

Experimental Stress Analysis
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Lecture – 03
Stress, Strain and Displacement Fields

Let us continue the discussion on overview of experimental stress analysis. We have seen in the last class what the experimental techniques give directly. We have seen photoelasticity gives the information of σ_1 - σ_2 and the orientation or the principles of direction θ . When you go to Moire, you get displacements. When you go to holography, you get the displacement vector and so on.

So, essentially what we saw was each of the experimental techniques give you a particular kind of information based on the physics that we exploit in getting the information. The other aspect is in a first level course in strength of materials, you try to understand what is stress and you have to understand stress is a tensor. When I say stress is a tensor, you would like to know state of stress at a point of interest. At a point of interest, you are focusing on what happens on all the infinite planes passing through the point of interest.

This is what you get in a study of Moiré circle. So, you understand stress is a tensor, although many times you get only the stress component, there is a danger that you could construe stress as a scalar. The moment you come to any of the optical techniques you have the advantage of getting the information on the entire field. So, you get the whole field information and you get this whole field information in the form of contours.

So, when we go further what we are going to do is how to do the experiment, what physics is employed and how to interpret we will study in detail later. To start with, it would be desirable to see these contours for a class of problems from known situations to unknown situation, that is what we are going to see.

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EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

Typical Results for Various Problems

The problems considered are

- ★ Beam under four point bending
 - Closed form solution by Strength of Materials is possible
- ★ Cantilever Beam
 - Engineering analysis possible by Strength of Materials.
- ★ Disc under diametral compression
 - Only Theory of Elasticity can provide closed form solution.
- ★ Clamped circular disc with a central load
 - $w = \frac{3\nu}{8} \frac{P^2}{D^2} + \frac{3\nu}{8} \frac{P^2}{D^2}$ obtainable from theory of elasticity
- ★ Spanner tightening a nut
 - Due to complex nature of the geometry only a numerical solution is possible

For all these cases relevant experimental fringe contours (recorded or simulated) are shown to appreciate the nature of fringe contours.

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We have discussed in the last class the problems considered are beam under 4-point bending, cantilever beam, disc under diametrical compression, clamped circular disc with a central load and finally spanner tightening a nut. So, this is based on increasing degree of complexity. Beam under 4-point bend easy to solve from strength of materials approach and you have a closed form expression.

The moment you come to cantilever beam, you have shear but you do an engineering analysis by strength of materials and when you come to disc, you cannot do by strength of materials but theory of elasticity can provide closed form solution and clamped circular disc, you could find out the displacements slope and curvature from a study of theory of elasticity, theory of plates and finally spanner tightening a nut, we have seen earlier that it is a down to earth problem.

Surprisingly, you cannot solve it by analytical methods, you have to solve it either by numerical or by experimental methods alone.

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EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

Typical results for various problems

Beam under pure bending – Analytical solution

$$\frac{M_b}{I_z} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$

$$\sigma_x = -\frac{M_b y}{I_z}$$

Stress tensor

$$[\sigma] = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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Now, we take up the problem of beam under 4-point bending and what you have here is, I have a beam and this is shown as X axis, this is Y axis and you have taken central portion of the beam which is subjected to bending moment M, B and in this you have the stress variation over the depth as like this which shows the variation is linear and central core does not contribute to load shearing.

You have a famous flexure formula here which is in slightly different form than what you might have seen in some of the books. Many times people simply say $M/A=F/Y=E/R$, it is written in different fashion, know the difference, it is one of the same. This is much more precise mathematically, we show that this is a bending moment indicated by a subscript B and the moment of inertia is indicated as I_{ZZ} depending on the axis that is chosen for the problem.

When you come to the second term, you also have a sign attached to sigma X. This is minus sigma X/Y, why this is so is the sign convention adopted is on a positive surface anticlockwise moment is taken as positive and for a loading like this, the top fibre of the beam is subjected to compression to indicate that you have a negative sign; and once you have this, it is easy to find out what is the value of stress component. You can write it, it is child's play.

Please write down the expression and this turns out to be $-\frac{M_b y}{I_z}$ (05:27) and what you get here, you get as a function of Y sigma X changes linearly and you also find that sigma X is not a

function of X . So, it is constant over the entire length of the beam which is taken slightly away from the points of loading and this is what you get in a first level course expression for σ_X . When you look at, it is only a component that you are looking at and what you have to understand is you have to understand that this is just not a number, you have to understand this as a tensor.

So, when you say tensor, I have to plug it in a 3×3 matrix appropriately this component and fill the other components. In this particular example what you have is you have the stress tensor, only the component σ_X is exist, all other components are 0. What (0) (06:33) important is the 0s are very important and you know if you look at the life history of Ramanujam, when he was a young student, he was sitting in a class of mathematics and he was observing 0s.

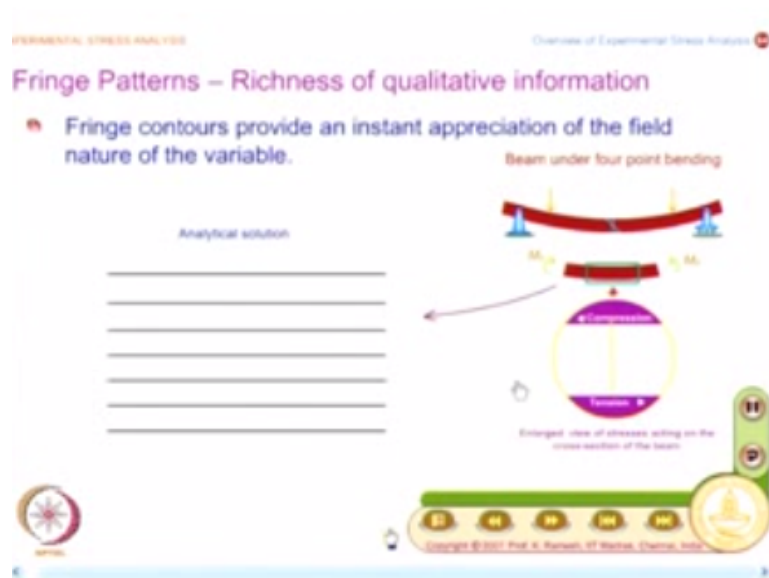
When you put the 0 before the number the value of the number decreases, when you put the 0 after the number the value of the number increases. You know, this is a very-very important and pertinent observation. 0s play a very important role and if you look at the history of science, Indians are credited with giving the concept of 0 to the entire world. So, though in this case you have 0s present, you have to respect those 0s.

They serve a very important purpose and when I have a failure analysis, then those 0s also play a very important role. The failure planes could be different depending on the material on hand. Once you have this, I have the analytical solution and it is possible for you to plot the value of the type of contours that you could get and we have seen earlier that we get only σ_1 - σ_2 in the case of photoelasticity. Since you have the stress tensor, you know how to find out the principle stresses.

Then, you can find out what is σ_1 - σ_2 and you can comfortably plot it and when I say contour what is that you mean. Along the contour the value or the variable is same. So, you plot discrete contours and I would like you to make some effort because the problem is very simple and find out how you can plot this contour, what way they look like. Let me give you few minutes of you time, take few minutes of your time and then try to see how you can plot this contour.

This is fairly highly straightforward because if you know how to plot contours, then you will know how to interpret the experimental results. Because you need to have a visual appreciation of the variable over the domain and if you have done that what I will do is I will show you, some of you must have done it. I see some of you must have done it already.

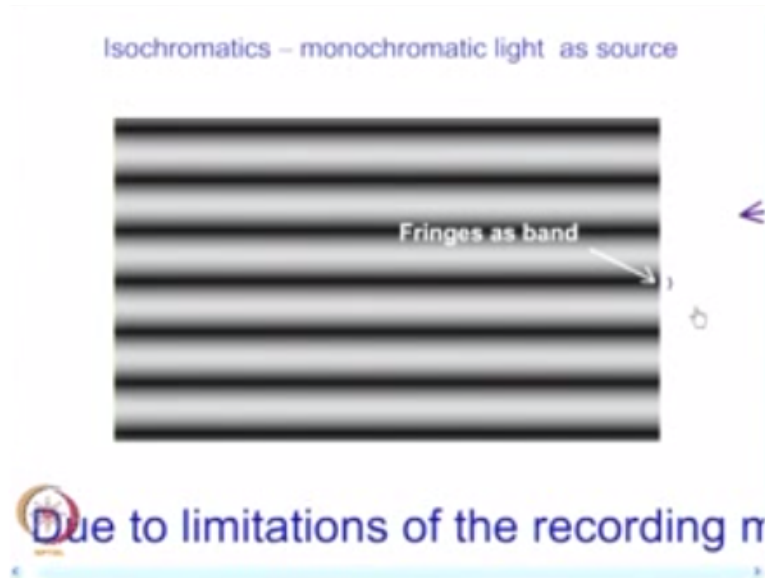
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If you look at I have in this zone the contours are essentially horizontal lines and in fact, if you go and look at many of the engineering problems you get very interesting contours, in some cases straight lines, some cases circles, concentric circles, circles touching at edge, so they give beautiful patterns. In fact, Prof. Durelli has written an article on art and science where he has shown how the fringe patterns could be related to a piece of art.

So, fringe pattern gives a lot of information. So, if you are an artist you look at the artistic value of it. If you are a stress analyst, you look at the value pertinent to your requirement and now what I do is that we look at an experiment, what does it give. I will also magnify this picture and you have this as an analytical solution.

