Experimental Stress Analysis
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Lecture – 21
Three Dimensional Photoelasticty

Till now, we have looked at aspects of 2 dimensional photo elasticity, we developed the stress optic law, we also saw that for any photo elastic experiment the first step one has to do is to evaluate the material stress fringe value and material stress fringe value is a very key parameter, it is the only parameter, which relates the experimental information to compare with analytical and numerical solution.

And we looked at there are circular disc under diametral compression is a suitable model to find out F sigma as accurately as possible. The next challenge was how to label the fringes, in fact we had a long discussion on how to find out the fringe order looked at the isoclinic fringe field; isoclinic fringe field features also aspects of 2 dimensional elasticity, what advantage you get from an understanding, what happens in a free outward corners how that knowledge could be effectively utilized in order in the fringes.

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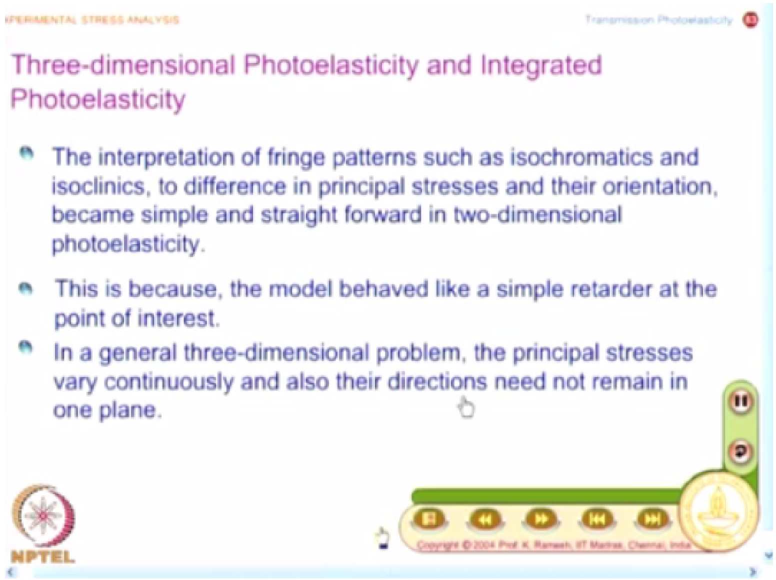

And we also found whatever the fringes we observed could be easily related to physical parameters. Now, let us look at how we can go and analyse 3 dimensional problems, so we have 3 dimensional photoelasticity and it is important to look at the interpretation of fringe patterns such as isochromatics and isoclinics to difference in principle stresses and their orientation became simple and straightforward in 2 dimensional photo elasticity.

We had no difficulty at all, why is it so? How we have been able to find out that isochromatics represents sigma 1 - sigma 2 contours and how isoclinics represent the principle stress direction contour, the basic idea was the model behaved like a simple retarder and what you had was; the thickness was considered as small as possible, we were looking at plane problems, so what you had was the principal stress direction remained same even though the thickness is about 3 to 6 millimetres.

So, because the model behaved like a simple retarder, you are able to interpret isoclinics as contours of principal stress direction and isochromatics as contours of principle stress difference. On the other hand, if you look at a general 3 dimensional problems, the principal stresses vary continuously not only this, the orientations also vary and they need not remain in one plane.

This is a very crucial aspect you know, just because of this the analysis of 3 dimensional problems becomes extremely complex and once you face a problem, you know engineers are smart enough to devise methodologies to overcome whatever the difficulties. So, we have 2 methods essentially, one intelligently uses aspects of 2 dimensional photo elasticity for interpreting 3 dimensional photo elastic results, how do we do that we look at.

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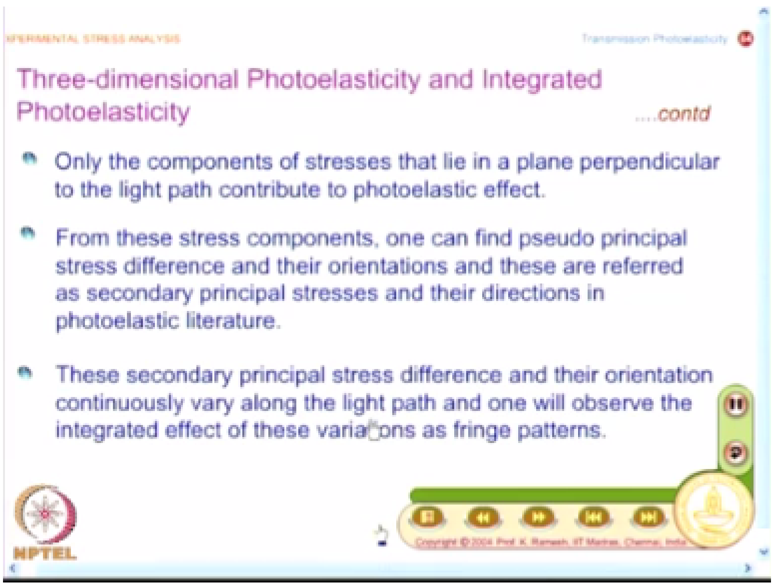

We will also look at another aspect, where we record the fringe pattern as such and go for a integrated photo elasticity, where you develop appropriate mathematics, which is quite complex we will see those details but before we get into it, we need to know another important aspect only the components of stresses that lie in a plane perpendicular to the light path contribute to photo elastic effect, this is very, very important.

Because what you have here is the moment you want to graduate to analyse 3 dimensional problems, we also need to develop certain additional terminologies. One of it what we will try to develop is what are known as secondary principal stresses, so in order to develop that we have to understand what causes photo elastic effect only those stress components that lie in a plane perpendicular to the light path contributes to it.

So, in a 3 dimensional problem, what I will try to do is; I will try to find out what are known as pseudo principal stress difference because you will have a stress tensor in general 3/3 matrix and you will have to evaluate depending on the direction of the light, so light path is also very important and we should find out stress components perpendicular to the light path and from solid mechanics point of view, you will have at the point of interest principal stresses and their orientation.

Those definition change when we want to apply photo elasticity, so I will essentially get only pseudo principal stresses in a plane perpendicular to the light path and in order to distinguish this from conventional understanding of principal stresses these are referred as secondary principal stresses and their directions. This is well understood and established in photo elastic literature and what we will also have is; as I had mentioned the stressors vary from point to point in a 3 dimensional model, the secondary principal stress also varies from point to point.

And that is what we will have to account for when we want to interpret what the fringes that we see, so the secondary principal stress difference and their orientation continuously vary along the light path and what you normally get on the screen is the integrated effect of these variations as fringe patterns because I do not get the information of one plane, I have a series of planes as the light passes through the model.

So, what I get as output is integrated effect of all these planes, that is why the interpretation becomes much more involved in a 3 dimensional photo elastic analysis but what is the greatest advantage? Photo elasticity offers the methodology to penetrate into the model and find out the stresses interior to the model, this is the speciality. Suppose, I have an inclusion and I want to analyse the stresses around it, photo elasticity is an ideal choice.

And when people, who are developing contacts stress problem, the maximum stress occurs beneath the surface it is not on the surface. If you look at any of the coating techniques, if I take a brittle coating or if I take photo elastic coating or if I take strain gauges or even digital image correlation, I essentially get only the surface information. If I have to find out the information interior in certain optical techniques it is possible, in strain gauge also it is possible.

