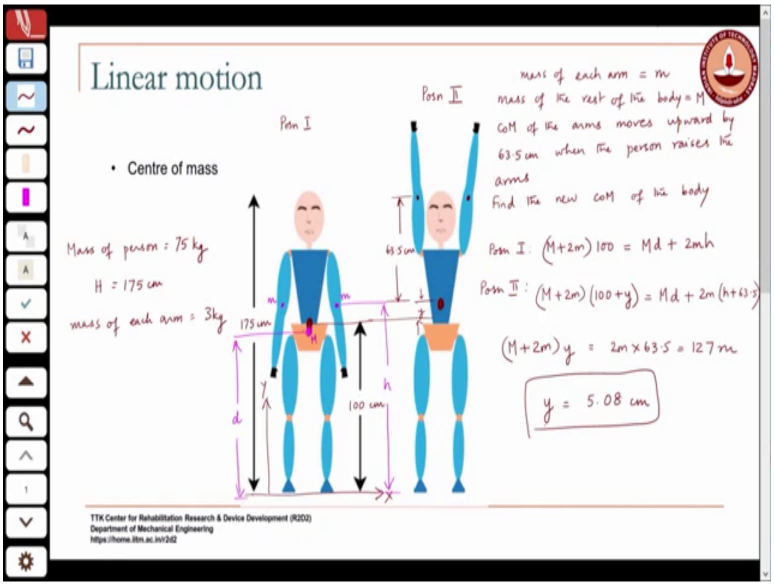
Mechanics of Human Movement
Prof. Sujatha Srinivasan
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture – 27
Kinetics: Linear Motion Part II

So, today we will look at the human body is a multi segmented system. And we are going to look at so, each segment has a mass, and we know that the weighted average position of this is the center of mass of the body. Since, the human body can change it is configuration, you can change your posture, you can move the individual limbs, they are all connected by joints and you can move them the center of mass of the body can change. So, today we look at an example, where we have a person who is initially standing, and then the person lifts their arms and we will look at the change in the center of mass that happens because of that ok.

(Refer Slide Time: 01:09)



So, let us look at we have position I, where the person is standing and we have position II, where the arms are lifted, let the mass of each arm we will call it m small m. And let us call the mass of the rest of the body that is excluding the arms equal to capital M ok. And what we are given is the center of mass of the arms moves upward by 63.5 centimeters, when the person raises the arms.

So, what we want to do is to find the new center of mass of the body. What we are given, the height of the person is 175 centimeter and the initial location and the person is standing straight, the location of the center of mass is if this is our coordinate system say, this is the X-axis and I have the Y-axis like that. So, it is located at a 100 centimeter above the ground initially ok. Now, what we want to find out is the new center of mass, when the person lifts their arms.

So, what we know is in position 1 ok, so in position I, let us divide the body into we have the mass of each arm and the mass of the rest of the body. And let us say that the initially the location of the center of mass of the arms is somewhere here, so located say at a distance h from the ground. And then let us say the center of mass of the rest of the body is located at some distance d above the ground.

So, this is this here denotes m and this here denotes m, so there is an m 1 this side also mass of the arms mass of them. So, in position 1, if I look at, I can write it as M plus 2 m that is the total mass of the body times 100 that is the location of the center of mass the total center of mass equals capital M times d plus 2 m times h.

And in the IInd position, now the center of mass of the arms has moved up from this initial position by a distance of 63.5 centimeters. So, for position II, I can say M plus 2 m, now I want to find out how much by what distance has this total center of mass moved right. So, this is the total center of mass of the body that has moved by say a distance y, so that is this distance that is the location of the new center of mass.

And this is equal to now the configuration of the rest of the body has not changed, there is a reason I split it into the mass of the arms and the mass of the rest of the body the configuration, so that will still remain M into d in the new configuration. And for the arms, I know that it is now at h plus 63.5. So, if I subtract equation 1 from 2, I can eliminate M plus 2 m into 100 M d all that goes away. So, I am left with M plus 2 m into y equal to 2 m into 63.5, which is equal to 127 m.

