

Experimental Stress Analysis
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology - Madras

Lecture - 37
Special Gauges

We have been looking at finer aspects of strain gauge instrumentation system. And in the last class, we looked at how to account for transverse sensitivity in a general case, then we looked at how you can effect these corrections in the case of a T rosette, rectangular rosette, as well as delta rosette. Then we moved on to how to make strain measurements, under hydrostatic pressure, nuclear radiation, temperature extremes and when you have cyclical loading.

And finally, we looked at environmental effects, I said water is elixir of life, but water is very bad for strain gauge installation, we saw strain gauge installation need to be protected against moisture

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PERIMENTAL STRESS ANALYSIS Strain Gauges

Effect of moisture and humiditycontd

- Apply a thin layer of microcrystalline wax or an air-drying polyurethane coating.
 - ★ Suitable for laboratory applications (readout time is short)
- Build up a seal out of soft wax, synthetic rubber, metal foil, and a final coat of rubber.
 - ★ Appropriate for severe applications (e.g.: Prolonged exposure to sea water)

The diagram illustrates a cross-section of a strain gauge installation. It shows a central 'Gauge specimen' and 'Terminal' embedded in a 'Wax' layer. Above this, there are layers of 'RTV Rubber', 'Aluminium foil', and 'Polysulphide rubber'. A 'Rubber insulated leadwire' is also shown connected to the terminal. The entire assembly is shown on a 'Gauge specimen' which is part of a larger structure.

So when you learn that it has to be protected against moisture, one of the simpler approaches is apply a thin layer of microcrystalline wax or an air-drying polyurethane coating, and this is good enough for laboratory measurements, and your read out time is essentially short. On the other hand, if we have to make strain measurements under prolonged exposure to sea water, you need to go for a very elaborate protection for the strain gauge installation.

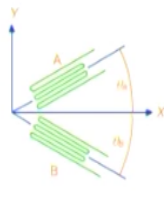
So you need to build up a seal out of soft wax, followed by synthetic rubber, then metal foil and finally a coat of rubber, and you could see this animation which shows the specimen first, then you have the gauge, then you have a seal of soft wax, then you have this RTV rubber, you have aluminum foil, and then followed by a coat of rubber. So when you have to make measurements on underwater pipe line or offshore steel structures below the water.

Then you need to have such well-developed protective arrangement for your strain gauge installation, and I would like you to have a neat sketch of this a figurative sketch. So what you essentially have is on the specimen you have the gauge, then you have a coat of wax, then you have a coat of rubber, then you have a aluminum foil and have a final coat of polysulphide rubber, and this is also you can see it as animation where you see one layer after the other.


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Plane-Shear or Torque Gauge

$$\epsilon_A = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{\epsilon_{xx} - \epsilon_{yy}}{2} \cos 2\theta_A + \frac{\gamma_{xy}}{2} \sin 2\theta_A$$

$$\epsilon_B = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{\epsilon_{xx} - \epsilon_{yy}}{2} \cos 2\theta_B + \frac{\gamma_{xy}}{2} \sin 2\theta_B$$


The shearing strain is

$$\gamma_{xy} = \frac{2(\epsilon_A - \epsilon_B) - (\epsilon_{xx} - \epsilon_{yy})(\cos 2\theta_A - \cos 2\theta_B)}{\sin 2\theta_A - \sin 2\theta_B}$$


You know now we move on to an interesting class of special gauges. First we take up a plane shear or torque gauge, and what we are going to do here is we are going to look at a very generic arrangement, and in this you have 2 strain gauge elements which are oriented at angles theta A and theta B, and you have the reference axis as X and Y. So you have 1 strain gauge element measuring strain along theta =theta A, and another strain gauge element which is measuring strain along theta =theta B.

We will develop equation for a very generic arrangement first, then we will substitute specific values of this theta A and theta B and simplify. And you know in all these approaches I have said you need to know the strain transformation law, if you know that analysis of strain gauge data is very simple. Here, the strain transformation law is written down in terms of cos 2 theta and sin 2 theta, instead of cos square theta, sin square theta in that form, we have written it in another convenient fashion.

So when I have a strain gauge oriented at theta =theta A, epsilon A is given as epsilon xx+ epsilon yy/2+epsilon xx-epsilon yy/2*cos 2 theta A, +gamma xy/2, sin 2 theta A, and you simply replace theta A*theta B in the second expression. And my focus is to find out the shear strain, so from these 2 expressions I can solve for the shear strain gamma xy and that is finally given as gamma xy=2*epsilon A-epsilon B-epsilon xx-epsilon yy multiplied by cos 2 theta A-cos 2 theta B/sin 2 theta A-sin 2 theta B.

And what I am going to do is, I am going to take the first step I will make theta A=theta B in magnitude, and if I have theta A=- theta B, then you will find cos 2 theta A-cos 2 theta B will go to 0. So you will have only 1 term in the expression for gamma xy.

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EXPERIMENTAL STRESS ANALYSIS Strain Gauges

Plane-Shear or Torque Gaugecontd

- The gauges are oriented such that $\theta_A = -\theta_B$

$$\cos 2\theta_A = \cos(-2\theta_B) = \cos 2\theta_B$$

$$\gamma_{xy} = \frac{(\epsilon_A - \epsilon_B)}{\sin 2\theta_A - \sin 2\theta_B} = \frac{(\epsilon_A - \epsilon_B)}{2 \sin 2\theta_A}$$

- Thus, the shearing strain is proportional to the difference between the normal strains experienced by the gauges A and B when they are oriented with respect to the x axis.

So when I have theta A=- theta B I get gamma xy as epsilon A-epsilon B/2 sin 2 theta A. And you know, in normal strain gauge measurements you connect 1 strain gauge to a Wheatstone

bridge, only in special applications you may connect more than 1 strain gauge, suppose I use a delta rosette or a rectangular rosette, essentially I would connect each of the strain gauges in a rosette to separate Wheatstone bridges for strain measurement.

Now here what you are looking at is I will correct 2 strain gauges they may form a T rosette, and when I connect them on adjacent arms, we have already seen adjacent arms cancel each other, and if the angles are oriented at $\theta_A = -\theta_B$, then you get by connecting 2 strain gauges on adjacent arms of the bridge, the bridge automatically gives you $\epsilon_A - \epsilon_B$. So what you essentially get is, you get shearing strain at the point of interest.

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PERIMENTAL STRESS ANALYSIS Strain Gaug

Plane-Shear or Torque Gaugecontd

- For the angle, $\theta_A = 45^\circ$,

$$\gamma_{xy} = \frac{(\epsilon_A - \epsilon_B)}{2}$$

- The subtraction $(\epsilon_A - \epsilon_B)$ will be performed automatically by the bridge and the output will be $2\gamma_{xy}$ directly.
- Shearing strain can also be measured with a two-element rectangular rosette by orienting the gauges at 45° and -45° with respect to the x axis.



