Experimental Stress Analysis
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology - Madras

Lecture - 20
Miscellaneous Topics in Transmission Photoelasticity

We have been discussing on transmission photoelasticity and I said one of the very key equation in transmission photoelasticity is a stress optic law wherein you get sigma 1-sigma 2 as NF sigma/thickness of the model and I said F sigma is a material stress fringe value, which need to be evaluated with as much accuracy as possible. We have seen how to evaluate this using a circular disc and a diametral compression.

And I said one of the most crucial data that you need to collect from an experiment is the fringe order and fringe order needs to be evaluated correctly from the fringe field. You had seen circular disc, which is a very simple fringe field where you have 0th fringe order on the outer boundary and you are able to go and increase it towards the load application point.

We also saw another example where ring under diametral compression, which showed almost all features of a generic fringe field. You had a source, you had a sink, you had a saddle point, you had singular point, you have isotropic point and that presented a very complex fringe pattern and when you have a complex fringe pattern if you resort to color code, you get very good colors.

And based on the color it is possible for you to identify the 0th fringe order and also identify the gradient and what is important here is today we are going to discuss certain aspects, which use the color code in a different sense. I said color code could be used for finding out the gradient and you can also find if the changes are very small, the color sequence helps you to say whether the fringe order is increasing at a point of interest or decreases at a point of interest.

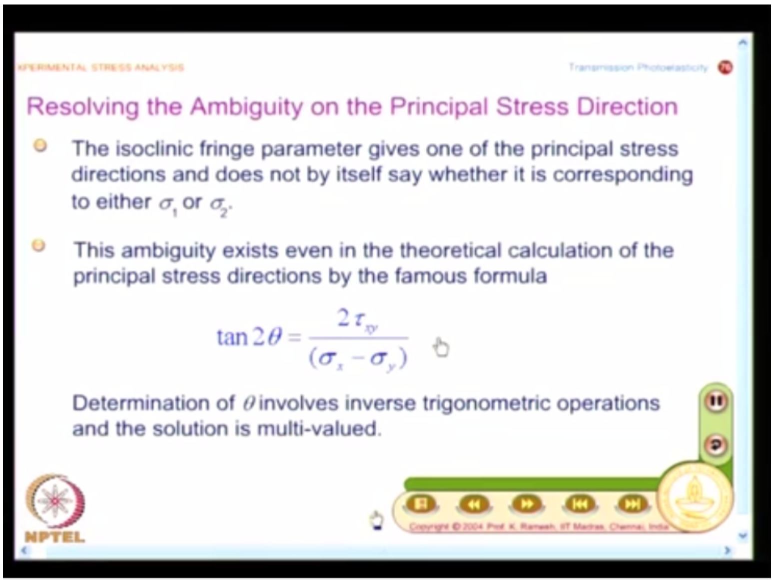
(Refer Slide Time: 02:22)

So it is better that we go to the color code and recapitulate what we have learnt. You have a black, which I labeled as 0th fringe order and we have the first transition as tint of passage we have fringe order 1 and if you look at the color sequence, it goes from black to grey, white, yellow, orange, dull red and tint of passage.

See in actual model situation if the load changes are small, you know you will not be able to see appearance of a new fringe order when you use the monochromatic light source. On the other hand, if you use a white light source, it is possible for you to observe the color change so your eyes need to be tuned because the changes may be very small and subtle. So the color change is the only way to identify whether the fringe order has increased or decreased.

So the color sequence is very important and that is what we see here, I have from 0 to 1 you have a color sequence like this, from 1 to 2 you have a color sequence like this, from 1 to 2 you have blue, blue-green, green yellow, orange and rose red. So if you are focusing on a particular point of interest and when we do some modification on the optical arrangement, the color sequence will help you whether the fringe order increases at the point of interest or decreases at the point of interest.

(Refer Slide Time: 04:02)



And we are going to take up a very important topic which I had said earlier also that when you are actually finding out the principal stress direction, photoelasticity gives you sigma 1-sigma 2 as well as the principal stress direction at the point of interest. Many of the failure theories you know you do not want the orientation whether it corresponds a sigma 1 direction or sigma 2 direction.

I said even in conventional photoelasticity such a distinction is not generally required, but when you are having an experimental arrangement it is all the more desirable that you also find out whether it represents principal stress 1 direction or principal stress 2 direction. I had mentioned long time back that you need a calibration of the polariscope to do this. Whether this is the problem only in experimental approach?

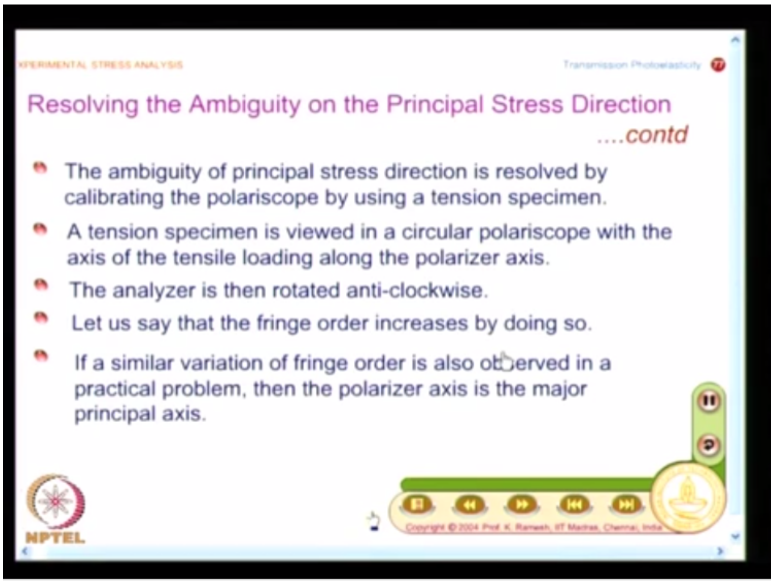
No, we have also seen the ambiguity exists even in the theoretical calculation or the principal stress directions by the famous formula you all know tan 2 theta=2 tau xy/sigma x-sigma y. When you find out theta from this because theta evaluation uses inverse trigonometric operations, the solution is multivalued. You do not know whether it represents theta 1 direction or theta 2 direction.

We have already seen principal stresses are arranged algebraically, so you have a maximum principal stress is given the value of sigma 1 labeling of sigma 1 and algebraically the next smaller one sigma 2 and a smallest you call it as sigma 3. So when I say theta when I want to know the maximum principal stress direction. How do you associate this?

We have seen when the equation is ambiguous in your simple strength of materials, one can always take recourse to Mohr's circle and resolve whether the theta evaluated indeed principal stress direction 1 or direction 2. So you need to have some kind of an auxiliary information. Suppose you want to evaluate it mathematically and we have also seen that you have to have this problem post as Eigen vector and Eigen value problem.

In that case, for every value of principal stress you will get the corresponding Eigen vector that will fix the theta value to the associated principal stress direction. So this is what we had seen mathematically. So now what we will look at is what is the kind of calibration that do we have to do for the polariscope?

