Mechanics of Human Movement
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Lecture - 26
Kinetics: Linear Motion

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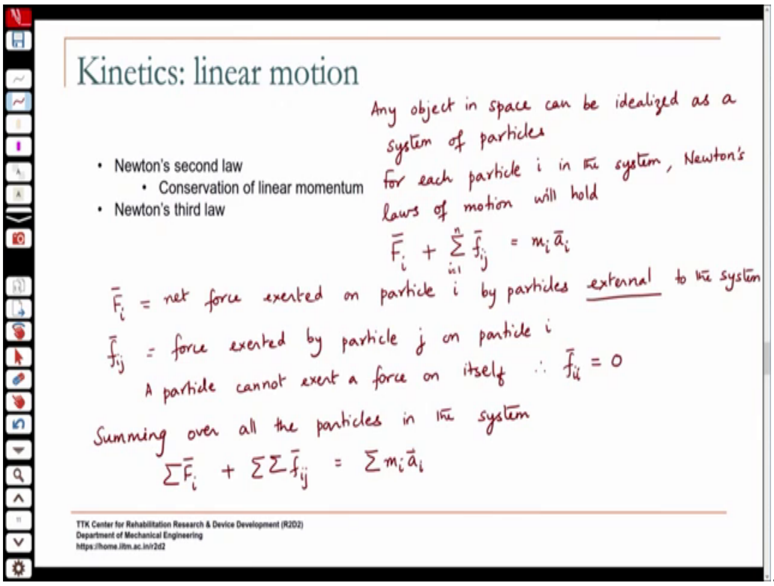

So, we know that all of you have done a course in mechanics. So, we will be you know first we will look at linear motion modelling some human movements as linear motion, then we will look modelling it as angular motion and then we will look at generating some equations of motion as we heard so, that we can look at the time progression of different types of motion. So, if you look at any object in space can be thought of collection of a system of particles as a system of particles right it can be idealized.

So, essentially we take them as small mass elements that can be idealized as a system of particles, now for each particle in the system particle i in the system. So, if you take a system of n particles for each particle i in the system, Newton's laws of motion will hold right, Newton's laws of motion will hold. So, I can say that F i plus sigma over i equal to 1 to n small f ij equal to m i a ok, when I put a bar I am denoting them as vectors there F i is the net force exerted on particle i by particles external to the system to the system.

So, these could be contact forces, gravitational forces etcetera, but these are forces that are external to the system, these are the forces that are that the net force exerted on particle i, because of external reasons. And my f ij is the force exerted by particle j on particle i.

So, a particle cannot exert a force on itself so, f ii equal to 0. Now if I sum this over all the particles in the system ok, I get sigma F i plus sigma f ij equals sigma m i a i for the system I get this.

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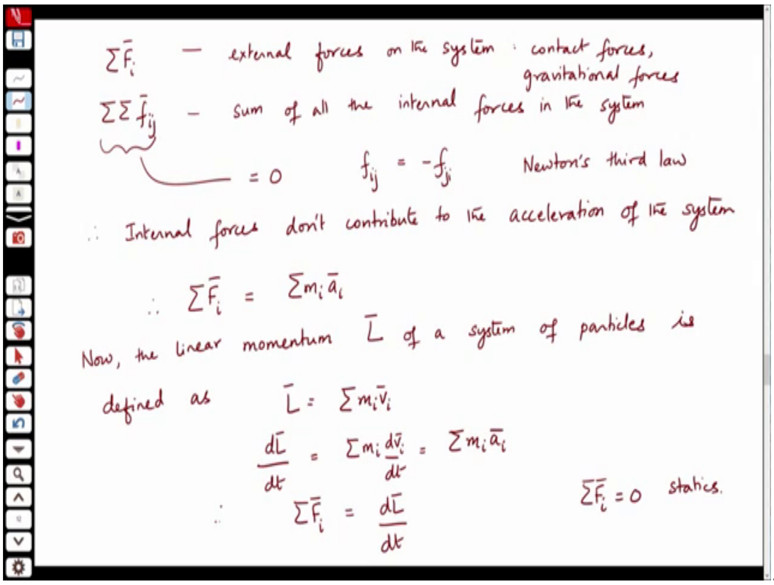

Sigma F i here is you know all the external forces on the system yes ok, external forces on the system. So, these could be contact forces, gravitational forces and then this is the sum of all the internal forces in the system.

