Experimental Stress Analysis
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Lecture – 16
Tardy's Method of Compensation

In the last class, we had seen what are all the ways to find out fringe order and theta at any arbitrary point. What you need to keep in mind is, all the optical techniques provide you whole-field information. The whole-field information you can make certain general appreciation of the stress field or the displacement field from the way the fringes are arranged, the density of the fringes gives one kind of information.

The thickness of the (()) (00:48) gives another kind of information. So, you can make certain assessment about the problem by visually looking at the fringe patterns, this is one aspect of it. The other aspect is you need to make quantitative evaluation of the parameters involved. Only then, you will be able to compare whatever the numerical values that you get from an experiment to a parallel analytical or a numerical comparison.

So, you need to also know how to get these quantities quantitatively and what we saw was finding out theta at an arbitrary point is very, very simple. You go to a plane polariscope, keep rotating the polarizer-analyzer combination and when an isoclinic passes through the point of interest, you find out what is the value of isoclinic. The only key difficulty there was the isoclinic fringes are very broad.

Because they are very broad, unless you carefully stop your polarizer-analyzer combination at the point of minimum intensity, there could be error introduced. The moment you come to finding out fringe order N, a simple way is to take a photograph and then processes the data by collecting points along the particular line and if your point of interest does not lie exactly on the fringe order by plotting the graphic and get the fringe order; however, this is time consumed.

So, we also need to have techniques, which help you to find out the fringe order at the point of interest quickly and what we said was these are all compensation techniques. Though, your interest is to find out only the fringe order N, even to find out the find out the fringe order N, the first step is to identify the isoclinic at the point of interest, this is the key point. Once you understand this, you can also appreciate what we do in digital photoelasticity later.

And, what you saw was we had Babinet-Soleil compensation. In this, you had an external compensator. You had a gadget and this was aligned by finding out the isoclinic angle; in this case, it was 30 degrees and then, this was kept and what you did was? Then, you rotated the knob until a fringe order passes through the point of interest and I cautioned that this is the field of view within the compensator.

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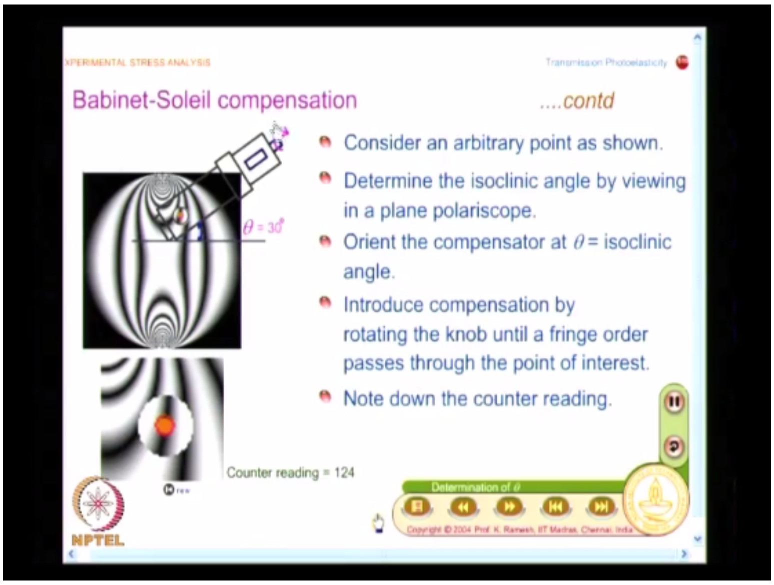

Within this, whatever the modification that you have done is valid only for the point of interest from the point of view of data interpretation and I also said that you need to find out whether the higher fringe order comes and coincides with the point of interest or a lower fringe order comes and coincide with the point of interest. In this case, it is a higher fringe order that has come and coincide with the point of interest and the manufacturers give it in various ways.

In this case, the manufacturer has given directly note down the counter reading, then he has also supplied you a graph and based on the graph, you can find out the fringe order at the point of interest. Then, we moved on to what is known as Tardy's method of compensation? The focus here is instead of using an external compensator, can one of the elements in the polariscope itself could be used as a compensator.

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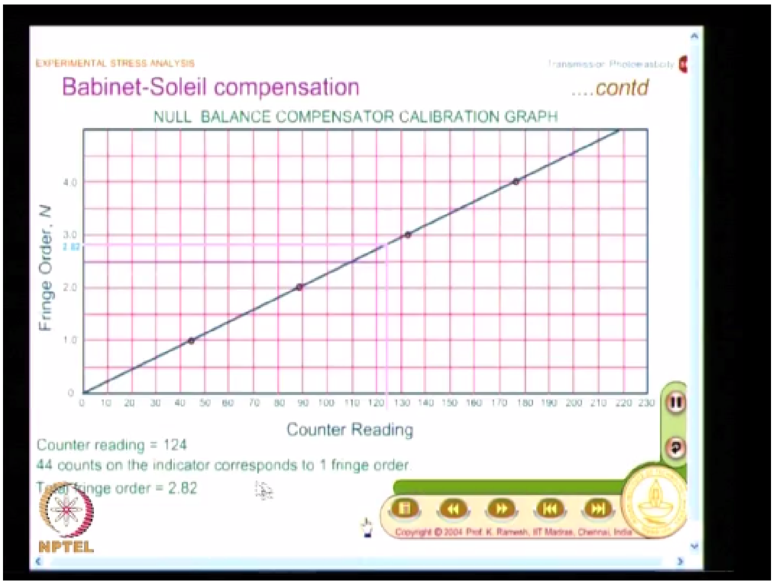

In this case, it is a analyzer which is to be rotated to an appropriate angle to function as a compensator and what we will do is, we will see the procedure first, then we will go and develop the mathematical analysis and convenience ourself whatever the procedure that we have stated indeed coincides with our analytical development that is the way we will proceed and as an any compensation technique, you will have to find out the principal stress direction at the point of interest using a plane polariscope.

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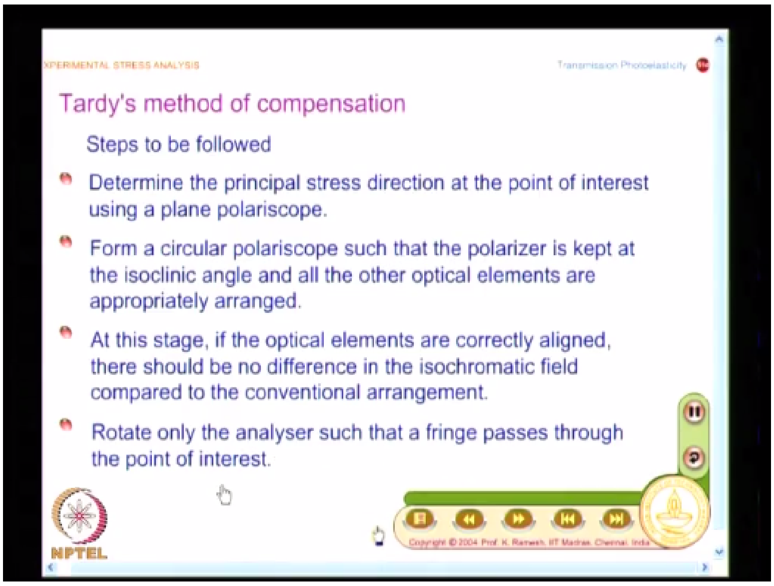

Your interest is only to find out fringe order N, but you also have to find out theta. The second step is I have to form a circular polariscope such that the polarizer is kept at the isoclinic angle and all the other optical elements are appropriately arranged and if you do not align the elements properly, there would be distortion in the isochromatic field.