(Refer Slide Time: 07:00)



So what I have here is I can do this by taking tension specimen. So when I take a tension specimen what do I know? When I have a tension specimen like this I know the principal stress direction because that becomes the major principal stress direction. I am pulling it like this perpendicular to this there is no stress. So the maximum principal stress direction is the direction of the pulley.

And what I do here is the tension specimen is viewed in a circular polariscope with the axis of the tensile loading along the polarizer axis. I take up a very simple problem and then I align it with a polarizer axis. Then I do we have already seen I can use analyzer as a compensator. So when I rotate the analyzer anti-clockwise or clockwise, the fringe order at the point of interest will change.

And that is precisely what is going to happen and what is going to happen is the retardation introduced would be so small I would not have a full fringe order come and occupies this. If I know the color code and the color sequence, it is possible for me to assess whether the fringe order has increased or decreased by a particular rotation of the analyzer. So the calibration is looked at from that point of view.

So what I do this is I have the analyzer rotated anti-clockwise. Then I observe how the fringe order changes. Let us say that the fringe order increases by doing so. Then what I understand, when the fringe order increases by doing so, I know the original major principal stress direction was along the polarizer axis and when I rotate the analyzer anti-clockwise, the fringe order has increased.

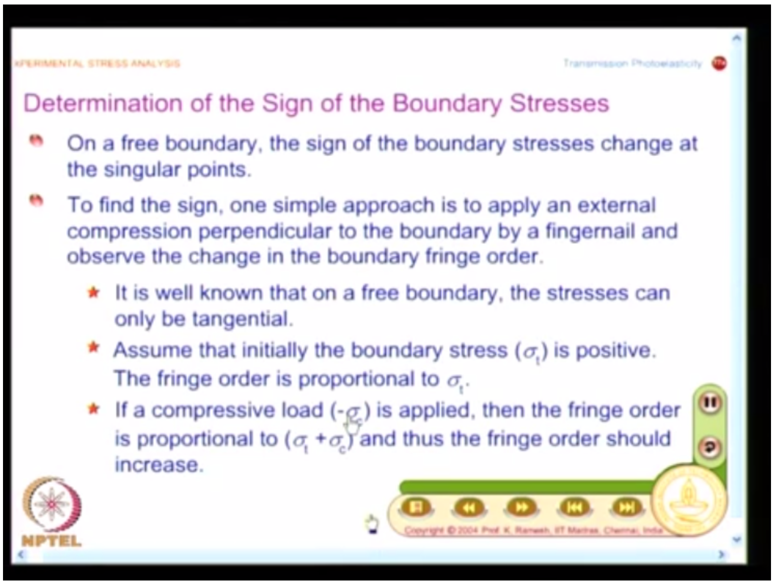
So this is keep it as a base information. I go to an actual experimental model and I identify at a point of interest and then rotate the analyzer, if the fringe order increases then I say polarizer axis coincides with the major principal stress direction. So this is the kind of calibration. So what is summarized here is if a similar variation of fringe order is also observed in a practical problem, then the polarizer axis is a major principle axis.

So what you need to do is you need to have some kind of a calibration to the polariscope. I said this is part and parcel of any experimentation. If you want theta for the major principal stress direction and minor principal stress direction to be determined, that kind of association needs to be done, it is possible to do experimentally and in fact some of the difficulties in digital photoelasticity was how to specify that this corresponds to major principal stress direction or minor principal stress direction.

This has caused what is known as inconsistency in the isoclinic phase map and also caused ambiguity in the determination of fractional fringe order. So though in a conventional photoelasticity we are not worry about major or minor principal stress direction. In digital photoelasticity, you need to worry about and they approach the problem slightly differently.

But the fundamental question here is from experimental point of view, is it possible to say the major principal stress direction at the point of interest and minor principal stress direction at the point of interest, you can do that if you calibrate the polariscope and we have seen what the calibration that we need to do.

(Refer Slide Time: 11:10)



Then what we move on is another simple information in many of the problems you know we want to know what is the sign of the boundary stress. Determination of the sign of the boundary stresses is also a very important aspect and how do we do that? If I want to find out the sign of the boundary stresses how do, we do that? In the case of beam under 4-point bending, we knew which fiber is under compression, which fiber is under tension.

You do not need experiments to tell you that, even before you go to the experiment you know the way the deformation of the beam, you know which is the fiber subjected to compression, which is subjected to tension, but nevertheless we need to know how to find out the sign of the boundary stresses and what I can do is to find the sign a simple approach is to apply an external compression perpendicular to the boundary by a fingernail and observe the change in the boundary fringe order.

Here again the boundary fringe order will change by a very small amount and it is advantageous that you view the model in white light and see the color sequence when the model is compressed by a fingernail and you know you have to come back and then say look at the model very carefully and we will have to identify depending on the change in the fringe order whether it is positive or negative.

First principle is it is well known that on a free boundary the stresses can only be tangential. So this is the first point that you need to keep in mind and for our discussion let us assume that initially the boundary stress is positive. So I label it as sigma t. We do not know what is the sign of the boundary stress, but we take up with a simple situation where the boundary stress is positive to start with.

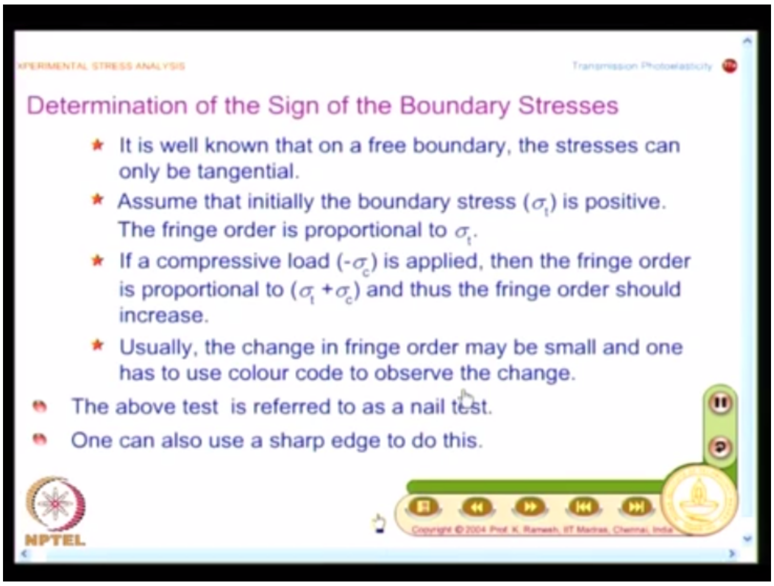
And whatever the fringe order that you see in the circular polariscope is proportional to sigma t because the perpendicular direction the stress is 0 because it is a free surface. So I see that a sigma t whatever the fringe order is proportional to sigma t. Suppose I go and apply a compressive load-sigma c by a fingernail, then what happens to the fringe order? We are only looking at sigma 1-sigma 2.

