Mechanics of Human Movement
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Lecture - 29
Kinetics: Angular Motion Part I

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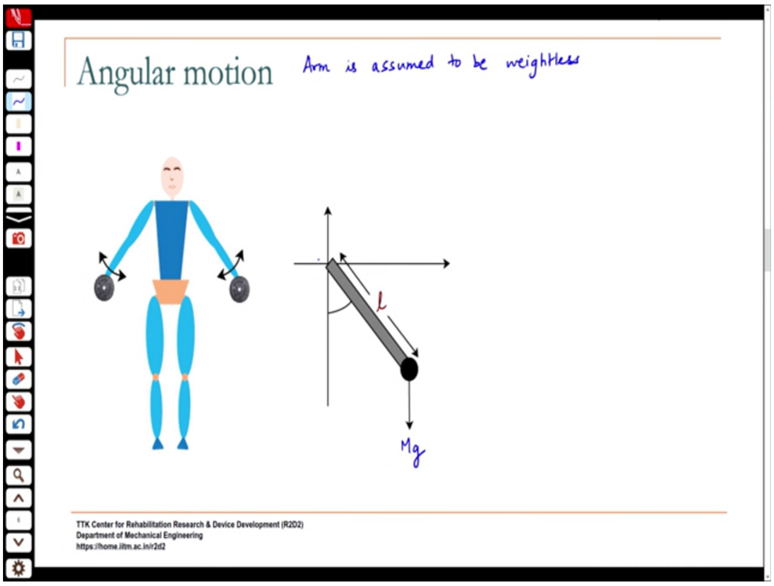

Ok.

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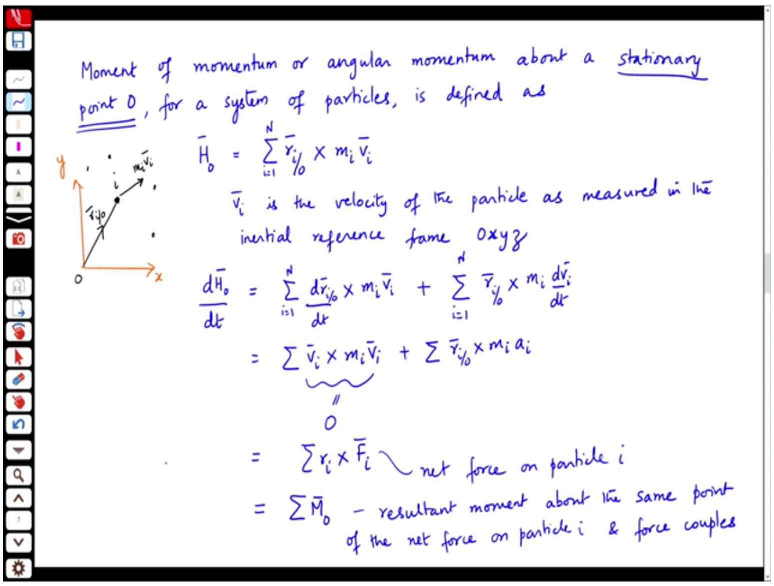

So, before we do that let me just talk about the angular momentum or moment of momentum about a stationary point. Moment of momentum or angular momentum about a stationary point O, for a system of particles is defined as H naught equal to sigma cross m i V i

Where so, if I have y axis and I have a particle I may have a bunch of particles for this i-th particle. So, this is a fixed in a it is an inertial reference frame xy is an inertial reference frame, r is the position vector of the i-th particle in this reference frame And this particle i has some velocity V i. So, it is momentum is m i V i then where V i is the velocity measured in the inertial reference frame o xyz.

Now, if I take the derivative of this angular momentum, d H naught by dt equal to product rule. So, this is over all the particles i equal to 1 to N ok. So, I can say first sigma d ri by dt cross m i V i plus now.

So, I differentiate the first term and the second one r i cross m i does not change with time and I get d V i by dt is ai d V i by dt. So, this is just sigma now this is nothing but.

Student: V i.

V i. So, I have V i cross m i V i plus sigma of ri cross m i ai. Now, this cross product this is 0, V i parallel to m i V i. So, this cross product is 0 and this is essentially the resultant of the net force on particle i; m i a i the acceleration of particle i m i mi times the acceleration of particle i a is the result of the net force on ri cross fi.

So, this is F i is the net force on particle i and this therefore, I can write as ri cross F i is a sigma of that is nothing, but the sum of the moments about point O. So, this is the resultant moment about the same point about which I am computing the angular momentum of the net force on particle i. And any force couples because if I have two forces that cancel each other out, but which create a couple I would have to add those as well ok.

So, this is the principle of angular momentum about a fixed point it says d H naught. So, it is important that you keep this in mind this is about the fixed point. Sigma M naught equals d H naught by d t for the angular momentum about a fixed point. Now, let us look at the angular momentum principle about a point that is other than a fixed point some arbitrary point.