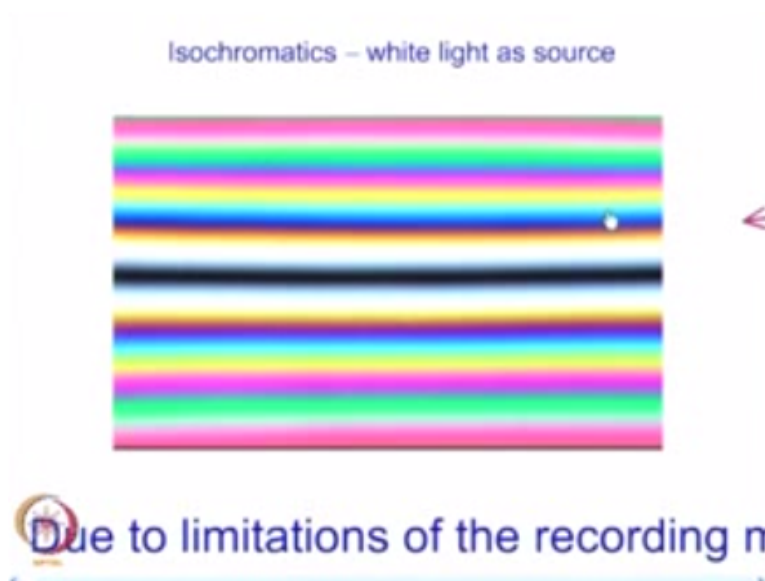
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Now, what I do is I take the model put it on optics and find out what the contours look like. I do not get straight lines as thin as possible, I get only a band. I get essentially straight bands. So, this result matches with what you have seen in analytical solution with a difference. The difference is I see the fringes as a band rather than a single line and I see this as black-and-white contours mainly because I have used monochromatic light source for elimination.

In most of the optical techniques, it is desirable to use monochromatic light source because you have single wavelength and data interpretation is far more simple.

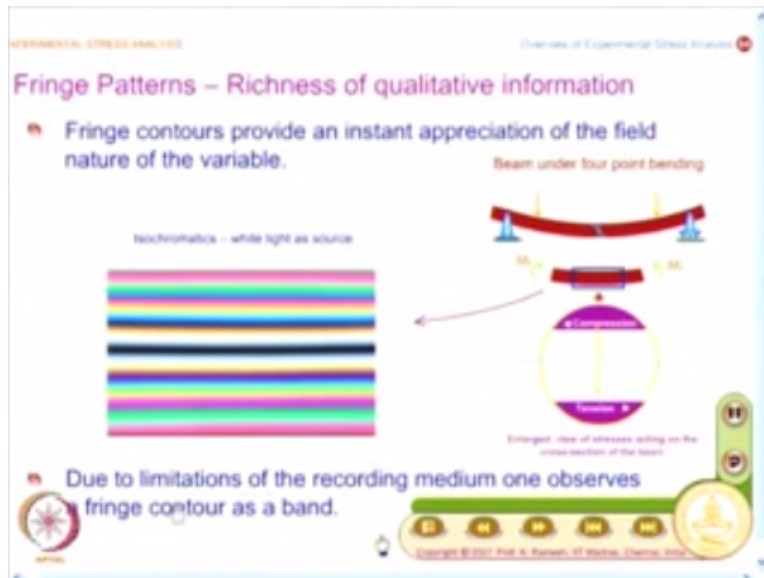
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On the other hand, if I use white light which is the uniqueness of photoelasticity, I get the same

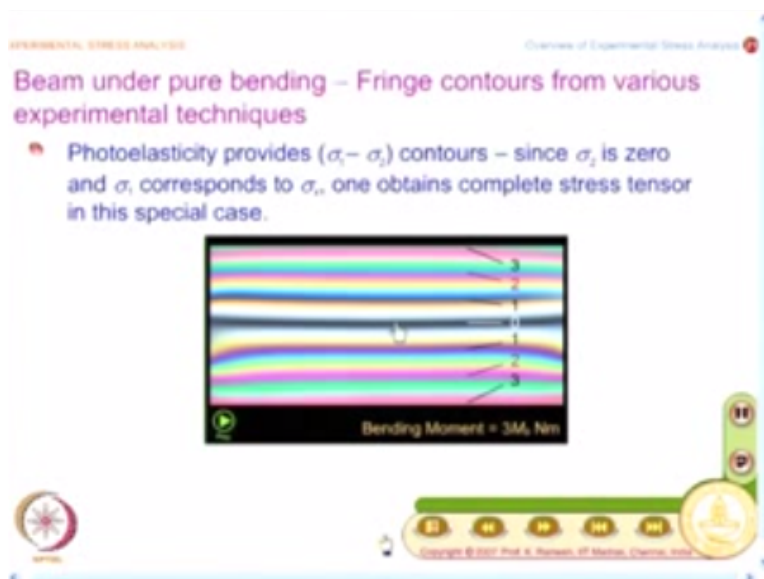
fringes appearing as beautiful play of colours and the advantage with colour is using the colour it is possible to label and odd with the fringes and why do you seek the contours as bands. You see contours as band mainly because of that deficiency in the recording medium. You observe this as a band because of the deficiency in the recording medium.

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If I use a very fine digital camera, you would be able to find out the variation in the greyscale and pick out minimum intensity points as a (0) (11:58) and now what we will do is, it is also desirable that we go and look at how to number these fringes.

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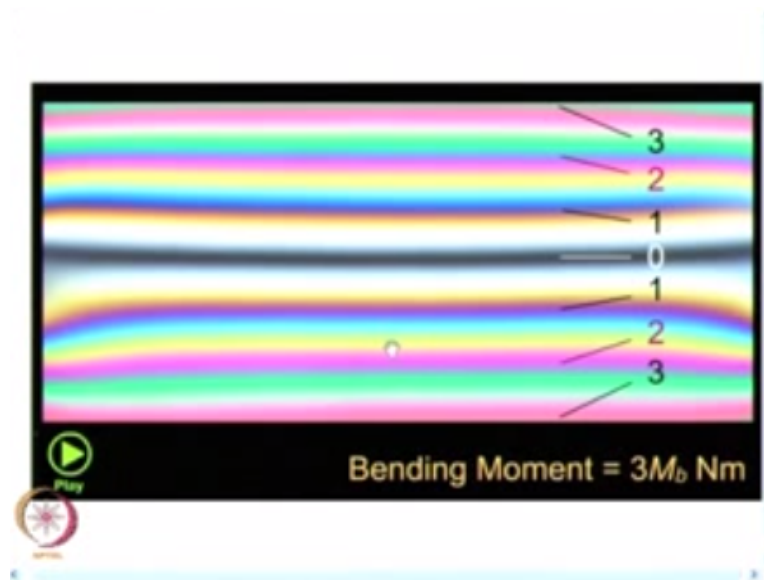


What we do is, we now go back and see what photoelasticity gives. It gives you contours of

$\sigma_1 - \sigma_2$. In the bottom half of the beam, you have σ_1 corresponds to σ_x and σ_2 is 0 and $\sigma_1 - \sigma_2$ contours appear horizontal and you see some deviation from straight line in this zone because this is closer to the load application point in the experiment and what you see here is I have increased the loading in steps of one and you find that this is fringe order 0, fringe order one, fringe order one at the bottom and when I double the load, I see the fringes double.

So, it is a function of load and I can also label them and when I triple the load, the fringes are increasing from 0 to 3 on either side.

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What you can do is by looking at the colour it is possible for you to identify the number associated with the fringe in photoelasticity reasonably well with I and with developments in digital image processing, you have techniques like 3 fringe photoelasticity has been developed where you could pick out this (0) (13:32) values very comfortably and then label these numbers.

One of the challenging task in any optical technique is how do I label these number. This is not something trivial and this is where many of the automated techniques people want to have where they would like to minimise the human intervention. Now, come back to the stress field analytical approach. Now, you have got the stress field, you have also seen the stress tensor. With your knowledge of mechanics of solids, it is possible for you to find out the strain tensor.

It is not difficult at all. Can I request you to look at, brush your old memories and find out how do you find out the strain tensor? because here the stress tensor is available. From stress tensor, you can easily find out the strain tensor and you have to be very careful while applying this step and what you find here is stress tensor, you have only one component, and it is a uniaxial state of stress. The moment I go to strain tensor; I find something different.

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EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

Typical results for various problems

Beam under pure bending - Analytical solution

- Using stress-strain relation one can find the strain tensor

Strain tensor

$$[\epsilon] = \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = \epsilon_z = -\nu \frac{\sigma_x}{E}$$

I find strain tensor is no longer uniaxial but it is triaxial and can you say what is the value of epsilon XX, epsilon YY and epsilon ZZ. In epsilon XX, there is no problem. You will be able to directly write from your understanding of even simple tension test and what many people may ignore is the effect of Poisson's ratio. They may ignore the presence of epsilon YY and epsilon ZZ. So, you have to be very careful about that. There is a possibility that you may ignore the presence of these 2 strain components.

So, what you have is you have a stress tensor which gives you uniaxial state of stress. On the other hand, when I go to strain tensor, I have a triaxial state of strength. You may ask, suppose I make a mistake and I go and interpret the experiments, will experiments understand this or not. I have always been saying experiment is truth. You may make approximations in your formulation to simplify your mathematics; and in some cases if the values are reasonably small.

We may also ignore it but the moment you come to experiments, they always reveal the truth. What is the role of these strain components? ϵ_{YY} and ϵ_{ZZ} .

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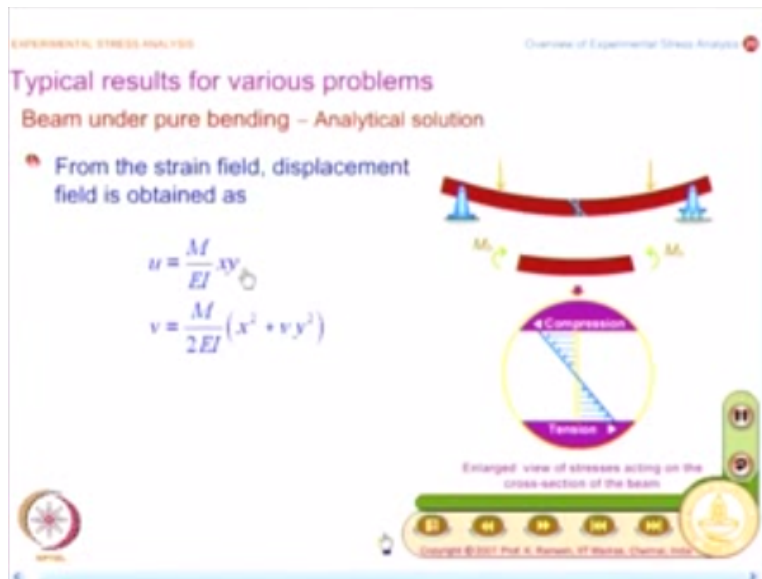


You have a Poisson ratio here and when you have the beam under bending, I would like to show you the model, if I take the model and bend it and what I have here is the beam bends here and this is one curvature you have and this is what you do it in your simple strength of material approach. I have said that in addition to this, you also have strain because of Poisson effect and what you find is if you look at sideways, if you look at it sideways, what I find is I have a bulge at the bottom and it is not bulging out in the top.

