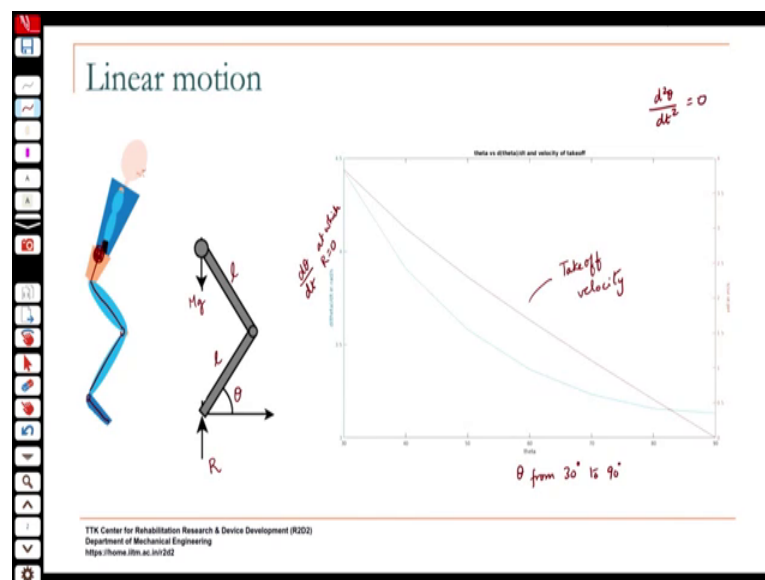


Mechanics of Human Movement
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Lecture – 28
Kinetics: Linear Motion Part III

So, last class we looked at the simple jumping model. And you know it is a question about the initial condition. So, I just went ahead and plotted for different initial conditions.

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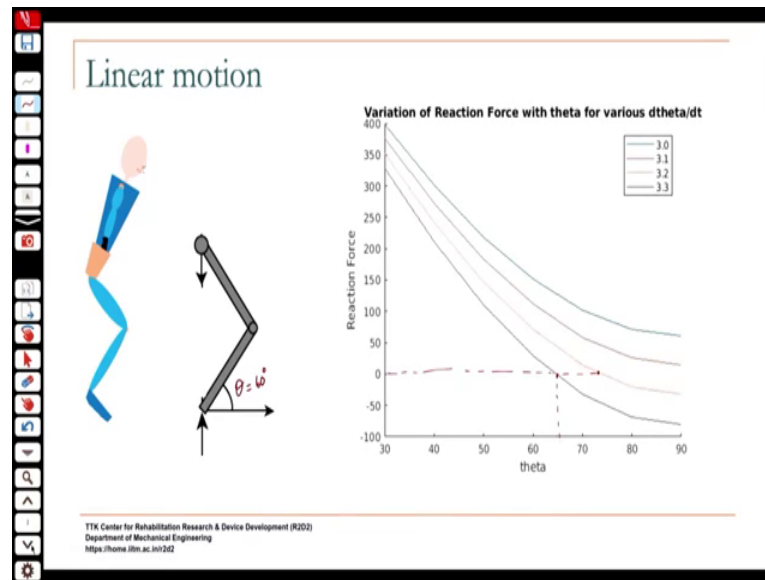


What at what $\frac{d\theta}{dt}$ the reaction force becomes 0? So, say θ I varied θ , θ from 30 degrees to 90 degrees in this simple model, and then I computed $\frac{d\theta}{dt}$ in radians per second at which R becomes 0 so, at take off ok. And then I computed the corresponding take off velocity, using the equations that we derived that ok.

So, this shows that if I have θ equal to 90 as saturate had pointed out, $\frac{d\theta}{dt}$ velocity of take off becomes 0. So, for with this simple model, this says that if your legs are straight you cannot jump ok. If this shank makes an angle of 90 degrees, but that is more a limitation of the model because in actuality you also have the foot segment ok, and the ankle joint. And a more accurate model would take into would be more like this. So, you would have this, ankle this, this and then you know perhaps this with this center of mass moving.

So, that would be a; this is a very simplistic model. What that tells you this is a very simplistic model. But it still gives you an idea of what is required in terms of jumping. I also looked at if you say this was also for the condition $d^2\theta/dt^2$, I took that to be 0.

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The second thing I looked at was ok, how does the reaction force change with theta for various $d\theta/dt$, ok?

So, if I am extending at different angular velocities, starting from a with the if my initial condition at the time of jumping is different, and I am extending with what is my likelihood of successfully taking off. Because I want the reaction force to be 0 for me to be able to take off. So, you can see here for instance at 60 degrees, if my initial you know if my inclination is 60 degrees, then I need to have actually with this I am not even able to take off for this particular condition.