Now I apply a compression through the fingernail so minus of minus becomes positive, so I get this as sigma t+sigma c and thus the fringe order should increase. So what we are looking at is if the boundary stress was positive, a compression by a fingernail on the surface would result in increase in the fringe order at the point of interest.

So what we will do is if the fringe order increases by a compression, you will say boundary stress was positive. On the other hand, if the fringe order decreases, boundary stress is negative. Then the question comes can I always apply using a fingernail? You know we will also have a short discussion on various photoelastic model materials. We will see for which kind of problem fingernail is good enough.

If the fingernail is not sufficient, we may have to apply the force by external means that is one way of doing it.

(Refer Slide Time: 15:26)

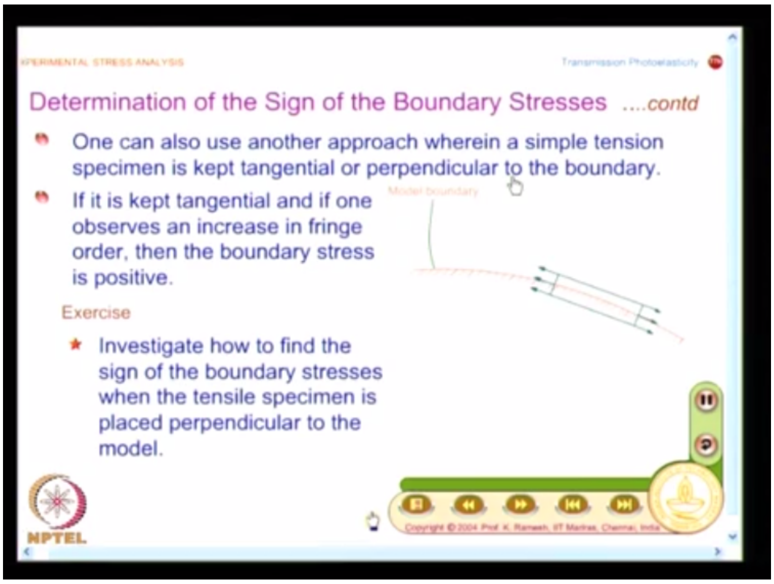


And that is what is summarized here. One can also use a sharp edge if you are not able to apply the compression by a fingernail because we use a finger nail to do this test this is also called as a nail test and as I have mentioned the change in fringe order may be small and one has to use color code to observe the change whether the fringe order has increased or decreased, you will get the information by knowing the color sequence.

That is the best way because these changes are very small and only a color code can provide you this information. So from this point of view also knowledge of color code really helps. So what we have seen is I can do it by a nail test, go and apply the compression and if I am not able to do with the nail, use a sharp edge, but we can also think of other ways to do it.

Once you understand the principle, they have already seen what is the Babinet–Soleil compensator? We have put a compensation after the model and added compensation or subtracted compensation. So on a similar fashion I can use an external member to assist you in doing this kind of a test and that is what we will see now.

(Refer Slide Time: 17:04)



For us to do that I can even use a simple tension specimen, I can take a specimen subjected to tension. I can do it either keeping it tangential to the boundary or keeping it perpendicular to the boundary. So what I can also do is I can adopt another approach wherein a simple tension specimen is kept tangential or perpendicular to the boundary. So if it is kept tangential what will happen?

If I keep it tangential to the boundary, I am adding retardation. The original retardation was positive if I add retardation, the fringe order will increase. When the fringe order increases, I would say the boundary stress is positive. So you also can do a similar approach when I keep it perpendicular to it. So that you take it as a home exercise because you know here you have to be very alert in your logical development of the argument.

Because if you miss the way you have added the retardation you may end up with wrong result. You cannot say if the fringe order increases it is always positive, fringe order decreases is always negative, that kind of a generic conclusion we cannot arrive at. We have to look at whether the model was kept tangential to the boundary or perpendicular to the boundary.

If it is kept perpendicular to the boundary, the reasoning will be different because you are adding or subtracting retardation in a direction perpendicular to it. So what you will have to do is you have to be very careful when you do this when it is tangential to the boundary and when it is perpendicular to the boundary. So develop the reasoning when the tension specimen is kept perpendicular to the boundary.

So what we have seen, we can find out the sign at the boundary stresses either by using a nail test or by a sharp edge or even by having a simple tension member kept in front of the model and in fact if you see stress freezing you can also stress freeze the tension specimen and keep it ready and just keep it tangential to the boundary or perpendicular to the boundary, you do not even have to pull it.

So that idea you will get after looking at what is the way people employ 3-dimensional photoelastic analysis wherein they use stress freezing followed by slicing, similar concept can also be extended to simplify your experimentation, but the principle comes from your basic understanding of your compensation technique and what happens on the free boundary.

On the free boundary, you must always keep in mind that stresses can at best be tangential to the boundary and you should never conclude by looking at the circular disc on a free boundary stresses are 0. If stresses are 0 in the case of circular disc was a special case. We had beam under bending you had maximum stress on the top and bottom surface. They are all again free boundaries and you had another example, you had a ring under diametral compression.

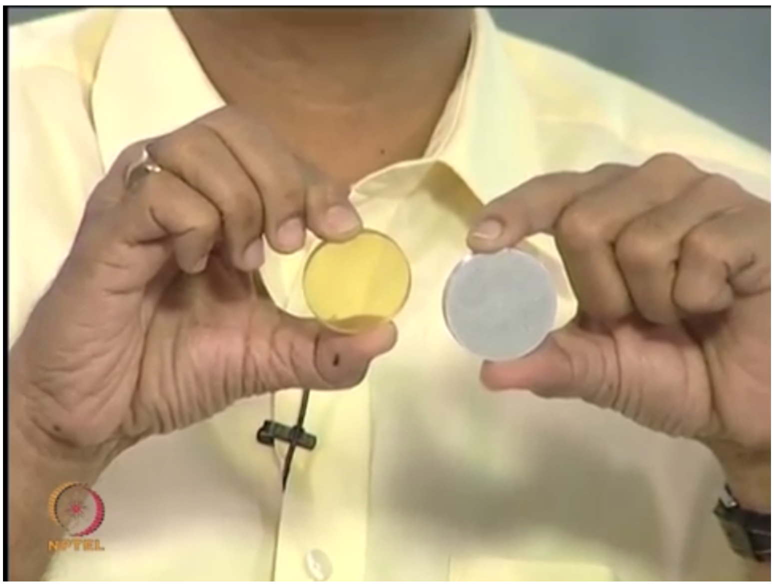
You had fringe orders varying on the outer as well as the inner boundary and I cautioned whenever the fringe order crosses a 0th fringe order, the sign at the boundary stress also changes and you will also have to recall if you have done a course in theory of elasticity, you know many times you talk about stress concentration. Suppose I have a plate with a hole and pull it, people know the maximum stress reaches 3 times the nominal stress.

