Experimental Stress Analysis
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Lecture – 17
Calibration of Photoelastic Materials

We have been discussing about transmission photoelasticity and I said one of the key parameters that needs to be determined is material-stress fringe value and as we use polymers, the material-stress fringe value changes from batch to batch as well as over a period of time, there may be small changes and as material-stress fringe value is the only parameter that relates the experimental measurement for comparison with analytical or numerical methods.

You must take sufficient care to determine it with as much accuracy as possible and I said one of the common models that is widely used is disc under diametral compression and we have also seen why we choose a disc under diametral compression, the first aspect is it is simple to machine and easy to load and since we have this stress field from theory of elasticity, it is also possible to compare the experimental result with analytical solution.

So, in the process we used this analytical solution to find out the material-stress fringe value. Because we need a model for which analytical solution is available so that you perform an experiment find out the fringe order, use the stress optic law instead of finding out the stresses from the analytical computation; plug in the value of the stresses. From that you find out the material-stress fringe value.

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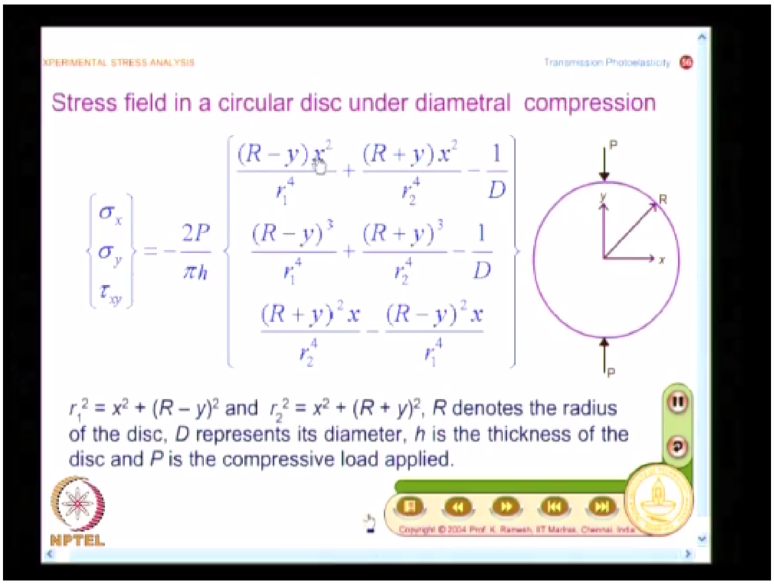

And in the last class, we saw the stress field for the disc under diametral compression and I have sigma x, sigma y and tau xy and these are the expressions we have also noted down earlier and we take the center of the disc as the origin and r is the radius and what I said in the last class was to find out from these expressions, the value of sigma 1 - sigma 2. In fact, what we want is, we want to see principal-stress difference.

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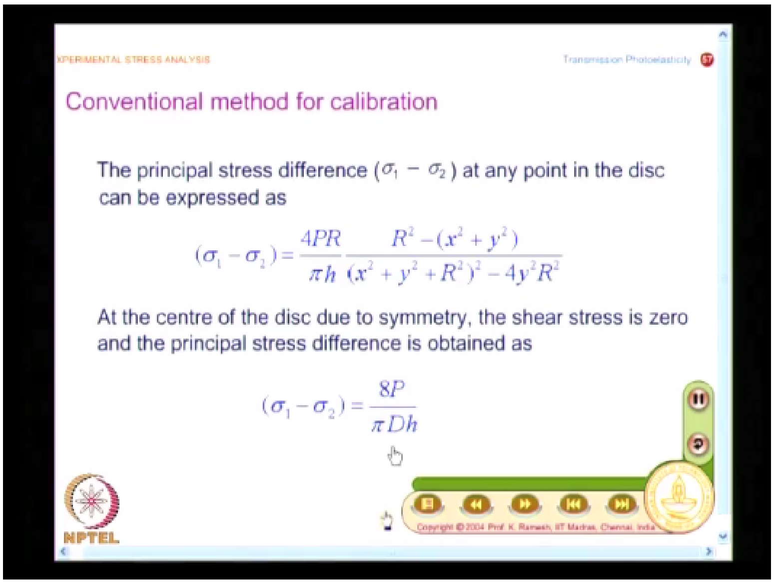

And since, I have the expressions for sigma x, sigma y and tau xy, it is simple for me to find out expression for sigma 1 – sigma 2 and although, for conventional method, I need only the values at the center of the disc, I give this as a generic expression in x and y with the purpose in mind. The idea is we will do the conventional method for calibration and later, we will also elaborate on a method which will use as many data points as possible from the field.

This is particularly useful with developments in image processing techniques where acquisition of data becomes lot more simpler and also in 3 dimensional photoelasticity where they have a stress freezing process. At the end of the process, you may get either one or 2 discs with stresses locked in. So, instead of just using the center, which will give only 2 data points, you would like to augment the data points.

And from that point of view also you need to find out a methodology which uses several data point from the field. So keeping that in mind, I am going to have the expression of sigma 1 – sigma 2 as a function of x, y and that is given as 4PR divided by pi h * r square – x square + y square divided by x square + y square + r square whole square - 4 y square r square and once you have this expression, it is very simple to find out what is the principal-stress difference at the center of the disc.

You just put x = 0 and y = 0, you will get an expression and I want you simplify and also the expression is popularly written in terms of diameter of the disc. So, instead of putting the value as r, the radius you express it as d/2. So that you have a very popular expression and that is obtained as sigma 1 – sigma 2 = 8p/pi dh as simple as that now what we have done here is popularly the diameter is used in this expression.

So, I wanted you to replace r as d/2 and from stress optical law, we know what is the expression of sigma 1 – sigma 2 in terms of fringe order and material-stress fringe value. Now, the focus here is not to find out the stresses at the center of the disc, but to find out the material parameter. So that is my focus that is why I have taken a problem for which I have an analytical solution.

The key point here is if you machine the disc perfectly circular, which is easy to do if you have a lathe and it is also load that properly then this comparison can be almost exact, so that the focus is to find out the f sigma as accurately as possible. So, you have a good model and you will also have to evaluate N and you have already seen compensation techniques to find out N with at least second decimal place accuracy.

So, we can find out N accurately and we can also find out f sigma accurately form the experiment and we will modify this expression in a manner to directly find out what is f sigma and this what I said the stress-optical law gives you NF sigma/h. These you have written it earlier and now, combining these 2 expression, I get an expression for f sigma as what is the famous relationship? What is that you are getting it.

