

Experimental Stress Analysis
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology – Madras

Lecture – 24
Correction Factors for Photoelastic Coatings

We have started our discussion on photo elastic coatings in the last class and I mentioned photo elastic coatings can be called as birefringence coatings, taking to account that we paste a birefringent material on the specimen surface and we also saw the material become birefringence, when the model is loaded. So, whatever the polarization optics as well as the crystal optics basics that we are developed in transmission photoelasticity are directly applicable to reflection photoelasticity with certain minor changes.

And I also mentioned because we analysed the reflected light, this is also called as reflection photoelasticity and one of the main challenge here is how to interpret the specimen stresses from observing fringe patterns that is seen on the coating. So, from the optics first, we find out what are the coating stresses and from the coating stresses, using the principles of mechanics of solids, we go and find out what are the specimen stresses.

(Refer Slide Time: 01:34)

Coating stresses

Stress strain relations

$$\epsilon_1^s = \frac{1}{E_s} (\sigma_1^s - \nu_s \sigma_2^s) \qquad \epsilon_1^c = \frac{1}{E_c} (\sigma_1^c - \nu_c \sigma_2^c)$$

$$\epsilon_2^s = \frac{1}{E_s} (\sigma_2^s - \nu_s \sigma_1^s) \qquad \epsilon_2^c = \frac{1}{E_c} (\sigma_2^c - \nu_c \sigma_1^c)$$

Coating stress

$$\sigma_1^c = \frac{E_c}{E_s(1-\nu_c^2)} \left[(1-\nu_c\nu_s)\sigma_1^s + (\nu_c - \nu_s)\sigma_2^s \right]$$

$$\sigma_2^c = \frac{E_c}{E_s(1-\nu_c^2)} \left[(1-\nu_c\nu_s)\sigma_2^s + (\nu_c - \nu_s)\sigma_1^s \right]$$

NPTEL
Copyright © 2014 Prof. K. Ramesh, IIT Madras, Chennai, India

And what we had seen in the last class was whatever the strain developed on the specimen is faithfully transmitted to the coating and individually, you can write the stress strain relations for the specimen as well as the coating and the idea was using these set of relations, it is possible

for you to estimate the coating stress σ_{1c} and σ_{2c} and this I had asked you to work it out in your rooms.

And I think some of you have done it and the expression looks like this and what we have here is σ_{1c} as given as $E_c/E_s \cdot (1 - \nu_c^2)^{-1} \cdot (\nu_c \sigma_{1s} + \nu_c - \nu_s) \sigma_{2s}$ and σ_{2c} as given as $E_c/E_s \cdot (1 - \nu_c^2)^{-1} \cdot (\nu_c \sigma_{2s} + \nu_c - \nu_s) \sigma_{1s}$ and I said when we go and look at what is brittle coating, there again the expression for coating stress remains same.

There we will find out individual stress components by analysing isointatics but as far as photo elastic coatings is concerned, though we develop individually the expression for σ_{1c} and σ_{2c} , essentially you get only difference in principal stresses or principal strains, so that is what we are going to look at, we will look at the difference in the principal stresses. First, we will find out how I can express specimen stress difference as a function of the coating stress difference.

(Refer Slide Time: 03:52)

Specimen stresses

- The inter relationship between the difference in principal stresses of the specimen and the coating is obtained as

$$\sigma_1^s - \sigma_2^s = \frac{E_s}{E_c} \frac{1 + \nu_c}{1 + \nu_s} (\sigma_1^c - \sigma_2^c)$$

- The principal stress difference in the specimen can be found as

$$\sigma_1^s - \sigma_2^s = \frac{E_s}{1 + \nu_s} \frac{N}{2 h_c K}$$

- It is interesting to note that the elastic properties of the coating material do not appear in the above equation.

Then coating stress difference can be replaced by your principles of optics, you will have a strain optic relation and then plug it in and do it. So, the specimen stresses are expressed as follows, you have Young's modulus of the specimen divided by the Young's modulus of the coating multiplied by $(1 + \nu_c) / (1 + \nu_s) \cdot (\sigma_{1c} - \sigma_{2c})$, whatever the expression that we have got individually for σ_{1c} and σ_{2c} can be recast in a form, which relates the specimen stress difference to coating stress difference.

And now what we will do; we will replace the coating stress difference in terms of the fringe orders. If it is a transmission photoelasticity, we will simply have this as $NF \sigma / h$, in reflection photoelasticity, I said you have to have the light which is entering the model and coming out of the model, so you have twice the distance or the thickness the light travels the path. So, one modification is you will have 2 times the thickness coming in the denominator.

Other one is instead of expressing it as material stress fringe value, it is lot more convenient to express in terms of material strain fringe value. So, we lose all those internal relationships, we have also looked at what is interrelationship between $F \sigma$ and $F \epsilon$ and we have also seen $F \epsilon$ is λ / K , so what I am going to do is I am going to write the final expression, which involves the fringe order and K .

Because K is supplied by the manufacturer for whatever the photo elastic coating material, they normally supply the strain coefficient K as part of the supply of the material but it is also desirable that you calibrate it because all polymers slowly change their properties for a period of time. So, it is desirable that you also evaluate it, so now what we are going to look at is; we will look at the principle such difference of the specimen in terms of the fringe order and the strain optic coefficient K .

So, what I have here is; I have the elastic constants of the specimen come into the picture, which was not the case in the case of transmission photoelasticity. I had only an $NF \sigma / h$, here it is $NF \epsilon / 2hc$, $F \epsilon$ is replaced by λ / K but in addition, you also have Young's modulus of the specimen divided by $1 + \nu$. So, what I have in photo elastic coatings is our focus is to find out the stresses developed on this specimen.