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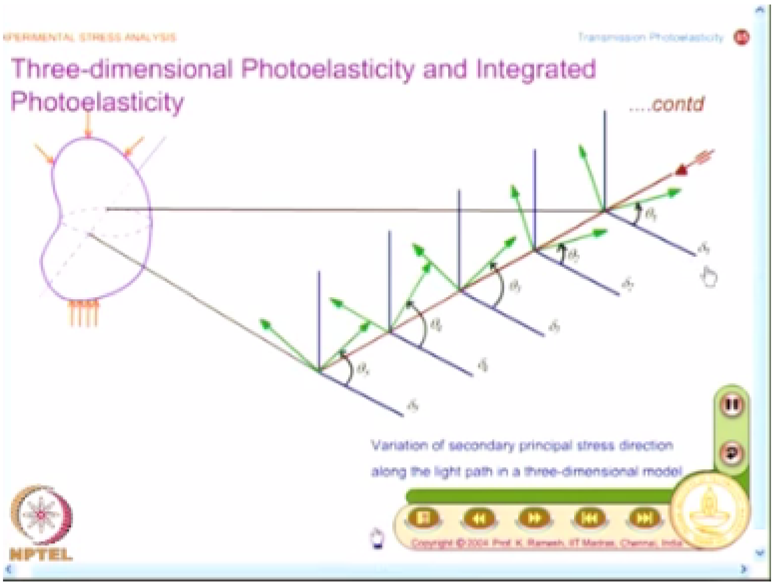

But you can do it at 1 or 2 selected points with lot of effort from your point of view, on the other hand, photo elasticity offers you how to find out the stressor interior to the model, this is its greatest advantage. Now, let us look at what really complicates our analysis, so what you have here is a 3 dimensional model is shown and I have a light path that passes through the model and this passes through several planes of the model and this is expanded and a representation is given.

And what you have here is; suppose, I take the plane 1 because that behaves like a retarder, so now I have a series of retarders along the light path and we have always seen each retarder can be represented by the retardation delta and its orientation theta. In a 2 dimensional problem, you had only 1 plane, so whatever the light that you sent, whatever the fringe patterns you observed you could relate it to theta as well as delta without any difficulty.

But what complicates in a general 3 dimensional model is that this orientation changes as a function of the plane not only the orientation but also the retardation changes from delta 1 to delta 2, delta 3, delta 4, delta 5 and so on. I have only shown 5 representative planes for this light path incident, you will have many planes and you can consider all of them as individual retarders from a mathematical point of view, you will have several Jones matrices for these retarders.

And you can multiply them and get the integrated effect and in order to write this, we have already discussed that what are represented is actually secondary principal stress direction and the difference. We have already defined what is the secondary principal stress difference, so the moment you come to 3 dimensional photo elasticity, you will have to know how to handle variation of the principal stress direction along the light path as for as the variation of the magnitude.

And whenever you face a problem like this, you know mathematicians first try to solve the problem by looking at simpler cases. In one case, I may have a thick model but the stress distribution is such the principal stress direction does not change over the thickness only the magnitudes vary; this is one type of problem. There could be another class of problem, where the magnitude remains same but there is a rotation or the principal stress direction along the light path.

And problems of this nature people have found out even analytical solution, how to go about and evaluate the stress distribution. So, the moment you come to 3 dimensional photoelasticity, you cannot confine your knowledge development only to the plane of interest but you will have to bring in the stress distribution along the light path, so that makes the problem complicated and have a reasonable sketch of this diagram.

So, I have a 3 dimensional model and the essence here is the orientation as well as the retardation changes along the light path and obviously one single light path will not give you all the parameters of interest. I may have to have multiple light paths for me to find out the stress distribution, so you understand what kind of complexities that come in a 3 dimensional photo elastic analysis.

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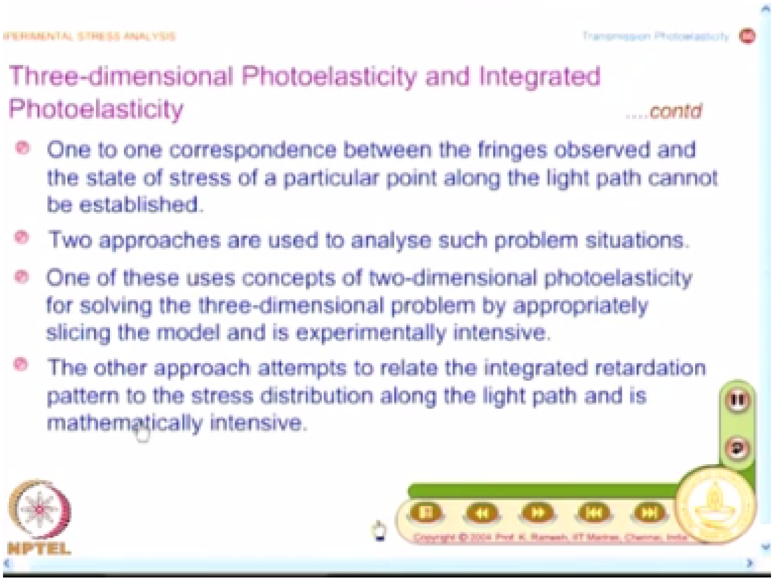

So, what you find here is one to one correspondence between the fringes observed and the state of stress of a particular point along the light path cannot be established straight away, which I can find out, if I solve the problem I will be able to get this answer, whereas in the case of 2 dimensional photo elastic analysis, we were able to get it directly, we had no difficulty in associating the isoclinics to principles stress direction and isochromatics to principle stress difference.

So, now you have 2 approaches that are used to analyse such situations. That is what I said when engineers look at a problem they tried to find out solution and one of these uses concepts of 2 dimensional photo elasticity for solving the 3 dimensional problem by appropriately slicing the model, this was possible because of a very unique phenomena called stress freezing. The method is experimentally intensive.

See, if you have a problem when you want to approach you have to pay a price for it, the price may come from either experimentally intensive, if you want to simplify the experimentation then it becomes mathematically demanding. If the problem is complex, you have to pay a price for it, there is no escape from it, you will have to do it, you can circumvent it only in a manner which is possible from the methodology that we use.

So, because of stress freezing people were able to extend 2 dimensional photo elastic analysis for analysing 3 dimensional problem intelligently but it is experimentally intensive that means, I have to do stress freezing then I have do slicing, all that we will see now. The other approaches directly record the fringe pattern and you have that as integrated retardation pattern and that needs to be interpreted.

Because I need to assume a stress distribution and obtain the parameters based on mathematically solving the problem, so the other approach becomes mathematically intensive either it has to be experimentally intensive or mathematically intensive. Mind you, anything interior is always difficult, what you will have to look at is; if I want to find out what are the stresses interior, I have a methodology, you have to look at it from a positive point of view.

In fact, many of the industrial problems in the 1930’s and 50’s have been solved by exhaustive 3 dimensional photo elastic analysis that is though, many of the fundamental designs were perfected. Even now, even the A3 89, the landing gear was done by 3 dimensional photo elastic analysis and photo elastic coating in fact, both Boeing and airbus use extensively photo elasticity and photo elastic coating for many of those aero structural components.