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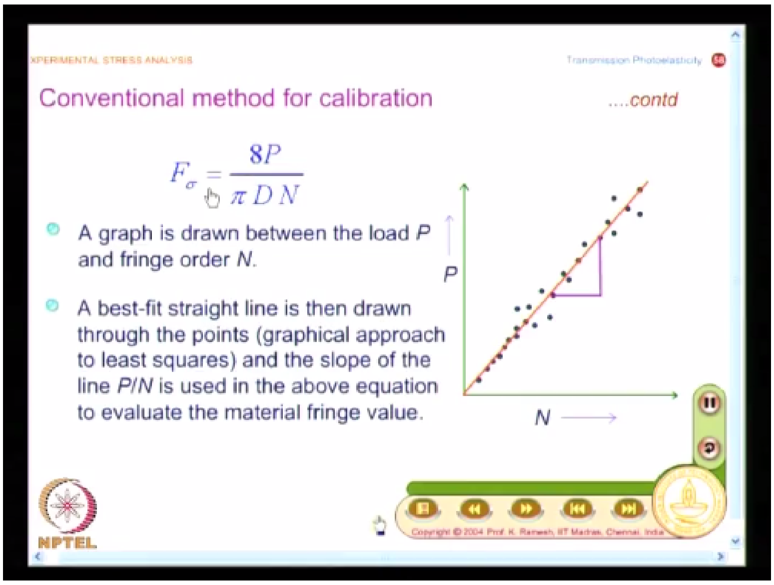

You work it out yourself. You work it out yourself, it is very simple and what you get here is 8p/pi DN and what I have here is the very famous expression and the thickness of the model does not come in to the final form of the expression. You find out the fringe order N and you know the load that is applied, you know the geometric parameter of the disc. So, I can find out f sigma.

And I have always mentioned that as experimentalist, you should not be satisfied with just one measurement. You must make as many measurements as possible, so that you are able to bring in some kind of the statistical data processing and finally arrived at the value and in this case, what you can do is, you can keep on varying the load and find out the N for all these loads and you can plot a graph.

And from the graph, you can determine the value of P/N, then you will have some kind of an averaging process in arriving at the value of f sigma and what we will do is, we have now plotted a graph between P and N and I have shown many data points. In practice, you may not do so many experiments, you may use 7 or 8 experiments and then, you will get 7 or 8 data points and this is shown to illustrate what I can do with this data point.

First thing what you find is there is scatter and scatter is possible in any experimentation. Scatter is in built in experimentation that is why this is emphasize, the scatter is emphasize little more you may not have so much scatter, some form of scatter will always exit in any experimentation. As experimentalist, what you need to do, you need to make the best value possible.

One simple approach is I can draw a graph such that the points lie on either side of the graph equally. So, when I do that what happens, I have a graph which is drawn and this is how you have the graph and I have points lying on either side and this process is nothing but a least square evaluation by a graphical approach. So, what I have done is I have drawn a graph, it is sensible to draw a graph such that points of scatter lie on either side of the graph evenly.

And what you have implicitly done is, you are actually done a graphical least squares analysis. So, what we have taken the advantage is we have taken the advantage of large number of data points. From a graphical approach, you find out what is P/N and once you plug in this P/N in this expression, I would get the value of f sigma as accurately as possible from a simple experiment.

And I can find out the N accurately by Tardy's method of compensation, where I have to just rotate the analyzer. For each load, I have to rotate the analyzer find out the fractional fringe order and I also mentioned earlier. In early days, people had a very complex loading mechanism, where in they will adjust the load, so that you a data point, you have the fringe passes through the center.

The data point here is the center of the disc. So, they have to adjust the load to make the fringe passes through the center. Instead if you are in a position to apply a compensation technique and Tardy's method of compensation is so simple, I just rotate the analyzer, I get N accurately. Our focus is to get f sigma accurately. For us to get, f sigma accurately, you must measure N accurately and then draw the graph and then from the graph, find out P/N and then use this expression.

This is so far so good when you are able to do a live load experiment, where I can keep changing the load and find out what is the fringe order at the center. Suppose, I do a stress freezing, which we would see later at this point in time, you understand there is a thermal cycling process by which I can lock in the stresses inside the model and when I lock in the stresses inside the model, I was also place a circular disc under diametral compression within the thermal cycling process.

Whatever, the oven that I use, I must keep this loading mechanism inside, allow also a circular disc to pass to the same thermal cycling. Then, finally I will take out the circular disc and in that I have the fringe information. I have the fringe order at the center even if a full fringe is not through the center, I can always find out the Tardy's method of compensation. But if I want to have additional data, then I need to have one more disc.

For every data, you need to have so many discs; when your oven will not space to keep so many discs under diametral compression. So, you need to think of a different strategy because I record whole-field information, why not I use the whole-field information? That is the focus and you have that given as method of linear least squares and we want to use whole-field data to evaluate material-stress fringe value.

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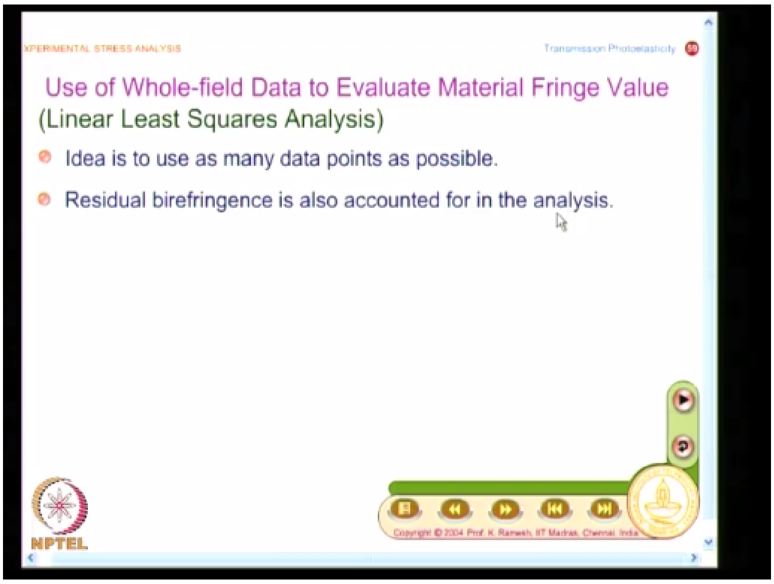

And in this case, the resulting equations are essentially linear and I called this as linear least squares analysis and the credit to introduce these kind of methodologies goes to professor (()) (13:33), he was the first person to initiate this kind of a thinking in experimental mechanics. This was initially obtained for finding out disc under diametral compression. Then, it was used for fracture mechanics problems, finding out stress-intensity factor and sigma not x.

We would confine for the time being, how you can find out the material-stress fringe value. Now, I said that you can use as many data as possible. What way we can go about essentially we are going to use a computer to do all this processing. Why not, we also looked at certain additional features to our analytical model.

See, one of the common problem in any one of this, we have also looked at what is the (()) (14:20) as a function of time, you have spurious fringes which we have to avoid for all the practical purposes, but you may also have some amount of (()) (14:30) locked in while casting a sheet. Since I am going to find out the value of f sigma by processing the field information, I can also improve my model by also incorporating the residual birefringence.