The difference in principal stresses on the specimen is related to the fringe order through this expression but even this expression is not complete because I have said while developing the theory, we have merged certain assumptions and all these assumptions you will not be able to fully satisfy in your experimentation. So, whenever we violate, this we bring in a correction factor to improve our result, so we will see that also.

In the case of transmission photoelasticity, the famous stress optic law was $\sigma_1 - \sigma_2 = NF \sigma / h$ that was the golden rule, still we are not come to look at what is the final expression that is of value in reflection photoelasticity, I need to have one more parameter, it is

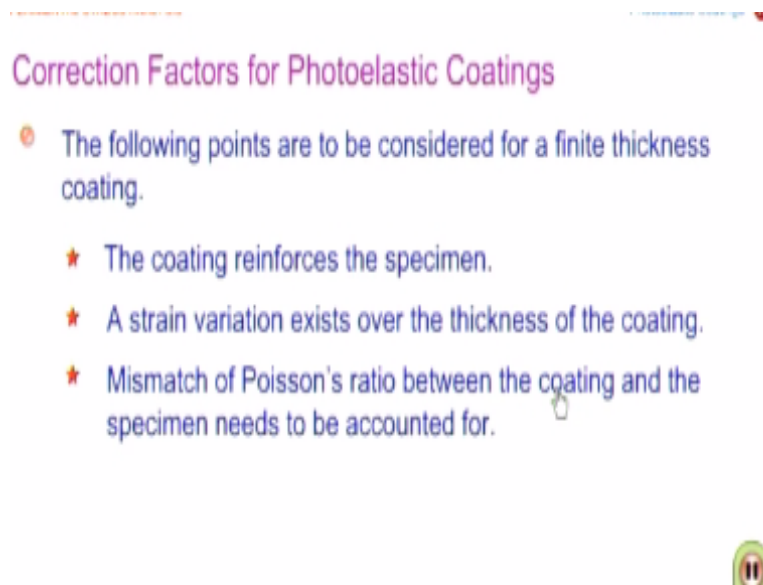
only an intermediate step. I cannot stop my photo elastic coating development of mathematics with this set of expressions.

I need to have one more factor that is needed and we will also see how that factor varies from different type of loading situations and another point of interest here is; if you look at this expression, the elastic properties of the coating do not appear in the above equation that is one general observation and I also mention you know it is prudent to see here, I said one of the important issues in photoelastic coating is, what is the maximum fringe order obtainable which question we never raised in transmission photoelasticity.

It was not critical because the loading is different, it is basically the strains that develop on the specimen surface are transmitted to the coating, the level of loading on the actual coating is very small and if you look at this expression, you can actually replace this by the yield stress and find out what is the maximum fringe order and you will find that number is small and now that number is going to be small that also we will take it up later.

Why I am reminding this is; there are certain peculiarities in reflection photoelasticity and if you understand those peculiarities whatever, you have learnt in transmission photoelasticity, the same comfort you have in interpreting, what our isoclinics, what our isochromatics all that knowledge is same, only the interpretation is slightly different here and you should know the peculiarities here, what are the peculiarities that you have.

(Refer Slide Time: 10:03)



Correction Factors for Photoelastic Coatings

- The following points are to be considered for a finite thickness coating.
 - ★ The coating reinforces the specimen.
 - ★ A strain variation exists over the thickness of the coating.
 - ★ Mismatch of Poisson's ratio between the coating and the specimen needs to be accounted for.

Then, we move on to what are the correction factors, why they are needed and I said the moment I talk about photo elastic coatings, the discussion is complete only when we talk about the correction factors and first consideration is the coating reinforces the specimen and why we have to look at this, we are having a finite thickness coating. Suppose, I have a magic material, I simply go and spray it on the actual specimen that is what done in the case of image correlation.

You have a white paint or the thermo elasticity, you put a black paint where the coating thickness is negligible in terms of microns, it will be but in the case of photo elastic coating, I have already said coatings of the order of 3 millimeters are not uncommon. So, you are handling a coating, which has a finite thickness. So, when you look at finite thickness, you should also look at whether the coating reinforces the specimen significantly.

And I also said there is a comfort, people started developing photo elastic coating with glass as the coating material and glass was having a Young's modulus of 70GPa, which is comparable to the base material that definitely reinforces, whereas the coatings that are employed on metals have Young's modulus of the order of 3 GPa, which is very negligible, when you compare to aluminium or steel or brass, it is very, very small.

And on the similar note, if you go for low strength alloys as well as rubber, then you need to develop a coating material, which has much less Young's modulus than the base material, so the issue here is you are having a finite thickness coating and definitely you have to appreciate that coating can reinforce the specimen, you must take corrective measures for that, either you bring in a correction factor or minimize the reinforcement of the specimen by the coating.

And another issue is a strain variation exists over the thickness of the coating, this is very important in which class of problems do you think that strain variation can exist. See, if the coating is very, very thin, you do not have to really worry but I have a thick coating, if I apply a tension absolutely no problem thickness does not really signify, you will have an average strain approximately over the thickness.

The moment I go for bending or torsion, where the distance from the neutral fiber dictates the strain developed, so definitely in problems where we have to do tackle or evaluate the loads due to bending or torsion, a strain variation exists over the thickness of the coating and this needs to

be addressed completely. See, when we looked at transmission photoelasticity, the famous specimen we used for calibration was a circular disc under diametral compression.

We also saw, what are the reasons for it? On the other hand, when I come to reflection photoelasticity, I would use a cantilever beam as the favourable calibration specimen for evaluating the strain coefficient K. So, that beam is subjected to bending and if I do not use the correction factor, my evaluation of K would be erroneous. So, even for a simple calibration test, I have to accommodate a correction factor.