Now, suppose we are given the mass of the person equals 75 kg's and the height of the person we are given as 175 centimeter although that is not something we have used here. And then the mass of each arm let us say is 3 kg's so, I if I substitute m and capital M and small m into this equation, I can solve for this to get y equal to 5.08 centimeters. So, you can solve that and say that we get this.

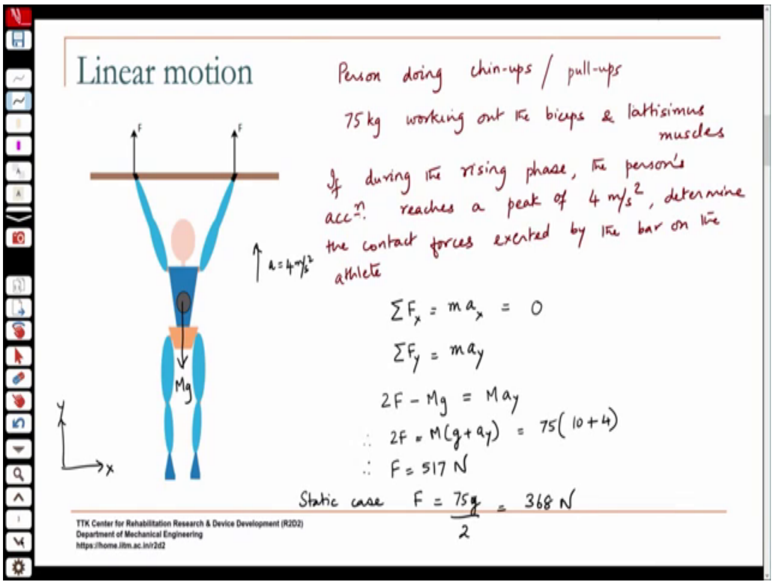
Now, one thing that we will encounter later so, this happens when the body is supported ok, this body is against some kind of a support. So, here the body is supported by the ground ok. You could have one arm against a wall ok and then if you change the configuration from that point, your body center of mass is going to change.

The reason I mentioned this is later on when we do when we go to dynamics, you will see that when the body is airborne, where it is not supported ok, it would not matter how much you change the configuration of the limbs. When the body is airborne, changing the configuration of the limbs will not change the center of mass of the body, we will see that later.

But, when the body is supported with so with respect to the support, you will see a change in the center of mass of the body, when the configuration of the body changes. Now, sometimes center of mass and center of gravity are used interchangeably that is because, in a uniform gravitational field, the location of the center of mass matches with the location of the center of gravity.

So, you can compute this for different configurations as well. Now, that you know in this case, we were not concerned, there was no change in the x coordinate of the center of mass here, because the body is symmetrical right. So, in this case whereas, if I lifted only one hand, then I am likely to see that there will be a change in the x coordinate of the center of mass as well, because the body is no longer symmetrical about the vertical axis ok, so that is how we look at calculating the center of mass for the multi segmented human body.

(Refer Slide Time: 10:27)



Now, let us look at some cases of linear motion. In many of the cases of linear motion, we will just look at the motion of the center of mass, because that will describe the motion of the entire body. So, let us take the case of a man or a person doing chin-up, are you aware of this exercise? You hold on to a bar, and you lift yourself up and sorry.

Student: Pull-ups.

Pull-ups are so, pull-ups you typically there is a slight difference between the terminology pull-ups. You hold the bar like this and you pull yourself up ok, and chin-ups, you hold the bar like this, and pull yourself up ok. So, you can have pull-ups or chin-ups, we are not really looking at which muscles are involved right now, so it is so, you can say pull-ups or chin-ups ok, but you would be working different muscles.

Because, if; because if you are if you want to work a biceps, then you would have you would have your hands would have to be in the supinated position to engage the biceps. Otherwise, it is the other elbow flexors that would be that you would be working. And of course, you would your also working your lattissimus dorsal the (Refer Time: 12:06) that muscle is also being worked in this exercise.