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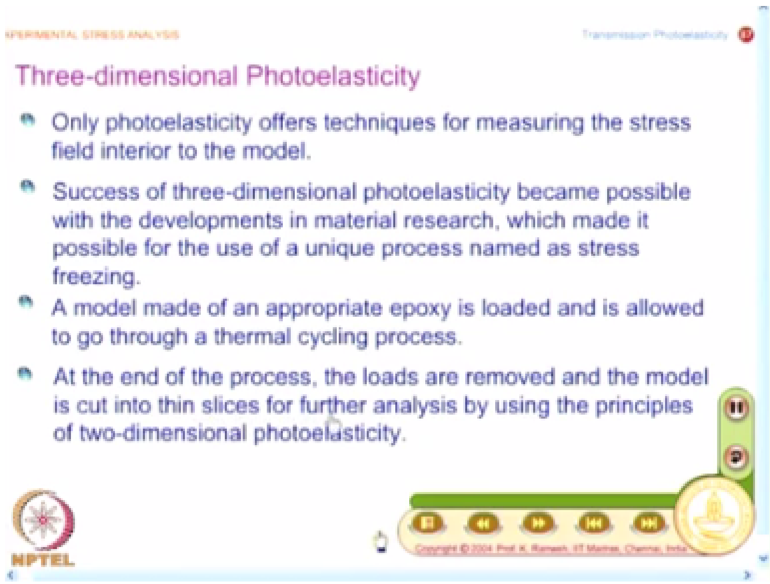

And also in space industry, people use this because in those are industries where any human error in design could be disastrous, so they have to be very clear about the performance of the structure and all these experimental methods really play a role in such situations. So, now we will look at what is the method of stress freezing and slicing. So, we have emphasized earlier also that only photo elasticity offers techniques for measuring the stress field interior to the model.

That is one of its advantages and this was possible because of a unique process named as stress freezing, so I have also mentioned many developments in engineering is because of advancements in material research, so you had developed epoxies and they exhibited a nice property when I go over a thermal cycling, the stress has get locked into it. So, it is a material research, which has contributed to the development of stress freezing.

So, what do you do in this? A model made of an appropriate epoxy is loaded and is allowed to go through a thermal cycling process, it is actually going through a thermal cycling process because at the end of the process, the stresses get locked you call this as stress freezing. We will also see an analogy, why we call it as stress freezing, so what you find is the model goes through a thermal cycling process with the loads applied and at the end of the process, the loads are removed.

And the model is carefully cut into thin slices, that is very important. If you do not slice it properly, the process of slicing can introduce machining stresses that should be avoided, you have to be very careful in doing it that is why I said, when you want to solve a 3 dimensional problem, slicing it what I am really looking at when I make it as a thin slice, I can invoke the model behaves like a 2 dimensional model, whatever the stresses locked in or from a 3 dimensional situation.

From mathematics point of view, I analyse it as a 2 dimensional model, so I take the advantage of 2 dimensional photo elasticity but in order to do that I have to slice it very precisely as thin as possible, so that the principal stress direction remain constant within the thickness and the stress field is actually from a 3 dimensional situation. So, the advantage here is analysis can be done by using the principles of 2 dimensional photoelasticity that is the greatest advantage.

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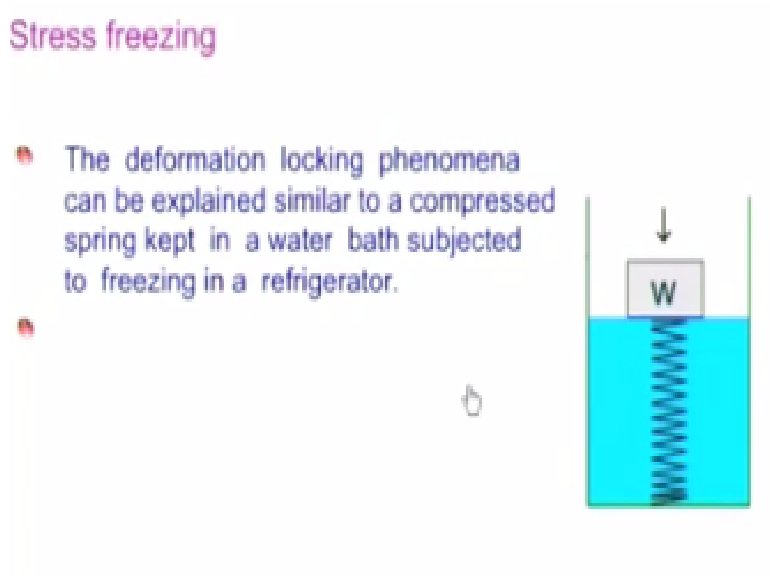

And what I do here this is what you have a stress freezing and I said that I will give an analogy so what I am doing is I am taking a spring, I compress it and put it in water and keep this in a fridge what will happen? I keep the spring compressed and I apply the load and I keep it in the fridge and what will happen is the water will solidify and become ice and it will prevent the spring to come back.

The compressed spring will remain compressed and something similar to that happens in the epoxy also and because I do a freezing to keep the spring compressed, the process of thermal cycling in fact, I heat the model I do not cool the model in this case, I cool water and it becomes ice. In actual stress freezing, I only heat the model but because the phenomenon is similar and it is also nice to call stress is frozen into the model, so I can; the stresses are locked in.

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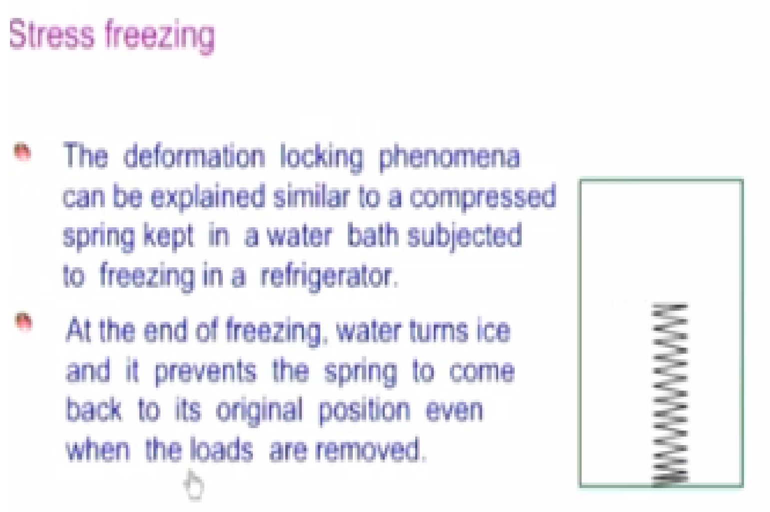

Now, I can take out slices carefully and analyse only those slices, so that is what you see here the water turns to ice and it prevents a spring to come back to its original position even when the loads are removed, I do not have to keep the load here, now I have the animation also shows that load is removed when this was water, load was necessary keep it compressed. The moment it becomes ice; you do not need to keep the load to keep this spring compressed.

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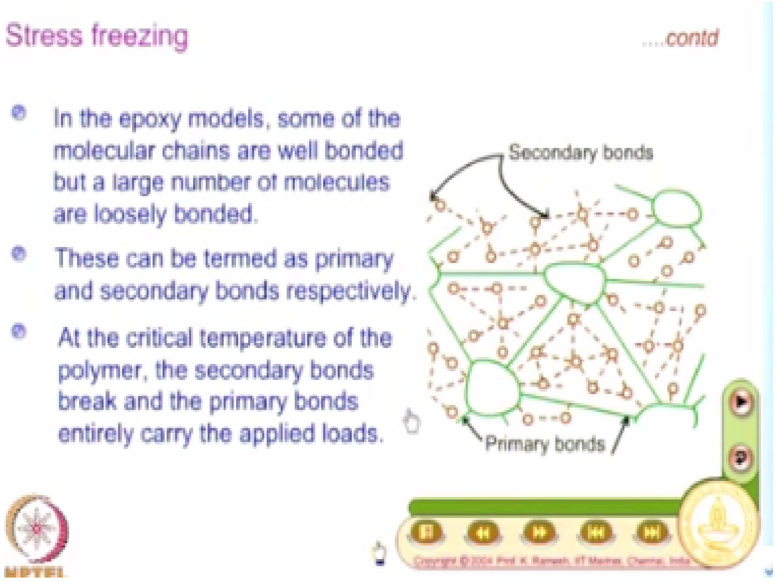

And something similar to that happens in the epoxy and we will also not just stop about explaining the process, I will also show by making a cut in the model, the essential stress distribution is not disturbed, if you cut it very carefully. If you cut it very carefully, the stress lock will not get disturbed and what you have here? I have this epoxy can be thought of as assembly of secondary bonds as well as primary bonds.