This is what is the peculiarity in reflection photoelasticity and finally I have always been saying in all the experimental technique, Poisson's ratio has a role to play and you have a mismatch of Poisson's ratio between the coating and the specimen and that needs to be accounted for and what we would do cleverly is; we will develop correction factors for many of the simple loading situations assuming that there is no Poisson's ratio mismatch.

So, you will have some kind of corrections and we separately address for certain class of problems, you could also accommodate Poisson's ratio mismatch, only those class of problems we will bring in how to accommodate mismatch of Poisson's ratio and we will also see how do you find out stress concentration factor, when I use a photoelastic coating test. There is a slight modification that you need to do.

(Refer Slide Time: 15:38)

EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings 11

Correction Factors for Photoelastic Coatings

....contd

- So the principal stress difference equation to be rewritten as

$$\sigma_1^s - \sigma_2^s = R_f \frac{E_s}{1 + \nu_s} \frac{N}{2h_c} \frac{\lambda}{K}$$

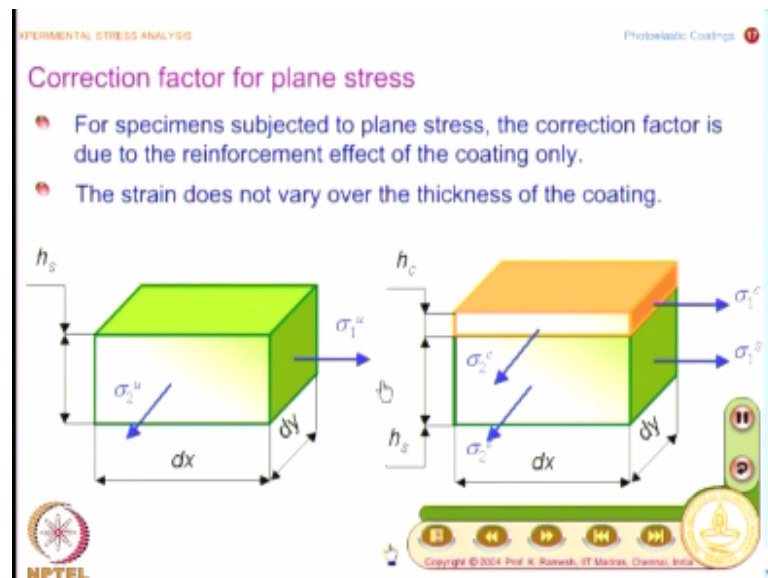
Where R_f is the correction factor.

It is not as simple as you saw in the case of transmission photoelasticity mainly because of mismatch of Poisson's ratio, which becomes significant in those class of problems and we will

take up one of the very simple problems to start with and before we go into that, as I said the final expression that you need to know in reflection photoelasticity is as follows. The difference and principle stresses of the specimen namely $\sigma_1^s - \sigma_2^s$ is given as $R_f \cdot \text{Young's modulus of the specimen} / (1 + \nu_s) \cdot N / 2hc\lambda / K$.

And whatever, the discussion we had on colour code is equally applicable in reflection photoelasticity, so I will have to find out what is the fringe order accurately at a point of interest. Once I know fringe order N , I know the specimen material properties and also the coating properties and if I know what is the wavelength of light that I have used and also the specimen we calibrate the coating property namely the strain coefficient.

(Refer Slide Time: 17:10)



I can find out $\sigma_1^s - \sigma_2^s$ and what we will have is; we will have this correction factor evaluated for in plane stress, bending, torsion and internal pressure so on and so forth that is how we will have. We will have different expressions for R_f and R_f is used because it is essentially a reinforcement factor and we will first take up; what are the correction factors for plane stress.

This will really establish; what is the basic procedure to evaluate the correction factor and the credit goes to Zandman and his co-workers. He really established the advancements of photo elastic coatings and for specimens subjected to plane stress, the correction factor is due to the reinforcement effect of the coating only. The strains do not vary over the thickness of the coating and what I am going to do is; I am going to take a uncoated specimen subjected to plane stress.

And I will consider the specimen with a coating, so what you will have to look at is; the moment I put a coating, the coating can also participate in load sharing. Now, the question is what is the share of load the coating takes in? So, in the process the specimen is loaded less, so I will essentially compare what are the strains developed for a given loading on the uncoated specimen and compare it to the coated specimen from which I will get the correction factor.

Because my requirement is by looking at the reading from a coated specimen, I should evaluate the strains that would have developed in an uncoated specimen but coated specimen only I can do experiment, uncoated specimen I cannot do experiment. So, I bring in a correction factor to do this and this shows the load sharing what you have, I have a coating for the discussion it is put very thick, so that you see the coating is sitting very prominently.

So, you have the specimen thickness as h_s , coating thickness as h_c and in an uncoated specimen, I will have the specimen taking the load of σ_1^u , the superscript u denotes that these are the stresses developed in a uncoated specimen. In a coated specimen, what do we say? I will have a stress developed as σ_1^s on the specimen and σ_1^c on the coating, so now I will have to essentially do a force balance.

(Refer Slide Time: 20:34)

The slide, titled "Correction factor for plane stress" (with a "....contd" label), presents the following content:

Force balance

$$\sigma_1^u h_s dy = \sigma_1^s h_s dy + \sigma_1^c h_c dy$$

$$\sigma_2^u h_s dx = \sigma_2^s h_s dx + \sigma_2^c h_c dx$$

On the right side of the slide, the following strain relationships are shown:

$$\epsilon_1^c = \epsilon_1^s$$

$$\epsilon_2^c = \epsilon_2^s$$

The strains in the uncoated and coated specimen are related by

$$\epsilon_1^u - \epsilon_2^u = \left[1 + \frac{h_c E_c (1 + \nu_s)}{h_s E_s (1 + \nu_c)} \right] (\epsilon_1^c - \epsilon_2^c)$$

The correction factor R_f^u is defined as:

$$R_f^u = 1 + \frac{h_c E_c (1 + \nu_s)}{h_s E_s (1 + \nu_c)}$$

The slide includes an NPTEL logo in the bottom left corner and a navigation bar with icons for back, forward, and search in the bottom right corner. A copyright notice at the bottom reads: "Copyright © 2014 Prof. K. Ramani, IIT Madras, Chennai, India".