So, if you doing that, you want to find the contact force exerted by this person on the bar so, you have a person 75 kg's working out the biceps and lattissimus muscle muscles ok. If during the rising phase, the person's acceleration reaches a peak of 4 meter per second square, determine the contact force forces exerted by the bar on the athlete, which is nothing but by Newton's 3rd law, the equal and opposite to the forces exerted by the athlete on the bar ok.

So, what are the forces acting on the person ok. So, if I already have the diagram partial free body diagram here, so you have the forces exerted by the bar on the athlete will be F.

Student: (Refer Time: 14:21).

So, what is the other external force?

Student: (Refer Time: 14:28).

You have M g, there are no horizontal external forces. So, you can assume that not assumed, there would not be any horizontal reaction forces, because there are no horizontal external forces. So, that sigma F X equal to 0 is already there, there is no acceleration happening in the if this is my X, Y, there is no acceleration happening in the X, Y. So, this person has an acceleration of 4 meter per second square at the instant that we are interested in ok.

So, for statics problems, we had sigma F X equal to 0, sigma F Y equal to 0. Here we are going to look at sigma F X equal to m a X ok. In this case, what is a X is there any acceleration in the horizontal direction, it is 0. In the vertical direction, you have sigma F Y equal to m a Y. The forces acting on the person or 2 F in the Y direction minus M g equal to M into a Y ok. So, what is F equal to straight forward, therefore 75, let us take g is 9.81 plus or you can approximate it to 10 plus 4.

Student: 517.

517. So, F equals 517 Newtons, in the static case, what is F equal to? So, if the person is just hanging from the bar, what is F equal to?

Student: (Refer Time: 16:59).

Which is equal to?

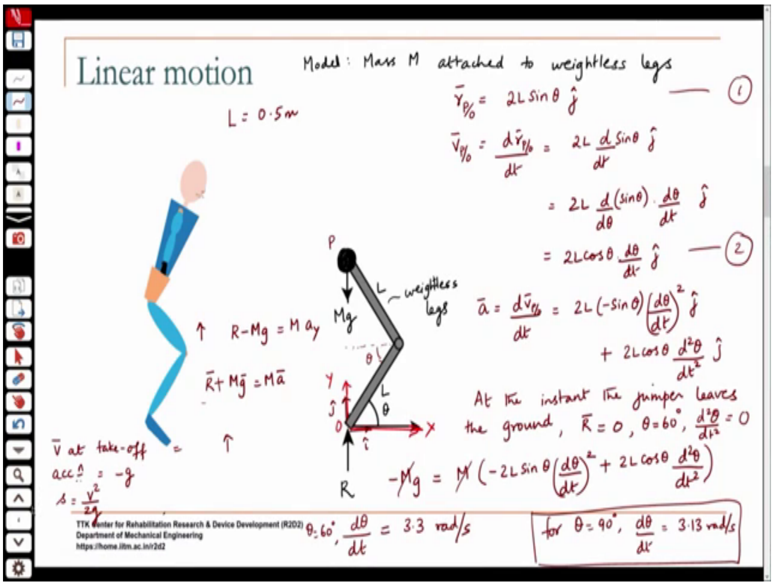
Student: 368.

60?

Student: 368.

368 Newtons.

(Refer Slide Time: 17:59)



Now, let us model a situation of jumping in a simple manner using you know treating the so, we can create a model, where I have weightless legs to which a particle of equal to the mass of the body is attached ok. Let us say each they up the thigh and the shank both are length L just to simplify, this is my reaction force from the ground.

So, I am modeling it model has a mass M attached to weightless legs. And there has to be there have to be muscles and tendons that are controlling the system, otherwise it would just collapse. But, we do not consider those forces in this model, because those are considered internal forces to the system that we are considering ok.

Now, this system will not work without having that without having your other structures in place. The muscles and the tendons in the control to do this in a controlled fashion, we are just looking at the overall motion of the system. And it is the what the muscles and all that do is, they change the angle between these two limbs in a controlled fashion ok. And we want to look at we are just going to look at the how that is happening.