And I can take a very special case, when I keep $\theta_A = 45$ degrees, I get a much simpler expression, and that is how you have most of these popularly available strain gauges are oriented at. So what I have is I get essentially $\gamma_{xy} = \epsilon_A - \epsilon_B / 2$. So I have a T rosette and I connect the 2 elements of this to 1 Wheatstone bridge. See this is the special application, here we are not measuring state of strain at the point of interest.

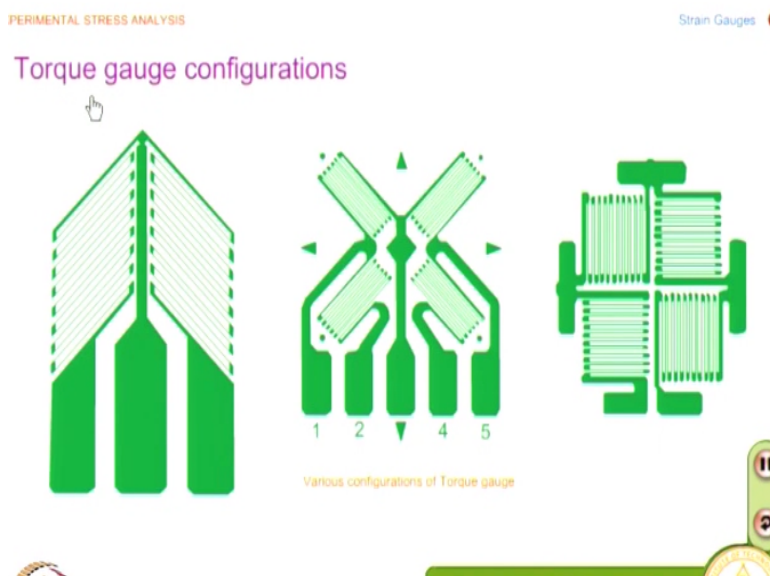
We are interested only in finding out shear strain, we are not interest in anything else, in such a case you can connect elements of the rosette to appropriate arms of the Wheatstone bridge. So this is the special application it is not a general rule, in a generic case you will connect each

strain gauge to Wheatstone bridge, only in transducer application where you design your spring element, and you know the state of strain on the spring element.

Your focus is to measure force, in order to amplify the signal, you try to have minimum full bridge configuration. So you have to distinguish in strain gauge instrumentation, how to connect the strain gauges, and how many channels you require, you need to have a planning, and you need to bring in concepts of strength of materials. So that you paste the strain gauge properly on the structure, as well as connect them correctly in your Wheatstone bridge.

So do not take it that in one class we saw the 2 strain gauges are connected to Wheatstone bridge, and I will do that for every other application.

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And you also have different configurations of this available instead of just 2 strain gauges you have four strain gauges which are oriented at 45-45 and 135 and so on. So here you will get 4 times a signal, and these are all special gauges which can be used to measure the value of torque, if you calibrated properly you can also measure the torque that is applied to the structure. So in these applications, you will connect all the strain gauges appropriately in a single Wheatstone bridge.

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Stress Gauge – Principle

- Usually transverse sensitivity of a strain gauge is a nuisance in strain measurement.
- In stress gauge, the foil design aims to increase the transverse sensitivity as high as the Poisson's ratio of the base material!
 - ★ In view of it, stress gauge will be different for different specimen material.
- This helps in simplifying the governing equation so that the strain gauge output could be directly related to stress.
- The output of any gauge is expressed by

$$\frac{\Delta R}{R} = S_a (\varepsilon_a + K_t \varepsilon_t) \quad \longrightarrow \quad (1)$$

We have seen how to measure shear strain? People also asked the question. In basic stress analysis, we are only looking at evaluation of stress components, so why not we also design a stress gauge instead of a strain gauge. So the focus is whatever the measurement I get should be proportional to stress, then it becomes a stress gauge. See we have looked at transverse sensitivity effects or a nuisance.

And I said, what you considered as nuisance in one application can become beneficial in another application. That is a case with friction, without friction you cannot walk on the street and you need to minimize as much friction as possible in many of your rotating components. And if you do not have friction you cannot apply breaks. So friction is advantageous, as well as disadvantageous.

On similar vein, we will see in the design of the stress gauge, how do we play with the transfer sensitivity? In fact we would like to have maximum transverse sensitivity. So that a given strain gauge functions like a stress gauge. So that is what is mentioned here, transverse sensitivity of a strain gauge is a nuisance in strain measurement. In stress gauge the foil design aims to increase the transfer sensitivity as high as the Poisson's ratio of the base material.

So what happens? In view of this stress gauge will be different for different specimen material, see that we have accepted in strain gauge technology. We have looked at self-temperature

compensated gauges, all these self-temperatures compensated gauges are meant for a particular specimen material. So we are accustomed to this kind of a scenario in strain gauge technology. So you extend a similar argument for the development of the special gauges too.

I have given the result in advance, but we will see how the result is arrived at. So let us look at the expression for $\Delta R/R$, what we looked at earlier we will recall here, that is given as $S_a \epsilon_a + K_t \epsilon_t$. We are not written it in terms of S_g , we have written the generic expression here. And now we are essentially trying to find out on the free surface, we are essentially finding out the state of strain or state of stress on a free surface.

And we consider that as a plane stress situation. So I can replace the strain quantities in terms of stress components by invoking stress strain relations that is what I am going to do.

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EXPERIMENTAL STRESS ANALYSIS Strain Gauges

Stress Gauge – Principlecontd

- The relationship between stress and strain for plane stress is given by

$$\epsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_t) \quad \text{and} \quad \epsilon_t = \frac{1}{E}(\sigma_t - \nu\sigma_a) \quad \longrightarrow (2)$$

Substituting Eq. (2) in Eq. (1) yields

$$\frac{\Delta R}{R} = \frac{\sigma_a S_a}{E}(1 - \nu K_t) + \frac{\sigma_t S_a}{E}(K_t - \nu) \quad \longrightarrow (3)$$

- From Eq.(3) it is clear that if $K_t = \nu$, the output of the gauge $\Delta R/R$ will be directly related to σ_a .

So when you look at the relationship I have $\epsilon_a = 1/E * \sigma_a - \nu \sigma_t$, and ϵ_t is given as $1/E * \sigma_t - \nu \sigma_a$. And when I substitute in the previous equation, I get $\Delta R/R = \sigma_a * S_a / E * (1 - \nu K_t) + \sigma_t * S_a / E * (K_t - \nu)$. So $\Delta R/R$ is a function of axial stress as well as transverse stress, suppose I make $K_t = \nu$ this term vanishes.