I will essentially do a force balance and find out what is the share of σ_1^c and what is a share of σ_1^s and this is how we will develop the methodology to find out the correction factor. See, the problem is simple enough, why we take a simple problem is; you understand the

physics behind how one goes about in evaluating the correction factor and you can easily write when I go to the force balance, I can write $\sigma_c h_c dy$.

Because this is the area and this could be a ; there could be a force component from the coating, that could be a force component from the specimen and you will have the expression written down like this; $\sigma_s h_s dy + \sigma_c h_c dy$. On similar fashion, I can also find out what is the force on the other direction and I get this as; so what we have here is; I have the force balance written down.

And we have assumed to start with the coating is well bonded to the specimen, so that the strains on the coating and the specimen are identical and this is possible only when the Poisson ratio is same fully, so that is the assumption that we make and when I do this and also find out what way the uncoated specimen strain difference is related to the coated strain difference. I have an expression like this.

This is easily obtainable from these set of force balance equation and making the strains equal, I can write this expression and what I call this factor, $1 + \frac{h_c}{h_s} \frac{E_c}{E_s} \frac{1 + \nu_s}{1 + \nu_c}$, I call this as a reinforcement factor and you will see a superscript coming here, I have said in general, it is R_f because it is axial loading, I have the symbol as, a . So, R_{fa} denotes the correction factor that is needed when I have to interpret the results from a simple axial test; simple axial test.

How do I interpret? I have to bring in a correction factor, there is no way. Then, we move on to more complex problem of; if the specimen is subjected to bending, how do I go about and evaluate the correction factor? So, what we will do is for illustration because you all know, bending reasonably, well we will find out how much of it you remember now, so that is what I have always been saying.

When you are learning a course in strength of materials, you should not forget it when you learn advanced courses; they form as the fundamentals for you to build up the theory in the higher level courses. The moment I put a coating what happens; it behaves like a composite beam, so that is what we are going to look at. So, our interest is; we are looked at what is the kind of correction factor that I need for in plane stress.

(Refer Slide Time: 23:57)

EXPERIMENTAL STRESS ANALYSIS

Procedural Controls

Correction factor for bending

Strain variations through the coating thickness exists.

$$\epsilon_1^c = \frac{y}{\rho} \text{ for } -(h_c - A) \leq y \leq A$$

$$\epsilon_2^c = -v_2 \epsilon_1^c$$

$$\epsilon_3^c = -v_3 \epsilon_1^c$$

$$\epsilon_1^c = \frac{y}{\rho} \text{ for } A \leq y \leq (A + h_s)$$

HPTEL

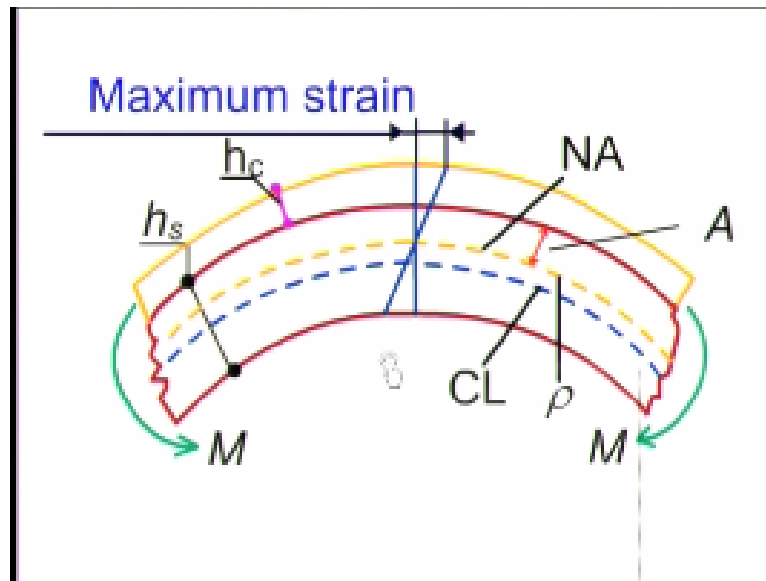
Copyright © 2004 Prof. S. Srinivasan, IIT Madras, Chennai, India.

Now, we will move and find out; how do I go about for a correction factor, when the specimen is subjected to bending. So, what you have here is; if I have a specimen, which is uncoated for a given bending moment, it would have its neutral axis like this and strains vary linearly over the depth of the beam. Now, what I have here is on top of this specimen, I put a coating material then this system behaves like a composite beam.

And what is the first knowledge in the case of a composite beam, what will happen to the neutral axis? Neutral axis will get shifted, so we need to find out what is the shift in the neutral axis and we will also have to look at and very prominently because you are looking at bending, when I have a finite thickness of the coating, strain is not going to be constant over the thickness, it varies.

It varies linearly, so I will also bring in the average strain over the thickness of the coating, so this is what I have to calculate. So, I have to find out; go back and find out from your conventional knowledge of how do you analyse bending of a beam, we will take up a very simple case of pure bending and we look at how do we find out the shift in the neutral axis that is what is shown here, there is a shift in the neutral axis.

(Refer Slide Time: 25:58)



And you can also have a clear picture of the beam and to label various quantities, so you look at; I have a thickness of the specimen, I have the thickness of the coating and the neutral axis, no longer coincides with the centroidal axis of the specimen cross section but it gets shifted and we have labelled the distance from one of the ends of the specimen top surface to the neutral axis as, a and you have a radius of curvature ρ .