So, in most problems in most when you are solving problems in dynamics, it is always good to start with deposition vector always start with defining the position vector. So, if I take let me draw this in a different color that is my coordinate system. In this coordinate system, the location of the particle is how can I describe it in vector form? If it is r is the this is the particle p r p relative to o is 2 L sin theta along the j-axis.

This X-axis the unit vector along that is Y unit vector along the Y-axis is j (Refer Time: 21:46). Now, if I differentiate, so the velocity of p is nothing but the rate of change of this dr p by dt and I get 2 L cos theta. So, remember, I have to apply the

Student: (Refer Time: 22:18).

Chain rule right, I am differentiate. So, this is d by dt of sin theta ok, 12 is a constant. This is equal to 2 L d by d theta of sin theta into d theta by dt ok. So, this gives me 2 L cos theta into d theta by dt along j, so that is the velocity of this mass.

Differentiate again, to get the acceleration, so again I apply this. So, 2 L say I differentiate cos theta first, now it is a product rule, there are multiple quantities. So, first I differentiate cos theta, I get minus sin theta and I also get a d theta by dt, so I have d theta by dt square also along j. Then I have another term this time 2 L cos theta difference differentiation of this gives me d square theta by dt square.

What happens at the instant? So, I want to find out what is happening at the instant, the jumper leaves the ground (Refer Time: 24:21). So, the x the forces acting on this system, you have the weight and you have the reaction force of the ground. At the instant, the jumper leaves the ground, what happens to R?

Student: 0.

R will be 0.

Student: (Refer Time: 25:02).

Sorry.

Student: That means, theta (Refer Time: 25:06) theta also become (Refer Time: 25:07).

No, theta is not 0; theta is not 0 as you will see. I can when you jump, is theta always 0? No.

Student: (Refer Time: 25:20).

No, theta is not 0 ok. See if I write the equation of you know they if I write like I did in the previous case, I have R minus M g in the Y direction right equals M a Y right ok. I found out a, a is in the Y direction, here everything is in the Y direction here or I can write this in vector form as R minus M g equal to M a ok. In this case, it so happens that again it everything is in the Y direction. So, at the instant, the jumper leaves the ground, R equals 0, I found out a.

Student: R is not a vector (Refer Time: 26:16).

R is a vector, it is a force, so it is a vector. R equal to 0 so, I have now this equation reduces to minus M g, everything is in the j direction, so I will leave out the unit vector equals M into minus 2 L sin theta d theta by dt square plus 2 L cos theta d square theta by dt square.

So, at the instance, let us also say that the initial condition ok. Let us say theta equal to 60 degrees and let me say that the (Refer Time: 27:27) d square theta by dt square is 0, it is an initial condition ok. Let us say it is d theta by dt is changing uniformly. So, d square theta by dt square is 0.

Now, and let me give you L as 0.5 meters, and M as actually M will cancel out on both sides, so it does not matter say L as 0.5 meters. So, what do I get for d theta by dt, can I solve for d theta by dt now? I know this, I know theta, I know d square theta by dt square, I know L, tell me please tell me what d theta by dts.

Student: That is (Refer Time: 28:32) 3.36.

Student: 3.36.

Student: (Refer Time: 28:38).

3.3 radians per second. So, what does this tell me, the angle between the shank and the thigh should increase at this rate right. So, d theta the rate of change of this angle has to be at this rate in order for the body to lift off. So, if I do it slowly, you can try that right. If I just extend my legs slowly, I am not going to get off the ground. I have to extend it with a certain d theta by dt in order to be able to lift off the ground in order to be able to generate the reaction force that will lift me off the ground. So, this

Student: Mam, possible only lift off when theta is equal to (Refer Time: 30:09) degree.

But no at the instant of lift off, after they lift off they will straighten out. You can lift off try jumping, why do not you try that, try jumping. Let us see if you can jump with a bent leg.

Student: It is straightens (Refer Time: 30:36).

