going to be able to stay straight because if you want to stay straight it means you are making the two angular accelerations equal you are rotating as one single body and that is not going to be possible without some muscular effort.

Let us see if the body remains straight from the beginning what will be the angular acceleration ok. So, if the body remains then you have a single rod of length 2 l mass 2 mg ok. And I can just compute this as if I take moments about o sigma m naught equal to i naught alpha. If alpha is the angular acceleration with which it rotates. So, then I get minus 2 mg l this is o equals what is my i naught mass m l square by 3 right length is 2 l by 3 alpha therefore, alpha is minus 3 g by 4 l in this case.

Now, let me look at I want to find out what is the moment that has to be applied in order to achieve this.

(Refer Slide Time: 35:10)




So, let us say this is O this is A this is E. I have 2 mg acting here. What would be the acceleration of the point E what would be the acceleration of point E it is just rotating about this fixed alpha into 3 l by 2 because that is the distance to point E alpha into 3 l by

2. Now what is a alpha minus 3 by 4 g by l into 3 l by 2 which gives me minus 9 by 8 g that is the acceleration of E.

This is in the negative y direction ok. Now let me draw the free body diagram of A B because this is where I want to see what is happening. I have mg acting at the point E. Let me say I have an A x A y , but now I am also applying some moment at let me assume it is the abdominal muscles applying that moment.

So, instead of actually looking at insertions and all that let me just compute the net moment that has to be applied at that joint in order to because muscles essentially apply moments ok. And muscles you may not have a single muscle acting in the previous studies we looked at we made the assumption that one muscle is a acting and then we computed the muscle force, but essentially the muscle is applying a moment about the joint ok.

So, you will see that in a lot of biomechanical analysis because a joint is not acted on by just one muscle it is usually it is always acted on by pairs of muscles you have the agonist and the antagonist acting at any joint. So, you will talk about net joint moments. So, the agonist applies a moment the antagonist applies a moments moment and the net moment will probably be in the direction of the agonist momentum ok. So, that is how we look at. So, here we will just look at the analysis we will just compute a moment that is being applied at that joint.

So, now if you look at this here is a case where the kinetic diagram becomes important ok. So, let us say if I want to compute the moment about A this will be. So, the kinetic diagram for this would be I have at the center of mass I have sorry m a E I am just assuming some arbitrary direction. And I have I E alpha moment of inertia about E because E is the center of mass for this rod. This is A this is B E is the COM of this rod.

So, I have sigma M a equal to I E alpha from the plus r of E sorry r of E relative to A cross m a E the moment of this force about a. Now the problem we directly applying so, in the case of a being a fixed point the acceleration of E is straightforward. It gives you r alpha right and. So, M r square you get that term which is the same as the moment of inertia of the by the parallel axis theorem moving.

In the case of this the acceleration of E comes from the movement from O right it is the movement from O that is causing the acceleration of E it is not going to be A E times alpha you have already seen that it is not l by 2 into alpha it is 3 l by 2 into alpha here ok. So, that is where it is important that you use the kinetic diagram to write this equation rather than just writing it as sigma M a equal to I A alpha that would just be wrong.

So, if you do this you get ml square by 12 about e alpha, alpha is minus 3 g by 4 l plus l by 2 along i in this case cross minus 9 by 8 mg along j that is my m into ae. So, this is equal to minus mg l by 16 minus 9 mg l by 60 that is equal to minus 5 by 8 mg l. So, although this example is from that book in the book there is an error in the computation of the dynamics. So, your values for alpha and these will not match in that just to warn you.

But the problem is from there. So, this is minus 5 by 8 mg l. And then this is equal to from the from this free body diagram sigma M a is equal to minus mg l by 2 plus m a b ok. So, using these 2 and this is equal to from here minus 5 by 8 mg l. So, I get M a b equal to minus 5 by 8 mg l plus half mg l I get it as minus mg l by 8 so that means, the net moment is a clockwise moment. So, which muscles are dominating, which muscles are applying the net moment.

The abdominals or the back muscles the net muscle is clockwise.

Student: (Refer Time: 43:30).

You have the body like this.

Student: Abdominal.

Your abdominal muscles are working. So, you have to contract your abdominal muscles to apply this net moment because your back muscles will also be contracting. So, overall you have to apply a net moment of mg l by 8. In order to be in order to keep your body straight while moving down. So, this is the net effect. So, that means, the abdominal muscles are that is what this tells you ok. So, this is actually the net effect of the agonist and antagonist muscles.

In this case the antagonists are the back muscles and there will be other see it is very difficult to isolate and say this muscle is because you have other you know you have ligaments you have. So, much soft tissue around in the body the best you can do is kind of compute these net moments. You need more sophisticated models which models say the individual ligaments and muscles and all that, but those are very hard to solve you have to use some kind of optimization techniques to solve those models they called musculoskeletal models. It tells you how the different muscles behave and all that.

But they are computationally very intensive ok, but this kind of a simple analysis can give you insights into what is happening here ok. So, this is how you do a more complex problem in rotational motion. So, if you are interested in the interface moments then you have to ensure that that is modeled as a joint. So, that then you can compute the using that free body you can compute the moments at that interface at the joint all right we will continue in the next class.