So, one of the indirect check is if the optical elements are correctly aligned, there should be no difference in the isochromatic field compared to the conventional arrangement. This is how you verify whether you have followed the second step correctly. Identify the isoclinic, second step is re-orient your polariscope such that the reference axis at the point of interest namely the principal-stress direction is the reference axis for formation of your circular polariscope.

And, this makes your analysis simple. See once you make the reference axis as the reference axis for your polariscope, what is the value of theta that you need to substitute? When I have to analyze the model, we always say gives the retardation delta, it has the orientation theta. Suppose, I take the reference axis as the principal-stress direction at the point of interest, theta becomes 0 and this makes your calculation very, very simple and fast.

So, the implication of making a circular polariscope also helps you in your mathematical analysis, the theta becomes 0. So, this is the important step that you need to understand. Then, finally you rotate only the analyzer such that a fringe passes through the point of interest. So from this axis, I should find out what is the angle by which analyzer is rotated. I should know what is the reference axis before I start rotating the analyzer that becomes the reference axis.

Then, I start rotating it until a fringe passes through. What are the possibilities? I can rotate the analyzer clockwise as well as anticlockwise. So, both will be of some numbers. So, we need to know how to interpret this. There is no preferred orientation that you should always rotate clockwise or you should always rotate anticlockwise. You have to do it depending on the problem on that.

And you know in experimentation, we always want to do some kind of an averaging, so you better do one measurement by anticlockwise, another measurement by clockwise. So, you take the average of these two and find out the actual fractional fringe order then the total fringe order and what I said was, suppose beta is the rotation given to the analyzer, then fractional retardation is given as in terms of fringe orders.

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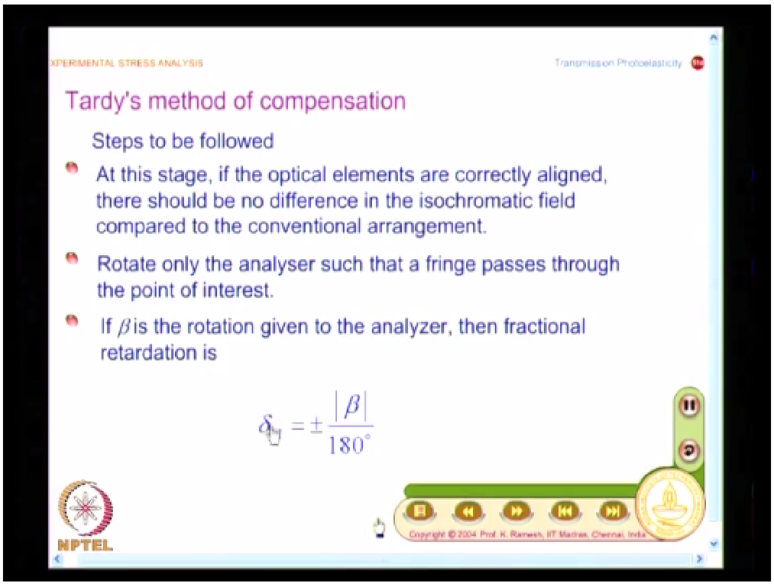

When I write it as simply delta, it is in radiance, if I write it delta suffix N, it is in fringe orders and this is given as beta/180 degrees and I have this as sign has to be fixed based on heuristic information. Sign does not come from your mathematical analysis. Sign comes from your observation whether the higher fringe order comes and occupies this point of interest or a lower fringe order moves and occupies a point of interest.

And, I have also cautioned whatever I do in a compensation technique it pertains to the point of interest from the point of view of data interpretation. That will become more clear when you look at Tardy's method of compensation and what we are going to do is rather than taking an arbitrary point, we will take a very convenient point for finding out the total fringe order and what I have taken is, I have taken a disc under diametral compression.

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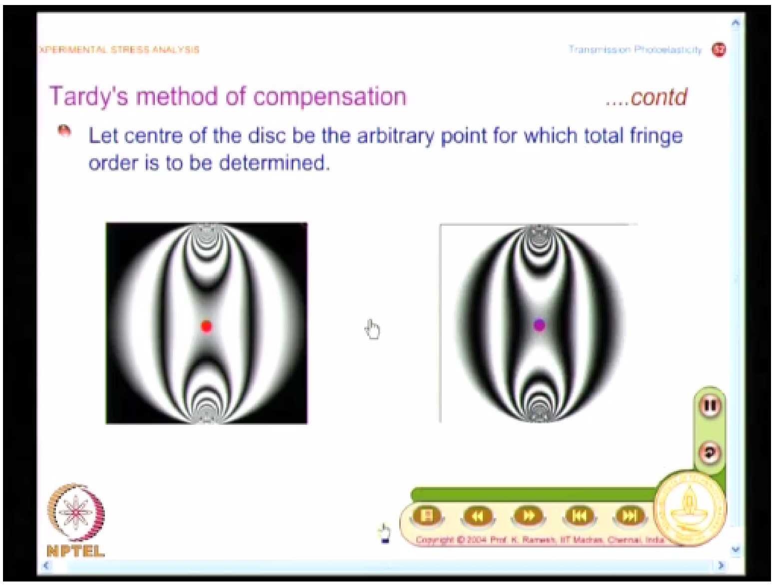

There is a particular reason behind it because the next topic we are going to discuss is how to find out material stress fringe value f sigma? And I would use circular disc under diametral compression as the basic model to be used for finding out f sigma and we will use Tardy's method of compensation to find out the fringe order accurately and what we are going to look at we are going to look at the center of the disc.

And, I have taken the loading in such a manner f neither the full fringe or half fringe that is here it is 0, 1 and 2. The fringe order lies between 1 and 2 and fringe order lies between 0.5, 1.5, 2.5 that is the way the fringes increase. So, it lies between 1.5 and 2.5. So, both in the dark field as well as the bright field, a fringe order does not pass through the point of interest. So, I need to find out the exact fringe order only by a compensation technique.

And, you know in some of the polariscopes built several years earlier, they used to have a control on loading the model, they used to have a water tank to load the model. So, they will adjust the level of water until a fringe occupies the center. There were also polariscopes like that. So, we do not need that kind of a loading arrangement if you know how to employ Tardy's method of compensation.

I can find out the fringe order very accurately with the existing loading frame without much difficulty in adjusting the load, so that a fringe passes through the point of interest and here, we have deliberately taken the center from your knowledge of solid mechanics and you have also looked at the analytical expression for the stress field. What is the principal-stress direction at the point of interest? This is exactly at the point of symmetry.

So, the x and y axis themselves function as principal-stress direction, we will verify. We will also see the isoclinic because in any compensation technique, my first step is to find out the isoclinic at the point of interest. Once I have isoclinic determine, the polarizer-analyzer are aligned to this as the base angle, quarter-wave plates are appropriately rotated. So, that you form a circular polariscope with the reference axis at the point of interest as x and y.

Let us go and see, how I getting it. So what I find here is, I have been able to get a fringe order passing through the point of interest, this animation we will see a quite a few times and you should also look at what is the shape when it passes through the point of interest I am moving the cursor what is it shape, it is like figure of eight. So, if you are able to observe this, is it necessary to mark the point at the center? It is not necessary.