You it straightens, once you leave the ground. At the instant, I can still lift off with a bent leg ok so, I can say that theta is you can do this. If you take theta equal to 90, you can also solve for d theta by dt depends on your initial condition. And usually you can you know this kind of kinematics is can be measured in a lab, you know they have ways of capturing these kinematics to determine these kind of conditions initial conditions.

So, if the legs straighten as 12 over speed, then jumping cannot occur ok. So, it has to be at least this much, for it to generate that propulsive force. Why do not you do it for theta equal to 90 and see what you get.

Student: 3.13.

3 point.

Student: 13.

13. For theta equal to 90, d theta by dt is 3.13 radians per second.

Student: One thing instant theta becomes 90 (Refer Time: 32:01).

Ok.

Student: So, that is when R is equal to 0 (Refer Time: 32:08) until then we have some reactions (Refer Time: 32:12).

I am not sure, it has to be 90 at the instant that you leave the ground. It is so, let us look a actually for theta equal to 90, you are getting a lower d theta by dt. So, it is likely that you will leave the ground that is a better initial condition than theta equal to 60 degrees, because you are accelerating right, you are increasing the angular velocity.

Student: After which we (Refer Time: 32:56).

So, if theta equal to 90, it is easy if you if you straighten out to theta equal to 90, you need a smaller d theta by dt to leave the ground.

Student: After which 90, we cannot change the angle movement.

Angle more than.

Student: 90.

No, because your leg would not we are talking about your leg cannot extend beyond your straightening out right, so you are lifting straight up.

Student: But, model will allow.

Model will allow, but that is where you know you have to give appropriate boundary conditions, mathematically I can do anything right. So, I can do theta equal to 0, will that make sense so, that is where you have to apply your you know see what conditions are practical for the problem ok. So, it is possible that you know it is more likely that you will take-off at theta equal to 90.

But, if you are able to if your muscles are able to generate, see but this is you know it is it is a pair, I could have. If theta is say instead of 60, you know say I am jumping from a seated position ok, then it is not necessary that when it is this theta equal to 90.

Student: Mam, but in a if you talking about initial condition (Refer Time: 34:55).

But, see it is a combination right, theta and d theta have to match for you to leave the ground. Does that make sense? Suppose, you have theta equal to 60 and d theta by dt equal to 3.13, you are not going to leave the ground. Do you see what I am saying? So, if you if theta equal to 60, your d theta by dt at that instance needs to be higher for you to leave the ground. But, if my theta is 95 already straightened out, then this is enough to leave the ground, is that clear.

So, now because I can do this you know I have taken it as a particle, they have taken the entire body mass as a particle, now I can analyze this. When it leaves the ground, how far is it going to travel ok, what kind of a problem does it become, it is a simple problem of constant acceleration. So, what you what do you have to find out, you first know do you know the initial velocity at the time of leaving the ground? How would you compute that? I have my.

Student: (Refer Time: 36:42) if we above equation (Refer Time: 36:47).

From this equation, so you have the velocity equation here 12 cos theta. I know theta, I know d theta by dt at that instant, so I know the velocity at take-off from the ground. Now, once it is off the ground, what is the acceleration acting on it?

Student: Gravity (Refer Time: 37:14).

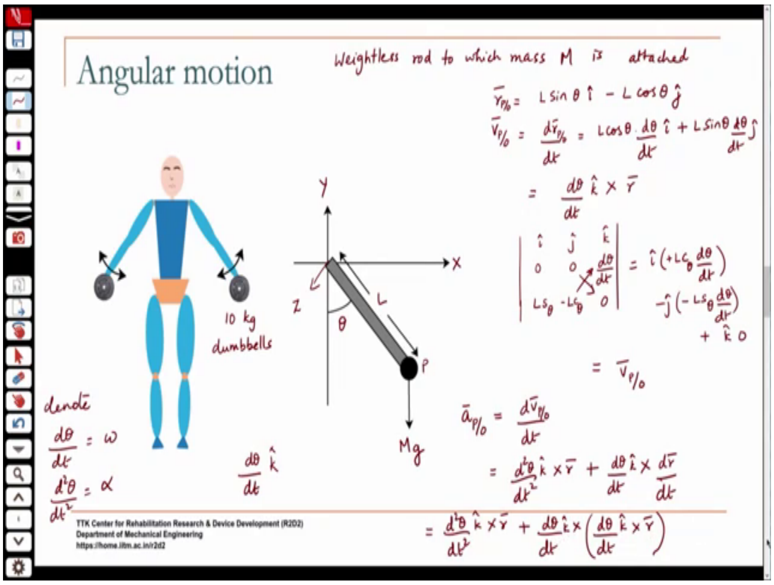
It is only gravity right, which is what we got here minus M g equal to M a. So, when R equals 0, a becomes g or minus g actually, I should just say R plus no R minus M g, I am taking M g. So, in the vector form, I should just say R plus M g yeah, because I am including the direction in that. It is only when I put it into the components along the Y directions, then I call it minus g, because my Y is positive upward. This is the sum of all the external forces on the particle R plus M g.