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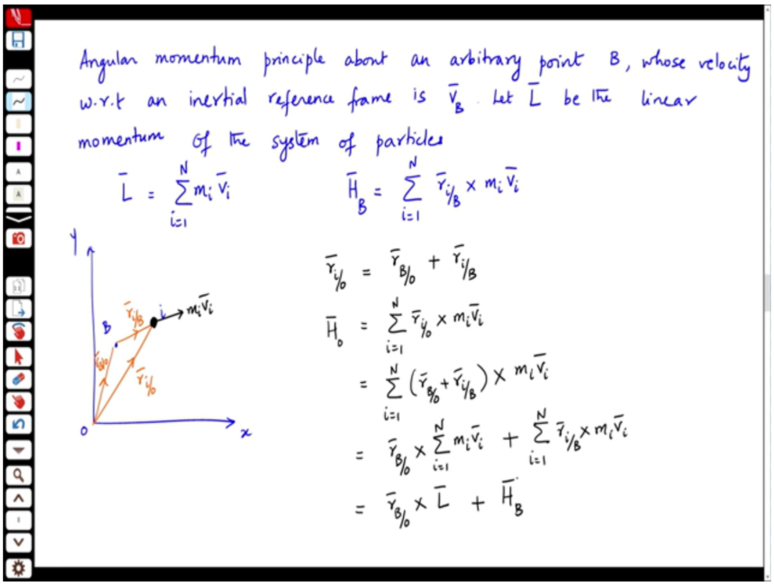

So, about an arbitrary point B whose velocity with respect to an inertial reference frame is v B. And let us say let L denote the linear momentum of the system of particles. So, L equals sigma m i V i and HB then the angular momentum about point B.

I define it as i equal to 1 to N r of i relative to B cross m i V i it is still m i V i here. It is not the, I am looking at the moment of the momentum of the particles but about a point B. So, only this vector is the relative vector ok. So, if I look at

Student: V i is the absolute velocity about (Refer Time: 10:08).

V i is the velocity in the inertial reference frame. Yes it is not about you know relative to B need to be clear about that. So, if I have i and I have a point B ok. So, this is O, this is r of i relative to O. This is r of B relative to O and this vector here is r of i relative to B.

So, I am trying to find the angular momentum about point B and this has momentum m i V i. The point B the i-th point yes.

Student: When i goes to 1 to N is B. (Refer Time: 11:23) Once it apart from the correction of particles.

B is not a particle; B is an arbitrary point in the plane. B is not a particle ok. B is not. So, the N particles do not. So, B could coincide with some other particle, but it is just a point in the plane.

So, I can write r of i relative to O as r of B relative to O plus r of i relative to B ok. You can see from the vector addition here that this is a valid. So, I know that I know H naught H naught is defined as sigma i equal to 1 to N, r of i relative to O cos m i sorry cross m i V i.

So, in this I can substitute I can say this is for r i O, I can write it as r i B plus r B O cross m i V i. So, what is the first term? R B o is a constant right. So, it is you can come outside the summation cross sigma i equal to 1 to N m i V i plus sigma i equal to 1 to N r i B cross m i V i.

So, this is equal to the angular moment sorry the linear momentum r B O cross the linear momentum of the body sigma mi V i plus.

Student: H B.

This is.

Student: H B.

H B angular momentum about point B ok.

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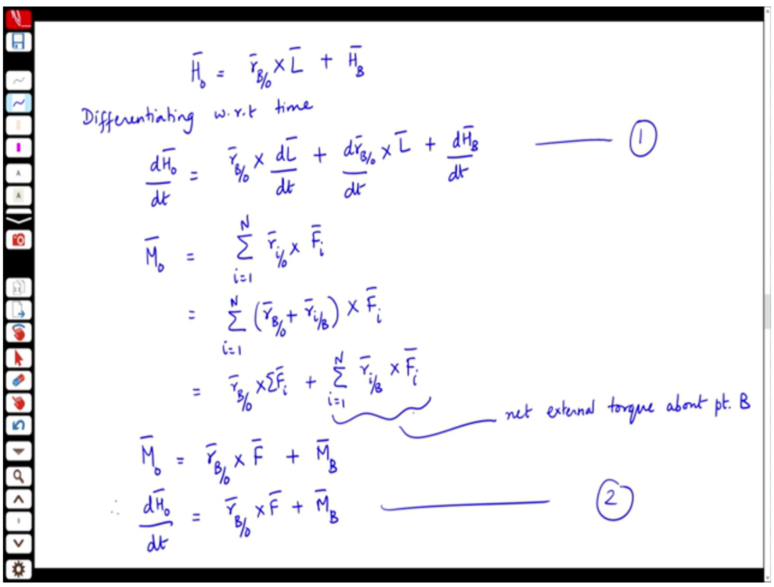

So, this is for. So, now let us differentiate this equation ok, r B relative to O cross L plus H B. If I differentiate this with respect to time I have d H naught by d t equal to r of this does not change right. So, I have r B naught cross d L by dt plus.