Why this has happened is in the way I have bent, this fibre is subjected to tension and the inner fibre is subjected to compression and what happens because of the Poisson effect, the bottom portion bulges out and top portion narrows down; and you also see one more interesting thing, you have additional curvature here because of this and this curvature is known as anticlastic curvature and none of this you might have heard in your first level in strength of materials, if people have not gone deeper into the analysis of beams.

Only when you go deeper into analysis of beams, you will find that these concepts you have to look at it and what happens the experiment looks at it, experiment reveals this and that we will see very closely when you see the fringe pattern.

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We will see the fringe pattern. It is will a second for it to come and what you will see is when I magnify it, you will see in a very subtle fashion what you have on the top fibre. In this particular fibre, the top fibre is subjected to compression and bottom fibre is subjected to tension. So, you will have the fringe order slightly greater than what is there at the bottom. So, it is not perfect symmetry. So, the experiment captures this,

You may call it as anticlastic curvature, but experiment reveals this information. So, you will not have perfect symmetry, there will be a small deviation and you shall not come back and say I have done the experiment and the experiment is wrong, it is not so. In your analytical evaluation of stresses, if you also bring in the effect of Poisson ratio effect, then the stress patterns will be like this. So, experiment always gives you truth and that you should never forget it, okay.

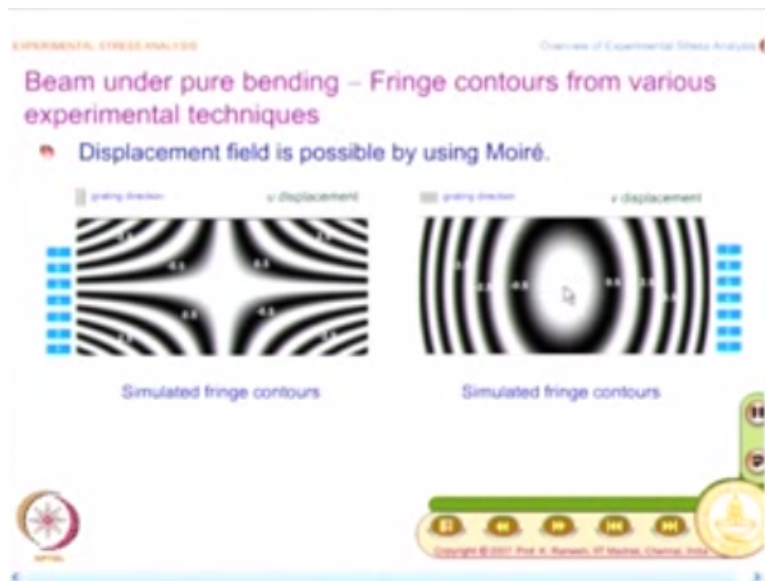
Now, what we are going to do is we will again go back to the slide and what we will do is we have seen the stress tensor, we have seen the display of strain tensor. Now, with that information using mechanics of solids, that is using the equations of theory of elasticity, you could integrate the strain quantities and get you the displacement values and I do not think, we would have done it in a first level course because you are happy with the stresses, you are happy that the in-bending central core does not contribute to load shearing, that is all you focus on that and you do some exercise on finding out would beam deflection, beyond that you do not go and study

deeper.

Now, when I look at this, I can get the displacement field and displacement field is like this, it is fairly simple. I have this $U=M/EI*XY$ and this is nothing but an equation of a hyperbola. So, when you plot the contours, you will have the contours looking like hyperbola and you also have an expression for the V displacement which is given as $M/2EI*X^2 + V*Y$ square. In your deflection calculation, you may not have calculated U displacement at all. You would have only found out the V displacement.

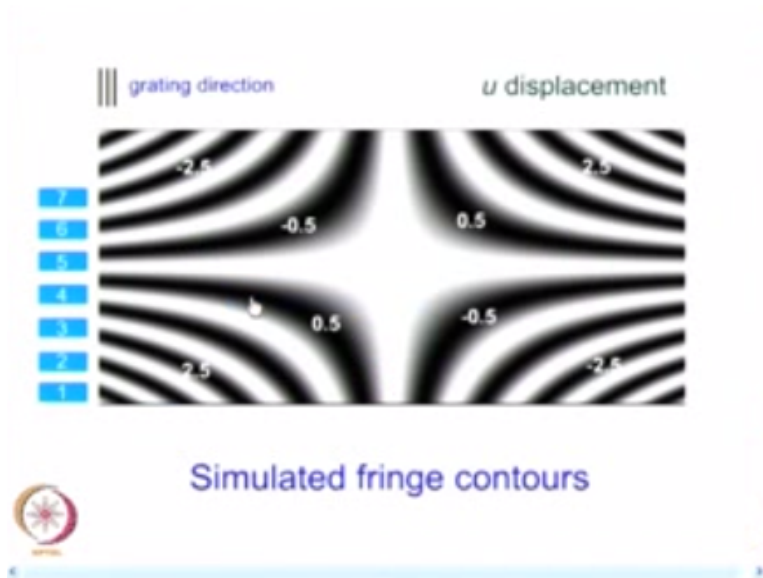
You would not have found out the U displacement, you would have determined only the V displacement. Now, what we will do is we have seen the fringe pattern from photoelasticity which gives essentially contours of the $\sigma_1 - \sigma_2$. Now, our interest is to see other contours.

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What I can do is I can do a contour, I can look at geometric Moire and find out how the displacement field is and this is what the U displacement and this is what the V displacement and we would see this little closely.

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What you see here, I have the fringes labeled and you have 0.5, 1.5, 2.5 like this and you have -0.5, -2.5. So, once I go to displacement I have both positive and negative numbering of the fringes. This is first observation. The second observation is the fringe pattern comes with an indication on what is the grating direction that has been used. When you go to Moire for me to reveal the displacement information, I have to use a grating and grating direction dictates what competent of displacement I do get.

I would like you to have a reasonable sketch of this and when you do this, you will have an idea how the whole field pattern looks like.

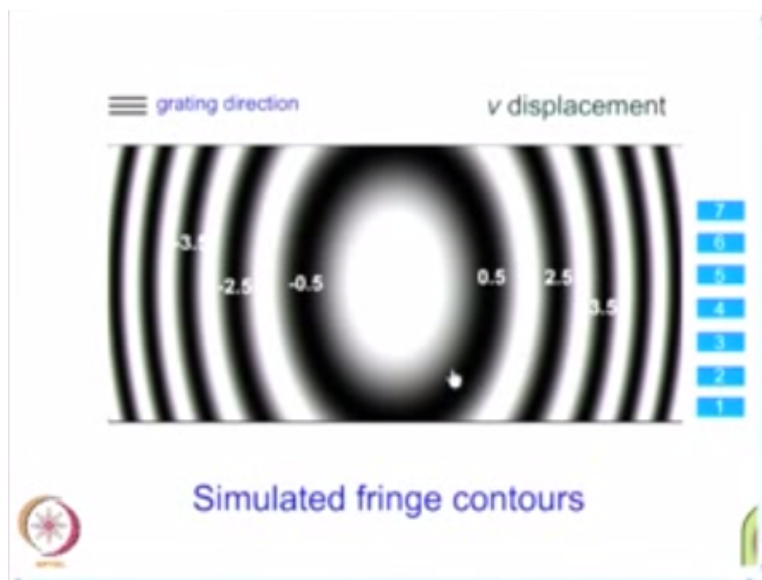
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There is many indirect learning you learn, one is suppose, I change load and you see this is how the fringes are developed. As the load is increased more and more, fringes are developed and this is a very key information in most of the experimental techniques to label the fringe orders as an auxiliary method to do that. Another one what you find is the fringe thickness is different. This also carry some information. This we will see later part of the course.

So, you have a familiarity when I want to look at a displacement field in the case of a beam under 4-point bending, it would look like this and we will also have a look at the V displacement, have you been able to make a reasonable sketch of this, okay.

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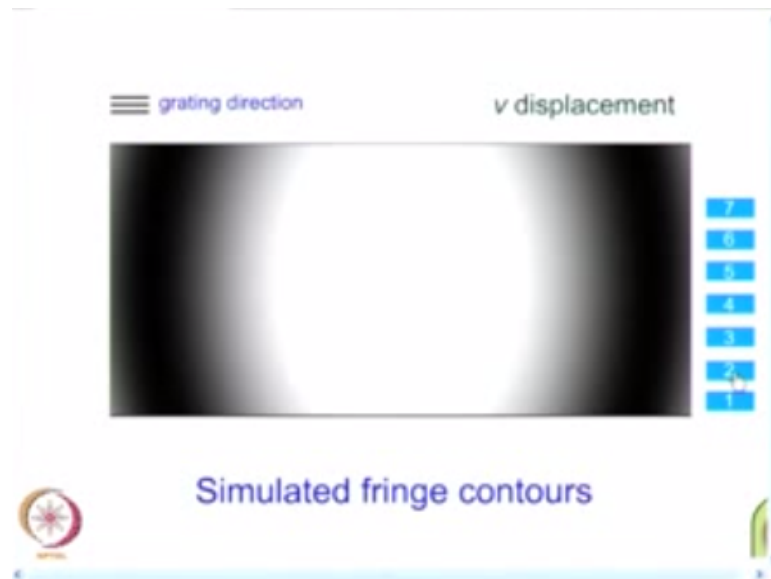
So, what I will do is I will see the V displacement and V displacement is like this and here you find, I have the grating direction horizontal. So, by comparing these 2 figures you can easily understand what I say as displacement is a component perpendicular to the grating direction and what I want you to note down is when I have grating direction in one way, I need to have one optical arrangement to record it.