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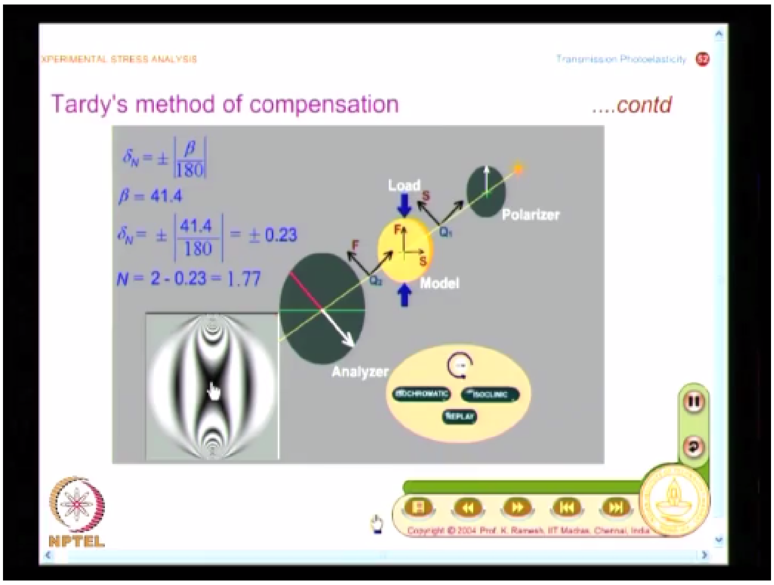

See, if I want to do any evaluation quantitatively at a point of interest, I must mark the point and then find out the fringe order. If I take the center because the fringe occupies the position as eight, it is easy to identify that I have got the fringe order at the center and this is one of the advantages when I want to go and find out the f sigma, I have to simply take a circular disc, make it as a figure of eight by rotating the analyzer, you find out the fringe order accurately.

You do not even have to mark the point, so it eliminates one more step in finding out comparison between experimental calculation and analytical comparison. Because the focus is to find out f sigma, focus is not to find out the fringe order. Fringe order you need to know and based on your analytical computation, find out what is the material-stress fringe value. So, this eliminates one more step in your evaluation of material-stress fringe value.

So, note down in the earlier slide what we saw was both the bright and dark field fringes were not passing through the center. Now, I have rotated the analyzer and I have made this fringe passes through this point of interest and I have made an observation that this forms a figure of eight and just to convenience you, you know we will see the isoclinic. Isoclinic at the point of interest is 0 degree isoclinic.

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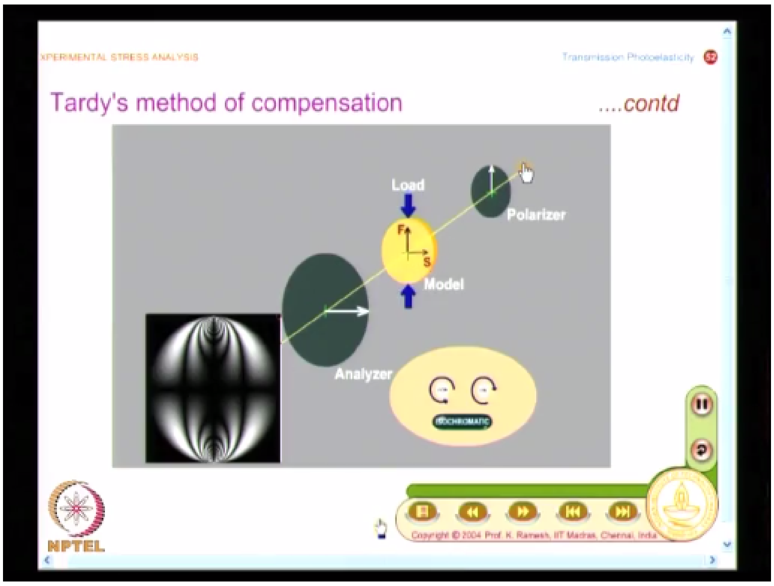

I have a plane polariscope because I have to go to a plane polariscope and then do it. I have the source of light, then I have the polarizer, I have the model and this is what I said, at the point of interest, the reference axis itself is the base reference axis for the final circular polariscope also. In this case, the point is selected as a center of the disc, the horizontal and vertical axis themselves are principal-stress directions.

So, I have the polarizer-analyzer coincide with this and normally, we also make a polariscope with polarizer vertical and analyzer horizontal. So, another advantage why I use circular disc as a model for f sigma calculation, I do not have to do anything for the polariscope even if I want to find out fringe order with second decimal place accuracy, it is enough, I keep the model properly loaded and simply rotate the analyzer.

Though, in a generic Tardy's method of compensation, my first step is to find out isoclinic at the point of interest, second step is rotate all the optical elements such that you form a circular polariscope with the reference axis at the point of interest. So, these two steps are redundant when I want to use circular disc and use the center to find out the f sigma calculation.

So, you are learning both Tardy's method of compensation as well as why a circular disc is so popular in photoelastic analysis because I said when you are learning strength of materials, you are torn from a simple tension member, then you go to bending and you never touch circular disc. Whereas in photoelasticity, you touch only a circular disc because unless you do a course in theory of elasticity, you cannot find out the stress field in a circular disc.

For circular disc, has very simple fringe contours, you can illustrate both isoclinic and isochromatic in a plane polariscope and also helps you in finding out material stress fringe value very comfortably. So, I can develop how to identify fringes using circular disc. I have also used it to find out f sigma that is why a circular disc is very popular. So the first step is, I need to find out the isoclinic angle at the point of interest.

And we find that in a plane polariscope, you find the 0th degree isoclinic passes through the center and if I look at the isochromatic to convenience yourself that I have this as the dark field, I have fringe order 0, fringe order 1 and fringe order 2. So, you can say the fringe order at the center is between 1 and 2. So, I need to find out the fractional part and I have already said, I can rotate the analyzer clockwise or anticlockwise.

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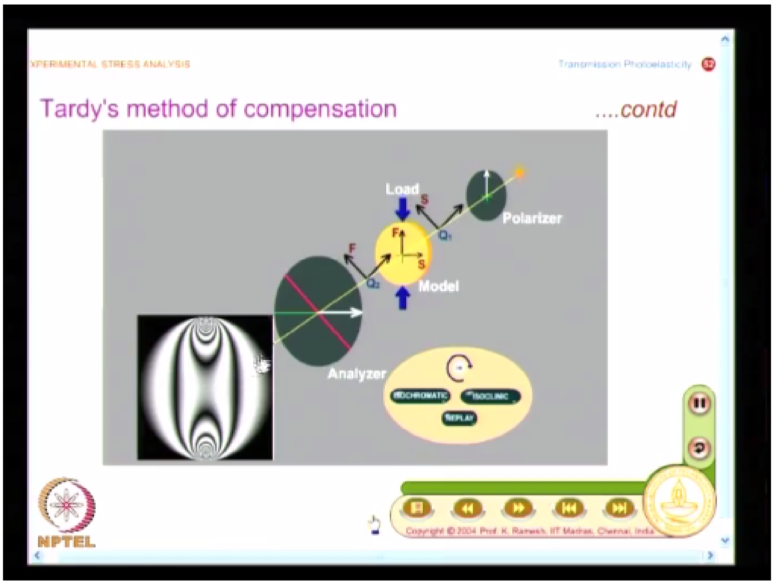

While doing so, I should also see which fringe moves and occupies the center. Because I have a very good animation, I can show repeatedly either clockwise or anticlockwise and observe. First observe, then make a note of it. I will also give you sufficient time for you to make a sketch of it because the concept you will have to understand. You have to form a figure of eight mainly because even without marking the point, you can ensure that you find out the fringe order at the center.

So, that is why I identified because when you rotate the optical elements you have to visually check whether you have reached the minimum intensity. So, one check is it forms the figure of eight. Even if you rotate the elements slightly this way or that way, the figure of eight will get disturb. There is one indirect check. So, what we will do is, we will rotate it in a clockwise direction.