Your mathematics may finalize the residual birefringence very close to 0 that is welcome, but since we lose the luxury of processing of just one data point, we are going to work with the computer and why not we try to have a better model which is logically fine. So, we bring in the residual fringe effect also and that is what we will do and what is that we will do, engineers are very happy with straight lines.

If you want to do anything first, we will first find out whether I can fit a linear graph. If I am able to do that, if my results come I am happy, only the result does not come, I go for nonlinear. So, one of the very simple aspect what we can do is when I say, I will also moral the residual birefringence in the analysis. A simplest assumption possible is ideally this as a linear variation. It will be a function of x, y.

My focus is only to find out f sigma, but I bring in one more aspect which I feel which is logically sound. So that I have a better analytical model to handle even situations where I may have residual birefringence inadvertently introduced. So, what I have is I defined Nr as a residual birefringence which is a function of x, y various from point to point. I make this as Ax + By + C.

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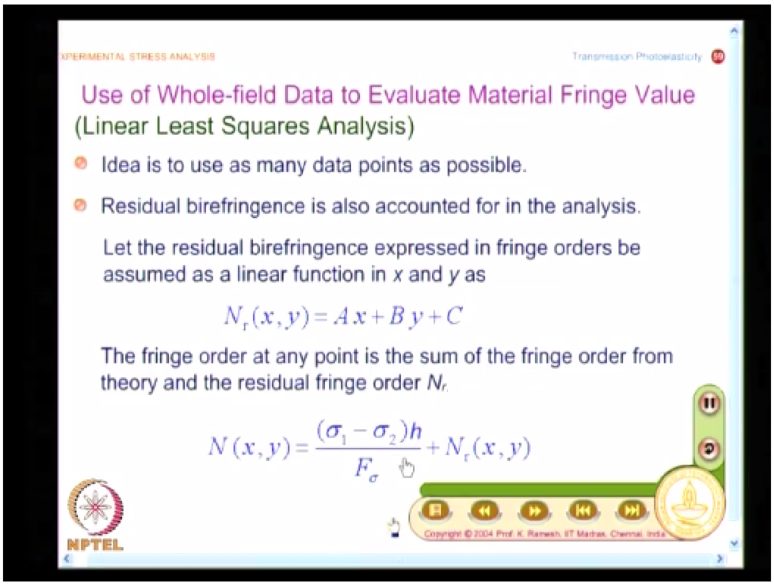

So, I am introducing a new function in that case, what are the unknowns, I have f sigma is an unknown that is the primary unknown that I have to find out and in the process of refining our methodology, we have also introduced the unknowns A, B and C. This you have to understand. So, what I have now done is instead of evaluating one parameter, we need to find out 4 parameters.

And from your understanding of solving simultaneous equation, if I have 4 unknowns, I need how many equations, I need 4, as may number of unknowns, as many number of equations I should have. If I have less number of equations, it is a problem. If I have more number of equations, then also it is a problem. I must have equal number of unknowns and equal number of equations.

Suppose, I have more number of equations and less number of unknowns, we also have methodologies to identify only the number of equations matching with the number of unknowns that we do by method of least squares that is what we will employ it here. See, one way of approach is from the field, you randomly collect lot of data points and simply take average.

The average may not be the right way to do it. That is why I emphasize when you have done a calibration by simple method, you have drawn a graph without your knowledge, you have a done a graphical least squares analysis. With a similar thing, we will also do in a situation where I collect large number of data points. That is the way I will develop the methodology and get the equations.

So, now what I am going to do, we already have an analytical expression what is the value of fringe order at a point of interest when x and y is specified. Now, what we say to that you need to add a residual birefringence N r x, y. So, that is how we will recast the basic equation. So, if I have fringe order at a position x, y, it will have 2 terms, one term contributing it from your analytical expression, which is completely known.

If I know the material-stress fringe value and if I know the h, the value is known. Our focus is to find out f sigma and you can find out an expression from an analytical method, what is the expression for sigma 1 – sigma 2 and to this, we add at the point of interest, a residual birefringence N r x, y and what I am going to do is, we already have an expression for this, we will replace this as a function of x, y from the generic expression of sigma 1 – sigma 2.

That will be the first step. Then, we will see how to coin the equations, so that we will do a least squares analysis. So, what I have is we already know sigma 1 – sigma 2 has a long expression and we rechristened as S as a function of x, y which is nothing but sigma 1 – sigma 2 * h. So, I have this as 4PR divided by pi multiplied by R square - x square + y square divided by x square + y square + R square whole square – 4 y square R square.

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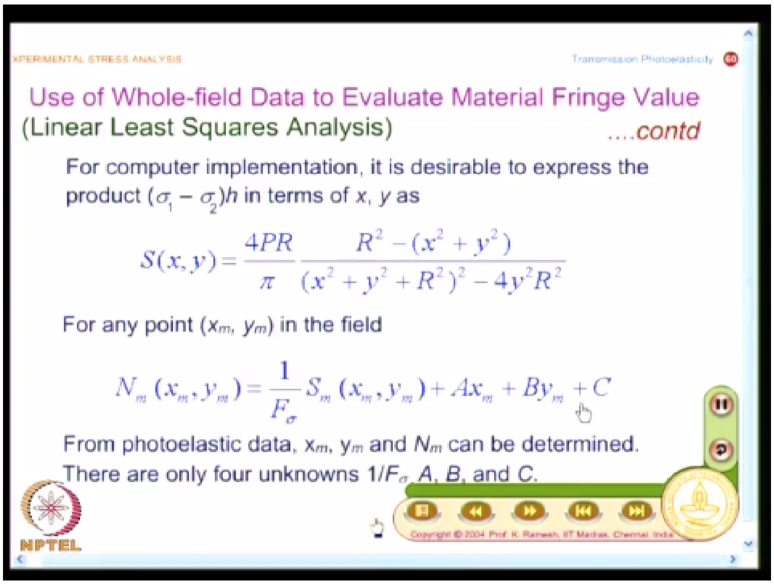

Instead of writing this complete expression every time, I will simply label it as S as a function of x, y. So, for the point m, I will write this as at the point m, Nm is defined as 1 contribution from analytical expression, other contribution from assumed residual stress field. It is assumed residual stress field and from photoelastic data, what you can find out? I can find out at every point of interest, the value of fringe order N that is what written here.

The coordinates x and y and the fringe order the point m can be determined from the experiments and if you do a conventional analysis, you have to take a photograph and find out xy accurately and also find out the fringe order. But, instead the method becomes advantages only when I go for digital photoelastic analysis. So, keeping that in mind, we will also have a brief discussion on how I can go about and extract these data by using digital photoelasticity.