When I have the grating direction, in another way I need another optical arrangement to it and this is what I have said earlier experiments will reveal information of a particular kind. They may have the capacity to reveal more than one information but they may require 2 different measurements scenario and when I have 2 different measurement scenario what I need to do is I

need to have, there also methods where they combine and try to get more than one information in one go.

When we you look at Moire, we will find out how you can get both U and V displacement simultaneously and what you should keep in mind is if I get 2 information together, it is always a nuisance and here again we can see as a function of load increase, how the contours appear.

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This is very important and this will give you a sort of understanding. It also gives you a feeling of doing the experiment right in the class. When I increase the load, I see fringes are formed and here again, the labeling I have positive and negative quantities. If you have noticed very carefully in photoelasticity we have not labeled negative fringe orders. We have labeled all of them as positive and this is an important point that you have to note. Suppose I want to find out the strain, what do I do.

Now, I have the displacement. I differentiate the displacement; it is possible for me to find out the strain.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Beam under pure bending – Fringe contours from various experimental techniques

- Strain field is obtainable from the displacement fields – however differentiation in experimentation is prone to error.
- Use of Moiré interferometry is suggested to get the strain field.
- Strain at discrete points could be evaluated by strain gauges and along lines of interest by strip gauges.

But I may not want to do that if I have my record from geometric Moire because geometric Moire gives with less accuracy and I have to go to Moire interferometry and then do the differentiation and get the strain field. Strain at discrete points could also be evaluated by strain gauges. Suppose I want for a series of points, then I have what is known as strip gauge in one backing several strain gauges are available at fixed lengths given by the manufacturer.

This is done by at the time of manufacture itself, one grid containing several elements of strain gauge. So, I can get the strain from strain gauges from point to point or along the line of interest.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Cantilever beam with an end load – Analytical solution

Stress field

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{-Pxy}{I} \\ 0 \\ \frac{-P(c^2 - y^2)}{2I} \end{Bmatrix}$$

Strain field

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{-Pxy}{EI} \\ \frac{\nu Pxy}{EI} \\ \frac{-P(c^2 - y^2)}{2IG} \end{Bmatrix}$$

So, we have seen reasonably various contours that is possible from experiment for the problem

of beam under 4-point bending. Now, let us move onto cantilever beam and what is the difference, because of the load here you have bending moment varying across the length of the beam, that is what is seen in the first term here. You do not have a bending moment but the bending moment is a function of X, that is what you find here and in addition you also have a shear stress.

For convenience, the stress tensor is simply given as stress field where I give only the component sigma X, Sigma Y and (()) (27:22) XY, fairly straightforward. This you have solution from strength of materials. What you may not have done is you may not have calculated the strains and the strain field looks like this. I have minus Pxy/EI, epsilon Y is nu times Pxy/EI, gamma XY is -P times C square/2IG and obviously here the stress field is little more complex, strain field is little more complex.

And you will also anticipate the displacement is going to be complex than what have seen in the case of a beam and we will have a look at it. The point to note here is when I do an analytical method, I am able to get the stress field, I am able to get the strain field, I am also able to get the displacement field and that is what you see here.

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The slide contains the following text and equations:

EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

Cantilever beam with an end load - Analytical solution

Displacement field

$$u = -\frac{Px^2y}{2EI} - \frac{vPy^2}{6EI} + \frac{Py^2}{6IG} + \left(\frac{Pl^2}{2EI} - \frac{Pc^2}{2IG} \right) y$$

$$v = \frac{vPxy^2}{2EI} + \frac{Px^2}{6EI} - \frac{Pl^2x}{2EI} + \frac{Pl^2}{3EI}$$

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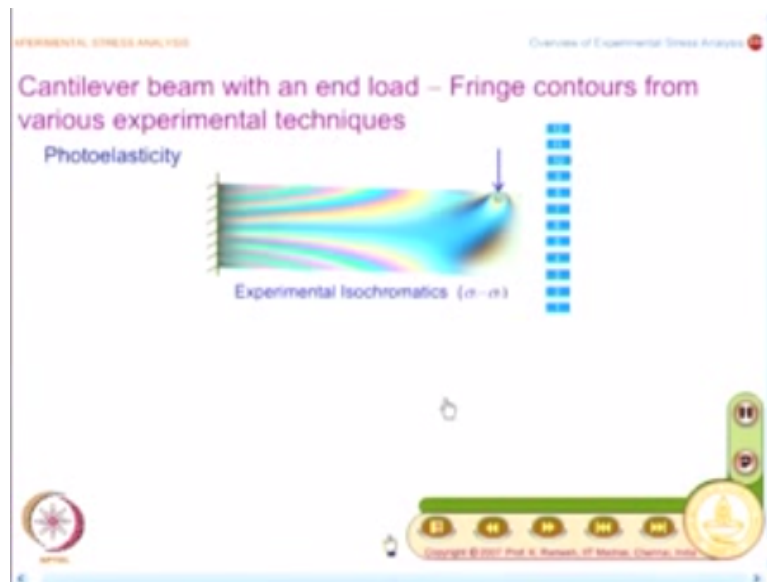
Displacement field is slightly more involved and you would have worried in your basic course in strength of materials only on the V displacement and when X is 0, you get PL cube/3EI and you

do not find out what happens for the V displacement as a function of Y and what you need to keep in mind is you have simply taken a 2-dimensional beam as simply a line and then analysed it. You have not considered this as a beam and then analysed it.

When you consider it as a beam, you will also have to have a variation over the depth Y and that is why you have the U expression if you look at it is $-Px^2y/2EI - \nu Py^3/6EI + Py^3/6IG + (PL^2x/2EI - PC^2x/2IG)y$. What is interesting thing about this expression. When y is 0, u is 0. So, you do not find out u displacement at all in strength of materials mainly because you are considering a 2-dimensional problem as a one-dimensional problem.

So, when you come to theory of elasticity, you have made an improvement and obviously now we will go and look at what the experimental methods give as contours.

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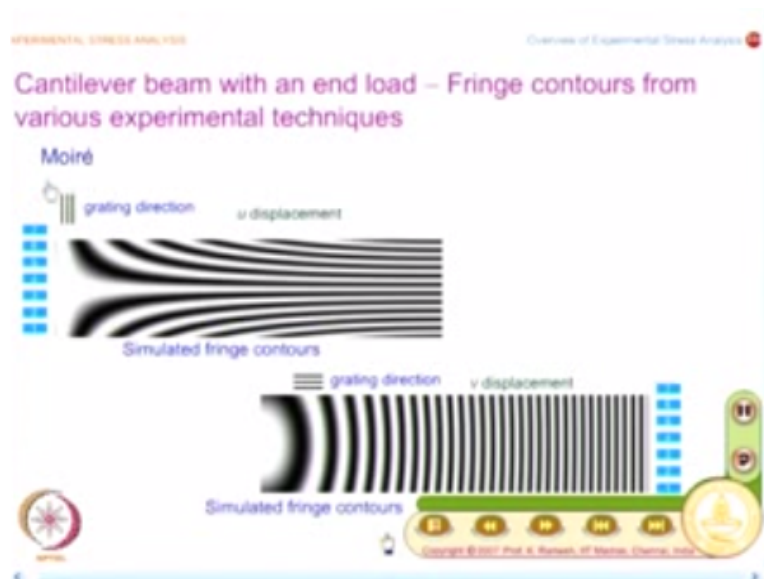
So, first thing what we have always been doing is photoelasticity and you all know that it gives you contours of $\sigma_1 - \sigma_2$ and these contours you would not be able to calculate right at the class. It is better that you have computer software to evaluate it and also have a post processing where you can go and plot it. As before, I would like you to have a reasonable sketch of these fringe patterns.

What you find here is these fringe patterns are definitely different from what you have seen in a

beam under 4-point bending. So, what you find is the stress field is different which your contours directly bring out. You have the effect of shear which is embedded in this stress field and another important point to note is on the centre of the beam you have a light shade of blue. It is not black like what you have in your 4-point bending.

This is the load application point where you have fringes concentrated and this is clamped in and this is how you get experimental isochromatics. So, experimental isochromatics are different from what you have seen in the case of beam under 4-point bending and naturally the displacement contours is going to be still more complicated. So, you have U displacement, you have V displacement.

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What you have here is I again have a grating direction and knowing this grating direction, it is possible for you to say it is U displacement. Knowing this grating direction, it is possible for you to say this is V displacement and you also have a feel of doing the experiment by seeing what happens as a function of load. I gradually increase the load and more and more fringes get formed and you could make a simple sketch of it, try to get only the skeleton, you do not have to draw the band.

You get the geometric shape and a few fringes will give you an idea how do I get the fringes in the case of a cantilever beam subjected to end load and I can increase the load, more and more

fringes get formed. Then, I can also look at the V displacement. This also I can look at as a function of applying load and you will see as the load is increased, you notice this zone more and more fringes develop and they become denser in appearance. I increase the load 2, I increase the load 3, increase to 4, 5, 6 and so on.

So, what you find here is it became almost like parallel lines, more fringes are seen in this zone and when you draw the sketch it is not necessary that you draw all these lines. You just draw a few lines indicative that you have fringes like this and slightly changes curvature as it goes close to the clamped end. These are simulated fringe contours as I have said, some of you who have an exposure to computer graphics can make an attempt, you have those equations.

You can go and make a contour plot and bring it to me and you will have a doubt how to get the fringe width. Let me keep that as a secret for the moment and we will see how to get this band also when you do the plotting.

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The slide is titled "EXPERIMENTAL STRESS ANALYSIS" and "Overview of Experimental Stress Analysis". The main heading is "Disc under diametral compression – Analytical solution". It contains two bullet points: "The famous assumption of plane sections remain plane before and after loading is not possible and one has to resort to the method of theory of elasticity to solve the problem." and "Closed form solution for stress field is as follows". To the right is a diagram of a circular disc with a vertical load P applied at the top and bottom. A coordinate system is centered at the origin with the x -axis horizontal and the y -axis vertical. A radius R is shown from the origin to the edge of the disc.