I rotate it in a clockwise direction (()) (19:28) you may not have noted all the aspects; it has form the figure of eight. Now look at, I will replay this, look at which fringe order has moved and occupied the position. Only now look at, which fringe order has moved and occupied the position. So what we find, what we find is, the fringe order 2 has moved and occupied this position. So, I need to have this information for me to assign the sign.

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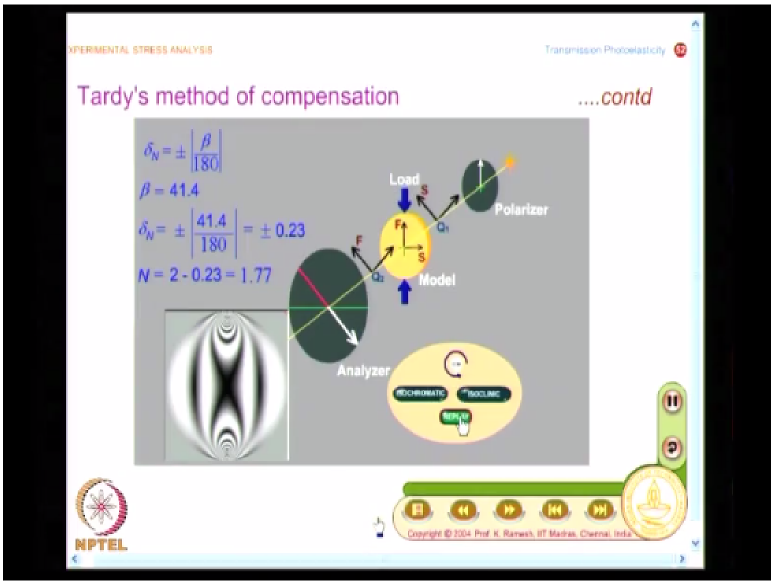

I have needed to note down two things. I need to find out by what angle I have rotated, I need to find out that angle, the second one is which fringe order has moved and occupied the position and we have said, the fringe order is nothing but the fractional fringe order is nothing but beta/180 degrees. So, now I have that information. So what I have here is, I have delta N = + or - beta/180 and this angle is like 41.4 degrees.

I could do this because my horizontal axis itself is a reference. See, in some other point in the disc, you may find some other angle will be the reference, so you must start from that reference. You should not always start from 0. The isoclinic angle at the point of interest forms the reference. Because in this case, the center horizontal vertical coincides, I measured from horizontal.

So, there is the possibility that you may misinterpret find out the angle from horizontal, it is not so, find out the angle from the reference axis at the point of interest. So, I have beta =

41.4, so I can find out what is delta N. I have still not attached the sign, I still say it is + or - 41.4/180 degrees and that gives you + or - 0.23. So now I know, I know the fringe order was between 1 and 2, so I do this as 2 - 0.23 = 1.77.

So, I can find out the fringe order at the point of interest very precisely. It depends on your skill because if I using human eye to stop your analyzer rotation, you need to see it without error of parallax and have sensitivity in stopping it at the point of minimum intensity. All that you have to take the precaution and also note down the angle correctly, then you attached the sign. Okay, let me ask you a question.

Suppose, I rotate it in anticlockwise direction, I will get some angle from that also. Can you find the relationship between the rotation I do it in clockwise and rotation I do it in anti-clockwise. Suppose, I have beta 1 as a rotation in clockwise and beta 2 as the angle rotated in anticlockwise, can you anticipate what would be the final relation. Beta 1 + beta 2 will be 180 degrees, so that is another check.

But, you do not have to make it as 180, you have to make the experimental measurement, so if I take two measurements because the fringe order at the point of interest is only one unique value whether I rotate the analyzer clockwise or anticlockwise, it should reflect the same value, but because of experimental uncertainties, you may find out 1 degree error this way or that way. So, you take both the measurements and take an average.

So, you also have the advantage of applying statistical principal in data processing. So, let us look at now that I will rotate it by anticlockwise. I rotate it anticlockwise, so I find that I have to rotate by a longer angle number one and I will replay and then ensure which fringe order has moved and occupied the position. So, which fringe order has moved, fringe order 1. In the earlier case, we saw fringe order 2 came and occupied the position.

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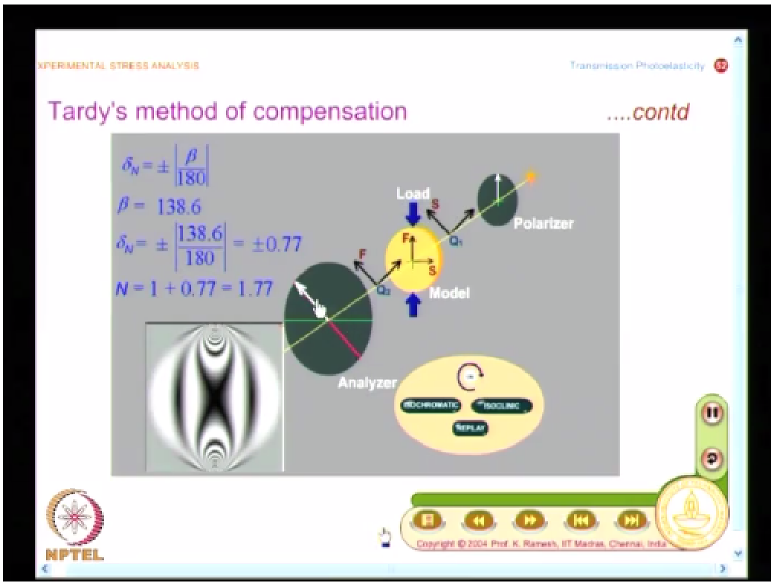

In the second case, a fringe order 1 has moved and occupied. This is the very important information. In fact, this is the advantage in a conventional photoelastic analysis, you get the heuristic information in fixing the sign of the fractional retardation. In fact, this was a very difficult task in digital photoelasticity. Fixing the sign of the fractional fringe order was not a simple task.

It took almost a decade for people to figure out completely all aspects related to this. It was not a simple problem. So, what I find? I find the angle as 138.6 degrees, then I am able to calculate delta N, this is + or - 0.77, because the fringe order 1 has moved, I attached the sign +. So, I make it as N as 1 + 0.77 = 1.77. So, now what we will do is, we will look at the complete procedure.

The first step what I need to do is I need to find out the isoclinic. So, I find out the isoclinic in a plane polariscope even though while starting the discussion, I said because it is at the point of symmetry, the axis coincides with the principal-stress direction, the reference axis themselves are principal-stress direction. You also verify experimentally, where in I put polarizer-analyzer and I ensure that I have a 0 degree isoclinic passing through the point of interest.

Then, we have also seen that at the point of interest in the dark field, no fringe actually passes through and we get information, it is between 1 and 2 and I rotate the analyzer, I am able to directly rotate the analyzer why? Because the basic polariscope arrangement is with respect to the horizontal and vertical, I do not have to do the step of oriented in the polariscope to the reference axis, the basic polariscope itself is oriented to that.

Now, when I rotated clockwise what happens, I am able to make the intensity transmitter at the point of interest as 0, this is another way of looking at it. I have a fringe with shape of eight, other way of look at is, intensity passing through the point of interest is 0, fine. I also see semblance of fringe pattern elsewhere. When you rotate the analyzer, I see the complete model. Can you attach any physical interpretation to those fringe branches?

Because I see a fringe, you cannot assign any interpretation to those points in conventional photoelasticity. The interpretation is valid only at the point of interest where we have done the compensation. This was not very clear when you looked at Babinet-Soleil compensator because the field of view itself was very small, here the field of view is the complete model you can see what happens when the analyzer is rotated arbitrarily.

