

Experimental Stress Analysis
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Lecture - 04
Completeness of a numerical solution

Let us continue our discussion on overview of experimental analysis, and what we have primarily focused in the previous class was for typical problems, for which you know the solution we have looked at the kind of patterns you could get from some of the experimental techniques.

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The slide is titled "Typical Results for Various Problems" and lists several mechanical problems with their corresponding solution methods. The problems listed are:

- ★ Beam under four point bending
– Closed form solution by Strength of Materials is possible
- ★ Cantilever Beam
– Engineering analysis possible by Strength of Materials.
- ★ Disc under diametral compression
– Only Theory of Elasticity can provide closed form solution.
- ★ Clamped circular disc with a central load
– $w, \frac{\partial w}{\partial r}, \frac{\partial^2 w}{\partial r^2}, \frac{\partial w}{\partial \theta}, \frac{\partial^2 w}{\partial \theta^2}$ obtainable from theory of elasticity
- ★ Spanner tightening a nut
– Due to complex nature of the geometry only a numerical solution is possible

At the bottom of the slide, it states: "In all these cases relevant experimental (recorded or simulated) are shown to appreciate the nature of fringe contours." There is also a copyright notice: "Copyright © 2007 Prof. K. Ramesh, IIT Madras, Chennai, India."

The problems considered were beam under 4-point bending, cantilever beam, disc under diametral compression, and also clamped circular disc with a central load. In all these cases, you have analytical solution possible for the stress field. So you have got a closed form expression for the stress field, you got the strained field and also the displacement field. And what we did was, we did a sample of experimental methods.

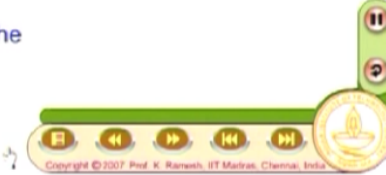

Some of them were directly from the experimental methods, some of them were simulated result to give the feel of how the whole field information looks like. So now your eyes get tuned to how to interpret whole field information to an extent possible, and finally what we will do is we will go to the problem of spanner tightening a nut. As I told you earlier due to complex nature of the geometry, only a numerical solution is possible for this problem, and that is what we are going to see.

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Spanner tightening a nut

- Consider one of the day to day application of the problem of a spanner tightening a nut.
- Let us consider the evaluation of the stress field in the spanner.
- You will be surprised to know that the problem can't be solved by Strength of Materials or even by Theory of Elasticity.
- This is because the shape of the spanner is quite complicated.
- Only a numerical solution is possible.



And what I am going to look at is, we have already looked at day to day application of the problem and surprisingly you do not have solution from strength of the materials or even by theory of elasticity. This is finally because the shape of the spanner is complicated and you cannot define the outer boundary in a convenient fashion for you to do a theory of elasticity solution. And in this case only a numerical solution is possible.


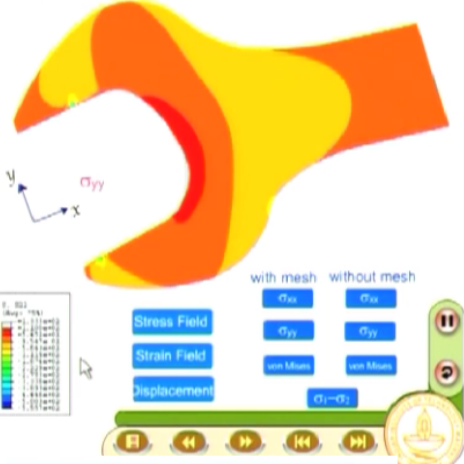
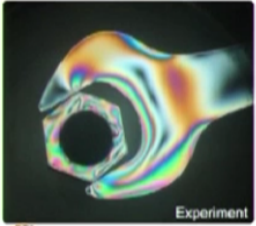
I do not have a closed form expression so I have to solve this problem numerically, and let us see how the numerical solution is.

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Spanner tightening a nut – Numerical solution

From a numerical analysis one can get displacement, strain and stress fields.



You have very rich set of results that you get from a numerical solution, and in this case the numerical method adopted is finite element method, and what you have done is, you have

meshed the spanner and what you find here is, this is the experimental fringe pattern, and this is the simulated finite element result of $\sigma_1 - \sigma_2$. And here I want to point out a few things, when I do a numerical analysis I can get the stress field, I can get the strained field, I can also get the displacements.

And if you go to commercial packages you would be able to plot specific contours. I have a σ_x contour, and the software chooses its own colours and then gives you an indication a red means a maximum and a shade of blue what you have here it goes to less value of the stress information. You could see this without mesh, you could also see this with mesh, and for this you know, you do not have to take the trouble of sketching it, because it is too complicated for you to sketch.

The idea is to visualise what way you have the information available from the numerical solution. I have the mesh here and you have a nicely done mesh. These are all quadrilateral elements and I have the σ_{xx} stress contour. Similarly, I can also get the σ_{yy} stress contour, and I can also get von Mises stress contour. And if you look at von Mises in certain aspects it captures certain geometry features of the photo-elastic fringe.

But the colours are totally different, because the colour is dictated by the standard finite element software package. And what was done was we have developed an in-house software what it will do is it will evaluate $\sigma_1 - \sigma_2$, and also mimic the colours that you get in an experiment and this is plotted. And approach like this helps you to quickly come to a understanding that you have very good comparison between what you observe in the experiment and what you observe in the numerical method.

And if you look at very closely I have this is stress concentration region, and what you have got and what you see in the screen, you know my students have taken a little time to apply the boundary condition appropriately until the experimental fringe pattern matches closely with the numerically simulated results. So both the choice of elements, discretization and also the boundary conditions are improved until you get a close match between experiment and numerical solution.