When I have an epoxy schematically, this is shown. In epoxy models, some of the chains are well bonded but a large number of molecules are loosely bonded, so the well bonded ones you call it as primary bonds and the chains that are loosely bonded you call them as secondary bonds and what happens is when I apply the load and I also raised the temperature I keep it at what is known as a critical temperature of the polymer.

The critical temperature is something like about 110 to 120 degrees and people also now operate at a lower temperature for various reasons, you do not even have to go up to 120 degree centigrade, you can also operate with 90 degrees, there are requirements for certain problems people also operate at slightly lower temperature than this but what happens at the critical temperature.

Though, it is not visually seen the secondary bonds break at the critical temperature and the primary bonds entirely carry the applied loads. This happens at the molecular scale, you will not see that at the critical temperature there is going to be liquid plastic, no; whatever the secondary bond break is similar to the fluid and the primary bonds gets compressed or elongated depending on whatever the load that you have applied.

And even the loading design for stress freezing has to be carefully chosen because the material properties change at the critical temperature, so if you have to maintain the load, essentially you will find the loading is obtained through a deadweight load because the load will remain constant, load should not change and if I have to have internal pressure they may have a mercury column to do that and people also have done very complex objects like turbine blades.

When it is rotating at high speed, people have frozen stress field because of centrifugal forces by a stress freezing process, so complex objects have been analysed by 3 dimensional photo elastic analysis and this is a very successful process. Though, it is experimental intensive there are technicians and skilled workers, who can do this comfortably. So, at the critical temperature of the polymer, the secondary bonds break and the primary bonds entirely carry the applied loads.

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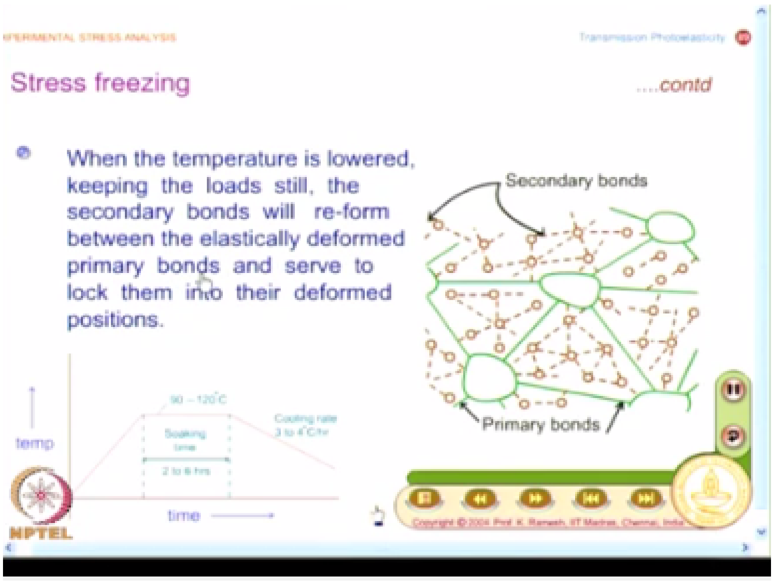

And this is what your thermal cycling process very similar to what I do in the case of keeping a spring compressed, when the temperature is lowered, keeping the load still the secondary bonds will reform and prevent the elastically deformed primary bonds to come back to its original position and hence the stresses are locked and at the end of the process, I remove the load like what I have in the case of spring put in water and frozen into ice.

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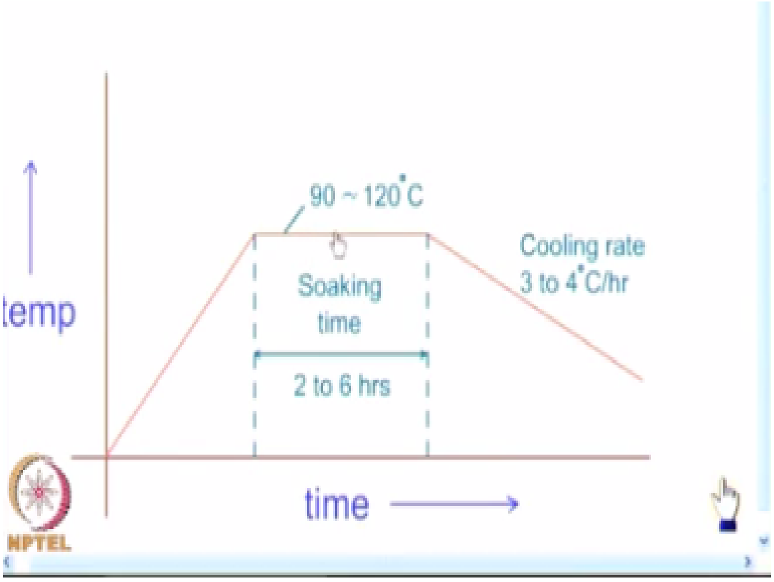

At the end of the process, I remove the load still the spring remains compressed, so similarly in a stress freezing process, I essentially go through a thermal cycling. At the end of the process, I remove the load and let us see, what is the thermal cycling process and this is the thermal cycling process. I have a gradual heating; I reach the critical temperature then I soak it at the temperature for several hours depending on the complexity of the model until the entire model reaches the particular temperature.

Then cool it at a rate of 3 to 4 degree centigrade per hour, so gradually you cool it and this is temperature as a function of time, so the stresses are locked and in the deformed position, the primary bonds remain. So, when I remove the load, the stresses indeed get locked, we will see that by an example, we will take the circular disc under diametral compression, we look at how the fringes develop when the model has gone through a thermal cycling process.

Then I put a slot and show the original stress distribution remains unaltered, so that is the proof of the pending that stress freezing you can do and you can also do slicing and slicing has to be done very, very carefully that is very important and in order to do the cooling, what you normally do is; you just switch off the furnace. Even the normal furnace cooling is good enough to mimic this and this is a very, very powerful process and people have adopted this.