We will first develop the mathematical procedure. For the mathematical procedure, to take advantage we need to collect data conveniently and for collecting data conveniently, digital photoelasticity is a must. Otherwise, this method is not attracted. So, what I have now is you should recognize the unknowns are 1/f sigma, coefficients A, B and C and what I have to do is, I have to write an error function.

See, in all our least square analysis, we need to write an error function and minimize that error function so that you get a result which is the best fit for the given data points. So, first step is we need to find out the error function. So, we defined the error now and this is again emphasized, we take several points in the field and you get over determined set of equations and the usual method to solve such a system of equation is to obtain a new set of equations using a least squares criteria.

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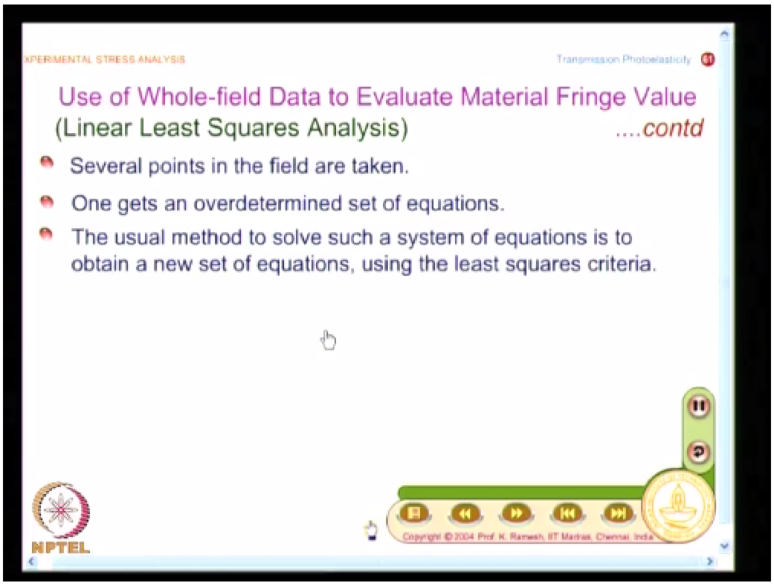

And for me to get a new set of equations, I must first write the error term. Because if I have found out all the coefficients correctly and then if I take a data point, the error would be 0. Because I have evaluated all the parameters correctly and it matches with the data point and I said in any experiment, there would be some sort of a scatter. So, instead of error being 0, we will only minimize the error.

So, the error function is very simple and straight forward. You simply subtract the actual fringe order at the point of interest, which is experimentally determined. So, I defined error as we have given the symbol as e and to illustrate the method better, we take m data points and we say m > 4 because I said you have 4 unknowns since I have whole-field information available.

I can take many data points and I will ensure that I have at least number of equations > the number of unknowns and we will also finally see a recommendation for this methodology to work how many data points are recommended. Though, our ultimate aim is to find out only 4 unknowns, we would use many data points. People have also developed methodology called sample released square methods.

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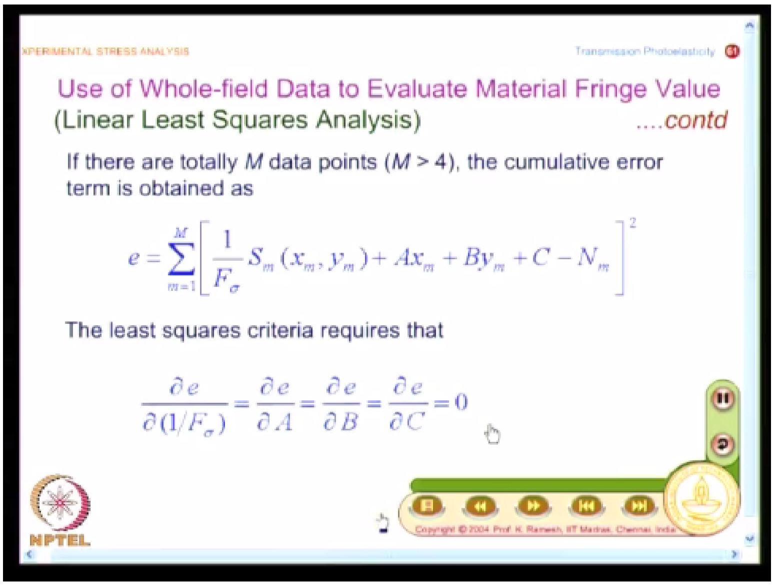

Because finally you should not get different values by taking different sets of data, so when you do a statistical analysis, you must also develop the statistical procedure that when you adopt the procedure, you get one unique value for a given experiment, you do not want to have multiple values. So, you take the statistical methodology to its logical conclusion, so that we will see later.

For us to write the error equation, we first ensure that m > 4. If m is = 4, there is no need to write this error equation at all and that is not what we want. Because I may select 4 points, somebody else may 4 other points and each one will end up with different result. Here, the question is not to get the result immediately. The question is try to get the result as accurately as possible using the whole-field information that is the focus.

So, I write the error equation and the error is nothing but I have this analytical expression, what I have said is this is from your analytical expression, this is a residual birefringence you have assume and this is the fringe order experimentally measured at the point of interest and what I do is, I have a difference that is why I put a – sign and I take a square of it. Suppose, I have m data points, I sum all these squares.

Because I am not worried about whether it is a positive error or negative error, I am only interested in the magnitude of the error. So, I have this as sigma m = 1 to M, 1/f sigma Sm which is a function of x, y, then Axm + Bym + C – Nm and suppose I want to construct, see this is only one equation. When I look at the error, error is only one equation. Suppose, I want to employ the least square criteria what is that I have to do?

The standard procedure is you differentiate this expression with respect to the unknowns and make it = 0. So, I have 4 unknowns, so I differentiate this expression with 4 of these unknowns and make them = 0. When you look at mathematically, the process may look complicated. But in reality, when you look at the final result, it is very easy to implement. See, the mathematics may appear complex.

But, if the final procedure is not simple, people will not use this because the final procedure is very simple and easy to do, this has become very popular. Nowadays, people find out the material-stress fringe value only by processing large volume of data. They do not just go by what you find out at the center alone. There were also reasons for it. See, scientist when they developed a methodology, they also come and out and then say in which class of problems, this methodology is appropriate.

Why you should adopt this kind of a methodology. So, what I have here my focus is to get the value as accurately as possible and I employ the least squares criteria and these criteria is nothing but (()) (29:08) 1/f sigma = 0. Similarly, (()) (29:13) = 0 and I can write this as an expression like this. So, what I have now obtained is I have a taken a large number of data points. I have written an error equation, I reduced this as just 4 equations.

I have 4 unknowns, I have 4 equations and these 4 equations are obtained by employing the least squares criteria. Now, it is very simple. Once you have these 4 equations, you use your simple Gauss elimination process. In one shot, you get all these 4 parameters. But, how to write the final expression, you have to differentiate. I want you to differentiate because once you do for one case, you will know how to do this for other cases.