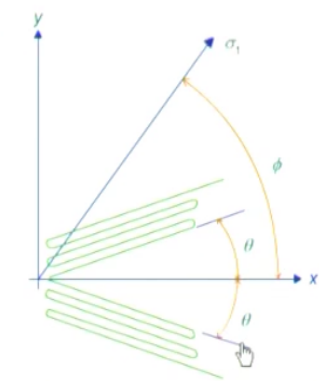
So what you are really looking at is in a strain measurement system, I essentially measure change in resistance, now we have looked at $\Delta R/R$ could be related to axial stress provided the gauge configuration has a transverse sensitivity of a very high value = the Poisson's ratio, see we are not taking a wire and making a strain gauge now, we have the luxury of etching any pattern of my design on a metal foil, because I have the etching process I can design complicated patterns.

In fact, the diaphragm gauge which you saw was the very complex pattern, which was used to measure pressure. So now we will have to go and investigate what type of grid pattern I should design, which will have transverse sensitivity of the order of Poisson's ratio that is what we are going to look at.

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PERIMENTAL STRESS ANALYSIS Strain Gauges

Stress gauge construction



- How to make a strain gauge such that its transverse sensitivity is as high as the Poisson's ratio of the test specimen?
- The answer is simple. The gauge has two elements oriented at an angle and the angle of orientation decides the function of the gauge as a stress gauge.
- This angle is

$$\theta = \tan^{-1} \sqrt{\nu}$$

Note the subtle difference between shear stress gauge and stress gauge.

Proof

So look at this diagram very carefully, and there is a subtle difference between what we saw as a torque gauge, and what we say as a configuration for the stress gauge. Here, also I have 2 elements orientated at $+\theta$ and $-\theta$, but what the subtle difference is these 2 elements are joined they are not independent elements. So I have 1 strain gauge which is made in a fashion having 2 elements which are joined, so the whole thing is 1 strain gauge.

So I will have 1 tab here, I will have another tab here, this has to be connected to the Wheatstone bridge. And I have also caution earlier you must develop a familiarity by looking at a pattern, what this gauge stands for? In fact, you will also be tested whether you have been able to identify

strain gauges' patterns of various types that will be tested in your examination, and you should not get confused between T rosettes or 2 element strain gauge or a stress gauge, there is a subtle difference, the subtle difference you should understand.

The elements are joined here, and this is how you have made a stress gauge, and what is this angle? Before looking into the proof, this angle is found to be $\theta = \tan^{-1} \nu$, that is a Poisson's ratio. We will go and look at the mathematical steps, and convince our self that when I have $\theta = \tan^{-1} \nu$, you will get a correct value for our delta R/R as a function of only the stress.

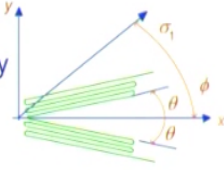
And what is indicated in this diagram is you have the x axis y axis, and in addition you have also located the principle stress direction at the point of interest. So I have principle stress σ_1 is oriented at an angle θ from the horizontal, and when I do the mathematical development I will refer these 2 angles with respect to the principles stress as the reference. So I will have one as $\phi - \theta$, other one as $\phi + \theta$.

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EXPERIMENTAL STRESS ANALYSIS Strain Gauges 01


Proof

- The strain along the top grid element is given by

$$\epsilon_{\phi-\theta} = \frac{1}{2}(\epsilon_1 + \epsilon_2) + \frac{1}{2}(\epsilon_1 - \epsilon_2) \cos 2(\phi - \theta) \quad \longrightarrow (1)$$


- The strain along the lower grid element is given by

$$\epsilon_{\phi+\theta} = \frac{1}{2}(\epsilon_1 + \epsilon_2) + \frac{1}{2}(\epsilon_1 - \epsilon_2) \cos 2(\phi + \theta) \quad \longrightarrow (2)$$



And again you have to go back and look at the strain transformation law that is what I am going to do. These equations are not difficult, these equations you already know, only aspect is we are applying it for a different strain gauge configuration. So I have 2 elements, what way we are

going to look at is, because they are joined together, essentially I am looking at $\epsilon_{\phi-\theta}$ + $\epsilon_{\phi+\theta}$ that is what I am going to finally do.

So I will look at what is the expression for $\epsilon_{\phi-\theta}$, since I have taken reference as the principle stress axis, I write it in terms of principle strains. So $\epsilon_{\phi-\theta} = \frac{1}{2}(\epsilon_1 + \epsilon_2) + \frac{1}{2}(\epsilon_1 - \epsilon_2) \cos 2\phi$, and this you can appreciate, because this is like a normal strain gauge which is sensitive to strain along the gauge length. So that you will call that as $\epsilon_{\phi-\theta}$.

Similarly, this is like another strain gauge, which will measure strain along the gauge length. So that is denoted as $\epsilon_{\phi+\theta}$. So you change θ from $-\theta$ to $+\theta$, see in strain gauge instrumentation if you look at we design complicated gauges, but finally we will also raise a question, can I do the stress measurement with a single conventional strain gauge? If you look at the theoretical development what I have done, I have cleverly taken to start with the principle stress axis, I have taken that axis as a reference.

And you note one more aspect, suppose I have $\phi = 0$, the expression for $\epsilon_{\phi-\theta}$ and $\epsilon_{\phi+\theta}$ will be exactly same. You keep this result in your mind, we will use this result when I go and develop whether a single element strain gauge could be used to measure stress. So we will develop a complicated gauge pattern to start with, in an unknown situation you will do this.

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Proof

....contd

- Summing up these two equations

$$\epsilon_{\phi+\theta} + \epsilon_{\phi-\theta} = (\epsilon_1 + \epsilon_2) + (\epsilon_1 - \epsilon_2) \cos 2\phi \cos 2\theta \quad \longrightarrow (3)$$



But you will also find out whether you could use a simple normal strain gauge for stress measurement. And when I add these 2 strain quantities, I get this as $\epsilon_1 + \epsilon_2 + \epsilon_1 - \epsilon_2 \cos 2\phi \cos 2\theta$, and I will further simplify. My focus is finally to get the addition of these 2 strains proportional to a single stress component that is what I am going to show, and we will not evaluate the angle θ .

But we will say $\theta = \tan^{-1} \sqrt{\nu}$ and then we will simplify the expression. Then we will say take $\theta = \tan^{-1} \sqrt{\nu}$ is what we have said.