Student: D by dt H B.

D by.

Student: D t H B.

Sorry d r i; I am changing I am saying point B has a velocity it is not a fixed point this time. So, this cross L plus d H B by dt that was the whole point of this exercise. It is an arbitrary point which has a velocity in that inertial frame ok. So, this is one equation.

Then yeah if I look at M naught and do the same thing r i cross F i right. So, this is equal to sigma i equal to 1 to N ri naught is r B plus r of i relative to B cross F of i r B naught cross F i plus cross F i. What is this term? This is the net torque external torque about point B.

Student: Summation.

Sorry.

Student: Summation (Refer Time: 17:17) sigma F i.

Sigma F i. So, sigma F i; I used F as the net external force know resultant force. Yeah ok. So, sigma F i let us let me call it just F. So, M naught equals r B naught cross F sum of plus M B. I know that M naught equals d H naught by dt we already showed that. So, therefore, d H naught by dt equals r cross F plus M B. Now, I substitute this 2 into 1.

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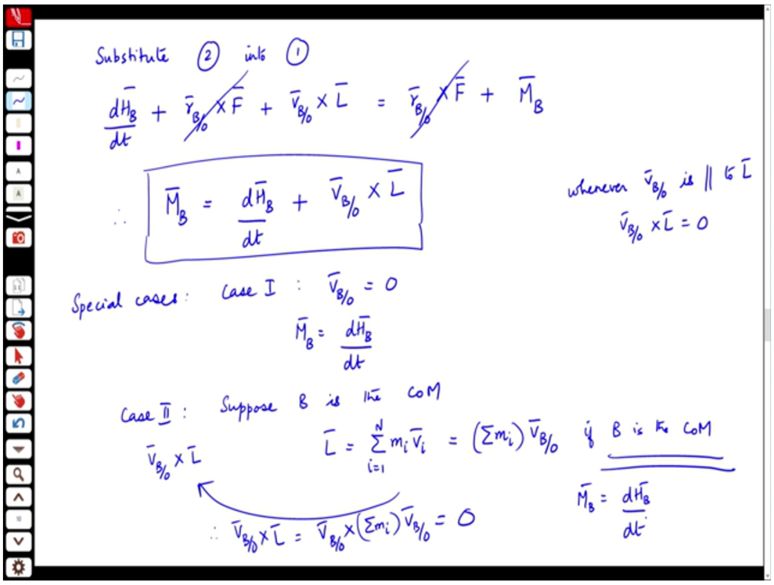

If I substitute equation 2 into equation 1, I get d H B by dt plus r B O cross F plus V B O cross L equals r B O cross F plus M B. So, this goes this goes.

Therefore, I have M B equals d H B by dt plus V B relative to O cross L. This is the angular momentum principle about a point that is.

Student: (Refer Time: 20:01).

A general point B ok, couple of special cases case I, velocity of B relative to O is 0.

Student: (Refer Time: 20:24).

It is a fixed point. Then it reduces to.

Student: (Refer Time: 20:29).

This is a fixed point this part becomes 0; it reduces to M B equal to d H B by dt which is what we showed earlier.

Case II: Suppose B is the centre of mass of the body. What happens to V B O cross? [Laughter]

What is L? L here equals sigma i equal to 1 to N m i V i this is equal to sigma m i times V B O. If B is the centre of mass therefore, this now becomes V B O cross sigma mi V B O again equal to 0.

If the velocity of this point B is parallel to this momentum, the linear momentum again it will be 0. But you are more likely to encounter these two cases ok. The mathematical relationship will hold whenever V B O whenever V BO is parallel to L ok. V B O cross L will be 0 ok. So, for all those cases M B will be equal to d H B by dt.

The special cases the cases that we are more likely to encounter involve the case where B is a fixed point, or B is the centre of mass. Then again you have M B equal to d HB by dt, but this is something that is important to remember.

Because in a lot of cases you tend to apply this equation M equal to d H by dt you will apply it to arbitrary points whose velocity may not be 0 or whose velocity who or points which may not be the centre of mass of the body. It is a common mistake that people make and that is the reason, I went through this derivation ok.

So, the thing that sticks from the undergrad angular momentum principle is M equal d H by dt ok. You do not really care about you know it works for a fixed point, you know it works for the centre of mass, but it does not work for an arbitrary point because there is this additional term ok. So, that is important to remember and because we will encounter this I wanted to do this derivation.

So, we will next class we will take up the case of this person doing this arm lifts with the dumbbell. The case of angular motion and we will proceed with that ok.