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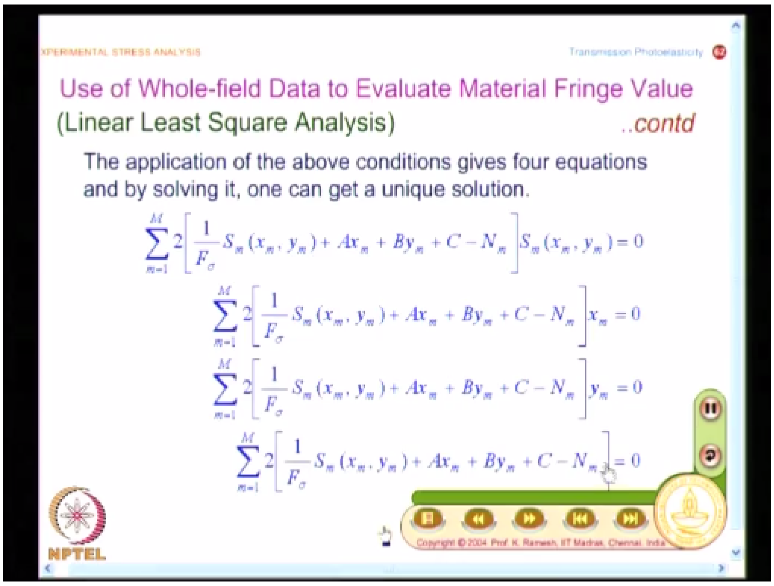

How to construct the equation that you need to know because the idea here is we want to get a unique solution. For us to get a unique solution for multiple data points, we construct only the number of equations = the number of unknowns. That is what we want to do it and this differentiation is very simple. See, normally you are differentiate with respect to x and with respect to y.

What you have to recognize here is the unknowns are A, B, C and 1/f sigma that is what you have to do. Otherwise, it is a child's play. It is a very simple expression and I am sure some of you have got it and this expression reduces to as simple as this. If you are recognized it, for every case, I will have this, this was a square. So, I will have 2 * this complete expression and differentiation of what is there inside.

And if I differentiate with respect to 1/f sigma, this becomes simply Sm x, y. All the other terms go to 0 and once you have seen this writing this for all the other 3 equations is simple and straight forward and what you will have to do is, if you have done a course on indicial notation by looking at this kind of expressions, you can recast this in a convenient matrix representation.

That is what will make the life simple. The equation as such look unwieldy, uncomfortable to handle. The equations are not simple to look at and the method would not have become popular, but for very simple implementation procedure and what are the other equations. When I differentiate with respect to A, what I have, I will have essentially this multiplied by xm as simple as that and if I do it with B, it will become ym.

But, what are you want you to think parallelly is when I have this 4 expressions, how do I represent this as matrix representation. Parallelly think about it, find out whether you are in a position to do it. Even if you are not in a position to do it, when I show the solution, go back and verify the solution is indeed correct. Do not accept it as it is. So, I have the third equation that is nothing but I have this (()) (33:12) ym and the 4th equation will be just this.

I will have the only one here. So, what are you found now is from M data points, I have written an error equation and I have done the least squares criteria by minimizing it to 0 and this results in 4 equations. Right now, this looks unwieldy because I have summation of this, I have so many terms in the series. But, on the other hand, if you look at as matrix representation, the matrix becomes very simple.

Because you should understand how the matrix are multiplied and if you have done an indicial notation, then you would be able to do it and I would show you the answer for the benefit of the class, but I want you to verify this solution. I want you to verify this in your rooms, how to get this final expression. The final expression is very simple I have this as b transpose b * u = b transpose N.

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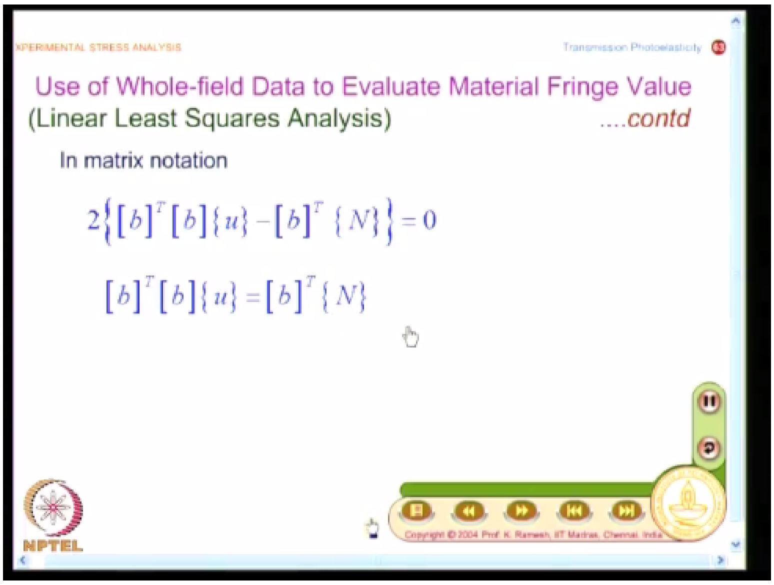

To recognize the set of 4 equations into a matrix notation like this, requires some reflection on your understanding of indicial, if you have understood indicial notations, you can quickly write it. If you have not done the indicial notation, you write several expressions and then see that this can be represented in a convenient matrix form and what is important here is the matrices what you have as b, b transpose, u, N are very simple.

They can be directly written down from your experimental measurement. There is no difficulty at all and you have N number of Gaussian elimination procedures available. So, solving this is also very simple. Computer time is hardly anything and because there is a linear equation you have, you do not need any iteration. Just one solution gives you the final answer.

But, recognizing the 4 equations into this matrix form is a bit involved, not difficult, but I wanted to verify this and what these matrices b, N etc at very simple like this. Please take down this b is nothing but S1, x1, y1 and 1. So, what I am going to do is, I will find out experimentally for the data point x1, y1, I will find out the N1. Similarly, for x2, y2, I will find out N2 and we already have an expression for S.

So, for each of this data points, I need to plug in what is S1, S2, S3, S4 and SN. So, that I can find out the matrix b comfortably, very simple and if you look at the vector u, I have 1/f sigma, I have a, b, c, these are all the 4 unknowns and the vector N is nothing but fringe order at several points. So, experimentally I need to find out the fringe order at several points and gets associated coordinates that is all I have to find out experimentally.