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Proof

....contd

- From Mohr's strain circle,

$$\epsilon_{xx} + \epsilon_{yy} = \epsilon_1 + \epsilon_2 \quad \longrightarrow (4)$$

$$\epsilon_{xx} - \epsilon_{yy} = (\epsilon_1 - \epsilon_2) \cos 2\phi \cos 2\theta \quad \longrightarrow (5)$$

- Substituting eqs. (4) & (5) into eq.(3) gives

$$\begin{aligned} \epsilon_{\phi+\theta} + \epsilon_{\phi-\theta} &= (\epsilon_{xx} + \epsilon_{yy}) + (\epsilon_{xx} - \epsilon_{yy}) \cos 2\theta \\ &= 2(\epsilon_{xx} \cos^2 \theta + \epsilon_{yy} \sin^2 \theta) \\ &= 2 \cos^2 \theta (\epsilon_{xx} + \epsilon_{yy} \tan^2 \theta) \quad \longrightarrow (6) \end{aligned}$$



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So that we will do it by induction, and we also know from Mohr's strain circle $\epsilon_{xx} + \epsilon_{yy} = \epsilon_1 + \epsilon_2$ and $\epsilon_{xx} - \epsilon_{yy} = (\epsilon_1 - \epsilon_2) \cos 2\theta$. And we will replace it in terms of ϵ_{xx} and ϵ_{yy} , so when you substitute these into the earlier equation I get $\epsilon_{\phi+\theta} + \epsilon_{\phi-\theta} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{xx} - \epsilon_{yy} \cos 2\theta$.

And finally it simplifies to $2 \cos^2 \theta \epsilon_{xx} + \epsilon_{yy} \tan^2 \theta$. See what we have done is we have simplified the expressions the way we want it, the focus is for what values of θ this expression gets simplified, and that is what we will take it up now.

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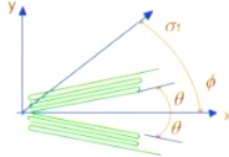
PERIMENTAL STRESS ANALYSIS Strain Gauges

Proofcontd

- If the gauge is manufactured so that $\theta = \tan^{-1} \sqrt{\nu}$

Then, $\tan^2 \theta = \nu$ $\cos^2 \theta = \frac{1}{1+\nu}$

- This simplifies Eq. (6) as

$$\epsilon_{\phi-\theta} + \epsilon_{\phi+\theta} = \frac{2}{1+\nu} (\epsilon_{xx} + \nu \epsilon_{yy}) \quad (7)$$


$$\sigma_{xx} = \frac{E}{1-\nu^2} (\epsilon_{xx} + \nu \epsilon_{yy})$$

This is the last slide for this link. To go to next/other chapters navigate through the main menu button.

$$\sigma_{xx} = \frac{E}{2(1-\nu)} (\epsilon_{\phi-\theta} + \epsilon_{\phi+\theta}) \quad (8)$$

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And we start with the premise if the gauge is manufactured, so that $\theta = \tan^{-1} \sqrt{\nu}$. I specify, what is the expression for θ ? So when I take $\theta = \tan^{-1} \sqrt{\nu}$, I get $\tan^2 \theta = \nu$ and $\cos^2 \theta = 1/(1+\nu)$, and this simplifies $\epsilon_{\phi-\theta} + \epsilon_{\phi+\theta} = 2/(1+\nu) \epsilon_{xx} + \nu \epsilon_{yy}$. And I get expression for $\sigma_{xx} = E/(1-\nu^2) (\epsilon_{xx} + \nu \epsilon_{yy})$.

So I finally, get σ_{xx} is related to $\epsilon_{\phi-\theta} + \epsilon_{\phi+\theta}$ throughout this factor $E/2(1-\nu)$. And what we have seen I have 2 elements these are joined, and when you connect this in one arm of the Wheatstone bridge, I measure essentially sum of the strains. And now I have an

expression to find out the normal stress σ_{xx} directly from strain measurement, so I call it as a stress gauge, only the multiplication factor has to be reset in your instrumentation system.

So what you have is if you design a foil like this having 2 elements, when you make the measurement it is possible for you to find out the normal stress component along the bisector. Now I said, I am not going to stop here, you cannot run for the stress gauge, and we have also seen the stress gauge is dependent on the base material, because of the factor the Poisson's ratio, because I have the factor as Poisson ratio it depends on the base material.

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PERIMENTAL STRESS ANALYSIS Strain Gauges

Single element strain gauge as a stress gauge

- This is possible when the principal stress directions are known.

$$\sigma_1 = \frac{E}{1-\nu} \epsilon_\theta$$

- Locate the gauge along a line which makes an angle $\theta = \tan^{-1} \sqrt{\nu}$ with respect to the principal axis.

$$\epsilon_{\phi-\theta} = \epsilon_{\phi+\theta} = \epsilon_\theta$$

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So I need to go and develop, how to use a single element strain gauge as a stress gauge. See in all this these are all gimmicks; you know once you have understood something you oriented at particular angle you get the result you want. For this I need to know the principle stress direction, without knowing the principle stress direction, I cannot do and use a single element strain gauge to measure stress.

There are certain applications where you can minimize the number of channels by just pasting 1 strain gauge appropriately, and if your design demands a stress quantity along a particular direction, then your job is done. So what you have here is the prerequisite is I need to know the σ_1 direction, and I will have to align a simple normal strain gauge at θ , and θ is given as $\tan^{-1} \sqrt{\nu}$.

And we have already noted, when you have $\phi = 0$, $\epsilon_{\phi-\theta} = \epsilon_{\phi+\theta}$ that $= \epsilon_{\theta}$. So your final expression reduces to $\sigma_1 = E/(1-\nu) \epsilon_{\theta}$. So the trick what we have done is a normal strain gauge can be used as a stress gauge, but it can measure only when it is align with respect to one of the principle stress directions. And the angle at which you have to align it is also specified.

And you know, you do not have to make a gauge which is dependent on the specimen material here. Here, I have to align it appropriately for different specimen material that is all I have to do. So now you have a via media, so if somebody asks you why are you doing only strain gauge you can also go back and say no, no I can also use and develop a stress gauge, it is one of a very common questions.

In interviews people can ask, do you know what is stress gauge? You should not see stars, you should say that there is way to do it, and these are all recent developments, you know people played with metal foil where you have to etch. If they have to live with wire probably they would not have proceeded in this direction, because you have the specialty to etch any pattern of your desire, whatever the design that you have, you are in a position to explore, and in fine innovative methods to improve your measurement.

And what we have done here if you look at stress gauge we have selected a particular parameter, so that one of the terms becomes 0. You have this idea borrowed even in fracture mechanics, my next focus is to find out how to get stress intensity factor using strain gauge technology. First we will see a very elaborate method, then we will simplify it, what we will do is in order to simplify we have to knock off terms.

And what we are going to do is we have to knock out 2 terms in a series. So I will have to work with θ as well as α , in this case we have to knock out only 1 term, so we decided that we align at angle θ . So the idea is similar, you know the people also played with Poisson ratio. So now once you got a queue, you know researchers try in that direction and find out what way this can be further exploited.

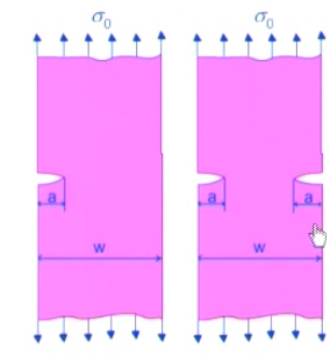
So you would see Poisson ratio is effectively utilized in the calculations, and the orientation decides the functionality of the strain gauge, and I have to knock of 2 terms, so I will have theta as well as alpha.