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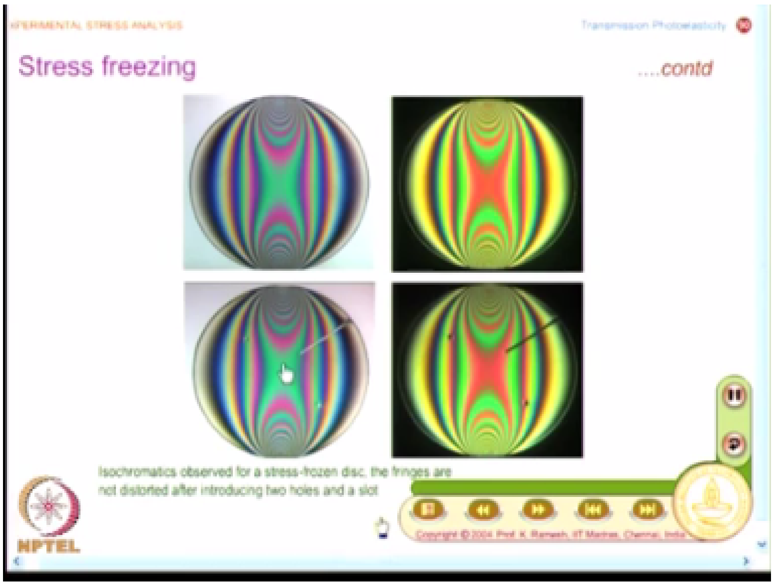

And here you have the fringe patterns, I have a bright field as well as dark field and this is the fringe patterns that you get after the thermal cycling process.

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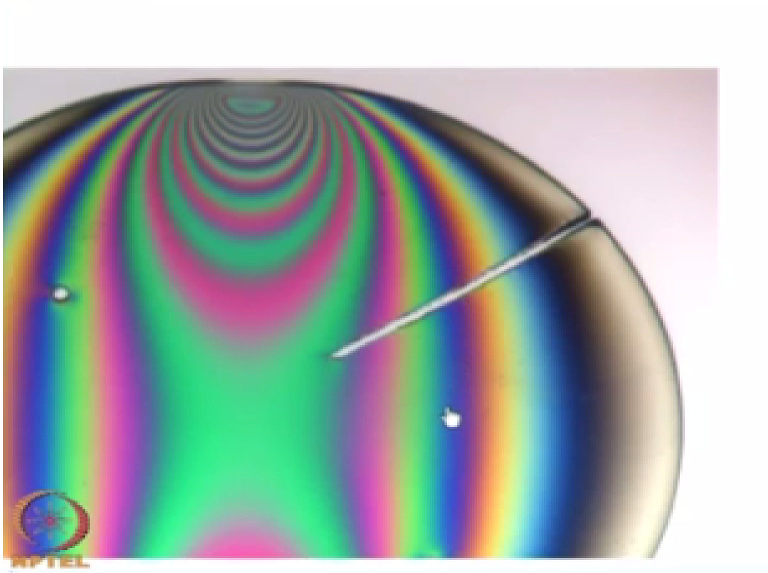

And what you see here, I will also enlarge it, what you see here is after the thermal cycling process, a slot is made, which is very clearly seen in a bright field and you could see very clearly there is no discontinuity in the stress distribution, it is only that this portion is removed and similarly if I do the slicing properly suppose, I make another cut here I will have a slice like this.

So, that slice will retain the original stresses that have been locked. So, when I do a machining process, I must take sufficient care that I do not introduce any kind of machining stresses and you have to also have a coolant, so that to take away the heat and you use a single point cutting tool, there are many restrictions on how you do it and you need a mechanic to be trained to handle this kind of problems.

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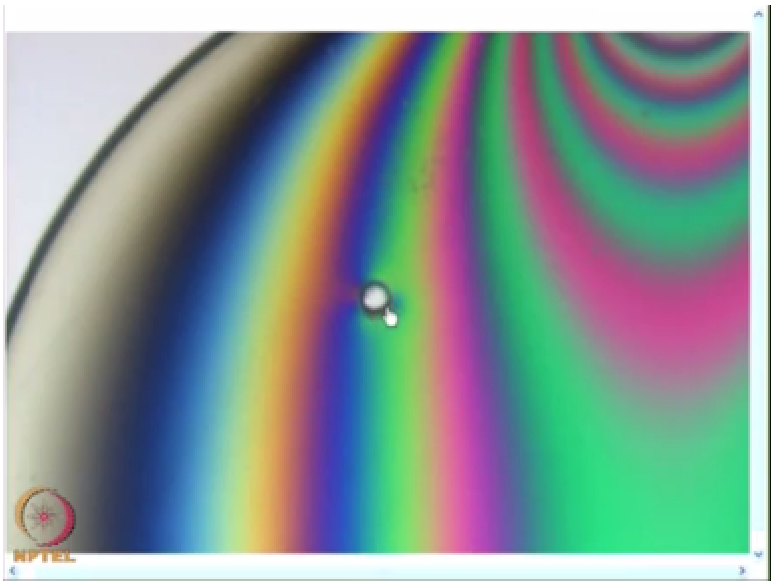

They will use a high speed tool, single point tool and then minimize generation of machining stressors and if you take that kind of a care what this fringe pattern shows is; when I have a slit made, the fringes are not disturbed and you have a slight difference here, this is a hole is made you know, you find small; though, the hole is made reasonably well, there is some small disturbance, it is not as good as what you had as a slot.

So, the person who has made the hole need to improve his process because you have some small disturbance, so even this could be eliminated by careful machining. So, what this slide shows is there is possibility to lock the stresses by a thermal cycling process and there is also a possibility to remove the portions of the model as slices and slices would retain the stresses that have been locked from a 3 dimensional loading.

Here I have taken a 2 dimensional model, I will also show you another example of a 3 dimensional model, which is very complex, I will show you that but the point is well made that a thermal cycling process can freeze the stresses that is very important. The second aspect is by careful machining, it is possible to cut slices, which retain the stress distribution and this is shown in colour.

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And you have also the patterns recorded in black and white.

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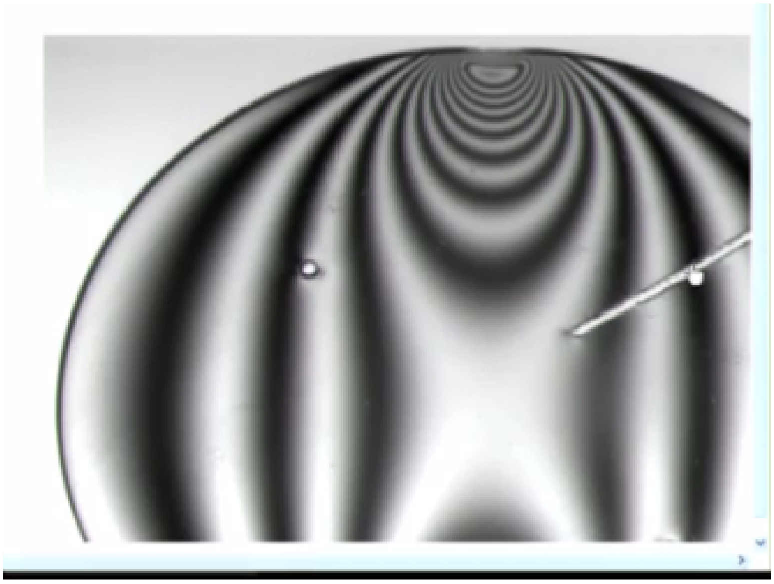

Because the black and white, even if I take this hole you know you would not see that subtle difference, I said that hole machining could have been improved slightly better and when you see a black and white fringe pattern, you will not notice that the hole has some small minor issues. When you see in a colour, you will see the colour change. In a black and white, you will not see that change, you will see that this is as good as the slot that you have made.

And that is the reason why I showed the fringe pattern in colour and colour variation we have also seen that has been used for calibrating the polariscope, used for finding out the sign of the boundary stresses, wherever you come across small variation in fringe orders, colour sequence is the best way to look at and I need you to make a sketch of this that fringe patterns are not disturbed, so you need to have only this sketch that when I make a slit, the fringe patterns are not affected if the machining is done carefully.