What is this equal to?

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This is equal to 0, why because of Newton's third law because for any f ij.

Student: (Refer Time: 06:10).

You have a minus f ji ok, because of Newton's third law you have. Therefore, this shows that the internal forces do not contribute to the acceleration of the system ok. Therefore, your equation reduces to sigma F i equal to sigma m i a i. Now the linear momentum L of a system of particles is defined as L equal to sigma m i v i ok, some of the products of the mass times mass and velocity.

So, if I differentiate this dL by dt is sigma m i dv i by dt which is nothing, but sigma because m does not change with time. So, I have this is equal to sigma m i a i and therefore, we have sigma F of i here is equal to essentially dL by dt. So, if a body is at rest or moving with a constant velocity then it is momentum does not change momentum is conserved right and that is the case where you have sigma F i equal to 0 that is the static situation that we looked at ok.

So, sigma F i equal to 0 is your is what we did for the last several classes where we said if the body is at rest we mainly looked at the case of bodies being at rest and we summed all the forces to be 0.

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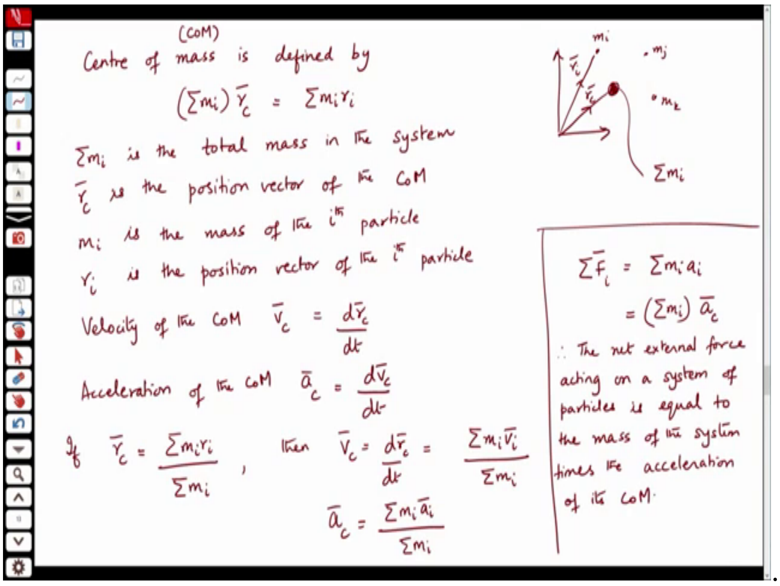

Now the centre of mass of a system is defined by and m i into r c equal to sigma m i r i. So, if I have a coordinate system and I have various particles ok, m i, m j etcetera. The centre of mass of the system is where the mass would be concentrated the effect is the same as concentrating the mass at that location and the location of that is defined by r c ok.

So, this is sigma of m i, this is the definition of the centre of mass. So, sigma m i is the total mass in the system, r c is the centre of mass is also usually denoted as COM is the position vector of the centre of mass, m i is the mass of the ith particle or element and r i is the position vector of the ith particle. The velocity of the centre of mass we see velocity of any point is just the rate of change of it is position vector right. So, it is dr c by dt and the acceleration is the rate of change of it is velocity.

So, if r c from here is sigma m i r i by sigma m i, then V c equal to dr c by dt is nothing, but sigma m i v i by if i differentiate both sides sigma m ok. And similarly the acceleration is sigma m i a i by sigma m i therefore, I got sigma F i equal to sigma m i a i ok. So, from here I can write that as this is equal to sigma of m i times the acceleration of the centre of mass. Therefore, the net external force acting on a system of particles is equal to the mass of the system times the acceleration of it is centre of COM.