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PERIMENTAL STRESS ANALYSIS Strain Gauges

Stress Intensity Factor

Edge-cracked tension strips



- The stability of the crack is determined by the opening mode stress intensity factor, K_I .
- The crack will be initiated when

$$K_I > K_{Ic}$$

Single-edge Crack
Double-edge Crack

So that is the way I am going to about, but before we going do it, we will also recall what we have learnt in our course on fracture mechanics. What is a stress intensity factor? Essentially the stress field in the vicinity of the crack is possible to write in terms of the stress intensity factor, and you may have multiple cracks in a system, the crack that is critical is going to propagate. And fracture mechanics says that when $K_I > K_{Ic}$ the crack will be initiated.

So the focus is for a given structure, you need to find out what is the stress intensity factor, here we are essentially looking at simple mode 1 situations. So the idea is there are occasions where you need to experimentally find out what is the stress intensity factor. There are many methods you can do it by numerical methods, you can do by method of cost aces, you can do by method of photoelasticity, you can do by method of Moire Holography.

And we will learn how strain gauge technology could be used. So what I need to do? I need to look at strain field here.

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Strain Field in the Vicinity of a Crack (Mode I)

$$E\varepsilon_{xx} = A_0 r^{-1/2} \cos \frac{\theta}{2} \left[(1-\nu) - (1+\nu) \sin^2 \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + 2B_0 + A_1 r^{1/2} \cos \frac{\theta}{2} \left[(1-\nu) - (1+\nu) \sin^2 \frac{\theta}{2} \right]$$
$$E\varepsilon_{yy} = A_0 r^{-1/2} \cos \frac{\theta}{2} \left[(1-\nu) - (1+\nu) \sin^2 \frac{\theta}{2} \sin \frac{3\theta}{2} \right] - 2\nu B_0 + A_1 r^{1/2} \cos \frac{\theta}{2} \left[(1-\nu) - (1+\nu) \sin^2 \frac{\theta}{2} \right]$$
$$2G\gamma_{xy} = A_0 r^{-1/2} \left[\sin \theta \cos \frac{3\theta}{2} \right] - A_1 r^{1/2} \left[\sin \theta \cos \frac{\theta}{2} \right]$$

Where A_0 , B_0 , A_1 are unknown coefficients which depend on the geometry of the specimen and the loading.



And what is the strain field in the vicinity of a crack. And you know you just look at this expression, you need not copy down this expression, you can copy down later simpler form of this, and what you have here is expression for a epsilon xx, epsilon yy and gamma xy is given. What you need to look at is I have 3 coefficients A_0 , B_0 as well as A_1 , and if you closely look at it is a first term which is related to the stress intensity factor.

And now you all know I am going to paste a strain gauge at an arbitrary angle. So I need find out for an arbitrary angle, what is the expression for strain? So if I know the expression for epsilon xx, epsilon yy and gamma xy, I can write on the strain expression along any axis of interest that is not a difficult aspect. So essentially you have look at this strain felid, and in order to have reasonable accuracy, I am taking a 3 terms solution.

In fact, if you look at I can write infinite number of terms, but those terms are will not be in a position to evaluate, and you should also not stop at only the first term. See in order to enhance accuracy, we want to take as many terms as possible, and we also look at the methodology should be simple enough to measure. So my focus is bring in 3 terms, but knock off 2 terms intelligently, by selecting appropriate angles that is what I am going to do.

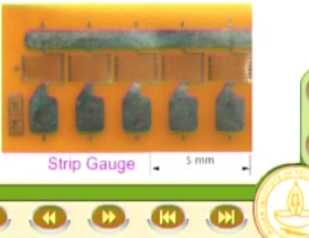
And this is what you summarized here you have A_0 , B_0 , A_1 are unknown coefficients, which depend on the geometry of the specimen and the loading.

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PERIMENTAL STRESS ANALYSIS Strain Gauges

SIF Evaluation by Strain Gauges

- The unknown coefficients A_0 , B_0 and A_1 can be determined if three or more strain gauges are placed at appropriate positions in the near field region.
- If A_0 is known, K_1 can be obtained from $A_0 = \frac{K_1}{\sqrt{2\pi}}$
- Strip gauges have been developed in which, on one backing several closely placed strain gauge elements are etched.
- From these strain gauges, strain information over a field could be evaluated.
- If sufficient data is available, it is possible to extend the overdeterministic least squares method to strain gauge data too.



And we will knock off other terms leaving only A_0 . If A_0 is known K_1 can be obtained from $A_0 = K_1/\sqrt{2\pi}$, this is what I am going to aim at. And if you are confronted with a situation like fracture mechanics, what you would like to do is you would like to measure at several points in the vicinity of the crack tip, find out strain quantities, and in such a case normally what you will have to do? You have to paste a strain gauge at different locations, and their alignment becomes difficult.

So in order to simplify for such applications, you also have what are known as strip gauges that is what is shown here, I think I will enlarge it and then show you what is the strip gauge. So what you have here is you have strain gauges pre-aligned available in one backing, very small strain gauge, and even to solder and connect and paste, all these are very complex issues. But the advantage is you are able to get strain at short distances very comfortably.

So I can essentially get the strain field, and in fact people have developed methodologies by using strip gauges, find out the strain field and solve this in an overdeterministic fashion using the principle of least squares, and you can actually evaluate several terms in the series, and

finally, evaluate only the stress intensity factor. So you could satisfy the strain field information by pasting several strip gauges in the near vicinity.

But the very thought is so difficult you know, because as available as one backing pasting may be simpler, but soldering is going to be very tricky issue, that is where you will find that whole field techniques are better, when I have to get a field information whole field techniques are lot more better, but you also need to have a via media when I use a strain gauge is it possible to get the strain field? If you ask that kind of a question, in principle it is possible, but you have to take lot of effort to do that.

And this is where you know researchers have stepped in, and then played with an equation and found a very simple approach, where a common general purpose strain gauge, a single strain gauge that too could be used to find out the value of K , how elevated it is we will have a look at it.

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EXPERIMENTAL STRESS ANALYSIS Strain Gauge

SIF Evaluation by Strain Gauges

- The number of strain gages required for the determination of K_I can be reduced to one by appropriate choice of strain gauge location and its orientation.
- The principle is to make the governing equation only as a function of A_0 .

So the idea here is the number of strain gauges required for the determination of K_I can be reduced to 1, by appropriate choice of strain gauge location and its orientation. See essentially my interest is to knock off B_0 and A_1 . So I want to knock off B_0 and A_1 terms, so I will look for 2 orientations. So you are trying to find out strain along a particular location, which is certain an angular locations, at that locations you will measure the strain along the particular orientation.