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If you go to your bandsaw and then cut it you will definitely generate large amount of stresses, you have to be very careful in doing the machining process and also show you another example, which is a much complex problem. This is just to give you a flavour, what kind of complex geometric shapes people have made and this is a model that is used in a rocket to transfer the thrust from thrust motor to the main body and you have a ball and socket joint here.

So, I have a spherical ball here and you have a threaded connection, you have a bush and the relative size is shown with the help of a figure of a gem clip here and what you do is their parts are fabricated and then assemble, loaded and stress frozen and for stress freezing, I said you have to apply a dead weight and that dead weight is done. After stress freezing, the slices are cut and the typical slices are shown.

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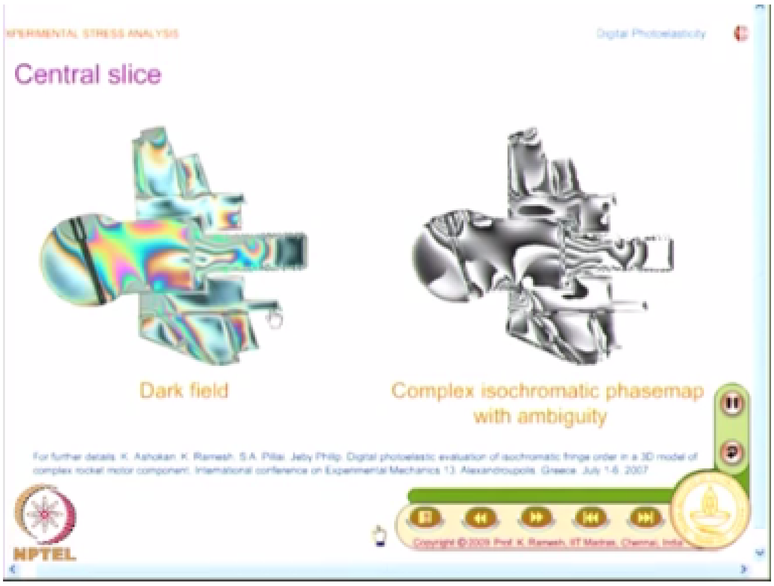

This is just to give you a flavour, how a complex geometry has been analysed and you will see a beautiful set of fringe pattern. This is the central slice and what you find here is; this is the ball and these are all; this is the bush and these are the outer plate and you see the threaded connection and you know, this also illustrates you have seen a 3 dimensional model. From a 3 dimensional model, a 2 dimensional slice is taken.

This is just to give you; see, I want to analyse the 3 dimensional problem, so I need to take a 3 dimensional model and then I apply the loads by following similar to equations and also model to prototype relation, I find out what is the actual load that should apply, do the stress freezing. Once you have done the stress freezing, stresses are locked from that whichever is a portion of importance, you make a slice and then apply 2 dimensional photo elastic analysis to interpret the data.

And mind you, in a problem like this; what is the slice that I should take, is one of the important questions that you should answer and what is the direction of light path that I should take. All these are issues, you should decide, what is the light path that I should investigate, what is the slice that I should investigate, how many slices I require and there has to be a very careful planning of the slice.

Stress freezing is a very elaborate procedure you know, people also you know; the F Sigma value that the material stress fringe value drastically comes down it may be around for a epoxy which is around 11 or 12 at room temperature, at stress freezing temperature it will reduce to

0.3 or so and in one of the challenges in 3 dimensional photo elastic analysis is what is the load that I should apply.

So, people calculate based on how many fringes are you want to see and you do not want to introduce large deformation because the material becomes very plastic at critical temperature, so that is why in similar 2D equation we have seen, we must try to maintain parity between the strains develop in the actual model, actual prototype and the model. So, many times you will find you make a complex model; the model breaks in the stress freezing furnace.

So, you waste 1 or 2 models, if you do not do the calculations correctly or if the problem is too complex and your estimation of loads have gone well beyond the capacity of the model to withstand, so in all the stress freezing process, model making is very difficult and finding out the actual loading to do that is also equally challenging and there are you know; now we have also shown by using a 3 dimensional finite element analysis.

How you can get guidance to select the load, guidance to see which slices have to be cut, all these people have looked at and you have a great boon with rapid prototyping, which has come in a big way in manufacturing. So, you have stereo lithography as a process which is fortunate that whatever the resins that you have used in stereo lithography or photo elastically sensitive and if you look at rapid prototyping, the model is made from a CAD drawing.

So, you have a CAD data, you translate that into a model you grow the model in fact, so that makes your model making simple in photo elastic analysis. The same model can be utilized for finding out the stresses by stress freezing and slicing and the same CAD model could be used for doing a finite element analysis also. So, now you can do a 3D finite element analysis as well as 3 dimensional photo elastic analysis starting from the same CAD model.

So, that way you have a combination of experimental and numerical approach to solve very complex problems, people have solved such complex problems and that is why this technique is very popular and I am sure some of you have taken down this fringe pattern and this shows what you have as a face map, we will have occasion to see when we discuss on digital photo elasticity.

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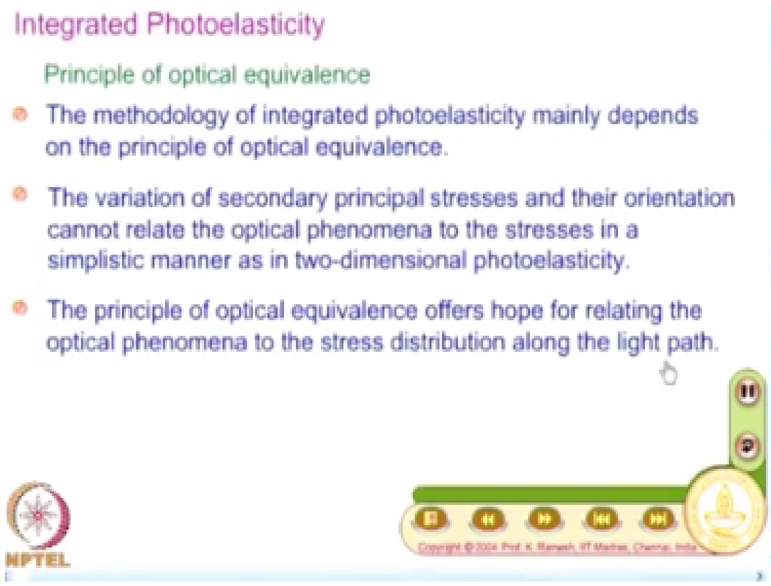

What is the difference between a fringe pattern and a face map? and the focus is to find out fringe order at every point in the model domain and this gives you how a complex 3 dimensional model could be stress frozen and slice for you to do the analysis. Now, we take a; we have seen that the conventional 3 dimensional photo elastic approach involves stress freezing and slicing.

And you must have really seen that it is mathematically; this is experimentally intensive, I am sorry it is not mathematically intensive, it is the experimentally intensive. Now, we are going to look at a process called integrated photo elasticity and this hinges on a very important principle called principle of optical equivalence. This is a very intelligent conception and we look at it more from optics point of view and the entire methodology of integrated photo elasticity depends on the principle of optical equivalence.

And let us see what is principle of optical equivalence and we have already seen that there is a variation of secondary principle stresses and also their orientation and that is a reason why you are not able to interpret by using simple concepts of 2 dimensional photo elasticity unless, you slice them. Suppose, I want to see the model as a whole, the principle of optical equivalence offers a hope, so here I need to determine the stress distribution along the light path.