And you could see very clearly within the coating, there is strain variation, it is not a constant but you know principal stress direction remains same, so I could also replace this for the purpose of mathematical analysis by an average strain, which is good enough to find out what kind of fringe pattern that you would get. So, from the mathematics, I will have to find out what is this shift in the neutral axis.

And we have; what we have done? We have done force balance in the case of in plane stress loading, the force balance was very simple and straightforward, it is mathematics is very simple. On the other hand, the mathematics slightly involved and you will have to go back to your fundamental understanding of how you analyse the beam under bending. Here, again I can write the force balance, I can also write the moment balance.

So, these are all the equations that are needed for you to find out the shift in the neutral axis and also estimate the average strain difference and related to an uncoated specimen. The mathematics is slightly involved, I am not going to derive every step but I am going to give you key intermediate results that will give you an idea, how we have gone about and what I have here is how do I write the expression for the specimen strain and coating strain.

If you go back to the way that you developed bending of beams, the distance from the neutral axis and the curvature if you know, I can write an expression for strain, I have to put the limits appropriately and that is what the expression looks like. So, I have y / ρ , this is a very famous expression and this specimen strain is from this fiber to this fiber, so that is given as $-h_s - A$, y lies between $-h_s - A$ to capital A and from A to $A + h_c$, you have the strain in the coating.

And we also know the strain in the transverse direction on the specimen is related to the strain in the longitudinal direction by $-\nu$ times ϵ but on the coating, it is related to $-\nu_c$ times ϵ_s . This expression is simple that is why I said, you should not afford to forget whatever that you have learnt in strength of materials. See, we build on fundamentals and if you want to understand advanced courses, you should know what you have done in your earlier courses.

(Refer Slide Time: 30:06)

Correction factor for bending ...contd

Equilibrium of forces acting along the axis of the beam (width b) requires

$$\frac{b}{\rho} \left(E_s \int_{A-h_s}^A y dy - E_c \int_A^{A+h_c} y dy \right) = 0$$

The position of the neutral axis is found to be

$$A = \frac{h_c}{2} \left[\frac{1 - \left(\frac{E_c}{E_s} \right) \left(\frac{h_c}{h_s} \right)^2}{1 + \left(\frac{E_c}{E_s} \right) \left(\frac{h_c}{h_s} \right)^2} \right]$$

NPTEL

This memory is very important, let it go a long way and while writing this expression, we do not know what is the value of A ; A has to be calculated from the mathematics and first let me write the force balance. So, the force balance you could write it in this fashion; straightforward on a beam under pure bending, the net force is 0, so I have one written down for this specimen. I have another expression written down for the coating.

And this reads like $b / \rho * \text{Young's modulus}$ or the specimen integral with the limits goes from $a - h_s$ to a , $y dy + \text{Young's modulus of the coating}$ and the integral goes from limits a to $a + h_c$ $y dy = 0$ and this expression when you solve, I finally get what is the expression for A , that turns

out to be $h_s/2 * 1 - E_c / E_s * h_c / h_s$ whole square, later we will replace this by convenience symbolism, I replaces this by e and probably will replace this by g and so on.

(Refer Slide Time: 31:59)

EXPERIMENTAL STRESS ANALYSIS

Photovoltaic Coatings

...contd

Correction factor for bending

Equilibrium of moments about the origin gives

$$\frac{b}{\rho} \left(E_s \int_{-h_s}^a y^2 dy + E_c \int_a^{a+h_c} y^2 dy \right) = M$$

The radius of curvature is obtained as

$$\frac{1}{\rho} = \left(\frac{12M}{bE_s h_s^3} \right) \left(\frac{1}{H} \right)$$

Where

$$H = \frac{4(1 + eg^3) - 3(1 - eg^2)^2}{(1 + eg)}$$

$$e = \frac{E_c}{E_s} ; g = \frac{h_c}{h_s}$$

NPTEL

Copyright ©2004 Prof. K. Ramani, IIT Madras, Chennai, India

And this is divided by $1 + E_c / E_s * h_c / h_s$ whole square, so by looking at the equilibrium of forces, I am able to find out what is the expression for; this gives the neutral axis position, it is measured from the top fiber of the specimen. Now, I have written the equilibrium of forces, then I have to write the equilibrium of the moments that will give me the curvature. The idea is to find out what is the curvature and so on and so forth.

I also mentioned that I will replace the ratio of Young's modulus as e and ratio of thickness as g that makes my life simple in expressing the expressions and the equilibrium of moments again has 2 terms; one corresponding to the specimen and one corresponds to the coating and if you have physically identified what are the limits of the specimen, it goes from a - h_s to a and coating goes from a to a + h_c , rest of the expression is very simple and straightforward.

And this is the bending moment applied and mind you we are considering a problem, where the specimen is subjected to pure bending and when I solve this equation, I get the radius of curvature is obtained as $1/\rho = 12 M / b * E_s * h_s^3 * 1/H$ and H is a very complex expression; H is given as $4 * 1 + eg^3 - 3 * 1 - eg^2$ whole square / $1 + eg$, you know this is for your understanding.

You know, in my examinations, I do not test your skill of memory you know but you also have provision for you to bring in A4 size sheet on either side, you can write important final

expressions of formulae and you filter out what formulae from the course to fill it in there that itself is a learning because once you have learned the subject thoroughly, you know what is important and what is not important.

Because once you function as engineers, you should know the methodology very well for details engineers look at handbooks and take out those expressions, so that comfort you will also have when you write the final examination. For the final examination, I allow a A4 size sheet, on 2 sides you know if you have a skill to write the entire set of expressions that is your left your choice.