So, do not make a wrong understanding that you can interrupt I rotate the analyzer and find out what happens at every other point. It is valid only for the point of interest. So, what I have here is, I find out what is the angle by which I have rotated, then I find out what is delta N, I finally attached the sign depending on which fringe order has moved and occupy that is the key point and we rotated counterclockwise, I have a fringe order 1 and moving and occupying.

So, now we have fairly understood what is the way the fringe orders move and how analyzer helps in making the intensity at the point of interest 0. Because you have always been looking, you have some intensity at the point of interest mainly because the model has not behaved like a full wave plate. So, for it behave like a full wave plate, I need to add some retardation or subtract some retardation depending on which way I want to proceed.

So that whatever, the additional retardation which I introduced is now done by the role of analyzer. It is very interesting because I will also have a brief discussion what has been the philosophy of digital photoelasticity, how people utilize this aspect in devising various algorithms. So, now you have a fairly good idea on how to proceed from Jones calculus point of view.

Because when I want to go with the Jones calculus, I need to represent the retardation matrix at the point of interest because the model behaves like the retarder and that matter becomes very simple because I take that as a reference axis. I simply make theta = 0. We have seen the procedure. Now, we have to validate yes that procedure was convenient to find out the fractional retardation. So, my interest in Jones calculus analysis is what?

I need to establish a relationship between delta and beta. Beta is the angle of rotation by which I rotate the analyzer, I must establish a relationship between beta and delta that is the focus and mathematics also fortunately becomes simple. Essentially, we are going to analyze a circular polariscope and you know very well when I go from plane polariscope to circular polariscope, the mathematics involved is quite lengthy.

But if you have theta = 0, the retardation matrix is very simple, then multiplication becomes a lot more simple and I would expect you to do that multiplication right in the class, okay. So, we will also look at the Jones calculus representation. So, my interest is to find out what happens along the analyzer axis and perpendicular to the analyzer axis and I think we will see this one after another. I will replay the animation.

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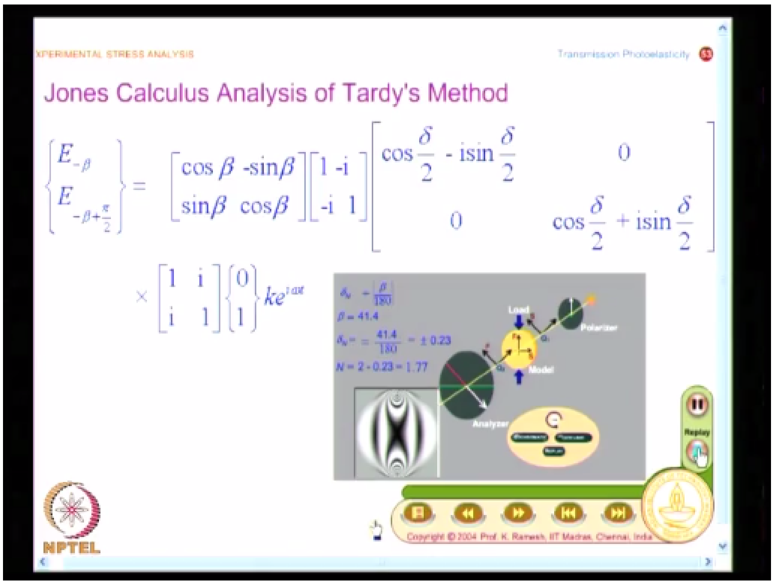

So what I am going to do, I want to find out this. So, I have what is a light that is coming out of the polarizer and then after the polarizer, I have a first quarter-wave plate, I have the model, I have the second quarter-wave plate, then I have the analyzer. Here, I also bring in the analyzer and I also made one more statement when a fringe passes through the point of interest, we are essentially saying intensity goes to 0.

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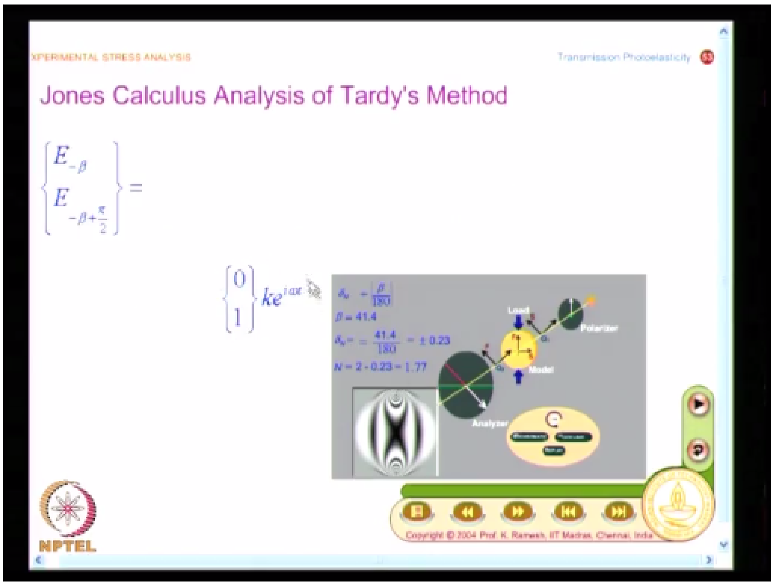

So, that is the condition I will use for me to establish a relationship between beta and delta. That is what I am going to do. See in all our earlier calculation, we start before the analyzer because I said that when I keep the analyzer horizontal and vertical, I can analyze for the dark field and bright field. So, it is advantage to stop the calculations just before the analyzer. But, here my interest is to find out the relationship between beta and delta.

So, I go up to the analyzer. After the analyzer only, I look at it. Now, I write the matrices for each of the optical elements and you know very well what is the Jones matrix for the first quarter-wave plate, very simple and the model because theta is 0, this matrix is very simple. I simply have cosine delta/2 - isine delta/2, and these terms go to 0 and I have cosine delta/2 + isine delta/2.

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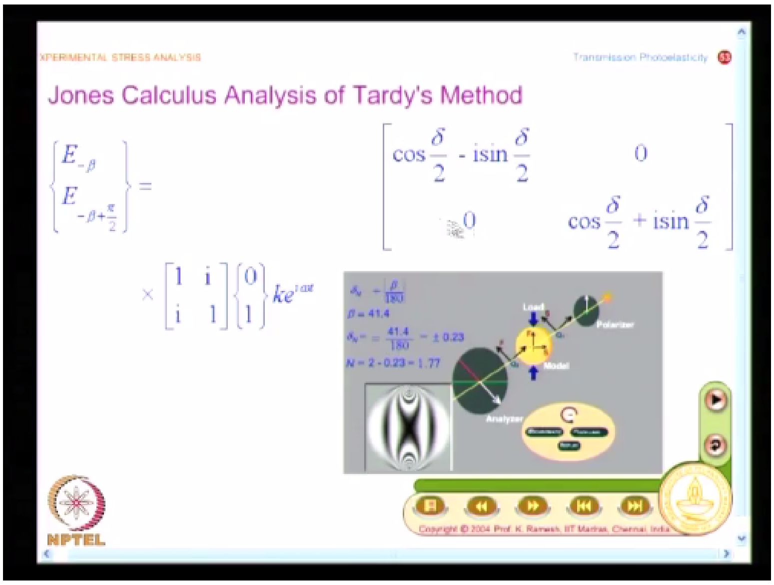

Now, you have the second quarter-wave plate and what is the difference between the first quarter-wave plate and second quarter-wave plate? I have not written 1/root 2 here, because I will write it finally for both the quarter wave plates product of 1/root 2 as 1/2. So, now I will write the second quarter-wave plate that is nothing but 1 - i - i 1 and I also put the rotation matrix for me to find out the component of light vector along the analyzer axis.