So I have 2 parameters to play with, so I can have the governing equation only as a function of A_0 . See people have done it, this is done out of research, you have result readily available and we are going to discuss in class of 10 minutes, in our 10 minutes we will find out what researches are done in 6 months or almost a year to arrive at implementation of this idea. The idea is I should have the governing equation only as a function of A_0 .

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EXPERIMENTAL STRESS ANALYSIS Strain Gauges

SIF Evaluation Using a Single Strain Gauge

- Commonly available strain gauge can be used to measure SIF provided it is put at a carefully selected point and at a suitable orientation.
- Poisson's ratio of the base material dictates the determination of θ and α .
- These are so selected that some terms in the strain field go to zero such that the strain reading could be easily related to the SIF.

And you will look at here, look at the animation carefully. So what I am going to do is I want use the common strain gauges, so I will have the orientation theta, but at orientation theta I will have the strain gauge aligned at angle alpha. So for this value only I would find out the strain along this direction, and now question is how to select theta and alpha? That is what you will look at it, you make a neat sketch of this diagram that illustrates how to use a single strain gauge for stress intensity factor measurement.

I have this as the crack tip and crack tip was taken as a origin, and you will locate the point P which is at an angle theta from the crack axis. At point P you measure strain along the direction alpha, that is very important, and I have already given you the clue. You know we have the Poisson's ratio of the base material which was used in designing the stress gauge, we will extrapolate those ideas.

And we will use the Poisson's ratio of the base material to find out appropriate values of theta and alpha, so that some terms in the strain field go to 0. So you have to evaluate theta, alpha intelligently to knock off the terms containing B0 and A1. So I will have expression only in terms of A0, then I can find out stress intensity factor from that.

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EXPERIMENTAL STRESS ANALYSIS Strain Gauges

Strain field

- For the rotated coordinates shown in the figure, the strain $\epsilon_{x'x'}$ is

$$2G\epsilon_{x'x'} = A_0 r^{-1/2} \left[k \cos \frac{\theta}{2} - \frac{1}{2} \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \cos 2\alpha \right. \\ \left. + \frac{1}{2} \sin \theta \cos \frac{3\theta}{2} \sin 2\alpha \right] + 2B_0 (k + \cos 2\alpha) \\ + A_1 r^{1/2} \cos \frac{\theta}{2} \left[k + \sin^2 \frac{\theta}{2} \cos 2\alpha - \frac{1}{2} \sin \theta \sin 2\alpha \right]$$

where $k = \frac{1-\nu}{1+\nu}$ G - shear modulus

- The values of θ and α are so selected that some terms in the strain field go to zero such that the strain reading could be easily related to the SIF.

And now for this I need to know the strain along any arbitrary direction, this expression you can write it down. So what I have here is $2G\epsilon_{x'x'} = A_0 r^{-1/2}$ multiplied by $k \cos \theta/2 - 1/2 \sin \theta/2 \sin 3\theta/2 \cos 2\alpha + 1/2 \sin \theta \cos 3\theta/2 \sin 2\alpha + 2B_0 (k + \cos 2\alpha)$. So now you have a clue, how to find out alpha? This should be -kappa, if I have this as -kappa then this term will go to 0.

So find out alpha and then find out theta, that is how you play with the equations. So I have this as $A_1 r^{1/2} \cos \theta/2 [k + \sin^2 \theta/2 \cos 2\alpha - 1/2 \sin \theta \sin 2\alpha]$. So once you look at the expression for the strain gauge which we have pasted, we have it in terms of theta and alpha. And you get a clue how to get the values of alpha as well as theta, and kappa is given as $1-\nu/1+\nu$ and G is the shear modulus.

See whatever the development that you have done in stress gauge, playing with the Poisson's ratio and orientating the strain gauge appropriately, that kind of an approach is extrapolated in

the measurement of stress intensive factor also. Once you strike an idea, you have also will find out whether this idea can work in other situations.

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Selection of alpha and theta

- The coefficient of B_0 term is eliminated by selecting the angle α as

$$\cos 2\alpha = -k = -\frac{1-\nu}{1+\nu}$$

- Next, coefficient of A_1 vanishes if angle θ is selected as

$$\tan \frac{\theta}{2} = -\cot 2\alpha$$



So now I have an expression, I can go and find out what is an expression for alpha and theta. So we have $\cos 2\alpha = -k$, so you have $-\frac{1-\nu}{1+\nu}$. So I can find out what is value of alpha. And if we look at the expression for A_1 that vanishes if angle theta is selected such that $\tan \frac{\theta}{2} = -\cot 2\alpha$. See you have the strain expression, and when you substitute these values of alpha and theta, you will find it is only a functions of A_0 , the terms containing the B_0 and A_1 are cleverly knocked off.

So you play with the angles and you are able to do it. So these are all finer aspects of strain gauge instrumentation, when you normally paste a strain gauge you are only measuring strain along a particular direction, whatever the strain that you measure is intelligently related to the stress intensity factor by your understanding of the strain field expression.

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Selection of alpha and theta

....contd

- By proper placement of a single strain gauge with angles α and θ determined to satisfy the above equations, the term $\epsilon_{x'x'}$ is related directly to the stress intensity factor K_I by:

$$2G\epsilon_{x'x'} = \frac{K_I}{\sqrt{2\pi r}} \left[k \cos \frac{\theta}{2} - \frac{1}{2} \sin \theta \sin \frac{3\theta}{2} \cos 2\alpha + \frac{1}{2} \sin \theta \cos \frac{3\theta}{2} \sin 2\alpha \right]$$

G – shear modulus

And you know if I am going to have alpha and theta comes as 22.32 degrees and 32.5 degrees nobody will use it.

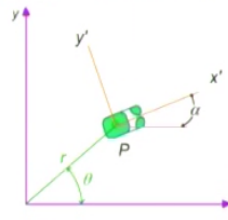
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SIF evaluation using a single strain gauge – Summary

$$\cos 2\alpha = -\frac{(1-\nu)}{(1+\nu)}$$

$$\tan \frac{\theta}{2} = -\cot 2\alpha$$

For aluminium ($\nu = 1/3$) $\theta = \alpha = 60^\circ$



$$K_I = E \sqrt{\frac{8}{3}} \pi r \epsilon_g$$

Where, r is the radial distance and ϵ_g is the measured strain.

This is the last slide for this chapter. To go to next/other chapters navigate through the main menu button.

See people find this as a great use mainly because most popular aerospace material is aluminum, for aluminum both angles reduce to 60 degrees, it makes your life very simple. So that is what we will see now, when I have aluminum $\nu=1/3$ and theta and alpha goes to 60 degrees, see the angle is very important. When the angle is very important if the angle is not possible to achieve, this technique would not have get taken off.