So, say I take 70 degrees ok, or somewhere here is where my reaction force will become 0. So, if the angular velocity I am able to generate is about 3.3, then I can take off when it is certain in at that angle of 65 degrees to the horizontal, the shank angle. At a different $d\theta/dt$, you know it happens at a different initial condition ok. So, this is just I just plotted the various initial conditions to show you what the reaction force looks like.

So, if the as long as the reaction force is positive, you cannot take off right. So, you can use a simple model to basically do some analysis like this.

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Linear motion

Assume the arms are weightless rods

Initial conditions
 $\theta = \phi = 45^\circ, \frac{d\theta}{dt} = 0, \frac{d\phi}{dt} = 0$
 Assume $\frac{d^2\phi}{dt^2} = 2 \text{ rad/s}^2$ @ $t=0$

$2F - mg = ma$

Differentiating w.r.t time
 $0 = -2l \sin \theta \cdot \ddot{\theta} - 2l \sin \phi \cdot \ddot{\phi}$
 $\therefore \ddot{\theta} = -\frac{\sin \phi}{\sin \theta} \ddot{\phi}$ (1)

Differentiate this again
 $\sin \theta \cdot \ddot{\theta} + \sin \phi \cdot \ddot{\phi} = 0$
 $\cos \theta \cdot \ddot{\theta}^2 + \sin \theta \cdot \ddot{\theta} + \cos \phi \cdot \ddot{\phi}^2 + \sin \phi \cdot \ddot{\phi} = 0$

$y_b = (l \sin \phi + l \sin \theta + k)(-\hat{j})$
 $\bar{v} = \frac{dy_b}{dt} = (-l \cos \phi \cdot \dot{\phi} - l \cos \theta \cdot \dot{\theta}) \hat{j}$
 $\bar{a} = \frac{d^2y_b}{dt^2} = (l \sin \phi \cdot \ddot{\phi} - l \cos \phi \cdot \dot{\phi}^2 + l \sin \theta \cdot \ddot{\theta} - l \cos \theta \cdot \dot{\theta}^2) \hat{j}$

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We will move on to linear motion will come back to this one. So, this is essentially the case that we did initially, where I told you that the acceleration of the center of mass is measured to be some value and we just computed the reaction forces ok. But if you look at how the human you know we are essentially influencing the motion of the center of mass by the movement of our arms.

So, a more sophisticated model would look at ok, if this is angle theta and this is angle phi ok. And say the length of each arm you know the 4 arm and the upper arm is l. Say, the distance between the shoulders say is d. So, this say this distance is d and say this distance where I am holding the bar, if that distance is capital D ok. Then I can look at how I am varying these angles, the rate at which I am varying these angles to influence the motion as I am doing the pull up or the chain up ok. How and what would be the equation the constraint equation I would start with?

If I look at this distance will not change right. So, I can say $d \cos \theta + d \cos \phi + 2l \cos \theta + 2l \cos \phi = D$ that is a constraint equation that I have ok. Let us assume the arms are weightless rods ok, arms are weightless rods ok. Then I can basically say $D - d \cos \theta - d \cos \phi = 2l \cos \theta + 2l \cos \phi$. This is my constraint equation; I can differentiate this to get a relationship between $d \phi$ by dt and $d \theta$ by dt ok. So, let us do that.

So, differentiating and get now D and d are constants respect to time. 0 equals minus $2l \sin \theta$ into $\dot{\theta}$ minus $2l \sin \phi$ into $\dot{\phi}$. Therefore, $d\theta$ by dt $\dot{\theta}$ equals $\sin \phi$ minus of $\sin \phi$ by $\sin \theta$ into $\dot{\phi}$. If I differentiate this equation again, I will minus $\cos \theta$.

So, I move it to this so, I say $\cos \theta \dot{\theta}$ plus $\sin \phi$ sorry, $\sin \theta \dot{\theta}$ plus $\sin \phi \dot{\phi}$ equal to 0 . If I differentiate this, I get $\cos \theta$ into $\dot{\theta}^2$ ok, the first term plus $\sin \theta$ into $\ddot{\theta}$, plus $\cos \phi$ I get a $\dot{\phi}$ from the differentiation. So, I get $\dot{\phi}^2$ plus $\sin \phi$, $\ddot{\phi}$ equal to 0 .

So now if I designate the location of the center of mass ok, let us say from the shoulder to that is we are assuming the arms are weightless. So, we are assuming with respect to the body the center of mass does not change. So, let us say this is some constant k , you know, it is k below the level of the shoulder ok. So, then this is the x axis. Let me call this y of the trunk. So, I have so, we are assuming this entire thing, the lowered the trunk and the lower body move up as one mass. So, I can write y equal to $l \sin \phi$ plus $l \cos \theta$ plus k , correct?

Student: (Refer Time: 10:41).

Sorry I do not know why I am partial to \cos . So, this is y is along the minus j direction. So, my velocity the velocity of my center of mass is y is let me just say body instead of t , instead of just the trunk is dy/dt $\dot{\phi}$ minus error, sorry plus oh because of that, $l \sin \theta \dot{\theta}$ along the j direction ok.