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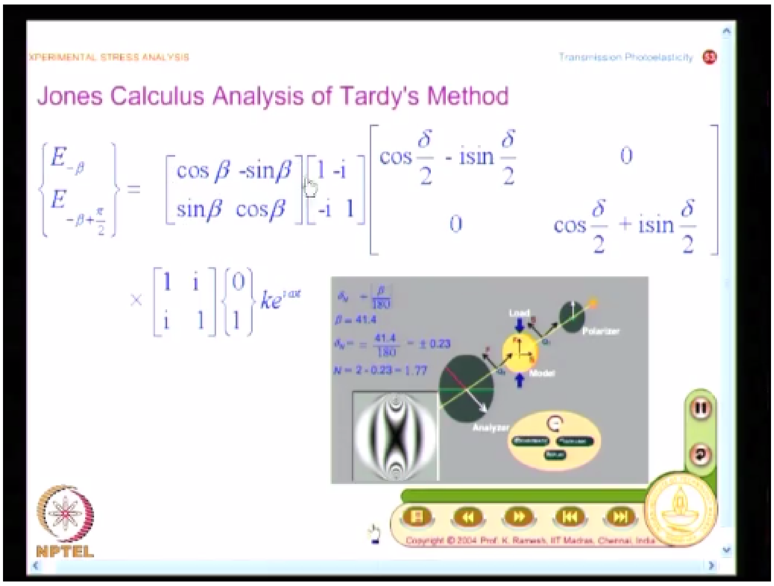

And perpendicular to the analyzer axis and I know I have taken this as clockwise, so I used this angle itself as a reference to write this and what I am going to do here? I am going to multiply all the elements to find out the component of light and what we are going to say is, the light along the analyzer axis is 0. So, that is the condition I am going to enforce. So now, I want to you to do the product of these matrices and try to get the expression.

I will give you few minutes of time for you to do this that would help for you to revise the notes comfortably and I am sure the mathematics is fairly simple and some of you have got the results reasonably and I will proceed to the next step and when I do the simplification, I get the final expression in this fashion. I have not multiplied this rotation matrix, the rest of the matrices reduced to sine delta/2 and cosine delta/2 ke power i omega k.

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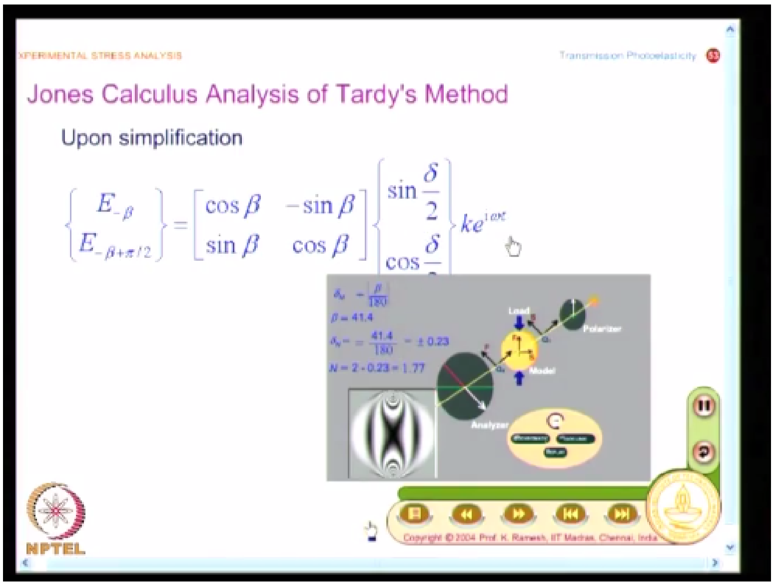

You can very your calculation and this is how you get the expression. Now, I am going to say that horizontal component goes to 0 and finding out that is fairly simple, that I fairly simple that you can work it out and when I do this, I have it like this. So, what I have here is for light extinction, the E - beta component should be 0. This occurs when this identity is satisfied, I get this as cosine beta sine delta/2 = sine beta cosine delta/2.

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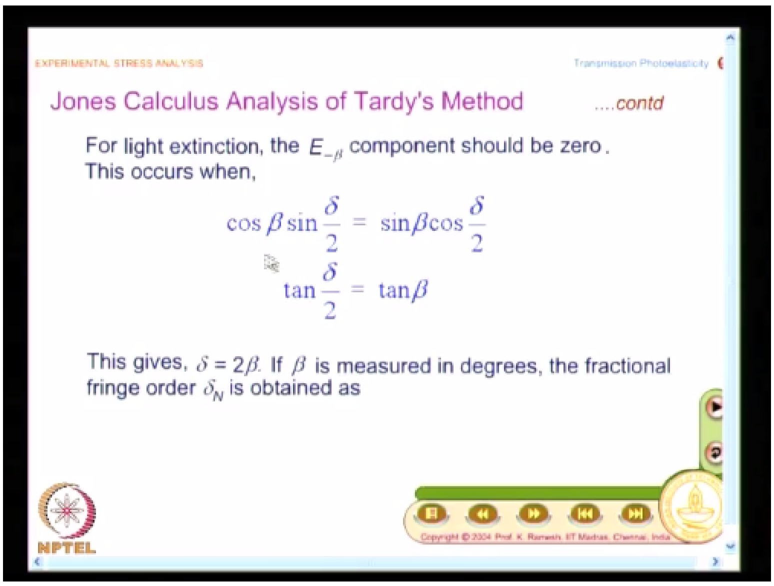

So, I get this as tangent delta/2 = tangent beta. So, now I find the rotation of the analyzer is linked to the retardation, fractional retardation introduced at the point of interest and you know though in all your expressions, this can be in radiance when you physically measure, you measure beta in degrees and I can express fractional fringe order simply as beta/180. I can simply do this as beta/180 degrees.

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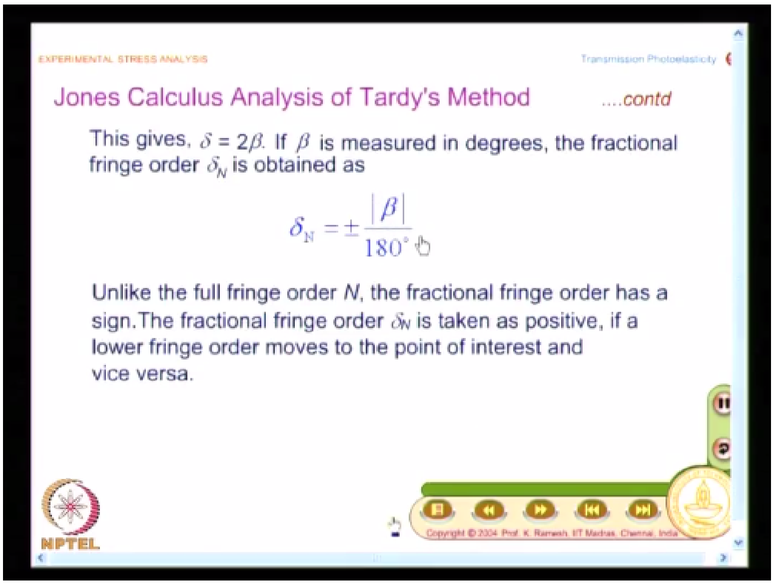

What we have seen is, we get delta = 2 beta and since beta is usually measured in degrees which is very convenient. I get the fractional fringe order as beta/180 degrees and I do not attach the sign. The sign has to be attached based on whether the higher fringe has moved to the point of interest or lower fringe order has moved to the point of interest and this has made conventional analysis very simply.