What many people do not observe is on the inner boundary of the hole at 90 degrees to the maximum stress, you have compressive stresses develop. So what you have here is on the boundary of the whole you need to find out what is the sign at the boundary stress and here again your singular point will help from isoclinic fringe field will help. I said ring under diametral compression and plate with a hole shares certain commonalities.

You can identify point of transition because from theory of elasticity we know you have maximum stress. Suppose I take a tension specimen and pull it vertically, on the horizontal diameter you have maximum stress developed, on the vertical diameter you have compressive stresses developed. So in between there has to be a transition. So on the inner boundary it is a free boundary sign at the boundary stresses changes.

And for such applications using an external member is convenient because if the hole is very small, I cannot go and put my nail. I can use a tension specimen and then investigate and reconfirm that your understanding of theory of elasticity is correct.

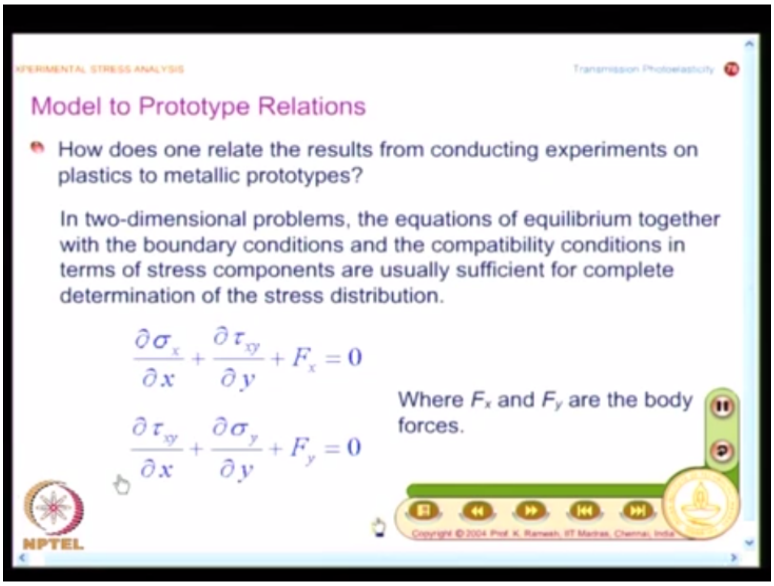
(Refer Slide Time: 22:33)



See now we come back to our famous problem, you know I have plastic model, this plastic model is polyurethane and I have an aluminium specimen and apply the same diametral compression load and what will happen to the plastic model? Plastic model will deform visibly because it has a lower Young's modulus whereas aluminium is about 70 GPa and plastic is around 3 GPa in general.

And this is polyurethane is much smaller than that. Now the question is I use only plastics in the case of photoelastic analysis. How am I justified in extrapolating the result from conducting an experiment on plastics to metallic prototypes? We will go in stages. First we will look at planar problems and when you look at planar problems, we will also simplify it further that we will not look at the body forces.

(Refer Slide Time: 23:35)



So what I have the important question that you will have to keep in mind is how does one relate the results from conducting experiments on plastics to metallic prototypes? You know the confidence comes by looking at the equations of theory of elasticity. If you look at the equations of theory of elasticity, what do the equation say? When I am looking at a planar problem, we will have to look at equilibrium equations.

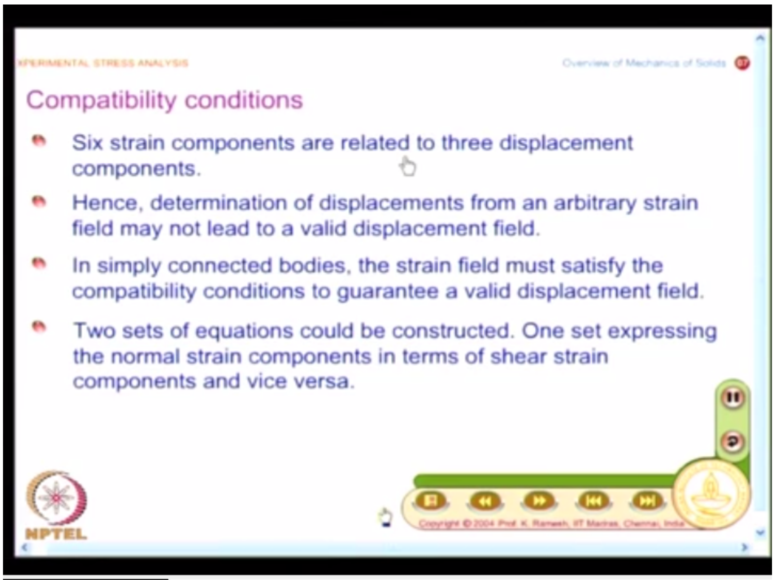
And if you are working on a stress formulation, we will also have to look at compatibility conditions. So what I have here, in 2-dimensional problems the equations of equilibrium together with the boundary conditions and the compatibility conditions in terms of stress components you must have stay in compatibility condition in terms of strained components. We will also spend 2 minutes on that.

These equations are usually sufficient for complete determination of stress distribution so what I need to have is I need to have the equilibrium conditions, I need to look at the compatibility conditions and satisfy the boundary conditions of the problem and how does the equilibrium condition looks like? The equilibrium condition is like this.

You all know, dou sigma x/dou x+dou tau xy/dou y+Fx=0 where this is the body force and you have dou tau xy/dou x+dou sigma y/dou y+xy=0. I have used the equality tau xy=tau yx while writing this expression. Does this expression have any elastic constants? This expression does not have any elastic constants. When this expression does not have any elastic constants, you do not need variation between a plastic and aluminium to behave when you satisfy this condition.

So the equilibrium condition does not have elastic constants fine and what are compatibility conditions? We will go back and then see.

(Refer Slide Time: 26:07)



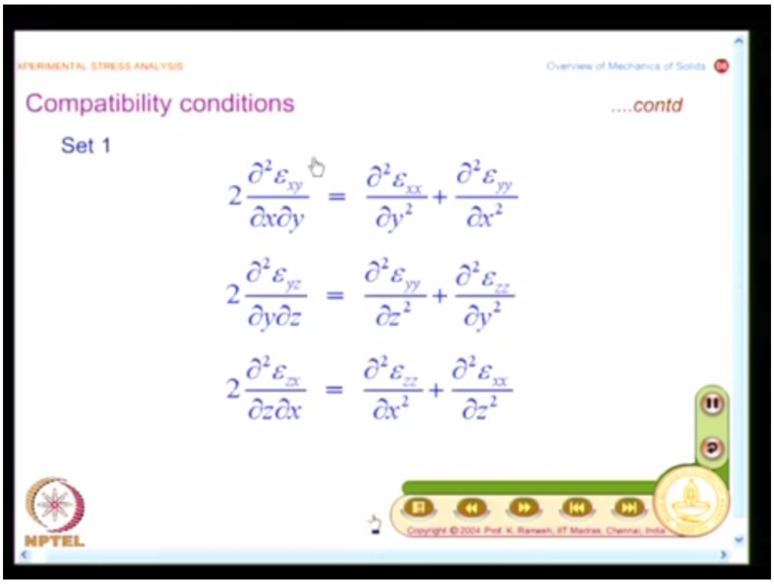
If you go back and then see from your theory of elasticity understanding, you have 6 strain components but you have only 3 displacement components. Suppose I find out the strains from displacements, no problem. From displacement to strain is a simpler exercise, but knowing the strain components, finding out displacement you have to bring in the equations of compatibility.