(Refer Slide Time: 34:55)

EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings

Correction factor for bending

...contd

- The average value of the principal strain difference in the coating is

$$(\epsilon_1^c - \epsilon_2^c)_{avg} = \frac{1 + \nu_c}{\rho h_c} \int_a^{a+h_c} y dy$$

$$(\epsilon_1^c - \epsilon_2^c)_{avg} = \frac{6M}{H} \left[\frac{(1 + \nu_c)}{b E_s h_c^2} \right] \left[\frac{(1 + g)}{(1 + e g)} \right]$$

No Xeroxing of the sheets is allowed, handwritten sheets are permitted, so you do not have to worry that I have such complex expressions, do you have to remember this you, do not have to remember them but you should know what to write in your final expression that will come only if you know how you are solve it. Now, what we do is we also define the average strain difference in the coating.

And that is given as $1 + \nu_c / \rho h_c$, the integral goes from the limit a to $a + h_c$ and I have y that is to be integrated and final expression for this takes the form like given here, so the average strain difference developed in the coating is as follows $6M / H * (1 + \nu_c) / b * E_s * h_c^2 * (1 + g) / (1 + e g)$, so in all these expressions you know, E denotes E_c / E_s , g denotes h_c / h_s that kind of symbolism we have developed and that makes your writing of expression simple.

(Refer Slide Time: 36:31)

EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings

Correction factor for bending

...contd

However,

$$(\varepsilon_1^u - \varepsilon_2^u) = 6M(1 + \nu_c) / bE_s h_s^2$$

$$(\varepsilon_1^u - \varepsilon_2^u) = R_f^b (\varepsilon_1^c - \varepsilon_2^c)_{average}$$

$$R_f^b = \frac{(1 + eg)}{(1 + g)} \left[4(1 + eg^3) - \frac{3(1 - eg^2)^2}{(1 + eg)} \right] \times \left(\frac{1 + \nu_s}{1 + \nu_c} \right)$$

This correction factor accounts for both reinforcing and strain variations through the coating.

NPTEL

Copyright © 2004 Prof. K. Ramani, IIT Madras, Chennai, India

So, now you have methodology how to go about and write the final expression for correction factor in bending. I have now have expression for average strain difference and I will also related to the uncoated strain difference, then my job is done and what you have as uncoated strain difference, this behaves like a simple beam. Whereas, when I have a coated specimen it behaves like a composite beam, so this is what I have here.

Finally, I have uncoated strain difference is given as $6M + 1 + \nu_c / b * E_s * h_s^2$ and this is related to the average strain difference by a correction factor R_{fb} , so here we have replaced the super script from a to b; a denotes axial loading and b denotes the bending loading and R_{fb} is finally obtained as $(1 + eg) / (1 + g) * [4 * (1 + eg^3) - 3 * (1 - eg^2)^2 / (1 + eg)] * (1 + \nu_s) / (1 + \nu_c)$, the whole of it multiplied by $(1 + \nu_s) / (1 + \nu_c)$.

And what does this correction factor do? In the development, we have seen there is a strain variation over the thickness of the coating, so whatever the correction factor that I have got accounts for both the strain variation in the coating as well as the reinforcement effect and you know this kind of expression is needed even when you want to calibrate the photo elastic coating material because there you are going to be a cantilever beam under the bending.

(Refer Slide Time: 38:45)

EXPERIMENTAL STRESS ANALYSIS Photoelastic Coatings

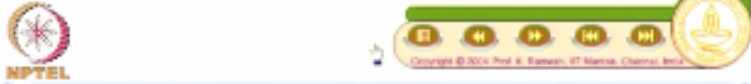
Correction factor for bending of plates

- For thin or medium thick plates, in regions of uniform stress, the correction factor is

$$R_i^{bp} = \frac{(1 + emg)}{(1 - g)} \left[4(1 + emg^3) - \frac{3(1 - emg^2)^2}{(1 + emg)} \right]$$

$$e = \frac{E_c}{E_s}, g = \frac{h_c}{h_s}, \text{ and } m = \frac{1 - \nu_s^2}{1 - \nu_c^2}$$

- For the restricted case of pure-bending loads, the correction factor may be applied for thick plates too.



And it is subjected to bending loads, so you have to use correction factor right from calibration of the coating material, so it is very important and what are all the ways that I can use this, from whatever the expression that I have got, I could also go and extrapolate it for bending of plates, we will see a little while later. For thin or medium thick plates in regions of uniform stress, the correction factor is given as R_i^{bp} denotes bending of plates.

And you have a very long expression for this and we bring in one more identity m ; m denotes $1 - \nu_s^2 / 1 - \nu_c^2$. We have already seen what is e , what is g and now we have introduced another parameter, these are all for convenience; for writing the expression in a convenient fashion. So, for bending of plates, R_i^{bp} is given as $(1 + emg) / (1 - g) * [4(1 + emg^3) - 3(1 - emg^2)^2 / (1 + emg)]$.

And this is for a generic expression for the restricted case of pure bending loads, the correction factor may be applied for thick plates too; you know in the initial development of photoelastic coating, people have analysed all this. They brought the correction factor and there are also comparison what is the correction factor if I use the simple bending equation and plate equation and that identity we will see.

(Refer Slide Time: 40:33)

Correction factor for bending of plates

...contd

- For $\nu_s = \nu_c$, $R^s = R^{sp}$ (for most engineering materials).
- Not applicable for common geometrical discontinuities such as a hole or a series of holes.
- Not applicable, where the plate has an abrupt change in thickness.

We will see now that when I have $\nu_s = \nu_c$, I do not have to separately go and find out correction factor for bending of plates, I can use simply the correction factor for bending what we had developed earlier, you know these are all the developments in the old days, where they had only slide rule to work with. Now, you have computer software available, you have to just put in your thickness of the specimen and thickness of the coating and coating and material properties.