You are not looking at stress components on a particular plane but stress components variation along the light path, so essentially it is a tensorial tomography. See, in medical imaging they have a CAT scanner, the CAT scanner essentially finds out in your hair skull, if there is a tumour it can go and find out locate where the tumour is, what is its size, so that surgeon can go and operate upon or take some corrective measures.

So, the focus is only to find out the location and the size, so they are called scalar tomography. The moment I want to find out stresses interior to the model at every point, I need 6 components and normal tomography itself is difficult, scalar tomography itself is difficult and if you are going to find out 6 components per point, it is called tensorial tomography and you can imagine what is the level of mathematics that you require to unravel this.

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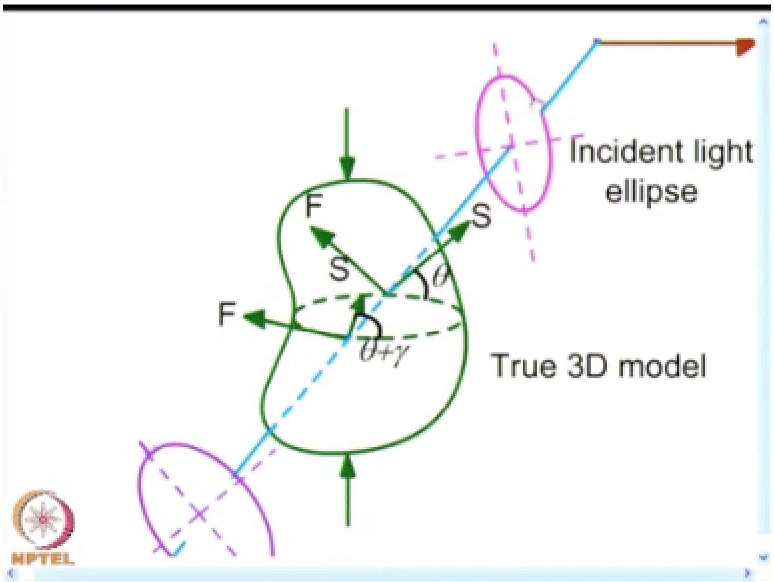

And in this case, even to approach a mathematical methodology, you need principle of optical equivalence. Without this principle, you cannot even formulate the mathematics behind it and let us see, what is this principle of optical equivalence? Here, also you need to make a neat sketch and let us hypothesize what we call it as optically equivalent and we also need to bring in new terminologies.

See, what I have here is; I have a 3 dimensional model and we have seen in general, you will have a light ellipse impinging on the model and because of the stress distribution along the light path whatever the incident light would get altered and essentially this will come out as some other light ellipse with some azimuth and orientation. So, this is well understood, so what you have here is when you send a light ellipse, it will come out as some other light ellipse.

In a 2 dimensional model, whatever the changes could be related to one single plane, in a 3 dimensional model it could be related to the stress distribution and in order for me to develop the mathematics, I need to introduce new mathematical entities, I need to recognize a difference what happens at the inlet point. I have a slow and fast axis and I call these as characteristic parameters; I bring in another terminology called characteristic parameters.

So, when the model; when the incident light enters the model, you have a orientation theta and you also have at the exit plane, another orientation theta + gamma, these are not coinciding with the principal stress direction at the inlet plane and exit plane. These are optically determined and this is called a primary characteristic direction and this is called the secondary characteristic direction.

And you call gamma as a characteristic rotation and you will also have 2 delta, you call that as characteristic retardation, so you attach a new terminology called characteristic parameters. Instead of 2 parameters, you will have 3 parameters now to define the 3 dimensional model, in a 2 dimensional model, you define retarder, you had a retardation and orientation. Here you have a retardation labelled as 2 delta, you call it to distinguish from 2 dimensional analysis, you call it as characteristic retardation.

Because this is the characteristic retardation of an optically equivalent model and you call theta as primary characteristic direction and theta + gamma as secondary characteristic direction and what I say as optically equivalent model? This is what I have in actual model. Now, I will replace the actual model by a set of optical elements, which would essentially modify the incident light ellipse to the same characteristic as the model modified.

And what you will see here is; I will have a retarder as well as a rotator required, I need to have 2 optical elements. The model has to be replaced by a retarder as well as a rotator, in a 2 dimensional problem you had only retarded; in a 3 dimensional problem I replace the actual stress distribution along the light path of interest by a retarder and a rotator, which gives the same exit light ellipse for the given instant light ellipse.

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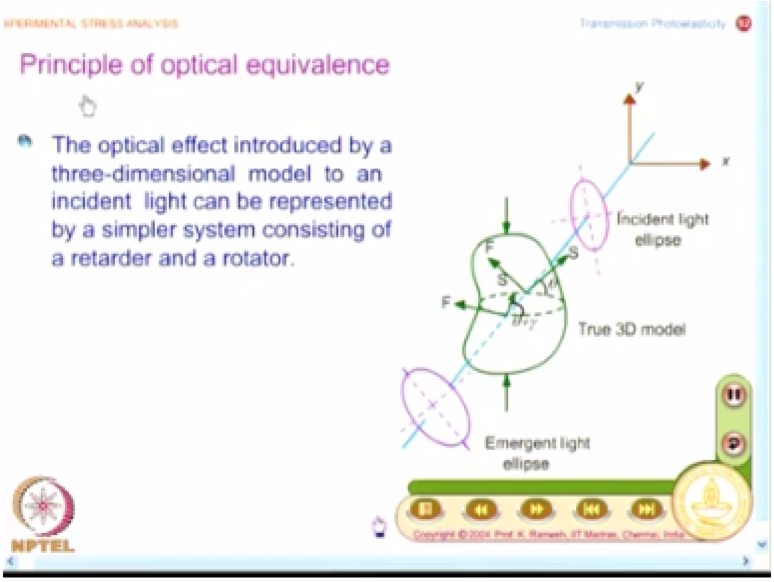

So, this gives you via media, instead of looking at stress distribution, now we have simplified the problem to find out the parameters of the retarder and the rotator that how the problem is rephrased, so we will have a look at the points whatever I have said is now summarized. So, it can replace by a simpler system consisting of a retarder and the rotator. A retarder will give a retardation and a rotator will rotate the light ellipse.

We have already seen when we are discussing a half wave plate in fact, rotates the incident plane polarized light weighted by an angle, so it is possible for you to conceive a generic concept of a rotator, which will rotate any light ellipse to any angle of your choice. So, the idea here is I want to find out an equivalence of what happens inside a 3 dimensional model optically because we are only providing an optical equivalence.

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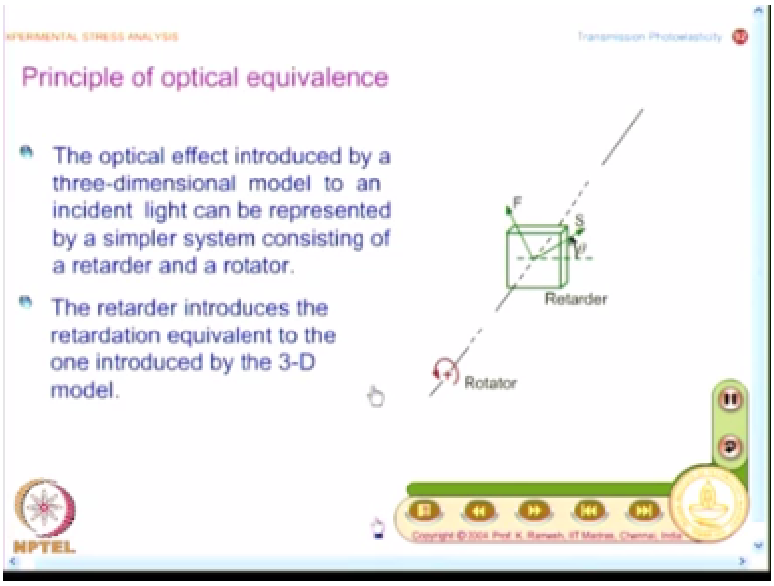

We are only looking at the incident and exit light ellipse, we match the incident and exit light ellipse by replacing the 3 dimensional model by a retarder and the rotator that is what is called a principle of optical equivalence and you see, what is that we are doing. I am putting a retarder and I am putting a rotator and let us see what happens you observe the animation now, let us see what happens when the incident light hits on the retarder.