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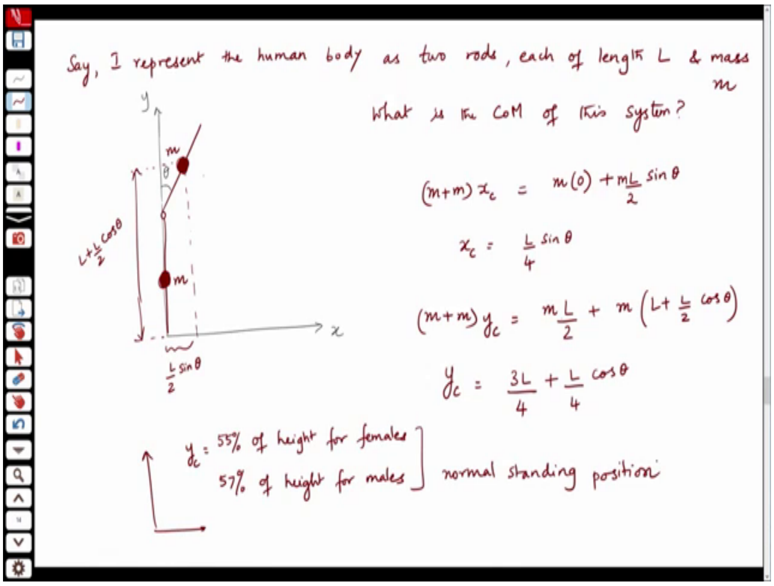

Say I represent the human body as two rods. So, for instance I say this is one particle connected by a joint. So, remember and in these kind of diagrams an open circle typically represents a joint whereas, a solid one like this usually represents the mass concentrated at that point. Let us say I have a rod of mass m 2 rods connected by a joint, let us say each one is of length L each of length L and mass m ok. Now let us say I choose a coordinate system xy coordinate system and I want to find the centre of mass, let us say this angle is theta what is the centre of mass of this system.

So, I can say that m plus m into the x coordinate of the centre of mass is essentially equal to sigma m i r i or the k the x coordinate right. So, let us say for the first mass my x coordinate is 0, for the second mass my x coordinate is I have this is L. So, this is L by 2 m L by 2 into sin theta this here sin theta. So, x c is essentially equal to l by 2 sin theta sorry L by 4.

Student: (Refer time: 19:29).

Now f for sin theta, similarly to find the y coordinate I have m plus m y coordinate is for this one for this particle the y coordinate is L by 2, for this the y coordinate is L plus L by 2 cos theta. So, my y coordinate is 3 L by plus L by.

Student: Cos theta.

Cos theta.

So, you can see that the centre of mass of the system is going to change as theta changes, say if you look at the human body you can talk about a centre of mass when you are in a particular posture you move your limbs or you know when you move from that particular position your centre of mass is going to change. So, you cannot say that there is one centre of mass for the body ok.

So, here you can see if I lean forward I lean more my centre of mass is going to change, if I think of this as this representing my lower body and that representing my upper body. The simplest case where I am modelling it as 2 particles ok, I can see that my centre of mass will change as the posture changes. Typically for a person who is standing in the normal you know standing position for adults I think for the centre of mass from the ground it is located at about 55 percent of the height for females.

So, from you know why see and about 57 percent of the body height for males this is the typical this is for the normal standing position. You remember so, it is located at the level of the second sacral vertebra and you remember when we talked about the anatomical planes we talked about the principle planes bisecting the centre of mass.

So, you the origin would be at the centre of mass that point where the 3 planes intersect, the sagittal the frontal and the transverse plane we talked about the principal planes bisect the centre of mass and of course, any planes that are parallel to them are also called you know by the so plane parallel to the sagittal plane is also a sagittal plane, but the principal sagittal plane is the one that bisects the centre of mass into left and right parts.