And aluminum is the one which is used in space structure, all aerospace structures that is where maximum amount of experimentation is done, because safety is the very important aspect. And you find fortunately the values of theta and alpha is as simple as 60 degrees, and this is a generic arrangement, you have to look at theta as well as alpha in generic situations. And when I have this the final expression is also available.

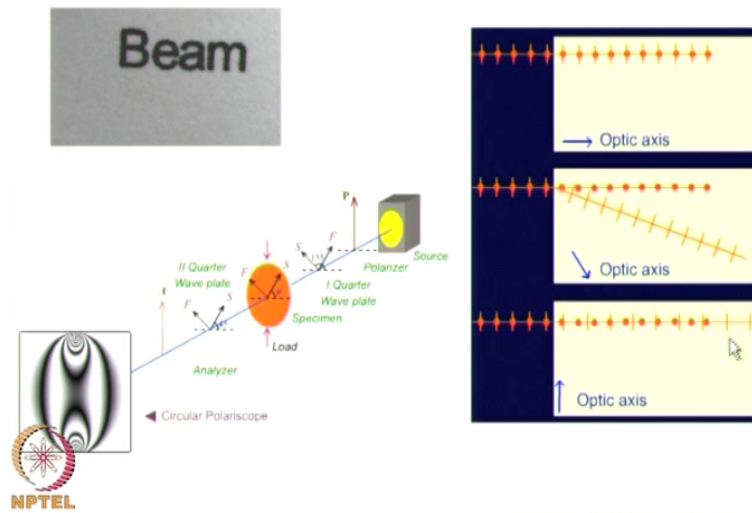
So what I have here is $K_I = \text{Young's modulus } E \cdot \sqrt{\frac{8}{3} \pi r \epsilon} g$, so epsilon is the strain along the particular direction for the case of aluminum, this is measured for the values of theta as well as alpha=60 degrees, for steel you will have to find out theta and alpha because you know the Poisson's ratio you can find out what is theta and alpha. So if you orient this, then you have an expression for stress intensity factor.

See fracture mechanics is becoming very important, earlier it was primarily used for aerospace structures, then people started using it for designing nuclear installations. Now even automobile manufacturers want to look at fracture mechanic aspects, in some aspects of the design. And in future it may seep into many of the day to day activities of design. So knowing how to measure fracture parameters is also very important.

So because strain gauge is a very versatile technique, it is also desirable that you have a method to find out stress intensity factor, approximately even if it is not very accurate. See if you look at in this problem how close the strain gauge should be pasted? It is problem dependent, what should be the value of r? What should be the value of the gauge length? Your accuracy depends on all that. Because when you have a crack, you are going to have a very steep strain gradient.

So those aspects are there, but from a design point of view even if you know what range the value exits that can give a direction on which way to take. And you know people also have a extended these methodologies to composites, even for composites how to find out stress intensity factor from a single strain gauge you have papers available. So with this we come to the close of this course, and we will also have a look at a brief overview of what we have been discussing in various chapters.

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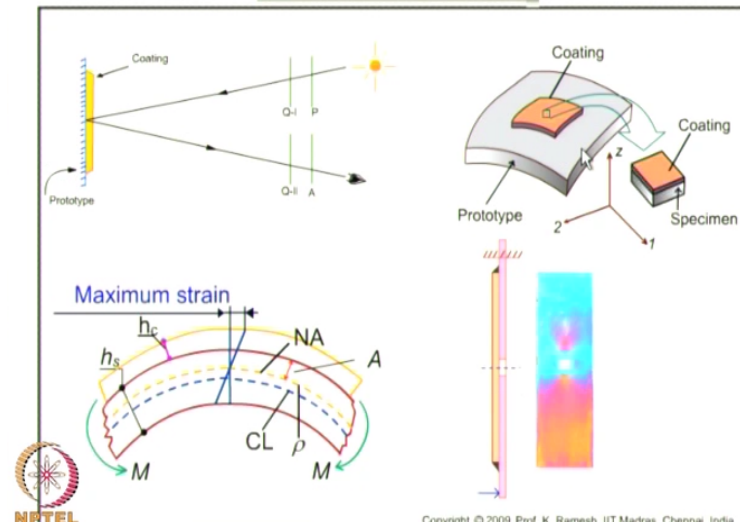


We started with transmission photoelasticity, I said the key concept was when you have model that is loaded, it behaves like a crystal, when it is loaded when the loads are removed it behaves like a normal material. And crystal exhibits a property of birefringence, and this is exploited in photoelasticity, and you are able to view fringe patterns in different optical arrangements, here you have a circular polariscope, you have both the dark field as well as bright field possible.

And we also discussed, what way the incident light should be oriented with respect to the optic axis of the crystal? When it is perpendicular it is useful in photoelasticity, and when you have incident ray, you have 2 refracted rays travel with different velocities. So that is the basic physics in transmission photoelasticity.

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PHOTOELASTIC COATINGS



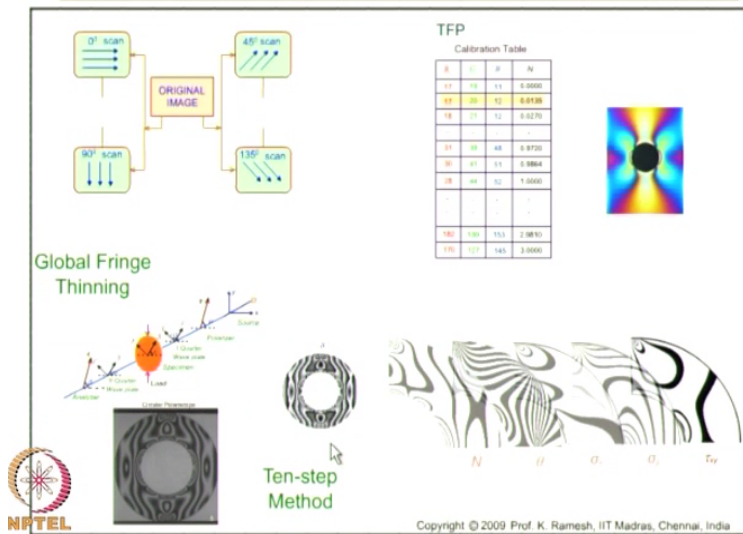
And we said, we also need to have methodologies to analysis prototypes that was discussed in photoelastic coatings. And I said all the coating techniques share a commonality, we have photoelastic coatings, we have brittle coating, and to an extent, you can also think of strain gauge as a coating applied on the material. And one of the key concept here is the strain in the prototype is faithfully transferred to the coating.

And in the case of photoelastic coating you have thickness of the order of 3 millimeter is pasted, so you need to necessarily have what are known as correction factors. Not only this being a industry friendly technique, I have also mentioned that engineering is approximation, and approximation starts right at the data collection stage in the reflection photoelasticity, you do not have a normal incidents, you have only a oblique incidents.