Similarly, I can find the acceleration I differentiate this, I get d^2/dt^2 $l \sin \phi$, $\ddot{\phi}$ square minus $l \cos \phi \ddot{\phi}$, plus $l \sin \theta \ddot{\theta}$, minus $l \cos \theta$ into $\ddot{\theta}$ along j . And I also have the relationship between $\dot{\theta}$ $\dot{\phi}$, difference. So, this equation is just this. These are together 1 and the differential of that when you differentiate that you get this.

So, from this I get θ , it take some initial conditions at let us say θ equal to ϕ equal to 45 degrees starting from the left at this position equal to 0 . And let us say assume $d^2\phi/dt^2$ equal to 2 radians per second square at t equal to 0 . So, if I want to find the forces exerted by.

So, if I look at the free body diagram of this of this guy ok. I have the reaction forces I have mg acting this way. So, I have $2f - mg = ma$; where a is given by this everything is in the j direction. So, I not use the vector form.

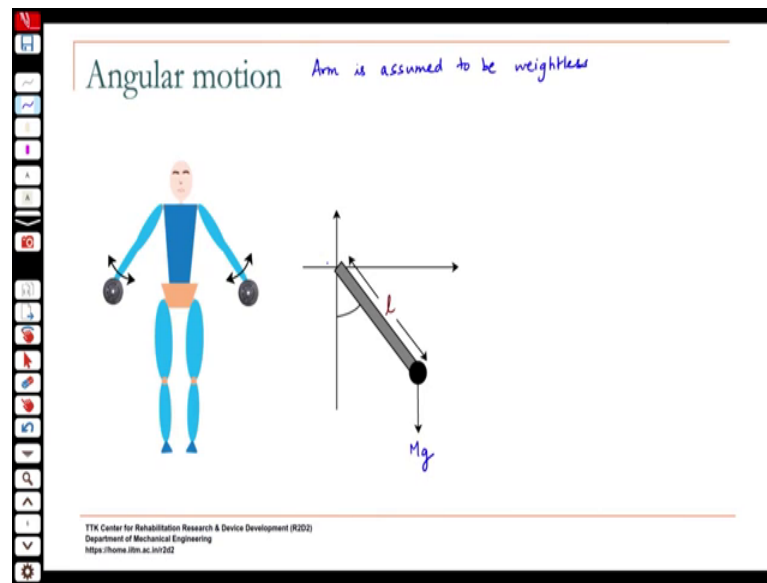
So, this kind of an analysis where knowing the kinematics. So, for instance these angles can be measured in the lab. You can you can look you can track θ as a function of time, ϕ as a function of time; there are ways to measure the kinematics ok. So, knowing the kinematics then you can compute the forces the unknown forces from this. This kind of an analysis is called an inverse dynamic analysis ok.

When you know all the forces that act on a body, and then you try to determine it is acceleration that is called forward dynamics ok. You are you are determining how that body is going to move ok; which is what in most of your engineering mechanics courses you will be given ok; I am pulling it with this force. This much friction is acting on it for instance ok. What is the acceleration going to be? Ok, those are called forward dynamics problems. In biomechanics in most cases we will be looking the kinematics are easy to measure, because in many cases internal forces for instance, if I take the free body diagram, which we will look at next for you know at the shoulder I cannot measure the forces at the shoulder directly ok.

So, I rely on kinematic measurements and then use the Newton Euler equations to find out what are the forces that are causing this particular motion. So, that is called an inverse dynamics analysis. And we will do we will see how we can do that over the yeah chain of the body segments as we go along ok. So, this is so, you can do an analysis like this now. So, you have determined the acceleration from these equations, and then you solve this equation to find the forces that are the reaction forces that are acting on the athletes hands ok.

So, this is again we looked at the case of linear motion.

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We will now move on to angular motion. And before we do that; so, in angular motion you also have the principle of angular momentum will also come into play in addition to Newton's laws, which tells you $\sum \mathbf{f} = m$ times the acceleration of the center of mass ok. We will also have to consider the rotational aspects of the motion and we will look at the angular momentum principle for that.

So, here is the case of an athlete; will first start off with the assumption that the arm is weightless, and the athlete is moving the arm with the arm extended at the elbow working out the shoulder muscles, the deltoids ok. So, this is not the case of just a pin joint here and this thing is moving freely. So, we will have to be we will have to look at how to formulate this model; say, for instance I want to assume that only the deltoids are acting ok. We done this in the static situations right.

We used $\sum \mathbf{f} = 0$, $\sum \mathbf{m} = 0$ and we have determined the reaction forces at the shoulder, and the force that would have to be exerted by the deltoid muscle to hold it in a particular position. Now we are looking at the dynamic case; where you have this mass that is moving, and you want to determine how the forces are going to be influenced in this case.