Conventional analysis became very simple because you have a way of finding out the sign of the fractional retardation and let me also give you a few ideas on digital photoelasticity here. See, if you look at one of the focus in digital photoelasticity was rather than finding out fringe order at an arbitrary point, they wanted to find out fringe order at every point in the model domain.

There are two issues, one issue is we have seen in a dark field arrangement whether it is 0, 2 pi, 4 pi, 6 pi, you will not be able to find out any distinction on how to order the fringes. You will have fringes at these discrete locations whether it is 0, 1, 2, 3, so this is one issue. Another issue is between fringe order 1 and 2 or 0 and 1, how to add the additional fractional fringe order.

So, these were the two issues they have to find out the fractional fringe order and they will also have to find out how to resolve adding the integer part because you are all trigonometric relations, all inverse trigonometric relations are multi-valued. When you do inverse trigonometric calculations, you get multi-valued functions, fixing up the integer part has always been difficult.

This is called unwrapping and this you know electrical engineers were doing it for a very, very long time, they were doing phase unwrapping. So, you need to borrow those concepts into photoelasticity and then do the unwrapping of the phase. The other issue was how to find out the fractional fringe order? And if you look at many of the interferometry techniques, we have looked at for example double exposure methodology in hologram interferometry.

We will just have a look at it. We will just have a look at that double exposure method and what we had was, we had one beam falling on the model and another beam goes to the mirror and then gets reflected and you called this as a reference beam. So, one of the techniques what people did was they wanted to record multiple images with known phase shifts.

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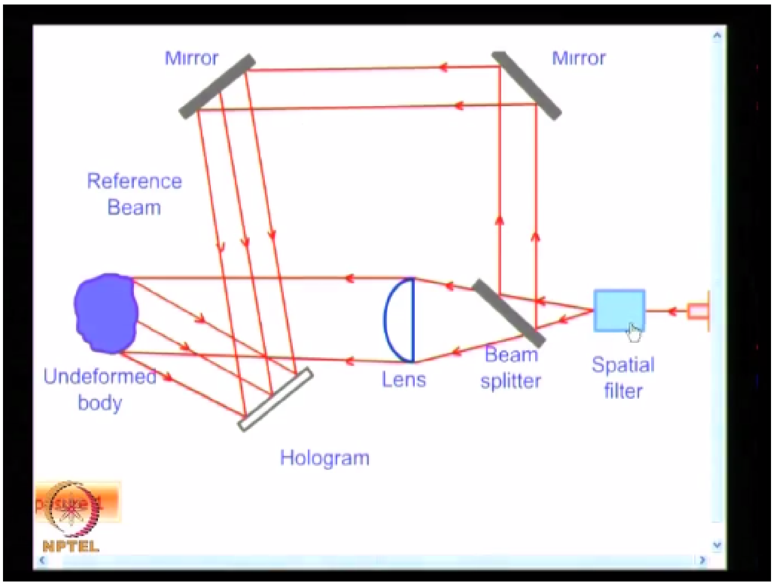

Because in order to resolve the ambiguity in fixing the sign, fixing the interior fringe order, they had recorded multiple images and the whole approach was processing the intrinsic information. The approach is not just looking at fringe order and then assigning it, you have to look at what is the intensity at the point of interest by processing the intrinsic information, they were trying to find out fringe order at every location.

And in all those experiments, you have only one phase that is the phase difference because of deformation and in order to get multiple images, the usual procedure was move this mirror at known steps and they also used piezoelectric actuators to do this. So, that means I will have one set of images with the reference beam and the actual model deformed and then I repeat the same thing by changing the reference beam alone with known phase shifts.

With this information and taking multiple images, people were able to fix the total fringe order and also automate the procedure and because I do this shift, they also called this as phase shifting techniques and what is the parallelism in photoelasticity? Photoelasticity, we saw one ray becomes two because it behaves like a crystal and you have a temporary birefringence.

So, whatever happens to one ray happens to the other ray also and this is where Tardy's method of compensation opened up a new perspective. So, in other interferometric techniques, they had external gadgets to introduce the shift in phase. In the case of photoelasticity, what I have is, I have established by rotating the analyzer, I am able to introduce the phase shift.

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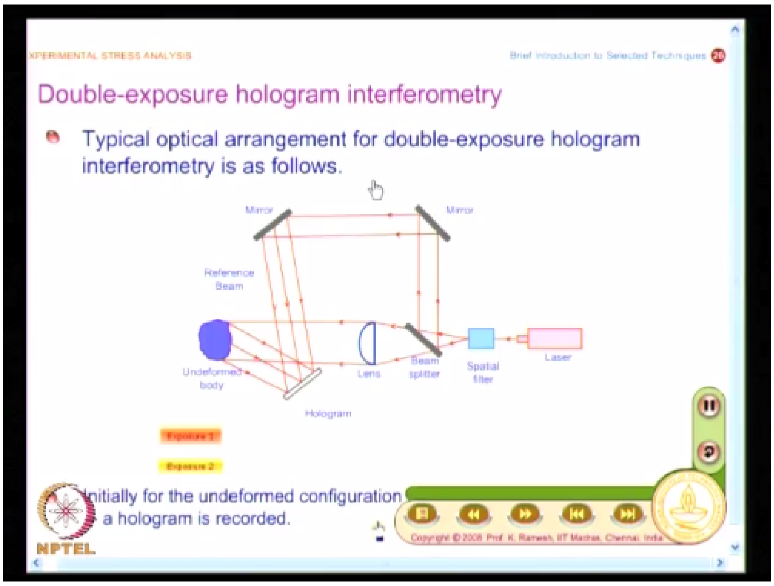

See, how a rotation of element is akin to giving a phase shift comes from understanding Tardy's method of compensation. So, this photoelasticity took it immediately. Then, they said I will rotate only the analyzer, I will rotate other optical elements also and many algorithms got developed. So, this is the basics behind it even if you want to go and understand digital photoelasticity.

In digital photoelasticity, people essentially process intrinsic information and for them to get phase shifted images, the via media was rotating the optical elements appropriately provided something equivalent to the phase shift, which was externally obtained in other interferometric techniques, so understanding Tardy's method of compensation will also help you to understand digital photoelasticity and also to verify digital photoelasticity.

Because you know you have algorithms to fix the fractional fringe order sign because those algorithms are developed based on certain assumptions, certain approximations, certain conditions, some of these may get violated in actual experimentation and when you compare it with Tardy's method of compensation, if there is a variation, you could use this as an input and improve your algorithm to find out the fringe order precisely. Now that is achieved.

If you look at in the last 20 years, digital photoelasticity has now matured to find out total fringe order at every point in the domain, no problems. People have solved this, but the genesis has come from understanding Tardy's method of compensation and initially if you look at Professor Asundi, he has only found out, I have been saying when I use Tardy's method of compensation, I find out the fringe order only at the point of interest.

He came up with the digital photoelastic technique, he would find out the fringe order on all the points lying in that isoclinic. See, when you are looking at the isoclinic at the point of interest rather than only at the point of interest when you rotated the optical elements, he extends that the first method was to find out the fractional fringe order at all points lying in the same isoclinic. So from one point, they have gone to multiple points.

Now, they have gone to the complete domain. So for all that Tardy's method of compensation is useful this gives an indication that rotation of an optical element is akin to introducing the phase shift that we have proved. Because we have now proved there is a relationship between delta and 2 beta and we have been able to convincingly show by Jones calculus analysis what we have been suggesting as a procedure is indeed correct.