If you do not bring in equations of compatibility, you will not have deformation consistent with the loading applied and what you have here is in simply connected bodies, the strain field must satisfy the compatibility conditions to guarantee a valid displacement field. Though in theory of elasticity, you have stress formulation as well as displacement formulation, the very popular airy stress function is basically a stress formulation.

In fact, we know stress function for a variety of problems. So you evaluate stresses, from stress-strain relations evaluate strains, from strain displacement relation evaluate the displacement, but when I go from strain to displacement, I need to invoke compatibility condition otherwise my displacement evaluation would not be compatible and what do you see here? You get 2 sets of equations could be constructed.

One set expressing the normal strain components in terms of shear strain components and vice versa and the equations appear like this.

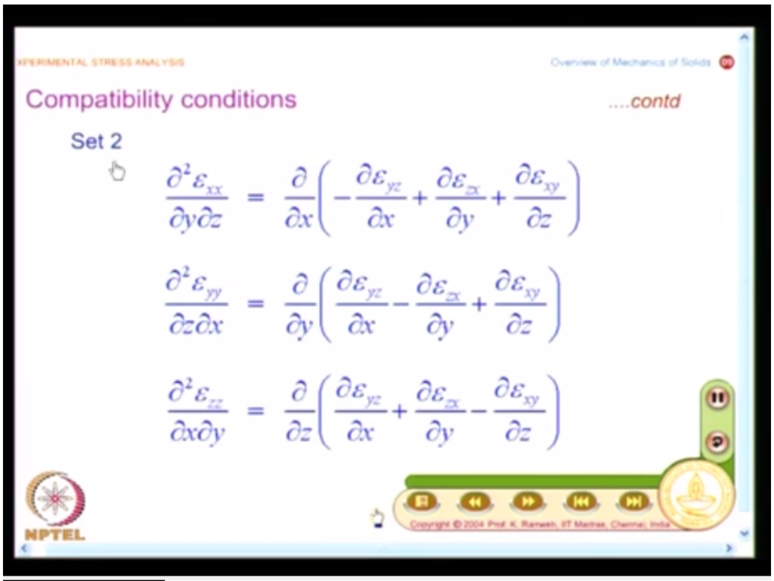
(Refer Slide Time: 28:01)



So when I relate these normal strain components to shear stress component, you have it like this. So on the left hand side you have only shear strain components, on the right hand side you have the normal strain components. If you go and plug in the expression for epsilon xx, epsilon yy, and epsilon xy, this equation will be completely satisfied and since we are confining our attention in our discussion only to 2-dimensional problem for the time being we need only the first equation.

So this is set 1. I have compatibility condition expressed in terms of strain components. Once I have this if I know the stress-strain relations or strain-stress relation I can replace the strain quantities in terms of stress components. In fact, it was one of the home exercise problems.

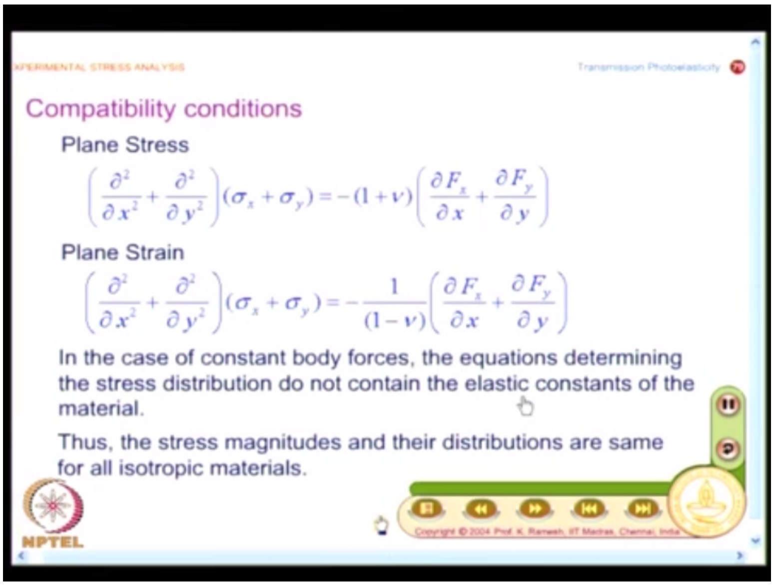
(Refer Slide Time: 29:25)



I am only reviewing solid mechanics for the sake of continuity and you have set 1 and you also have another set, which relates your shear strains to normal strain. The left hand side is only normal strain and right hand side you have only shear strain components. This you must have studied in a course in advanced mechanics of solids, it is nothing new, it is only looking at these old equations just for continuity.

And in fact we were more concerned with set 1 and that is good enough for our discussion.

(Refer Slide Time: 30:06)



And suppose I write this expression in terms of the stress components how do the equations look like? And we will look at plane stress conditions and plane strain condition separately. The compatibility condition in terms of stress components has the form like this for plane stress. On the right hand side what I have, I have -1+nu*dou Fx/dou x+dou Fy/dou y and these are body forces.

And one of the simplest assumptions what we make in most of our problems except civil engineering problems, we can consider the body force to be constant or 0. When it is constant also, the right hand side vanishes. Suppose the right hand side does not vanish, the compatibility condition is a function of what? It is a function of Poisson's ratio, which is a material property.

Whereas equilibrium condition was not a function of any of the material properties whereas compatibility condition is a function of the Poisson's ratio suppose you eliminate the consideration of body forces from the point of view of simplicity, you find the right hand side becomes completely 0. So this equation will be independent of elastic constants again and the right hand side will be slightly different when I go to plane strain.

When I go to plane strain and find out what is the compatibility in terms of stress components, only the first term on the right hand side changes instead of -1+nu it appears as -1/1-nu. So in the case of constant body forces, the equations determining the stress distribution do not contain the elastic constants of the material. So what is the implication?

The implication is the stress magnitudes and the distributions are same for all isotropic materials. So here you have the comfort, your theory of elasticity comes to your rescue because in most problems you know the body force if you are really looking at static problems is the dead weight and if it is constant you are not going to have any problem, only when you have rotating components where the body force is the function of the radius then you have to worry the Poisson's ratio plays it spoil sport.