So you have this, what is the difference here. Here when I have to find out I can get σ_{xx} , but I can do this only by interpolation and based on finite element formulation. I cannot go to

the location of the coordinates and plug simply x , y and get these values directly. So that advantage you had in the case of analytical solution. Analytical solution the main advantage is if you have a possibility to solve it analytically, there is nothing equal to that.

Because the amount of computational effort required is very small. I simply plug in x , y I get the value which I want. The moment I come to numerical techniques the greatest advantage is the shape of the geometry of the problem on hand does not pose any restriction, but you have to do a lot of computational effort, but you get whole field information. So I can also get the strain field and till now you have not seen strained field as a plot and this is with mesh.

And you can also see without mesh, this is ϵ_{xx} , similarly I can get ϵ_{yy} . I can also get shear strain ϵ_{xy} here. And I can also go and see the displacement field and this is what I get here, I have the U displacement, I also have the V displacement. So what I find here is when I have a numerical approach, I could get all the 15 quantities comfortably. But the very important aspect is I must match what I have in the experiment very closely by choosing the boundary conditions correctly.

Once I have done this, then I have solved the problem satisfactorily and a parametric analysis is very convenient when I go for a numerical methodology. And what is seen here is, here I have taken the effort of plotting fringe contours what you get in an experimental technique. And that requires special software to be developed. It is not readily available in standard packages, and we have this and this is the best way to compare results of photo elasticity with the actual experimentation.

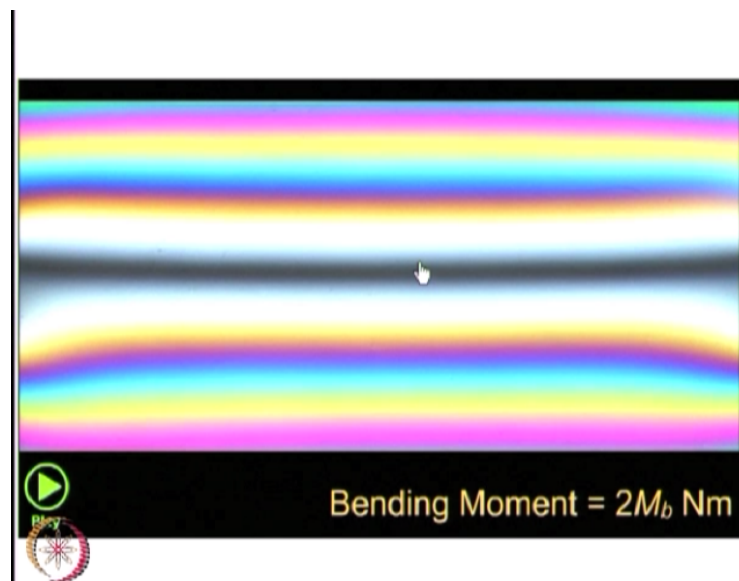
When you do a numerical analysis you can compare with the photo elasticity comfortably. And what I also want to emphasise at this stage is, you know though we have taken simple problems, we have gone and also studied in the process what are the approximations you do in your analytical modelling. You know we have taken a beam under 4-point bending, it bends like this and it is a 3 dimensional object, it has a cross section.

But what you have managed to do in strength of material is you just take it as a line, that is all you do the analysis. And when I go to theory of elasticity, you do not consider it as a line, but you consider that as a 2 dimensional object. But in reality, because you have flexing, what

you find here is, you also have the Poisson ratio effect becomes very prominent. And what you find here is this is the compression side, and this is the tension side.

And this compression side bulges out because of Poisson ratio effect, this may be very difficult to model from an analytical point of view, but experiment looks at all this. So when I get the fringe pattern, the fringe pattern is deflective of all the 3 dimensional effect that happens to the model, some you may have ignored it for the point of view of simplicity. So that is why I said that experiment is giving you truth.

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And we also notice the fringe pattern observed in the beam, there was slight variation on the tension side and the compression side if you look at it in a subtle fashion. For a quick look, it will appear as if it is symmetric, but for a very closer look it will have small deviation which could be neglected as a second order effects and carry on with it. So the point emphasized here is the moment you come to experiment do not discard the raw data.

Raw data is, very, very important. You may have an explanation to understand what the raw data means, if you do not have the explanation try to go and find out whether you have made any approximation, whether you can refine any of these, because in engineering what do we never want to solve a problem in 3 dimensional with all the complexities. I have also mentioned earlier the success of engineering is approximation.

And if this is possible, I would like to work with a 1 dimensional solution. If 1 dimensional solution is not feasible, I go for a 2 dimensional solution, only we are pushed to the wall that

without a 3 dimensional solution you will not get satisfactory result, we go and attempt 3 dimensional solution. The moment you go to analysis of plates and shells; it is actually 3 dimensional problems.

You bring in plates and shells theory approximation and try to live in 2 dimension, you do not want to go in 3 dimensions. So that is the knowledge that you will have to get, so what we have done is till now we have done 3 lectures on overview of experimental analysis. In the first lecture we essentially looked at what is an analytical method, what is a numerical method and what is an experimental method.

In the second lecture the primary focus was what is the information I get directly from an experimental technique. The idea is you may be able to combine more than 1 experimental technique and try to get all the 15 quantities, some quantities of your interest. But what you get directly is from the physics of the problem, physics of the experimental technique on which it is based.

And in the third lecture, we tried to look at what a whole field information is, because you have to graduate from stress as a tensor at a point of interest, you have to go and find out how the stresses vary over the domain of the model, that was the primary focus. So we will also discuss towards the end of this overview of experimental analysis a detailed discussion on how do you go about selecting different techniques very general guidelines.

You may not have, there is nothing like 1 to 1 matching for you to find out the technique for a particular problem. You can have multiple techniques. So it depends on various factors. Thank you.