And the key point here is how to find out the sign. For finding out the sign, in conventional photoelasticity you have the luxury of looking at the fringe pattern and seeing whether higher fringe order moves and occupies or lower fringe order moves and occupies, but in digital photoelasticity you record only intrinsic information and this also shows, they had flexibility in finding out which optical arrangement is suitable.

Because I said in a conventional photoelasticity, you use circular polariscope bright field and dark field, plane polariscope dark field and there even the association of whether the real fast and slow axis of the quarter-wave plate is considered even that was not critical. As long as quarter-wave plates are kept crossed, your job of conventional photoelasticity is done, but once you come to a digital photoelasticity, all these issues have to be looked at very closely.

And in fact, if time permits I will also give one lecture on what is bird's eye view of what digital photoelasticity is? And you have some advantage by understanding Tardy's method of compensation in that light and it can also verify your digital photoelastic algorithm because if I want to find out the fractional fringe order and then if I find out from my conventional approach that should match with the digital algorithm.

Now, having determined the fringe order, what is the next step? The very important step for me to do the photoelastic analysis is I need to have calibration of photoelastic model materials. What I have to do that? Because I use only polymers and when I use polymers, the stress-fringe values vary with time and also from batch to batch. They do not remind same, so I need to find out at the time of experiment, what is the material-stress fringe value for the model material that I have select.

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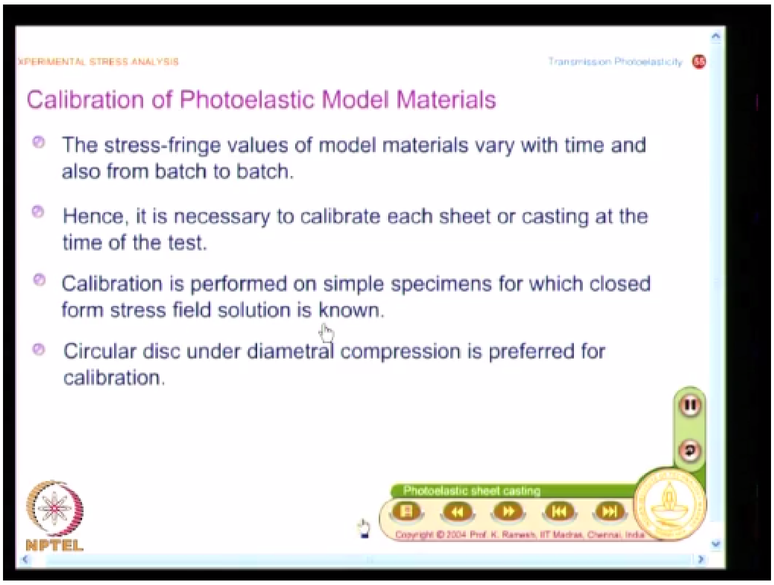

So, you need to calibrate each sheet or casting at the time of the test and what is the thumb rule? When I have to do this, my focus is on finding out f sigma. So, I need to do it on a model for which closed form stress field solution is known because I should conduct the experiment as accurately as possible. So that, I follow all the restrictions on loading and I find out what is the fringe order at the point of interest.

I used this information to compare it with my analytical solutions and find out the material stress fringe value. Remind you, the material stress fringe value has to be evaluated with sufficient accuracy. If you do not follow the step carefully, then comparison of any one of your quantitative information from a photoelastic analysis to the numerical or analytical approach would be (()) (48:29) such a very crucial step.

When it is a very crucial step, I should perform a very careful experiment to find out f sigma and for me to do that what is preferred? Circular disc under diametral compression is preferred for calibration. It is only said, it is preferred, you can also do it by other methods, but this gives the least amount of error and you know when you look at this, we will find out only one data point by a conventional approach.

Later on because I use a whole-field methodology, I will also extend, I will collect as much data as possible from the field and find out what is the f sigma value and that also opens up how people started employing computers in experimental mechanics. You will have a bird's eye view of that because you know experimental is wanted to get information quickly and they would be comfortable if I process one data point.

In fact, in fracture mechanics, Aravind was very intelligent to identify a one data point to find out K and sigma not x. So, in experimental mechanics, if you look at, you have on one hand, finding out the data from the field by sophisticated statistical methods; on the other extreme, people also find out by using only one data point or few data points for you to quickly assess what is happening.

You need to have both, you need to have quick solution as well as an elaborate methodology and in this case, we will look at both the quick solution as well as the elaborate methodology and this is just to brush your old memories. We have already seen this set of stress field equation. So, what you could do is, I have sigma x, sigma y and tau xy expressions are given where you have h is the thickness of the model, p is a diametral load applied.

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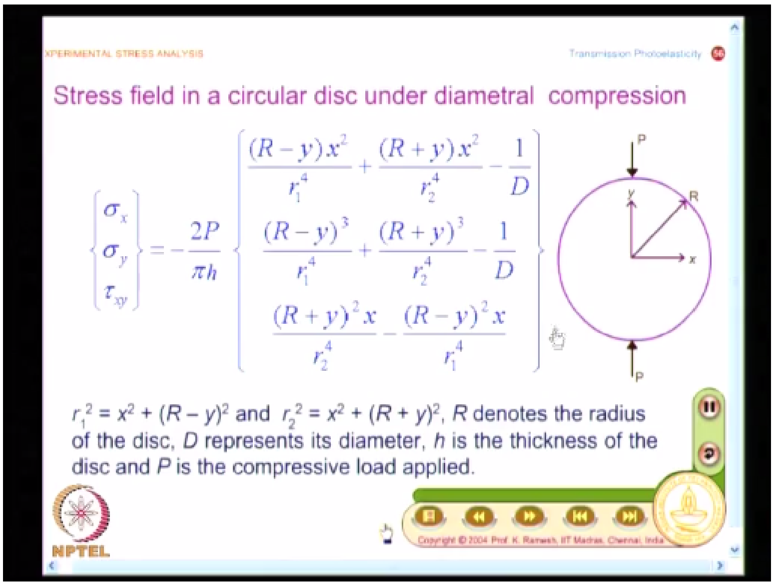

And once I have the expression for sigma x, sigma y and tau xy, it is possible for you to find out sigma 1 - sigma 2. In photoelasticity, we use only sigma 1 - sigma 2. So what I would appreciate is, since you have this expressions, go back, work I out in your rooms, find out the expression for sigma 1 - sigma 2, represented as a function of x, y. Then, later on we will do it for what happens at the center.

And we will also use it for whole-field determination for various values of x, y. So, the home exercise is to find out what is the principal-stress difference for a circular disc under diametric compression analytically. So in this class, what we had looked at was, we started with what are compensation techniques, we looked briefly Babinet-Soleil compensation, then we have looked at elaborately what is Tardy's method of compensation.

What are the steps involved, then we said these steps involved need to be verified by analytical development, we did the analysis by Jones calculus and established a relationship between rotation of the analyzer to the fractional retardation introduced and I pointed out Tardy's method of compensation as indirectly help development of digital photoelasticity.

Because people found akin to shifting a phase which is done external means in other interferometric techniques could be done in digital photoelasticity by appropriately rotating the optical element and people developed many different algorithms and towards the end, we saw that we need to find out the material-stress fringe value.

Because if I have wanted to do any quantitative analysis, I need to know the fringe order at the point of interest as well as material-stress fringe value and we have looked at how to find out fringe order at any arbitrary point and we will also look at how to find out the material-stress fringe value and is cautioned that this has to be determined with sufficient accuracy, all these details we would see in the next class.