Now, the next problem is we move onto disc under diametral compression and that is what shown here. I have the disc and the centre of the disc is taken as the origin. I have X and Y coordinates and R is the radius of the disc. I have a diametrical load P which is acting on it and we will call this diameter as either D or d and for this problem you have closed form solution from theory of elasticity.

You will not be able to approach and solve the problem from strength of materials but you will be able to solve only some theory of elasticity because plane sections do not remain plane before and after loading. You also have this problem in the case of a cantilever. When you have shear, the planes do not remain plane, they have a wrapping. Fortunately, there is no coupling between normal stress and shear stress.

So, you could live with flexure formula, that is why call it as engineering analysis. The moment you come to circular disc, you have to depend on theory of elasticity.

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Disc under diametral compression – Analytical solutioncontd

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = -\frac{2P}{\pi t} \begin{Bmatrix} \frac{(R-y)x^2}{r_1^4} + \frac{(R+y)x^2}{r_2^4} - \frac{1}{D} \\ \frac{(R-y)^3}{r_1^4} + \frac{(R+y)^3}{r_2^4} - \frac{1}{D} \\ \frac{(R+y)^2 x}{r_2^4} - \frac{(R-y)^2 x}{r_1^4} \end{Bmatrix}$$

$r_1^2 = x^2 + (R-y)^2$ and $r_2^2 = x^2 + (R+y)^2$, R denotes the radius of the disc, D represents its diameter, t is the thickness of the disc and P is the compressive load applied.

Fortunately, theory of elasticity provides the solution. Here again it is given in a convenient form, sigma X, sigma Y of XY and you have this as -2P/pi T and I would request all of you to take down these equations. Though these equations are very long, they are very valuable information. When we develop photoelasticity, we could directly use these equations for our interpretation and you have sigma X R-y x square/r1 power 4 and r1 is defined as r1 square=x square+R-y whole square.

And you have a second term is R+y x square/r2 power 4 and r2 square is defined as x square+r+y square. We have already seen R denotes radius, D represents diameter, t is the thickness and P is the compressive load applied. You have the expressions are slightly different for sigma y and ((

(36:23) and I would like you to have these equations written down and obviously the stresses what we see now they are much-much complex than what you have seen in the case of a cantilever beam or 4-point bending problem.

Naturally, you would not be able to calculate sigma 1-sigma 2 right away in the class and then plot the contours. So, we have to depend on computer graphics to do that job.

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EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

Disc under diametral compression – Analytical solutioncontd

The strain field is as follows

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{-2P}{\pi E t} \begin{Bmatrix} \alpha \frac{x^2 - \nu \alpha^2}{(x^2 + \alpha^2)^2} + \beta \frac{x^2 - \nu \beta^2}{(x^2 + \beta^2)^2} - \frac{1 - \nu}{D} \\ \alpha \frac{\alpha^2 - \nu x^2}{(x^2 + \alpha^2)^2} + \beta \frac{\beta^2 - \nu x^2}{(x^2 + \beta^2)^2} - \frac{1 - \nu}{D} \\ 2(1 + \nu) \left[\frac{\beta^2 x}{(x^2 + \beta^2)^2} - \frac{\alpha^2 x}{(x^2 + \alpha^2)^2} \right] \end{Bmatrix}$$

$\alpha = R - y; \beta = R + y;$
 $r_1^2 = x^2 + (R - y)^2 = x^2 + \alpha^2$
 $r_2^2 = x^2 + (R + y)^2 = x^2 + \beta^2$

When stress field is so complex, strain field is going to be much complex than this and this is how the expression look like and I have epsilon x, epsilon y, gamma xy=-2P/pi ET and here again I would like you to take down the notes for this expressions. I have this as alpha*x square-nu*alpha square/x square+a square whole square and alpha is defined as alpha=r-y, beta=r+y and alpha and beta are also related to r1 square and r2 squares as follows.

You have r1 square as x square+alpha square, r2 square as x square+beta square. You have second term here is beta*x square-nu*beta square/x square+beta square whole square-1-nu/D. So, this is the expression for epsilon x. You have a long expression for epsilon y and you also have a very long expression for gamma xy and though they are long, it is better that you have a copy of this in your notebooks.

Because this will help you because they are not readily available in the books that you have

access to. Have you been able to make a copy of this equation, okay. You have seen the stress field, strain field, both of them are longish and definitely the displacement field is going to be much longer than what you have seen all along.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Disc under diametral compression – Analytical solutioncontd

The displacement field is as follows

$$u = -\frac{2P}{\pi E} \left(\frac{1-\nu}{2} \left[\tan^{-1} \left(\frac{x}{R-y} \right) + \tan^{-1} \left(\frac{x}{R+y} \right) \right] - \frac{(1+\nu)}{2} \left[\frac{(R-y)x}{r_1^2} + \frac{(R+y)x}{r_2^2} \right] - (1-\nu) \frac{x}{d} \right)$$

$$v = -\frac{2P}{\pi E} \left(\frac{1}{2} \ln \left(\frac{x^2 + (R+y)^2}{x^2 + (R-y)^2} \right) - \frac{(1+\nu)}{2} \left(\frac{x^2}{x^2 + (R-y)^2} \right) + \frac{(1+\nu)}{2} \left(\frac{x^2}{x^2 + (R+y)^2} \right) - (1-\nu) \frac{y}{d} \right)$$

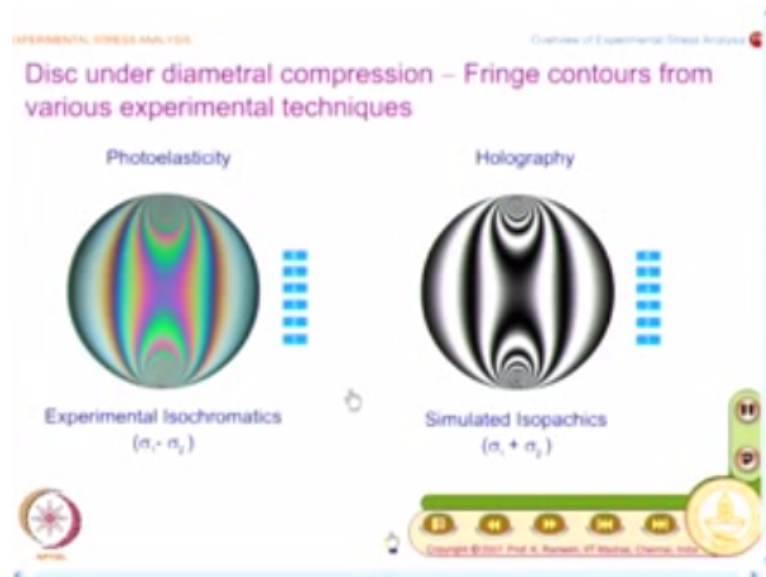
So, it is better please take your time to write it down and you have this as $u = -2P/\pi tE * 1-\nu/2$. You have a tan inverse $x/r-y + \tan$ inverse $x/\text{right} + y$. Then you have a minus of $1+\nu$ times $/2 * r-yx/r1$ square $+r+yx$ divided by $r2$ square $-1-\nu x/D$. On similar lines, you have a long expression for the V displacement. You have a term with natural logarithm x square $+r+y$ whole square divided by x square $+r-y$ whole square and the expression goes like this.

Obviously, it is very difficult to visualise what could be the nature of the displacement field by looking at this equation. If you have the plot, you have to go to a computer software, plug-in these equations, have a plotting software to cull out the numbers, collect them and then draw the contour. On the other hand, I take the model, I put it in the appropriate optics. I get stress information. I get displacement information.

The moment you want to go for strain, right now we do not have a technique which will give you whole field information. You can get from strain gauges and plot them or from displacement information, you can do numerical differentiation and plot the strain, but right now you do not have a whole field experimental technique which would give you whole field strain data

conveniently.

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Now, what we will look at is the circular disc under diametrical compression is a benchmark problem in photoelasticity. So, we will see the photoelastic contours and we will also see another set of experimental arrangement where you get contours of $\sigma_1 + \sigma_2$. In photoelasticity you get contours of $\sigma_1 - \sigma_2$ and I said one experimental technique will not give all the information. Suppose I want to find out information of individual stress components, one approach could be do a holographic experiment, gets iso-patch is recorded.

In this case, it is simulated but get the iso-patches recorded so when I have photoelasticity contours as well as holographic contours, I can process these 2 and find out individual magnitudes of σ_1 and σ_2 on the entire field. What you have to note it down is between photoelasticity and holography I have showed coloured contours for photoelasticity because we use white light and get information in colour which is unique to photoelasticity, although it also gives monochromatic information.

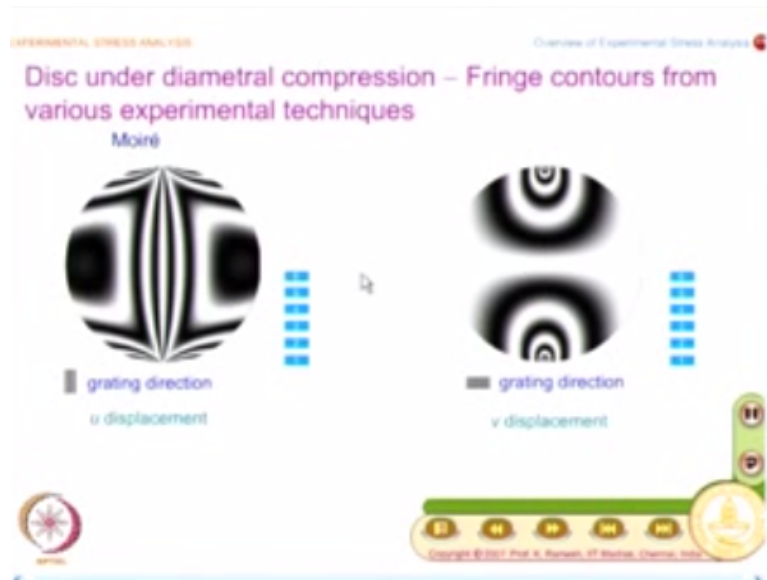
Many techniques depend on monochromatic light source and you get black-and-white information which is processed holography you do it only on a single wavelength and the contours are very similar and I would like you to have a reasonable sketch of this, anyone of it which gives you an indication how the fringes look like and here you have this as a shape of

eight. This has a shape of eight. This you have to note it down.