I have said in all experimental methods the Poisson's ratio will be a nuisance in one way or the other. We have to learn to live with that. So what we find here is even though I perform experiment on a plastic because of the strength of the equations from theory of elasticity for a

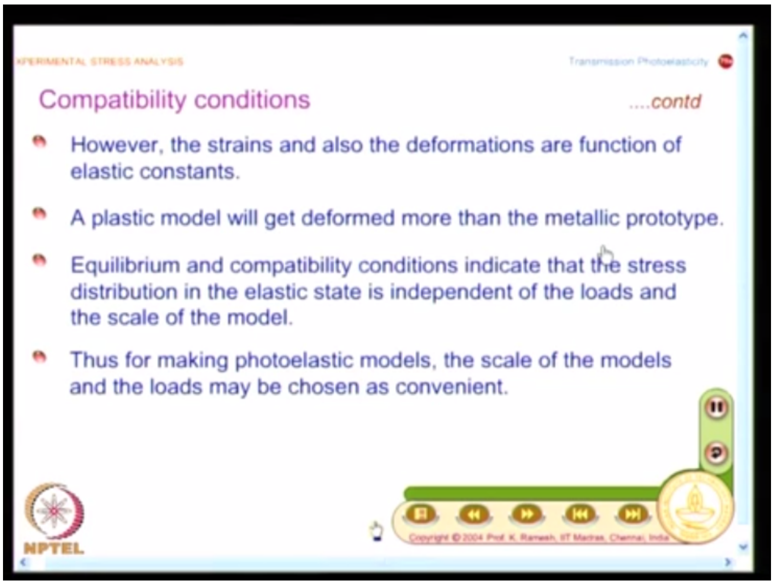
2-dimensional problem when the body forces are considered constant, elastic constants do not play a role.

The stress distribution is same but the displacement will be different because displacement or strain they are all dictated by the elastic constants. So you have to do the experiment very carefully, you cannot apply any load to the plastic model and then say that I correspond whatever the result that I do you will have to follow the loss of similitude that we will also see.

But what you find here is working on a plastic is convenient from optics point of view because it behaves like a crystal, I have birefringence and then I am able to visualize the stresses, all that advantage we have when you work with plastic and the results also can be correlated to metallic prototypes from the strength of equations of theory of elasticity. When we go to 3-dimensional what happens?

We will see that also. The comfort what you have in 2-dimension is not the same. Three-dimension is always difficult; you will have to live with problems.

(Refer Slide Time: 34:45)



And this is what is emphasized again what you will have to know is though the stress distribution and its magnitudes are same in a 2-dimensional model when we consider body force as constant, the strains and also the deformations are function of elastic constants. So in essence, a plastic model will get deformed more than the metallic prototype. That you would see.

But we will learn similitude equations and how do we load the model? What is the level of load that we should apply? So what we find here is equilibrium and compatibility conditions indicate at the stress distribution in the elastic state. All these are very important, we are not looking at the plastic condition, we are looking at elastic state and we are only discussing photoelasticity.

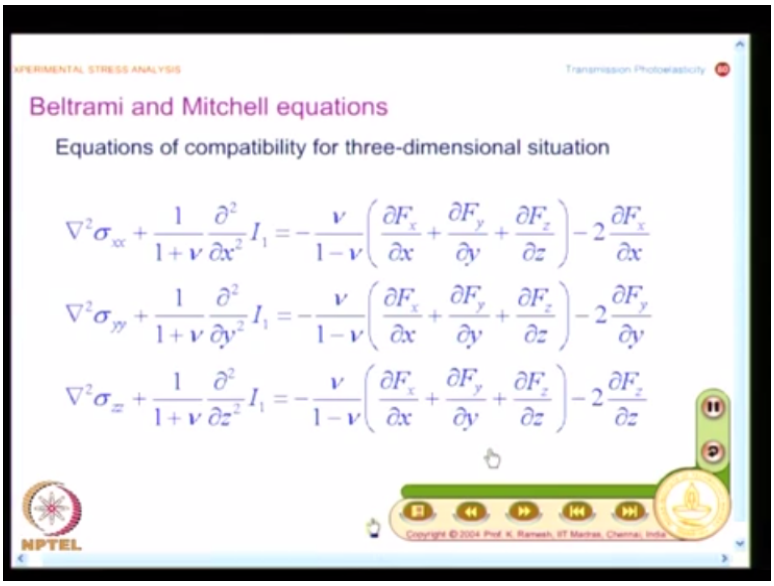
We are not looking at photoplasticity, so with such kind of modeling the stress distribution is independent of the loads and the scale of the model. So I do not have to worry if I do it on a plastic, I do not have to feel sorry I could do only in plastic and then compare it with the metal. You are doing it on plastic and the results are equally valid as far as stress and its distribution is concerned. The same is not true for displacement and strain.

They are dictated by elastic constants. This subtle difference you should understand very clearly and even if I have a rotating component, it may be a still a 2-dimensional problem, body first changes from point to point then your Poisson's ratio affects the distribution as well as magnitude. Then you will have to have a model material, which has same Poisson's ratio as the metallic prototypes, which is not possible always.

And what you have here, when we make photoelastic models, the scale of the models and the loads may be chosen as convenient and we will also keep in mind a plastic model will get deformed more so I should select the load such that I do not get into large deformation in plastic. The deformation and strain in the plastic model are matched more or less with the metallic prototype.

That also will see in the similitude conditions. The first and foremost advantage from elasticity equations is the stress distribution is independent of the loads and the scale of the model when we look at in plane problems with body force remaining constant.

(Refer Slide Time: 37:52)



Now let us look at how does the compatibility condition looks when I go to 3-dimensional situation and these are famously known as Beltrami and Mitchell equations. You may not find this in several books and experimentalist are concerned about this. So I have this as del squared sigma xx+1/1+nu dou squared/dou x squared I1, this is the first invariant that is equal to -nu/1-nu dou Fx/dou x+dou Fy/dou y+dou Fz/dou z-2 times dou Fx dou x.

What strikes you immediately when you look at this expression? First, it is long, very long expression than what you saw in 2-dimensional situation. In 2-dimensional situation, we could look at when body force is constant the equation gets simplified. Suppose body force is constant in 3-dimensional situations what happens? The right hand side still goes to 0, but the left hand side has the nuisance Poisson’s ratio.

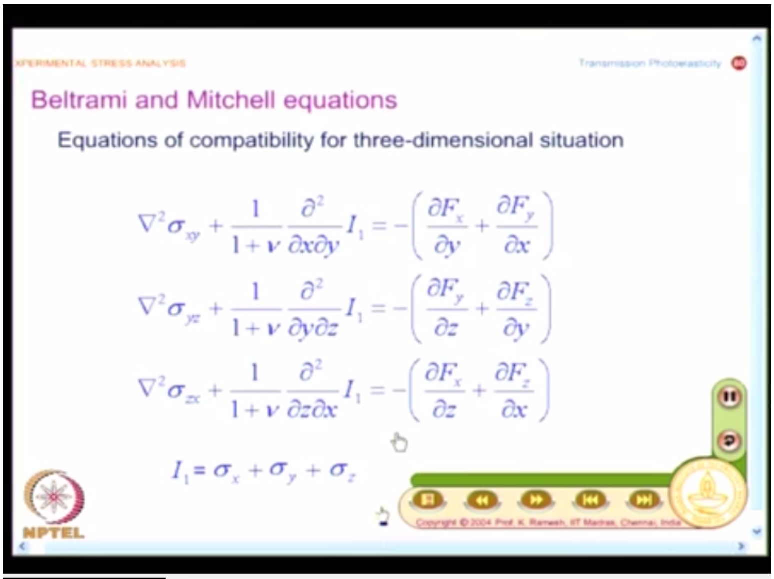
So the moment you come to the 3-dimensional problem, Poisson’s ratio change will affect the magnitude of the stress, not only the magnitude even the distribution. It is dictated by Poisson’s ratio, never forget this. In a practical situation you may say the influence of Poisson’s ratio is small, so let us gloss over it that is permitted because as engineers we have to live with approximation.