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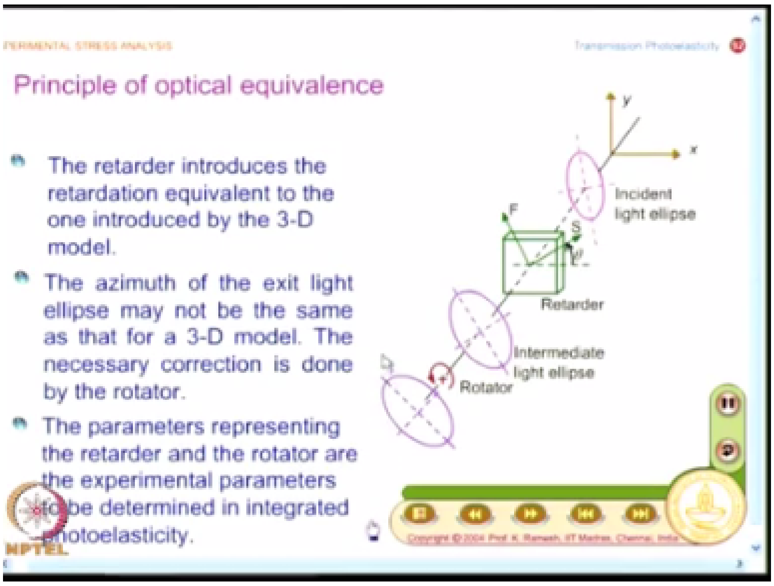

And after the retarder what it happens and after the rotator, how it behaves, just observe the animation, I have an incident light ellipse, so it gets transformed whatever the transformation by the retarder may not guarantee the same azimuth and rotator rotates it, so that the azimuth matches with the actual 3 dimensional model. So, whatever the 3 dimensional model you had and we are only confining to the particular light path, whatever that happens the light path is replaced by a rotator, a rotator here and a retarder.

What it does optically is whatever the incident light; the same exit light characteristic as that of a 3 dimensional model is maintained by this. So, experimentally what I am going to find out is; I will try to find out the parameters of the retarder and the rotator and theta refers to primary characteristic direction, theta + gamma is a secondary characteristic direction. So, what you find here is the retarder introduces the retardation equivalent to the one introduced by the 3D model.

The azimuth of the exit light ellipse may not be the same as that for a 3D model, so the retarder does the job of modifying the incident light ellipse to exit light ellipse but it may not guarantee the azimuth being same. Whatever the correction that is necessary to be done is done by the rotator, so experimentally you determine the parameters representing the retarder and the rotator.

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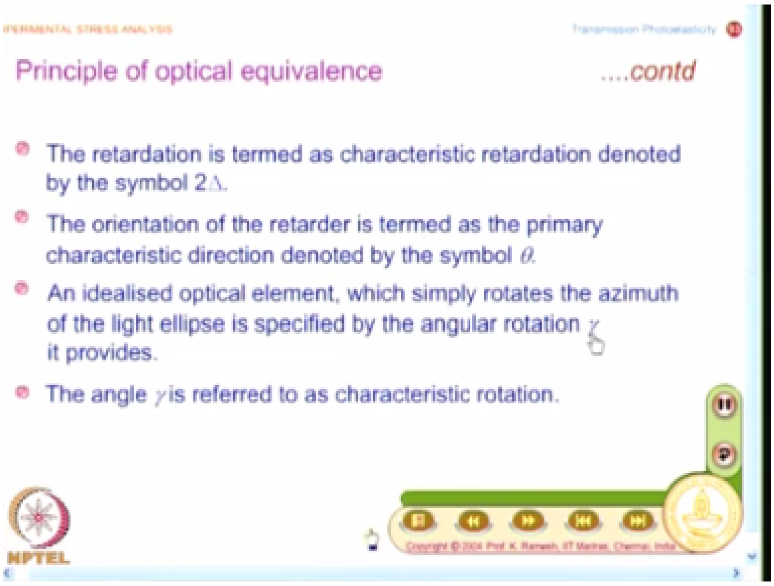

So, I will have to determine 3 quantities, when I go to 3 dimensional photo elastic analysis not one quantity and its interpretation as physical variable is very challenging. It is not simple and what you can think of is I can have a series of retarders and analyse it by Jones calculus and whatever I have said retardation is termed as characteristic retardation labelled as 2 delta. The orientation of the retarder is termed as the primary characteristic direction denoted by the symbol theta.

The rotation is given by gamma; the gamma is given as labelled as characteristic rotation, so these are all the 3 quantities I determined experimentally for a 3 dimensional photo elastic analysis. Suppose, you look at how do I go about? My intention is not to solve a problem in integrated photo elasticity, my interest is to give a flavour, what kind of mathematical complexities that come in an integrated photo elastic analysis.

Once I know the retarder and the rotator at a point of interest, I can construct Jones matrices for it, suppose I take up a problem because whenever we take up a problem or new methodology we solve a known problem. Suppose, I take a 3 dimensional problem and I know the stress distribution along the light path, then I can analytically write the Jones matrix for each of the planes, multiply all of them, I will get one final Jones matrix.

Compare these 2 and find out the parameters related to stress distribution. So, I need a assume stress distribution field, when I assume the stress distribution field reasonably, I will have several coefficients. If I have so many coefficients, I need so many equations, so I need to have multiple optical paths and multiple collections of the characteristic parameters, so you get a flavour that it is mathematically highly demanding; it is not going to be simple.

On the other hand, even though stress freezing is experimentally demanding, it is much simpler mathematically, you can see what is happening physically. In fact, if you look at glass manufacture they introduce deliberately residual stresses and because the process is well understood and also the glass thickness is very small, people have successfully employed integrated photo elastic analysis for such class of problems.

For a generic class of problems, it becomes extremely difficult, so what we have seen today was an overview of 3dimensional photo elastic analysis, we have essentially looked at what is the basic difficulty when we want to apply photo elasticity to a 3 dimensional problem. The principal stresses vary along the light path also their orientations vary along the light path, then we introduced concept of secondary principal stresses.

Then we said if you have a nice process of stress freezing, I can lock the stresses and take out the slices of my interest and extrapolate whatever you have understood in 2 dimensional photo elastic analysis to analyse the slice. The other approach was replaced whatever that happens along the light path optically by an optically equivalent model that gives you via media because that gives you what to measure experimentally.

Experimentally, I have to measure to 2 delta, theta and gamma and also give you an indication how complex the mathematical analysis could be in an integrated photo elastic analysis. So, the idea is to give you a flavour and this almost brings to a completion of transmission photo elastic analysis. Thank you.