So, I know the initial velocity at take-off. I know v at take-off, I know the acceleration becomes acceleration is now just. So, (Refer Time: 38:45) v at take-off is something in this direction, the acceleration is minus g ok. How do I know how much it travels?

Student: d square dt.

So, I say s is v square by 2g will give me the height it travels, because I am just considering it as a particle. So, for this particular case, if I take this is the condition theta equal to 60 and d theta by dt equal to 3.3, then I get s is 13 centimeter. So, you jump a height of 13 centimeters that is the motion of the center of mass.

(Refer Slide Time: 39:49)



We will next look at angular motion. So, for linear motion, I am basically just using F equal to M a to solve for the reactions are now, for angular motion, we will find that that is not enough. So, I have if you have a person exercising like this lifting a dumbbell, you know doing arm abduction adduction in a controlled fashion that can be modeled by again say a weightless rod to which let us say this mass M g is attached. So, let us say these are 10 kg dumbbells. What are the muscles he would be exercising?

Student: Deltoids.

The deltoids, not the triceps; the deltoids, you are working of the shoulder.

Student: (Refer Time: 41:50).

So, the your shoulder muscles are what you would be trying to strengthen using this exercise. So, you pull the weight up directly to the side and then lower it. And then now you want to look at the motion of this and the forces that it is generating at the shoulder ok, so again we start off with the position vector. So, if I take this as my X, Y. R is equal to L sin theta along i minus L cos theta along j position vector. The velocity is dr by dt, what do I get here, L cos theta to d theta by dt along i plus L sin theta d theta by dt along j.

For planar problems ok, d theta by dt can be denoted by described as a vector omega or the direction of d theta you take it as counterclockwise is positive, clockwise is negative.

And in vectorial form, you can write it as d theta by dt along k. Using the right hand rule, so I have so, z is coming out of the plane of the paper ok. So, this can be written as if I take d, I can write this in vector form as d theta by dt k cross r.

So, if I do the cross product i j k ok, this is 0 0 d theta by dt and this is L r is L cos sorry L sin theta, I will write it as L s theta for sin theta, this is L minus L cos theta and 0. See if you get the same thing, you will see that this is equal to i into 0 minus this, this, minus this, so I get plus L cos theta d theta by dt ok. Then minus j 0 minus L sin theta d theta by dt plus the k component, I get 0. So, this is the same as (Refer Time: 45:53) o.

Now, I can use this vectorial form to find the acceleration. This the derivative of this is d square theta by dt square k cross r plus the second term is d theta by dt cross dr by dt. Now, this term is (Refer Time: 46:46) k cross r plus d theta by dt k cross, what is dr by dt?

Student: (Refer Time: 46:57).

It is again this d theta by dt k cross r, this is your familiar alpha and omega. If I denote d theta by dt as omega and d square theta by dt square as alpha, I get the acceleration of p relative to o is alpha cross r plus omega cross omega cross r. We will continue in the next class.

Student: (Refer Time: 48:16).

Let us try that exercise. Can you not jump at all with your leg bend? You can, it is possible.

Student: (Refer Time: 48:25) question (Refer Time: 48:26).