This we will use it in our experimental interpretation later and you can also see as a function of load applied, how the fringes develop. So, what happens fringes developed here and move outwards that you could see from these simulations. This also gives you an indication of doing an experiment right at the laboratory. Same thing you can do for photoelasticity, fringes develop and move outward.

This also gives you an indication how you can go about in labelling fringes which is a very complicated exercise, we will see later. So, in this example we also shown results from holography and if you want to go for displacement information, what is the technique that you will use. We have been seeing Moire.

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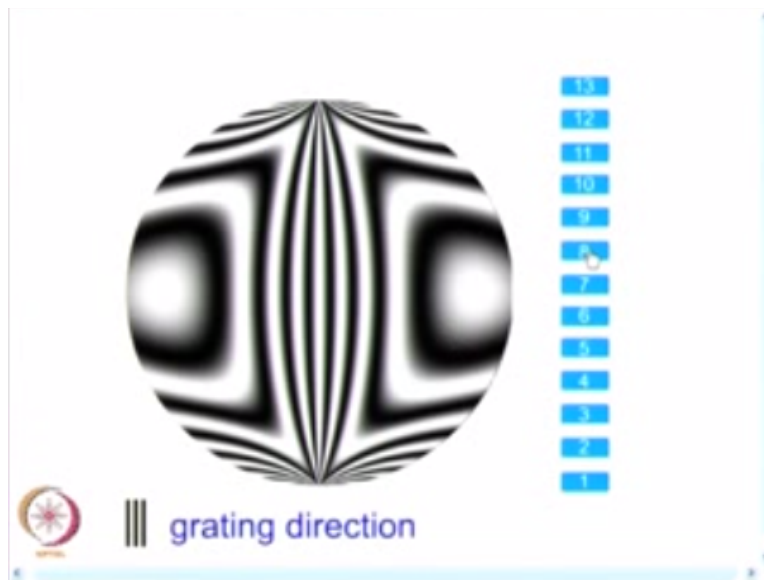


So, I would see the Moiré contours, what I get for U displacement as well as V displacement. How do I decide that? I have the grating direction with which it is recorded and you get this contours beautifully and we saw this very difficult expressions and obviously when you do it will be complicated like this and I can also change the load 1, 2 and gradually increase the load I see more and more fringes appear and probably this sketch you can make because you do not have many fringes but it gives you the geometric shape of the fingers reasonably well.

Because at a later point when you have occasion to see the experimental result, you could easily say I have seen these patterns earlier and they would be for a disc, it is only U displacement. Suppose somebody gives you a photograph without the grating direction on an unknown situation, if they want you to interpret, you will not be able to give an answer immediately unless you know how the experiment is conducted, how the values are recorded you will not be in a position to interpret the fringe patterns.

Interpretation requires additional information, labeling of fringes is not a simple task and for that we will have to know what are the various techniques available to label the fringes.

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Some of the lateral example I have not shown the fringe ordering deliberately, so that you can do this as an exercise and label the fringes after we have learnt the course. So, when I increase the load, more and more fringes appear and that is what I see in the case of disc under diametrical compression and this gives you U displacement.

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Now, similarly, we will also see the V displacement and this is what you have here, the grating direction is like this and as I increase the load, I go from this, gradually go from this, you find this is a load application point and all the fringes come from this and what you also find. You have a very broad fringe and as you increase the load, you have the fringes move and occupy different positions so it is possible for you to label them appropriately.

If you have this knowledge, what you find is this fringe has moved at a very high load. So, if you have a number attached, the number also will move along with it. So, that we will see later and here again you can make a reasonable sketch for an intermediate load to know the nature of the displacement field. So, that gives you certain level of familiarity and you feel closer to an experimental technique. You also get a knowledge how to appreciate visual information.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Disc under diametral compressioncontd

- Photoelasticity can also provide the direction of principal stresses.

Experimental Isoclinics

For this example, I also have another set of contours what you can get from photoelasticity. Photoelasticity can also give you principal stress direction and that is what you see here and we will see this closely.

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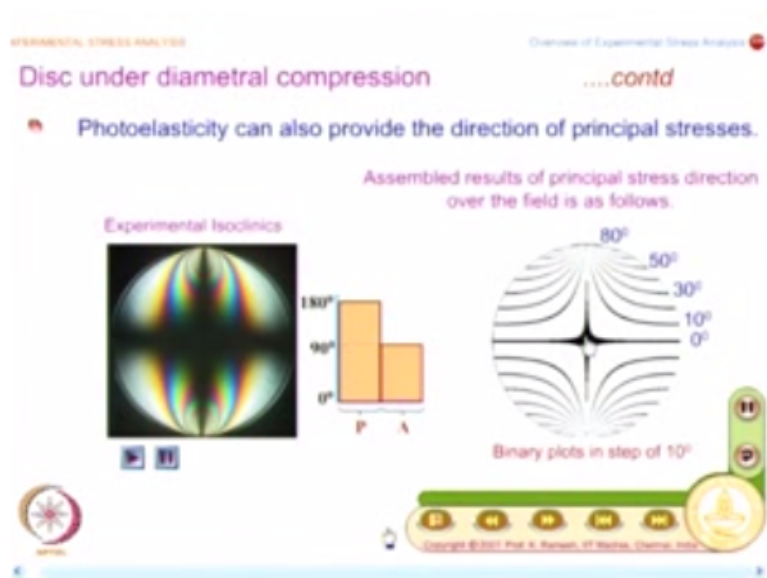
Experimental Isoclinics

So, what I find here is I have experimental isoclinics and what you find here is the whole image looks blur, here black contour moves over it that is what you see. A black set of contours move over coloured bands. Can you identify the coloured band? because you had seen these earlier Can you identify the coloured band. You can identify the coloured band. What the colour band show. They are contours of $\sigma_1 - \sigma_2$.

So, what you find here is in this optical arrangement, I get 2 information; one information is σ_1 - σ_3 , another information is the orientation of the principal stress plane and what you find. If you have 2 information superimposed the clarity is lost. So, you would always like to have independent information, then processing, data collection everything becomes much simpler and what you see in this example.

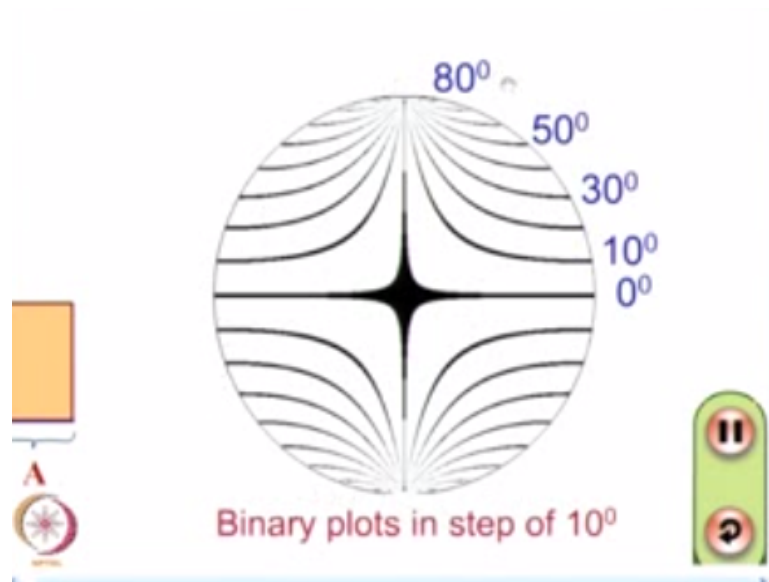
I have one set of contours that move and this you may not be able to understand at this stage what it indicates is the direction of polariser analyser, these are 2 optical elements. They are kept at mutually perpendicular degrees. For these angles, these contours move. So, what you will have to do is from a data collection point of view, I have to set it at fixed angles and try to make a reasonable sketch of what this contours are and that is what is shown here.

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So, what you have here is I have the set of contours which is a binary plot in steps of 10 degrees.

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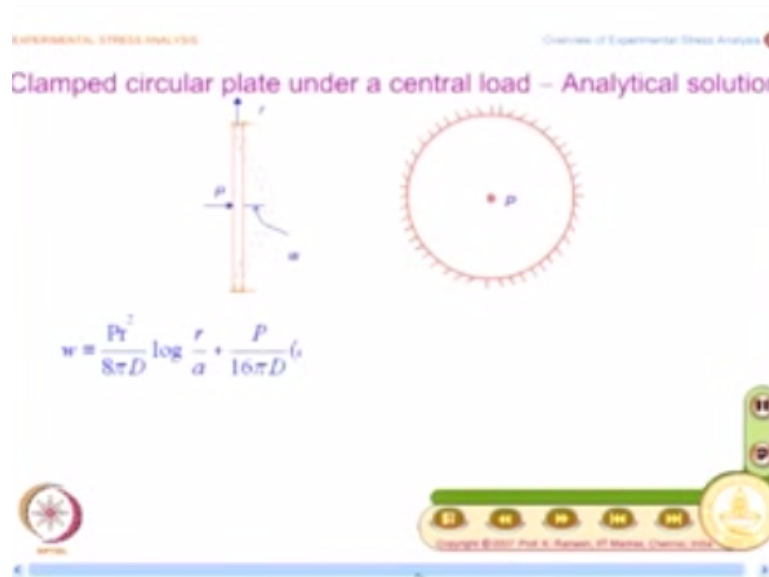


This you make a sketch of it. So, I have the process this information and extract this in this form. I do not have an optical arrangement which would give me this set of contours directly. So, I have a 0 degree isoclinic, 10 degree isoclinic, 20 degree isoclinic, 30 degree isoclinic and so on and we call this as isoclinics. In fact, when you do a chapter on photoelasticity, we would know in detail what these isoclinics are, iso means constant, clinic means inclination, isoclinic means contours of constant inclination.