You can even select it from a database you instantly, get the correction factors. Even if it is not available you can write your own code, so the days are different now. Though the expressions look very long and unwieldy, it is necessary even to do simple calculations in reflection photoelasticity, you cannot ignore it and I had already mentioned that whatever the correction factor that we do, they are not meant for common geometrical discontinuities such as a hole or a series of holes.

It is also not applicable when the plate has an abrupt change in thickness and what is the Via Media; what I have to do is; put a thick coating and find out which are all the stress concentration zones, remove the coating put a thin coating in those regions, where you want to find out the stress concentration area, so you do not need a correction factor for analysing those regions where you have stress concentration.

This is the way they solve the problem from an engineering point of view because the correction factor determination becomes difficult in fact, I am going to show you correction factor for torsion, I am going to give you only the final result. If you look at the development of

the necessary equations for correction very complex, when you go for pressure vessel it is going to be much complex than what we had seen earlier.

(Refer Slide Time: 42:55)

The slide is titled "Combined in-plane and bending loads" and is part of a presentation on "EXPERIMENTAL STRESS ANALYSIS" with a sub-topic of "Photoelastic Coating". It contains four bullet points:

- Frequently, a structural part is subjected to a combination of plane and flexural loads.
- For thick coatings, proper weighing of the correction factors is important.
- One approach could be to find a coating thickness such that the factors R^p , R^{pb} are equal.
- If done, one will observe that the thickness of the coating is larger than the thickness of the structure.

The slide also features the NPTEL logo in the bottom left corner and a set of navigation icons (back, forward, search, etc.) in the bottom right corner.

We have only developed a systematic procedure for in plane load and also for pure bending, for all other cases, we will write only the final expression. We will take the result from the literature as sacrosanct and use it and this is also discussed you know, these are all to show what kind of contradictions that you may have to face, you will have in general combined loads. I have a combined in plane and bending loads.

For thick coatings, proper weighing of the correction factors is important and one of the approaches could be to find a coating thickness such that the axial load correction factor and bending load correction factor are equal. Suppose, I do this thickness of the coating is larger than the thickness of the structure, so such contradictions exist. See, the whole idea here is we have looked at what is the correction factor.

Like I said, the finding out the maximum fringe order obtainable in a photoelastic coating testing is an issue, finding out a suitable thickness for a given photoelastic coating test is also a discussed issue. People use different philosophies, when I have to use correction factor, one philosophy, why not I use the thickness such that correction factor equal to 1, so that I do not have to apply the correction factor everywhere but I will modify the thickness of the coating.

So, you have such philosophies also, so some of these modification in thicknesses, you could do when you are using a countourable plastic, where you are going to cast and you can adjust

the thickness to your requirement and then paste it over the actual specimen. On the other hand, if you import or buy from your supplier, even that contourable plastic in the gel state, you do not have a choice on multiple thicknesses available from the supplier.

He will have certain discrete values, so you have to live with discrete values and another issue is when you are having a coating, when you are doing a contourable plastic if you go and press it with your hand, there is a possibility of small thickness variation. So, another philosophy for thickness determination could be small variation thickness should not affect the correction factor drastically.

So, there are multiple approaches in finding out the thickness of the coating; ideal thickness of the coating but finally it is all dictated by what is the coating readily available and you go paste it and then do the analysis but from an academic point of view, you should also look at from a holistic view that what are all the variations possible. So, when you have $R_{fa} = R_{fb}$, if I make it, I will have a thickness of the coating larger than the thickness of the structure.

(Refer Slide Time: 46:07)

PHENOLIC COATINGS

Combined in-plane and bending loads

...contd

- Alternately for analysing thin plates, a coating of convenient thickness be first applied to examine the general distribution of strains and to locate high strain regions.
- Then the coating is stripped off and replaced in the critical regions of high strain by a thin coating.

Obviously, not desirable that is what is implied here, what is implied here is; when you do this it is not desirable and this is what I have been saying off and on for analysing thin plates, a coating of convenient thickness be first applied to examine the general distribution and to locate high strain regions, then the coating is stripped off and replaced in the critical regions of high strain by a thin coating.

So, this is a generic procedure, for any problem if you use a very thin coating, you do not have to worry about the correction factor. Correction factors become very important, when the thickness is significant. Why you need a sufficient thickness? If I do not have sufficient thickness, I do not see fringe pattern, on the other hand if I have a very precise equipment, then I can go and use smaller thicknesses, so that is the advantage when I go for digital photoelasticity.

(Refer Slide Time: 47:36)

Correction factor for torsion

- Coating reinforces the member.
- Strain gradient exists through the thickness of the coating.

$$R_t^t = \frac{2}{(1+c)} \left[1 + \frac{G_c(c^4 - 1)}{G_s(1 - a^4)} \right]$$

G = shear modulus
 $a = r_i/r_o$
 $c = r_c/r_o$

- Not applicable for the regions such as keyways and abrupt changes in diameter.

NPTEL

Whatever, the digital photo elastic principles, which we have learnt are also equally applicable to reflection photoelasticity, so, when I go for digital reflection photoelasticity, I can afford to use a thin coating and do my experiment. So, my nuisance from correction factor effort becomes minimal. Now, we take up, what are the correction factors in the case of torsion and I am going to give you the expression directly.

Here again, strain gradient exists that is what you have to note down, both in bending and torsion, there is a strain variation and we have this denoted as R_t^t ; t denotes, it is torsion and this takes the form like this; $2 / 1 + c$ and c you define as ratio of the coating radius divided by the outer radius of the specimen and a , as inner radius of the specimen divided by outer radius of the specimen, the expression is lot more simpler here.