And you want to minimize this angle of oblique, so you keep the polariscope at least 2 to 3 meters away from this specimen, and you see rich fringe patterns, and these are possible to interpret for stress information.

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DIGITAL PHOTOELASTICITY



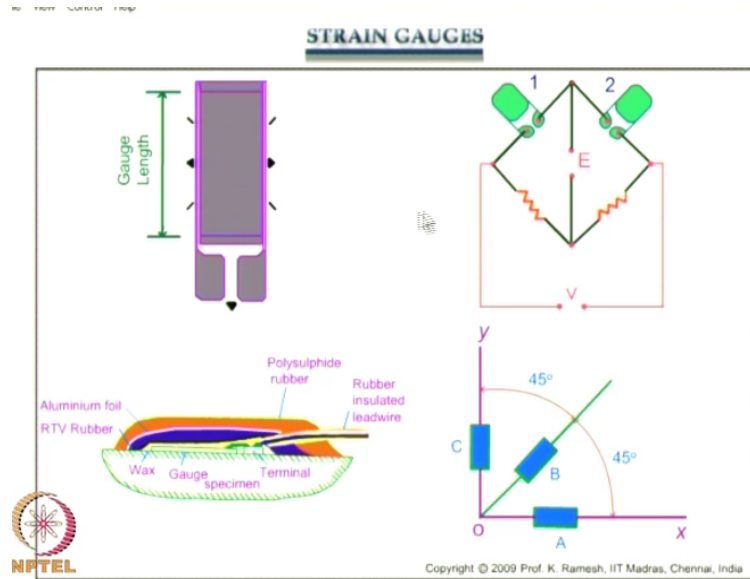
Then we moved on to another aspect of photoelasticity which is becoming very popular these days known as digital photoelasticity. An initial stage they had developed algorithms to thin the fringes that is what you have as the global fringe thinning algorithm. So it employed scan at 0 degree, 45 degree, 90 degree and 135 degree. And logical operators were used to get this skeleton free of noise.

Then I said that you also have extension of color code for quantitative measurement in digital photo elasticity, which is called as 3 fringe Photo elasticity where you are able to find out the R, G and D values and find out the fringe order at the point of interest. And when you want to do whole field analysis, what is possible now is the employment of what is known as a 10 step method, which assures you high quality isoclinic data, as well as isochromatic data.

And this high accurate values are needed for stress separation studies, we are not looked at in detail how to separate the stresses, but if you know N and theta you have methodologies available to get sigma x, sigma y, as well as tau xy. So what you have here is the modern methodologies not only stops at finding out N and theta accurate at every point in domain, you can also post process them, and even go to the extent of separating out the stresses.

And what we have found is among various phase shifting techniques, the 10 step method is found to be robust, and a commercial polariscope could be used to record 10 images. The normal mis elements do not have much influence in the 10 step method.

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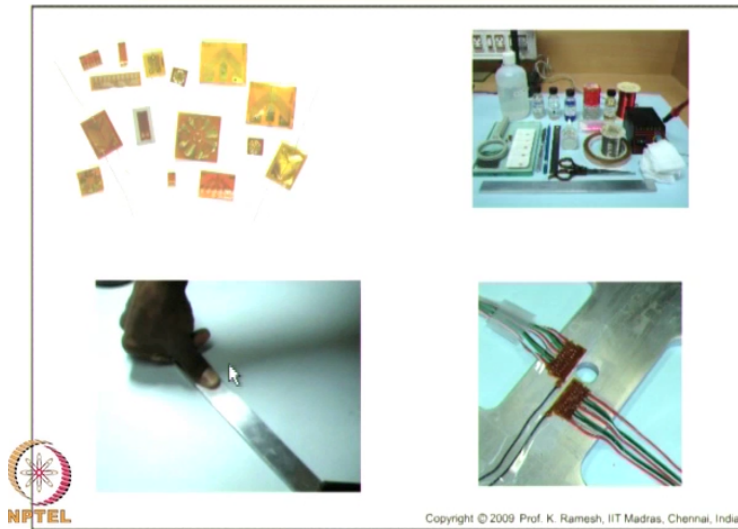


Then we looked at the method of strain gauges, and I said a strain gauge measures strain along the gauge length, it is only measuring a component of strain and for measuring the change in resistance the popular instrumentation is a Wheatstone bridge. And if you have to measure state of strain at a point, you need 3 strain gauges, they are pre-aligned available in one carrier known as a rosette.

And we discussed various finer aspects of strain gauge instrumentation, and we also finally looked at moisture is very bad, and this need to be protected. So you have a multiple layer form with the layer of wax, then layer of rubber, then aluminum foil and finally another layer of rubber.

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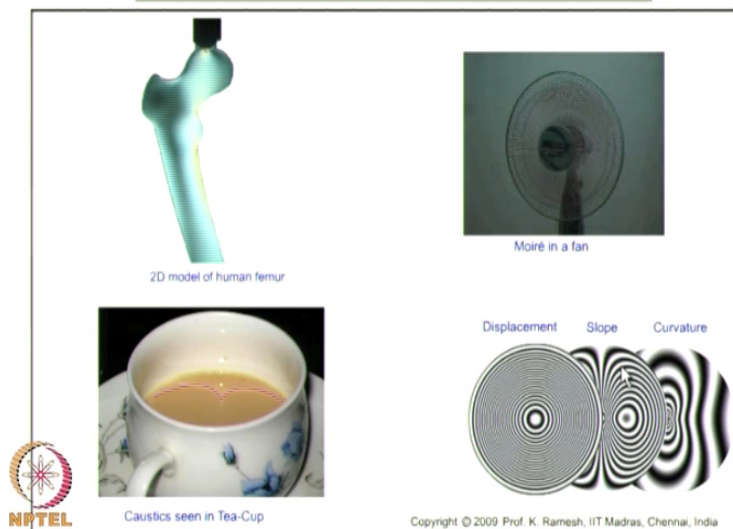
STRAIN GAUGE SELECTION AND INSTALLATION



And I said for any one of this strain gauge instrumentation, strain gauge selection and installation is very very crucial, we saw that variety of patterns exist, and there is a very elaborate kit available. And there is a procedure how to apply the bonding and a simple signoculte cement gets cured by a thump pressure that is what is illustrated here. And you press it with a gauss sponge and put it with a thump pressure and after about 2 minutes the bonding is completed. And this shows use of strip gauges.

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OVERVIEW OF EXPERIMENTAL STRESS ANALYSIS



And finally we come and see, what we learned in the first slide. The focus is I can get stress information, I can get displacement, slope as well as curvature, when I say, what is stress analysis. And what this slide shows is for every experimental technique, there is a physics behind

it, and this physics needs to be understood for you to interpret the result, so that is the focus. So I see a caustic cup here, and I see beautiful play of Moire in the case of the fan.

So you need to have physics understanding experimental methods. Whenever they come across any new physical phenomena, they try to develop a new experimental technique, thank you.