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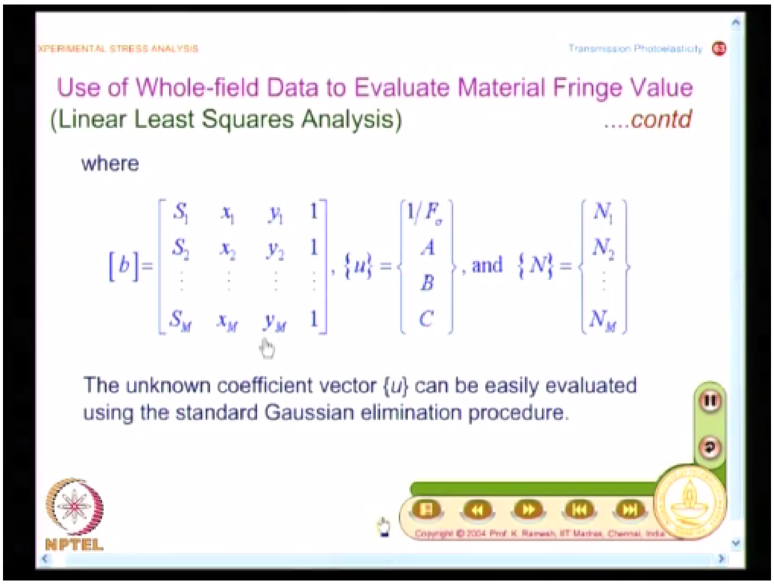

And from an experimental point of view, I can determine them conveniently if I have a digital photoelastic approach, but even manually you can do it in fact I have one of the assignment problems where I have a circular disc under diametral compression with the fringes and I would expect you to extract these data manually, so that you appreciate the advantage of a digital photoelastic approach.

You can also do it manually; you can also do it by digital photoelastic approach. Manual procedure will take time and also can introduce human errors. So, the idea here is you do not focus on only one data point and you take data points from the field and if you look at the literature, you see a contradiction. In the conventional method, you want to find out the fringe order at the center.

In the method where you use whole data point, they recommend because it was tailor-made for the stress freezing approach, where when you do the stress freezing because the material become reaches its critical temperature, the load application points will become flat and because of that in those applications, the center value does not match well with your analytical solution. So, you need to avoid the center and take data.

This is not case for live load model. If you are using a live load model, center is also acceptable. If you are not using a live load model, where using a stress frozen model because a stress freezing process, you get the load application points become flat, this method is advantage. When once people develop a method, they must also say under which conditions the method is required, why it is advantages.

You have to look at from that prospective. The method has become popular because I can construct matrix b, vector u, and N very easily for measurements and finding out the final result is very simple and straightforward. You can do it by several simple methods and I said when I am developing a statistical method, I must also ensure that I take advantage of the statistical processing completely.

So, that is why I called this as sampled least squares analysis because I want to know preferred selection of data points. Any set of data points which is select should yield me ne unique value. The focus is the final results are nearly independent of the choice of data points from the field. To achieve this, the least squares technique has to be combined with a random sampling process.

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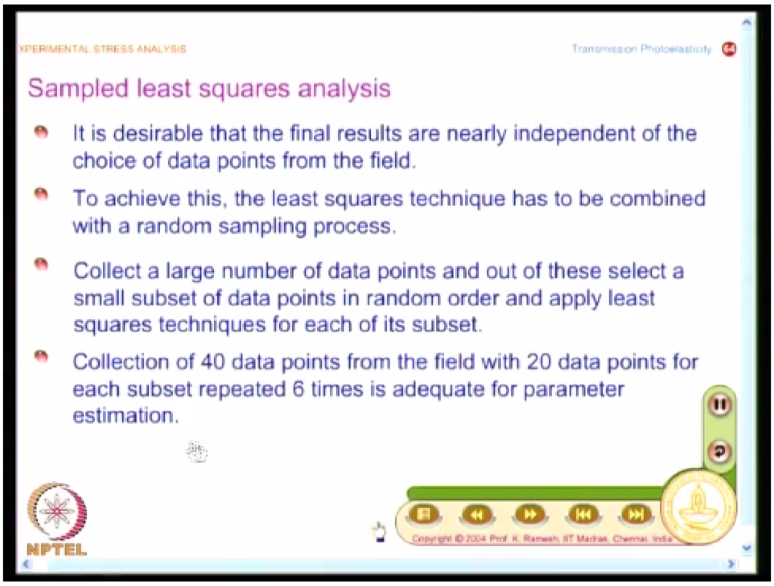

And this is very simple to implement. There are random number generators available, so you can easily do that and what you basically do is you collect large number of data points and out of this, select a small subset of data points, you do this in random order and apply least squares techniques for each of its subset. That is why you bring in your randomization. So, you collect large number of data points from that you take a small subset.

And this subset selects from this massive data points randomly and there were also recommendation how to statistically condition, so what they are recommend is collection of

40 data points from the field with 20 data points for each subset which is repeated 6 times is adequate for parameter estimation. Because in this case, we know only 4 unknowns had to be determined.

For 4 unknowns, they suggest based on experiment, collect 40 data points and from the 40 data points, at a time randomly select 20 of them and repeat this process 6 times and finally you take the average of this, you will have one unique value for the f sigma and this is called sampled least squares analysis and you know this I have said earlier, I have also emphasized many times f sigma has to be evaluated with desirable accuracy.

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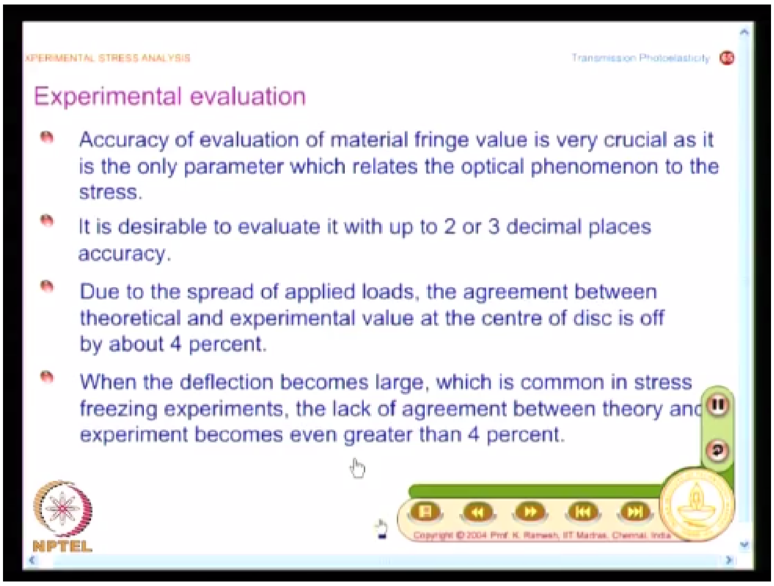

And I want to have 2 to 3 decimal places accuracy and I have also mentioned particularly in the case stress-freezing due to spread of applied loads, the agreement between the theoretical and experimental value at the center of disc is off by about 4 percent. So, in order to improve your agreement, you exclude the data from the center of the disc. There is a particular zone people have also identified.