Constant inclination of what, constant inclination of principal stress direction. So, from a photoelastic experiment it is possible for me to get σ_1 - σ_2 contours and you can also get by an appropriate optical arrangement, contours of principal stress directions. What I find is here I have σ_1 - σ_2 contours superimposed over isoclinic contour and I do not see all isoclinic in one shot. I have to scan the image and pick out this information, get this as a separate image of isoclinics.

So, without getting into the extramarital details, I have tried to project what is in store for you. In some experimental technique, you get the information separately. In the case of Moire, I can get U displacement, I can get V displacement separately and getting them separately is better. Though, I have to do one more experiment, getting them separately in an optical arrangement is far better from processing point of view than getting the image of σ_1 - σ_2 superimposed on the isoclinic pattern, that is what you have seen here.

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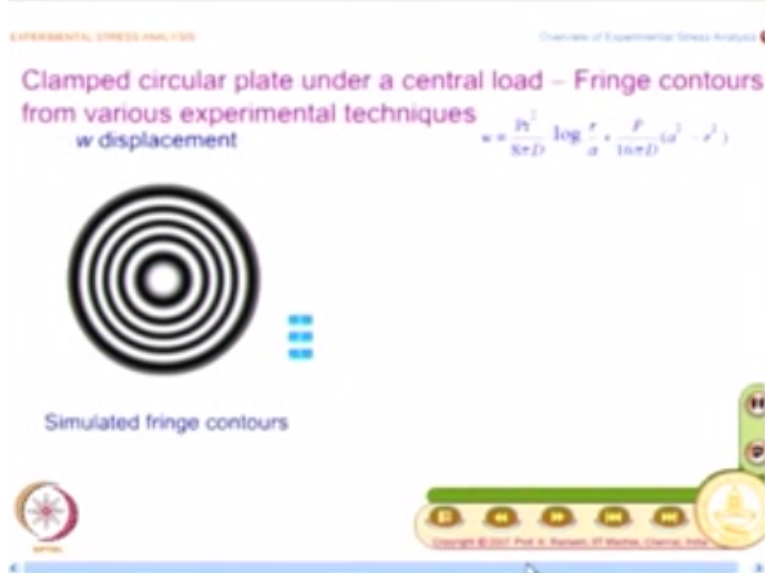


The next problem what we would take on is, we would take on the problem of a clamped circular plate under a central load and what I would appreciate is I have this clamped circular plate. This is clamped all around the periphery and is subjected to a central load. In fact, in the kind of problems where they do out-of-plane displacement, they take this as the benchmark example and using this only they testify their methods.

What you have here is your expression for w out-of-plane displacement, you have the expression for $\frac{dw}{dx}$, you also have the expression for the curvature $\frac{d^2w}{dx^2}$ and in all these cases you have logarithmic term appearing in this and the expressions are definitely complex and you would be able to find out this from a study on theory of plates where you have sufficient theories developed to find out how to get these expressions.

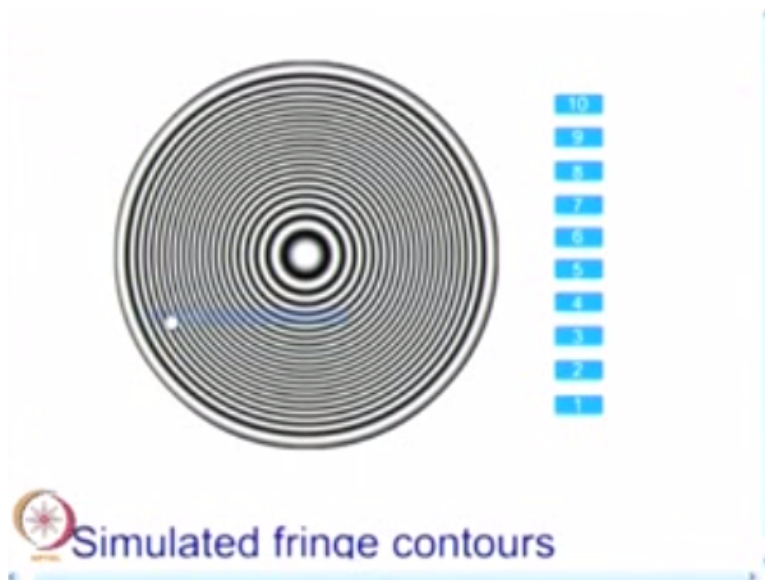
Essentially, when I want to go and find out whether I have at speckle interferometry working alright, I would not take a circular disc under diametrical compression, but I would rather take a clamped circular plate with a central load to test even my experimental setup. So, by doing that I would be able to get that information. Similarly, what I have here is I have shown the slope in direction X , you can also have the slopes in direction Y .

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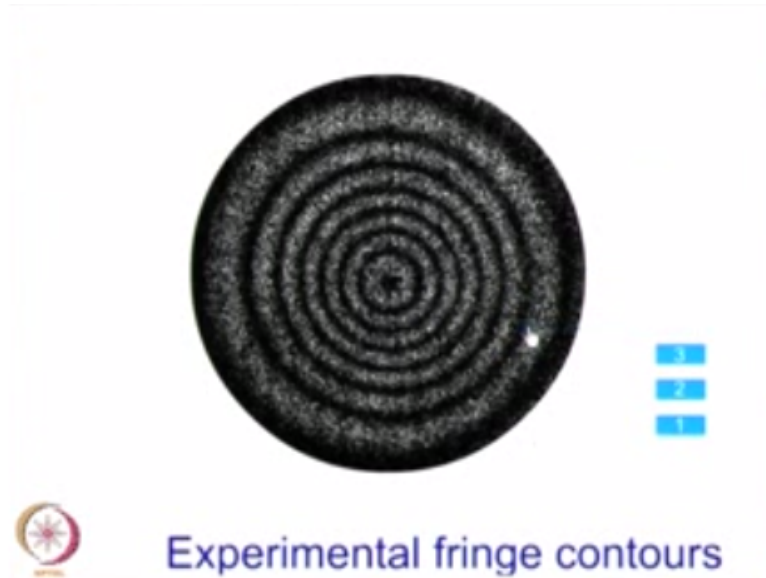
When I go and find out I get the patterns like this. So, what I have here is I said nature is so good. Nature is so good that it gives you.

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So, I have simulated fringe contours. I have concentric circles. Nature is so good, in the case of beam under bending, you saw horizontal lines. In the case of clamped circular plate, you see beautiful circles and this is again a function of load applied. So, you have concentric circles that they come and this is what you have and as the load is increased, you have more and more circle. What you find is if I want to go and look at an experiment, I can get this from speckle interferometry and I will magnify this and show this.

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Here you do not find the fringes as clearly marked dark and white. You have specular pattern and these are called correlation fringes and here again if the function of the load applied, I can apply the load 1, load 2, load 3 and so on and you do not have the level of contrast that you have in the case of photoelasticity. What is the advantage here, you are able to get out-of-plane displacement by whole field technique but they do not have high contrast?

So, if you look at any of this speckle interferometric methods, they spend lot of time on filtering. On the other hand, photoelastic fringes have very high contrast and you do need to do less post processing than what you can do in the case of speckle interferometry and this is the fringes you have and this is again the function of load, also make a neat reasonable sketch of this. Suppose I want to go and see how do the slope pattern look like.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Clamped circular plate under a central load – Fringe contours from various experimental techniques

Slope Fringes

$$\frac{\partial w}{\partial x} = \frac{4xw_{max}}{a^2} \log \left(\frac{\sqrt{x^2 + y^2}}{a} \right)$$

$$\frac{\partial w}{\partial y} = \frac{4yw_{max}}{a^2} \log \left(\frac{\sqrt{x^2 + y^2}}{a} \right)$$

Simulated fringe contours Simulated fringe contours

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The slope pattern looks like this. Slope fringe are like this. You have the expression here and you have the expression of slope in the direction X, slope in the direction Y and this is how the fringe patterns you have.

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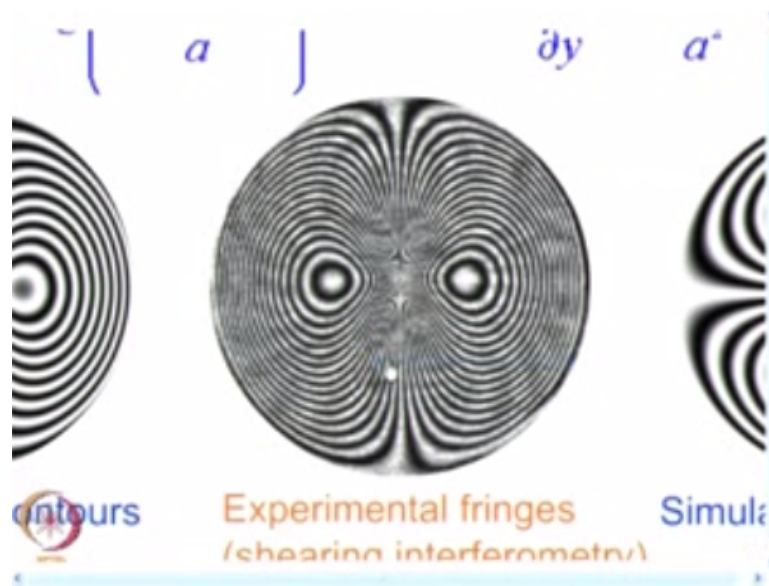
Simulated fringe contours

What you have here is I increase the load. So, people also say this as butterfly fringes, they look like the wings of a butterfly and they call this as butterfly fringes, particularly in the case of non-restrictive testing, when I want to do a honeycomb panel testing, any delamination you would be able to find out easily. If you see butterfly, you should not feel happy, you should feel disturbed that there is a delamination.

The butterfly pattern why they call it is in the case of commoners, you know they do not want to say that it is $\partial W/\partial X$ contours, they simply say it as butterfly fringes and you have a similar situation when you have the slope in the Y direction also. So, these are called butterfly fringe pattern and this you can have a look at it. It is oriented in a different direction and that is what you have and now what you will look at is, suppose is there any experiment which can reveal.