We will make the approximation which is reasonably good and sufficient for your practical application, but never forget the moment you come to 3-dimensions, Poisson’s ratio is the nuisance. Do not think that only photoelasticity has the stigma, you will see now all the coating techniques the Poisson’s ratio will play a spoil sport, but the grace is whatever the influence it will have it is very small.

It is all like second order effects but from an analytical understanding we should know such effects exist. When you are getting the result you should take it with a pinch of salt in a 3-dimensional problem that Poisson's ratio plays its role. So that you have to keep in mind and you know I have 3 such equations. Actually you can go back and fill those equations. I have 3 such equations, 1 is for sigma xx, another is for sigma yy.

And I think I leave that as a home exercise, you just have a look at it in the class but go and develop it because it is all cyclically repeating.

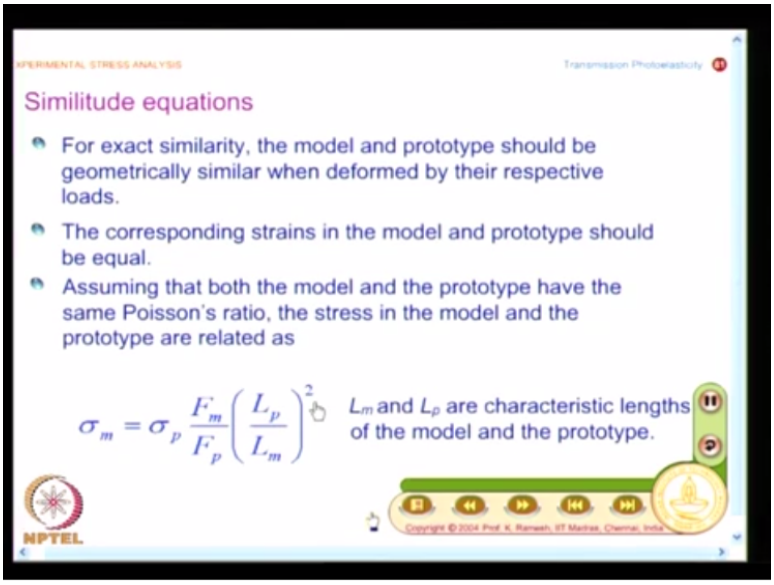
(Refer Slide Time: 41:10)



So I will have another 3 sets of equations, which are also cyclically repeated and the first equation looks in this form. I have del squared sigma xy+1/1+nu dou squared/dou x dou y I1=-dou Fx/dou y+dou Fy/dou x. So this also you will have 2 more equations, which you can look at them but you can easily fill it up from the symmetry of this symbols by looking at these cyclically changing this will be able to get this.

And you have this as I1 as sigma x+sigma y+sigma z.

(Refer Slide Time: 41:52)



See what is important in doing a model study is you will have to follow the similitude equations. For exact similarity, the model and prototype should be geometrically similar when deformed by the respective loads. See we have seen when you are working on in plane models for 2-dimensional problems, the stresses are not altered when I go from plastic to metallic prototype.

On the other hand, the displacement and strains are dictated by the material property. For the same load, a plastic will deform more and what we look in similitude equation is you do not apply the same load, if you apply the same load as service load in prototype, the model will break completely. So you have to apply a load much below the actual service load and you also develop a philosophy of how to apply the load.

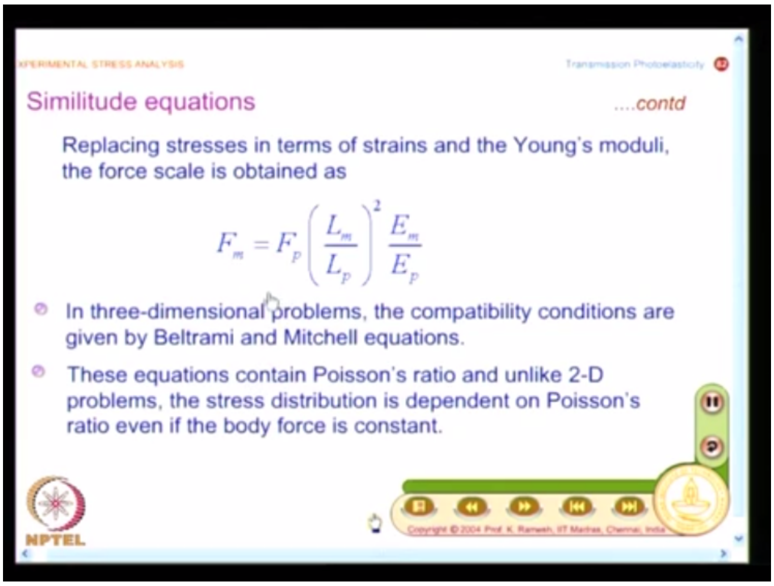
And what we are looking at here is the model and prototype should be geometrically similar when deformed. So that is the issue that you take it up to find out what is the load by which I can apply on the model. So what we look at here is the strains developed in the model and prototype should be equal.

This is the desirability and I said in all experimentation, Poisson's ratio is a nuisance assuming that both the model and the prototype have the same Poisson's ratio the stress in the model and the prototype are related, I have a symbolism that with suffix denotes whether it is a model or a prototype and m denotes the model and this is your actual prototype. So I have sigma p*Fm/Fp*Lp/Lm whole squared.

So what we look at here is we bring in characteristic lengths from the model as well as the prototype and then we find the stresses in the model is related to stresses in the prototype like this. Our goal is we want to find out the load whatever the load that I want to apply on the model, I want to do that in such a fashion when deformed the model and prototype should be having similar strains.

This is what we are looking at so that will bring in the elastic constants.

(Refer Slide Time: 45:02)



How does the equation look like? So what we do is we replace the stresses in terms of strains and the Young's moduli the force scale is obtained as the force that I need to apply on the model is related to the force that is coming on the prototype*Lm/Lp whole squared*Em/Ep so I bring in the elastic constants and obviously in a prototype of steel, it is about 210 GPa and model is about 3 to 4 GPa.

So that means Fm will be much smaller than what is the load that is coming on the prototype with the strength of this equation what we find the model as well as the prototype when they are loaded they will have similar values of strain as well as deformation. You do not want to have large deformation in the model otherwise the equations are not valid and here again we will have to bring in a distinction.