And when I haunt to do all this, it is desirable that I go for digital imaging processing methodologies. So, that is what I have here, so the recommended zone is r/R = 0.3 to 0.5, the reason why I do this is particularly in stress freezing experiments, there is lack of agreement between theory and experiment at the center, so I avoid the center of the disc and I take data in a region r/R = 0.3 to 0.5, this is the annular region in the circular disc.

And as I mentioned earlier, when I have to do all this when I collect large number of data points, see manually what we will have to do is, you will have to identify the center and then pick out data points, you may have to magnify the picture, pick out data points and then do the calculation. It will be very time consuming.

On the other hand, you simply go and click the cursor and select the data points and your computer automatically understands x, y position and also the fringe order, do not be think it is a very simple approach. But, in order to do that, you need to have some back ground on what this image processing. So, we need to use image processing techniques to identify fringe skeletons.

And mind you here, one of the earliest develop in digital photoelasticity, mimic what they did manually. They have not looked at fundamentally what is the requirement and how to go about. We were finding out the fringe skeleton manually. Now, let us find out the fringe skeleton by using a computer. That is the way people are looked at it and those methods are also useful in certain applications.

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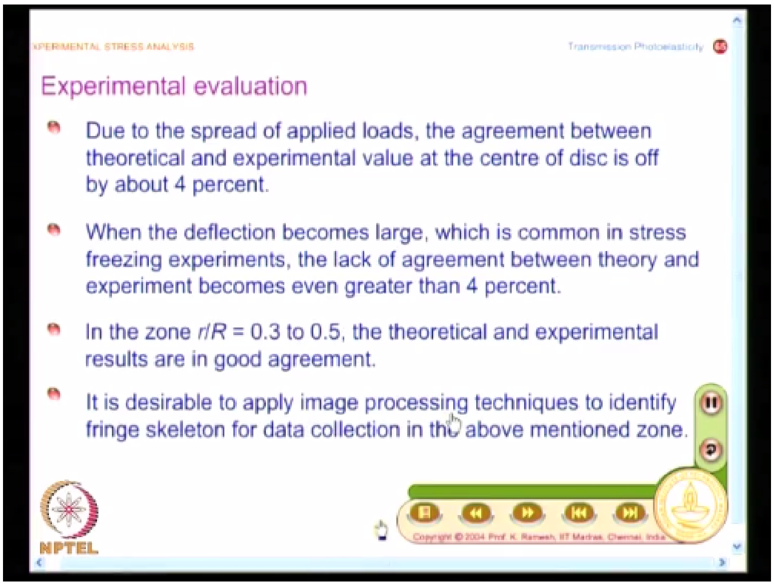

Though you have phase shifting techniques which give you fringe order at every point in the domain since skeletonization has its role in certain kind of problems. So, what we will now look at is, what is the basis of this image processing techniques? Our focus is to find out the data in this annular zone. But, for we to do that, we need to know certain elements of digital photoelasticity which would be of interest to us.

So, what we do in digital image processing, you replace the human eye by a digital camera and this is what I have here. What I have here is, I have a basic polariscope and instead of human being viewing the fringe pattern, I have the CCTV camera and I have this model plate with the hole and I see beautiful fringe patterns on the computer. So, what you see here is human eye is replaced by a digital camera.

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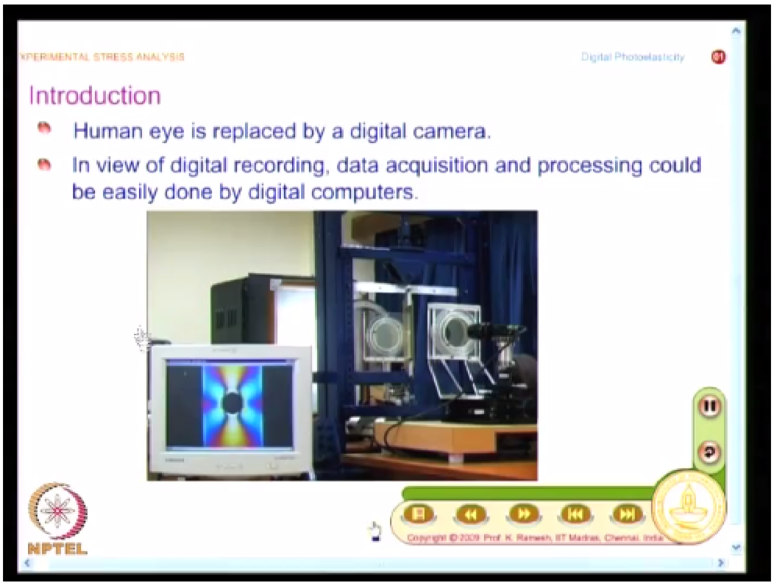

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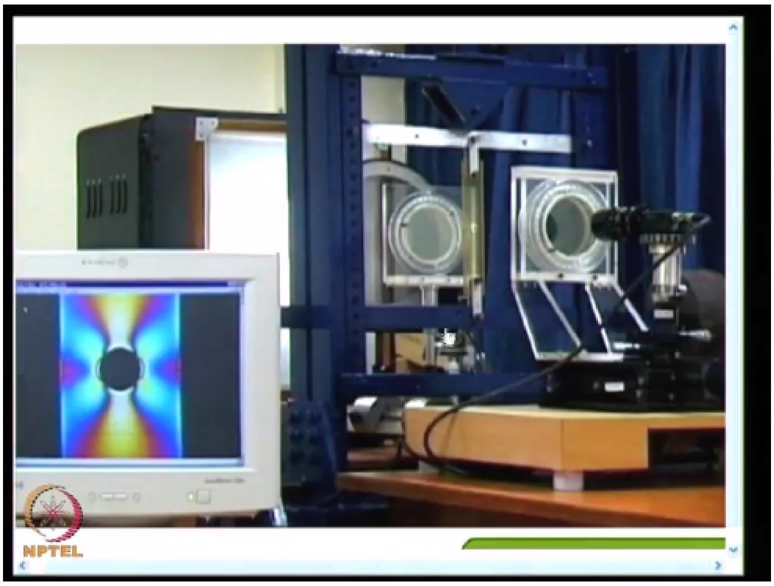

And, you do a digital recording and data acquisition and processing could be easily done by digital computers and you called this whole branch of photoelasticity as digital photoelasticity. But, now we have to understand, how an image is represented as array of numbers. I have a beautiful animation that animation itself tells you what is the sequence in digitizing the image and the greatest advantage is the hardware is so developed.

You can take these digitizations in real time, you have about even a normal camera can give you 30 frames per second and that is what you have here at video rates is called. So, what I see here is this is uniform sampling and quantization. So, we do a digitization of the spatial coordinate. When I do the digitization, I called that as image sampling. After image sampling, I do a quantization.