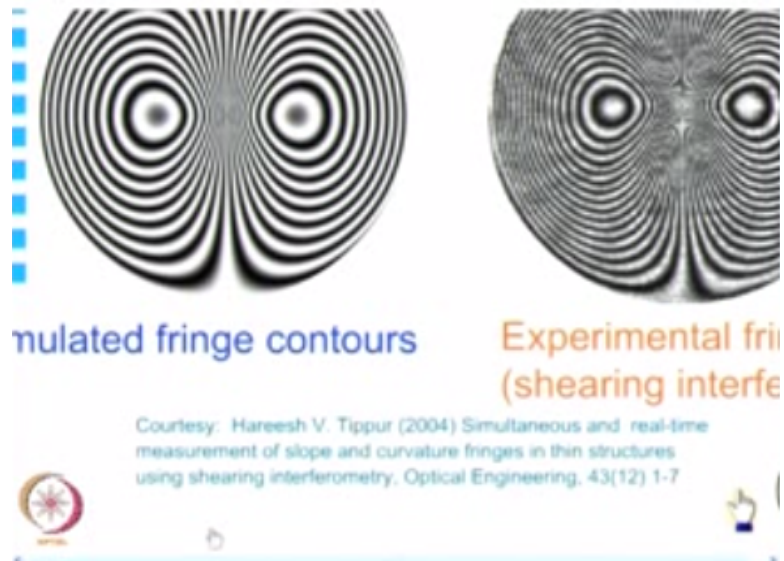
These are simulated contours; I have the expression here I have the simulated. When I increase the load I get that, and what you find here is a very recent experiment on speckle interferometry has given very nice set of contours on thin wafers.

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You know in fact it matches very well with what you have as the simulated patterns; this is how you see in the experiment.

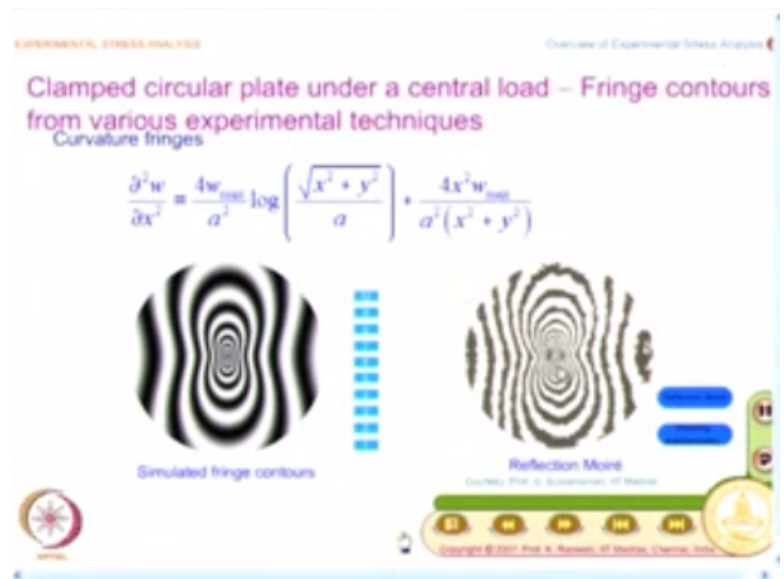
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This from the work of Prof. Hareesh Tippur and this has come in the Journal of Optical Engineering. You will get more details of these fringe patterns from this and courtesy goes Optical Engineering and The Society for Optical Engineers, SPOE and you can note down this reference. You could look for more details of the fringe pattern and this was very recent work. This is on silicon wafer.

So, it is a simultaneous and real-time measurement of slope and curvature, you see only slope fringes here.

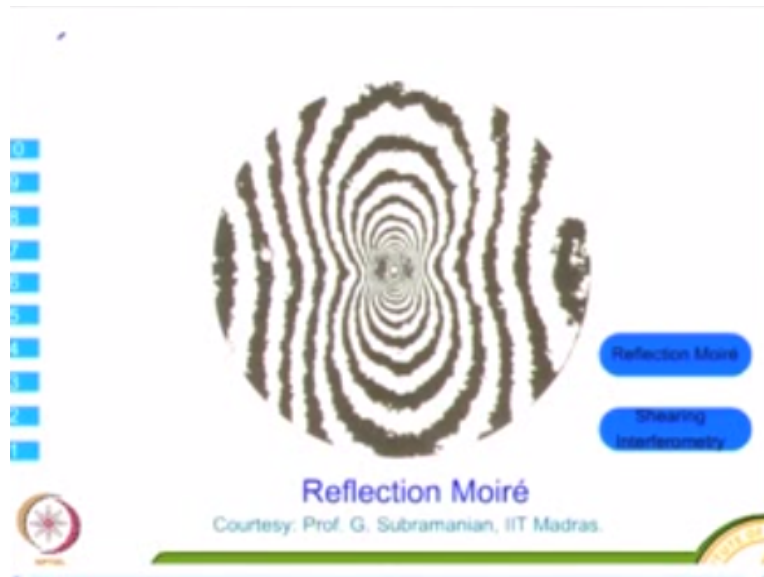
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What we would see now is, we would also see the curvature and curvature information looks like

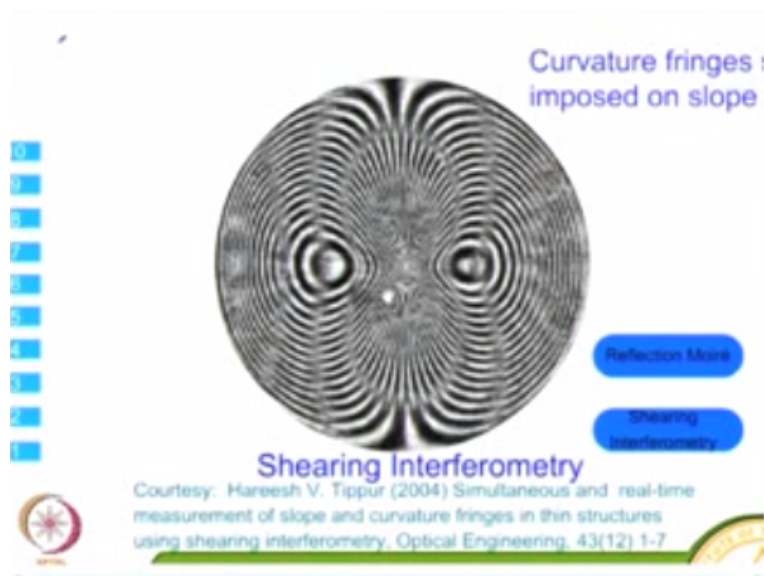
this and the simulated fringe contours are like this. If you go to experimental technique, you have a nice set of curvature fringes obtained and you may think from the experimental fringes we have seen earlier, these borders are not smooth.

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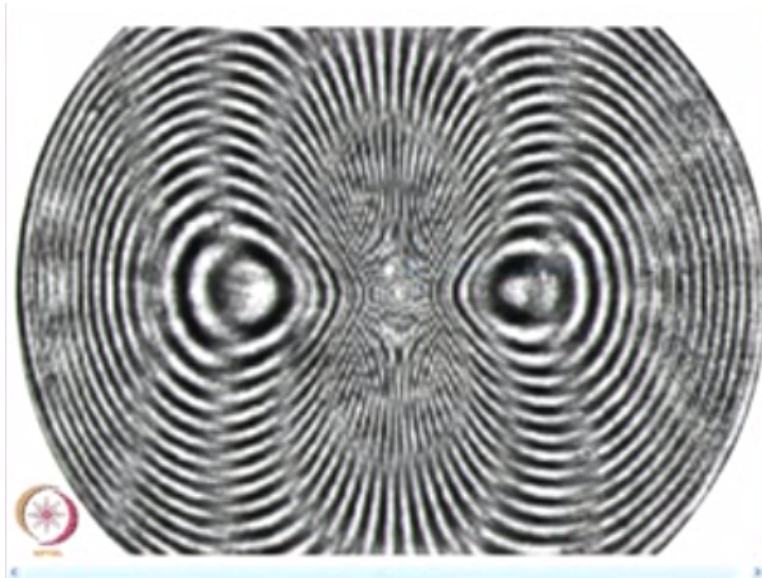
They are jagged but you see the shape as what you have seen here. This result is from Prof. G. Subramanian. He was an exponent on Moire interferometry in the county and this is from reflection Moire and what you will have to know is even to identify an optical technique to get this is a challenge and you will know the difference only when you want to find out the curvature in shearing interferometry.

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Shearing interferometry gives the information like this and what you see here. Do you see the curvature; I will magnify it further.

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What you see here is I have on the background slope fringes. On the slope fringes, you see faintly the curvature fringes, right. When I have shown σ_1 - σ_2 contour superimposed with isoclinics, you felt uncomfortable. Now, you will say that was much better because it had very good contrast. Here I get curvature information but the advantage here is though it looks little dull, the advantage is the optical arrangement used gives both slope and curvature information in one shot.

This is again the work from Prof. Tippur and this is from optical engineering. So, you can get more information on this from reading a paper like this. So, I can say comfortably now, you have a fairly good idea on how do the optical patterns look like. Optical patterns essentially gives certain physical contours are dictated by the physics exploited in the experimental technique. You have seen individual experimental contours.

You have also sample of superimposed contours and in some cases, you will get only superimposed information. Only in certain cases, you will be able to separate it and from an experimental point of view, you need both. Suppose I have a time varying phenomena I would like to record both. I would record both slope and curvature by a very different optical

arrangement, though the quality of information is slightly of a poor quality, I would record both the information together.

I am sure at the end of this lecture you have a fairly reasonable idea how do the whole field representation of stress field particularly σ_1 - σ_2 contours and displacement fields by and large look like, that gives a certain level of familiarity, certain level of affinity and as we go further, you would be able to find out how to get these contours by yourself, what is the principle that is used and also how to interpret not just be happy with the shapes of the contours but also get the actual magnitudes with a sense of confidence.