In 3-dimensional problems, the compatibility conditions are given by Beltrami and Mitchell equations, they are functions of Poisson's ratio and unlike 2-D problems, the stress distribution is dependent on Poisson's ration even if the body force is constant that is emphasized. Even if the body force is constant 3-dimensional problems are always difficult to handle.

And what you find here is by using this scale for finding out the force to act on the model, we maintain certain kind of similarity on the strain levels and mind you this is obtained with assumption that Poisson's ratios are same, so it is not going to be same between the model and the prototype. So we need to keep that also in mind when you are looking at the model study.

See we have looked at model to prototype relations. We have also looked at the similitude equations. Now let us look at what are the kind of photoelastic materials that you have? And I have a quite a variety of them.

(Refer Slide Time: 47:42)

We will also see them physically and also see their properties and what I have here is I have an epoxy disc, this is the disc made of epoxy and this will have particular elastic properties and I have another model, which is made of polycarbonate. This is of polycarbonate even by looking at the difference it has a slight yellowish tinge and these properties are very similar in characteristics.

(Refer Slide Time: 48:08)

On the other extreme, I have the polyurethane model and this also has a tinge of yellow. The difference is I can even compress it with my fingers. It has such a low elastic modulus and I can compress it. You can see the deformation when it is loaded, you can see the deformation when it is compressed, you can see my fingers are pressing that it is so soft. So it is very good for models for class illustration and also a good model to do the fingernail test.

So when I put the fingernail, it gets compressed very easily so I can find out the sign at the boundary stresses by nail test without resorting to any external gadget.

(Refer Slide Time: 49:03)

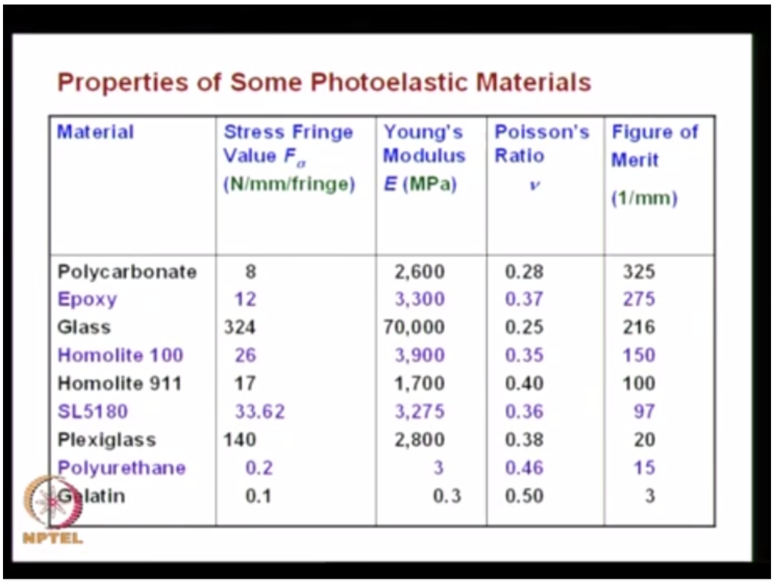
And you also have another model this is from a stereolithography process so this is something like SL5180 resin.

(Refer Slide Time: 49:14)

And I also have on the other extreme, this is Perspex. See if you look at Perspex and polycarbonate, they are both transparent and this is model of a plate with a crack and we will see the respective properties. You know long time back we had mentioned when you want to find out isoclinics it is desirable that you use a Perspex model because it has a very high material stress fringe value.

So you will see isoclinics lot more clearly because isochromatics would not develop for smaller loads and the properties are summarized here.

(Refer Slide Time: 49:48)



Properties of Some Photoelastic Materials

Material	Stress Fringe Value F_σ (N/mm/fringe)	Young's Modulus E (MPa)	Poisson's Ratio ν	Figure of Merit (1/mm)
Polycarbonate	8	2,600	0.28	325
Epoxy	12	3,300	0.37	275
Glass	324	70,000	0.25	216
Homolite 100	26	3,900	0.35	150
Homolite 911	17	1,700	0.40	100
SL5180	33.62	3,275	0.36	97
Plexiglass	140	2,800	0.38	20
Polyurethane	0.2	3	0.46	15
Gelatin	0.1	0.3	0.50	3

You have some properties are listed, I have the stress fringe value, I have the Young's modulus and I have the Poisson's ratio and I also have finally what is known as Figure of Merit? Figure of Merit is nothing but the Young's modulus/material stress fringe value and if the Figure of Merit is larger, it indicates that it is a good model for photoelastic analysis. So I have polycarbonate.

We have seen polycarbonate and epoxy are similar. Polycarbonate has 8 newton per millimeter per fringe and this is obtained for a sodium vapour source and we have seen that it is a function of the wavelength and it will also change from time to time. You know you will also have to keep in mind that you calibrate the material and then use the property. Then I have epoxy.

And glass has a very high value of F sigma and Plexiglass is comparable to the glass, it is about 140 and polyurethane which I said that I can even apply loads with the finger as a very low F sigma of 0.2 and its Young's modulus is also very, very small, it is only 3 MPa whereas for most plastics it is about 3 GPa. It is only 3 MPa and it is only 3 GPa for all the other plastics and glass is like almost like aluminium, it is 70 GPa.

And Figure of Merit is listed here, it varies from 325 to 3 and if you look at gelatin is a very nice candidate when you want to analyze the effect of body forces. If you are really looking at civil engineering construction, they use gelatin and make models out of it and bring in the body force component in the analysis and normally you know many experiments are conducted with polycarbonate and epoxy.

Polyurethane is good for teaching purposes and this is also used in civil engineering for modeling layered soil and with recent advancements stereolithography, you have this SL5180 that has compared to the common photoelastic material it has a high F sigma value and it has a Figure of Merit of 97 and if you look at the Poisson's ratio, the polycarbonate is much closer to metals.

So polycarbonate is one of the most preferred model material for photoelastic analysis and you can also use epoxy because it is easy to cast in a laboratory and then prepare models. So this gives you an idea of what are the various photoelastic materials and what we have seen in the class today was we looked at the importance of color code.

We said color code the sequence is advantageous to calibrate the polariscope to find out the maximum or minimum principal stress direction and also to find out the sign at the boundary stresses. We have looked at nail test, we have also looked at how to find out the sign by an external member. Then we moved on to model to prototype relations. Now one of the key learning, we learnt was in a 2-dimensional problem when the body forces are constant, the stress and its distribution is not affected by elastic constants.

Whereas when you have body force varying or when you go to any simple 3-dimensional problem, the Poisson's ratio of the model material dictates the stress and its distribution because it is always different from a metallic prototype, there would be influence of Poisson's ratio on the stresses evaluated and finally we looked at similitude equations.

Then we also had a look at what are the various photoelastic model materials that were commonly used and what are their relevant properties. Thank you.