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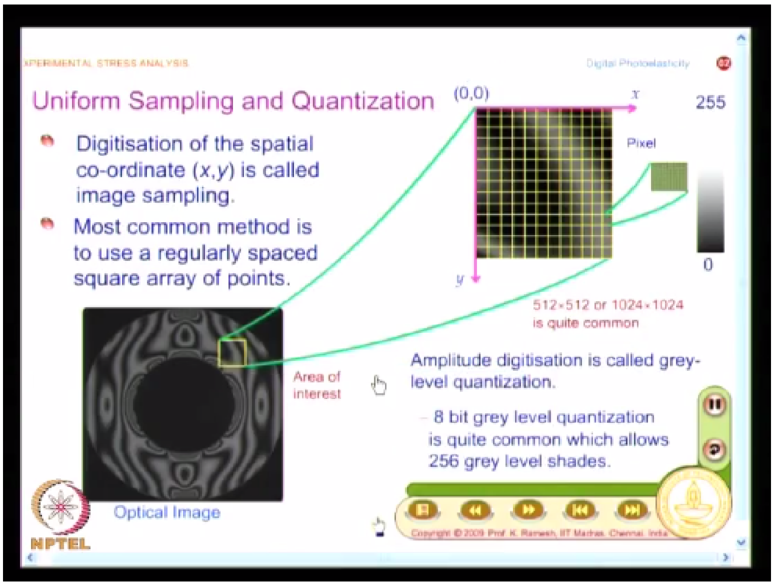

And what you have here is the most common method for digitization is a regular spaced square array of points and what I have here is an optical image and what you have here I have an optical image and for illustration, a small sample area is taken and what you do for this small sample area and if you look at this, I have this fringe pattern enlarged. This is further divided into smaller areas, which are further divided into small areas and this is called a pixel.

So, what I have here is a very small element you find out. So, I have a spatial discretization of the domain as assembly of pixels. And this pixel is very, very small. It is very small and what you also see here, a bar which goes from pitch black to pure white and you labeled this as 0 for pitch black and for white, you labeled it as 255. So, what you are going to do is, for each of this pixels, you are going to assign a number between 0 to 255.

So, what you get the result, the final result is the whole image will be available as a array of numbers and note down this axis. All the monitors, you have this as the origin and x is defined like this and y is defined like this. Y is positive downwards for all your graphic applications, this is how you will have the screen origin and when you design your own software to plot, you must take this into account and match it with you.

Suppose, I want to plot the circular disc under diametral compression, where center is taken as a origin. So, you should know what is the origin in digital screen, what is the origin in your physical problem and use it appropriately while plotting. They are very simple things, but these simple things also you should know, otherwise you get stuck, okay and what you have here is, the spatial discretization could be an array of 512/512 or 1024/1024.

Now, you have much higher spatial resolutions have come. What is quite common is, 512/512 or 1024/1024 and once you have a picture element, which is abbreviated as pixel, you are providing a number between 0 to 255 and this has come from 8-bit grey level quantization. If I have 16 bit, I will have much more division, but 8 bit is very common. So, the amplitude digitization is called grey-level quantization.

So, I will essentially have a number between 0 to 255 representing this image. So, that is what you have in the next slide. So, what I have is, when I use a digital camera, I essentially get a matrix consisting of integers. So, what I have is, I have the image available as numbers. So, then I do number crunching, I can extract the features, I can see the intensive variation much more closely, all that is possible.

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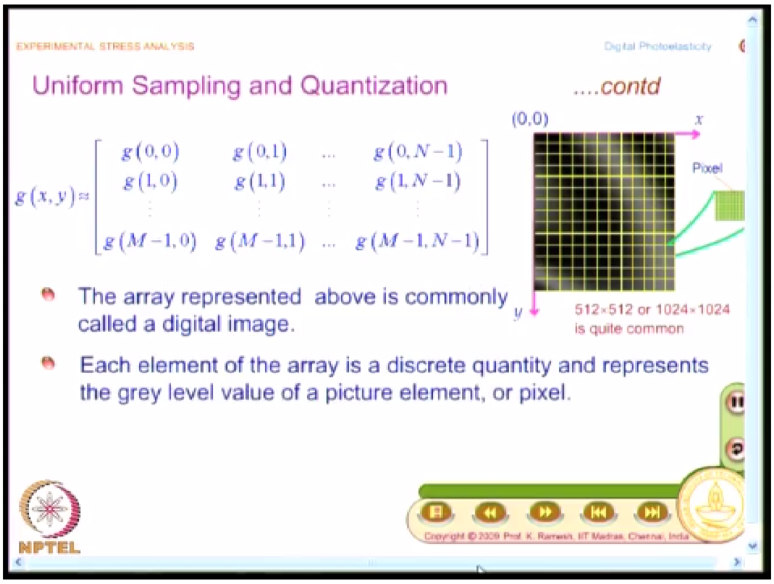

That is how the digitization is very key and I am also in a position to record the intensity data. Though, in initial development of digital photoelasticity, people only worried about fringe skeleton. Even in fringe skeleton, people had binary based methods and as well as intensity based algorithm and intensity based algorithm performed much better than binary based algorithm.

Later on, people had a paradigm shift, where they directly recorded intensity data process this intensity data to find out fractional retardation and theta every point in the domain. The key to all the development is first digitizing the image and this digitization you can do it in real time. That has made the technology very attractive for you to employ in photoelasticity and you have digital photoelasticity that came up.

So, in this lecture, what we had seen was we had looked at conventional method to find out material-stress fringe value. In that, we actually found out fringe order at the center of the disc for various loads and we collected the data in the form of a graph and we do a line which passes through the point such that the points lie on either side evenly and I mentioned this is the graphical least squares approach.

Then, we said we would not worry only about one data point from a disc since we record photoelastic fringe, which is basically a whole-field method, why not I used data from every point in the field. For that, we said we are getting into a over determined set of equations because the parameter to be determined is only one f sigma and if I collect large number of data points, we also felt why not to bring in one more aspect namely the residual birefringence.

Also, evaluated as part of you experiment. So, we brought in 3 more parameters a, b and c. So, finally we have to find out 4 parameters, but we may end up taking 40 data points. So the recommendation we saw and we also said that we will go for a sample least square analysis, so that the final value of sigma is independent of the choice of data points that I take and I also mentioned all this mathematical development looks fine.

But, from implementation point of view, if you do it manually, it looks cumbersome. On the other hand, if you go to digital photoelastic approach, collection of positional coordinates and fringe order becomes a lot more simpler and for us to appreciate how digital photoelasticity functions, the basic aspect you need to understand how an image is digitized. How an image is represented as an array of numbers?

So, we have looked at what is sampling and what is quantization. So, at the end you have the image available as a set of numbers. So, in the next class, we will see how to extract the skeleton from such digital images. We will have only a very quick overview of it, we will not get into much of the details, we will get into a quick overview of it, then proceed with conventional photoelasticity.

So, in between the lectures, I would try to give some aspects of digital photoelasticity and that is how we will also get introduced how conventional photoelasticity could be viewed from a different prospective. Thank you.