I have on $2/ 1 + c * 1 + G_c * c^4 - 1 / G_s * 1 - a^4$ and what do you get finally? You get finally not applicable for the regions such as keyways and abrupt changes in diameter. In fact, when you take any practical shaft, it will have key ways, it will have changes in diameter,

so in all those cases this correction factor is not applicable, so this is a contradiction. Correction factor you are able to find out for simple problems.

For complex problems, it is not applicable but the saving grace here is; because it is a region of stress concentration even a thin coating can provide do enough information. So, that is how it is solved from an engineering point of view, so when you initially do the test, identify zones of high strength concentration then, use a thin coating. If you look simple geometry, you know everything where is there is stress concentration, where there is reasonable value of strain, all this heuristic information you know.

The problem comes only when you have a very complex situation where in your experiment should rescue. Like I said in the case of concorde failure, photoelastic coating identified very precisely the zone of stress concentration, so identifying zone of stress concentration in a generic complex problem is challenging. So, from that point of if a coating test can really help, so you have the specimen here, you have the coating.

(Refer Slide Time: 50:49)

The slide, titled "Correction factor for pressure vessels", is part of a presentation on "EXPERIMENTAL STRESS ANALYSIS" and "Photoelastic Coatings". It contains the following text and diagrams:

- Coating restrains radial displacements of the vessel.
- Coating restrains longitudinal deformation of the member.

The diagrams illustrate the stress state in a pressure vessel. The first diagram shows a cross-section of a vessel with internal pressure P_i and external pressure P_o , with radial stress σ_r and hoop stress σ_θ indicated. The second diagram shows the stress distribution in a thin-walled vessel, with radial stress σ_r and hoop stress σ_θ indicated. The third diagram shows the stress distribution in a thick-walled vessel, with radial stress σ_r and hoop stress σ_θ indicated.

- The restraints are not equal for the transverse and longitudinal directions and the evaluation of correction factor is more complicated.

The slide also features the NPTEL logo and a copyright notice: "Copyright © 2004 Prof. K. Ravooji, DTU, Mumbai, Chennai, India".

And when I applied torsion, you have a strain variation the coating and all this is accommodated in this correction factor and another very important engineering problem is pressure vessels, you look at your household cylinder; gas cylinder, it is a pressure vessel, your cooker is a pressure vessel, your aircraft is a pressure vessel, so pressure vessel is a very important from engineering point of view.

And what you have here; I have an inner cylinder, which is put with coating, so it behaves like a composite cylinder and again have this specimen subjected to internal pressure as well as pressure from the coating and the coating is subjected to some amount of internal pressure. The evaluation of correction factor is more complicated in these classes of problems. So, you are going to have a very long expression, it is not going to be simple.

We are not going to look at the full derivation as such; nevertheless, you should have in your notes because pressure vessels are so important, you need to have the expression for correction factor. So, what you find is coating restrains radial displacements of the vessel that is why you have pressure acting on the specimen, coating also restrains longitudinal deformation of the member.

(Refer Slide Time: 52:40)

EXPERIMENTAL STRESS ANALYSIS Photoelastic Coati

Correction factor for pressure vesselscontd

- In the case of a long cylindrical pressure vessel with closed ends, the correction factor (R_f^p) is given by

$$\frac{1}{R_f^p} = \left[\frac{2(1-2\nu+c)(1-\nu)}{(1-2\nu+c^2)+P(1-2\nu+a^2)} - \frac{(1-2\nu)}{(1+P)} \right]$$

$$P = \frac{em(c^2-1)}{(1-a^2)}$$

- Valid only for regions in continuous vessels remote from closures, discontinuities, nozzles, etc.

But you always have a saving escape route that the coating is made of a low Young's modulus, so some of these effects are small, they are not very significant and you have this; this is slightly written down in a different fashion for convenience, you get only $1/r$ fp and I have this as $2*1 - 2 \text{ Nu} + c*1 - \text{Nu} / 1 - 2 \text{ Nu} + c \text{ squared} + P*1 - 2 \text{ Nu} + a \text{ squared} - 1 - 2 \text{ Nu} / 1 + P$, where P is given as $em*c \text{ squared} - 1 / 1 - a \text{ squared}$.

We have already seen how do you define c and a in the case of a torsional member, similar definition applies here too and like any other correction factor here again, it is valid only for regions in continuous vessels, remote from closures, discontinuities, nozzles etc. So, that is what you have to keep in mind, correction factors are very important and correction factors are developed for simpler loading situations.

And we have seen, if you have a stress concentration with a finite geometry even theory of elasticity cannot solve unless, I evaluate it analytically and then compare it, there is no way I can find out the correction factor with modern developments, you could possibly develop the correction factor by a finite element analysis. Any complex problem can be solved and then you could find out a correction factor.

But people have addressed it from engineering point of view, what they have said is; used the photo elastic coating to identify zones of high strain regions and in those regions use a thin coating and do away with the correction factor that is a very good strategy but why correction factors are needed? Correction factors are needed even for your beam under bending. See, beam under bending that is the basic experiment that we use for finding out the strain coefficient K .

And unless, you use the correction factor K will not be determined accurately, we will see that towards the end of the lectures on for elastic coatings. So, what we have seen in today's lecture was; I emphasized in photo elastic coatings, correction factors are very important. When you focus is to find out specimen stress difference, you also have correction factor coming on the right hand side and that is expressed in terms of the fringe order and strain coefficient.

And you also saw the elastic constants of the specimen appearing on the expression and we will see in a later class, what is the maximum fringe order obtainable from a photo elastic coating test, you can use this expression and find out and this is an issue, you cannot neglect the maximum fringe order obtainable in a photo elastic coating test because that is needed for planning your experiment.

You may not see very high fringes, even when the material yielded, so you will see only very less fringes in photo elastic coatings and that is the reason why people prefer white light as elimination for photo elastic coating. Thank you.