Experimental Stress Analysis

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Lecture - 13
Jones Calculus

Let us continue our discussion on transmission photoelasticity. We have looked at from crystal optics point of view. We need to find out how photoelasticity as a technique developed, and we looked at that the model is temporarily birefringent when the loads are applied. So we were able to relate aspects of crystal optics and we combine stress and optics and understood what is stress optic law? The stress optic law famously gives sigma 1 - sigma 2.

And if you want to find out that I need to find out the fringe order and the material stress fringe value F sigma. So we have looked at in the last class sigma 1 - sigma 2 is n f sigma/h and I said you need to find out n for finding out n, you need to go for optical arrangement and then you will also have to find out for the given model material what is the material stress fringe value if I know n and if I know f sigma and if I know the thickness of the plane model.

It is possible for me to find out using mechanics of solids in plane shear stress as well as normal stress difference if I also know the principal stress direction at the point of interest. So this we had seen. Now the next aspect was that we need to go and find out the fringe order and we need to have an optical arrangement. We started with the plane polariscope. In a plane polariscope what is that that we saw?

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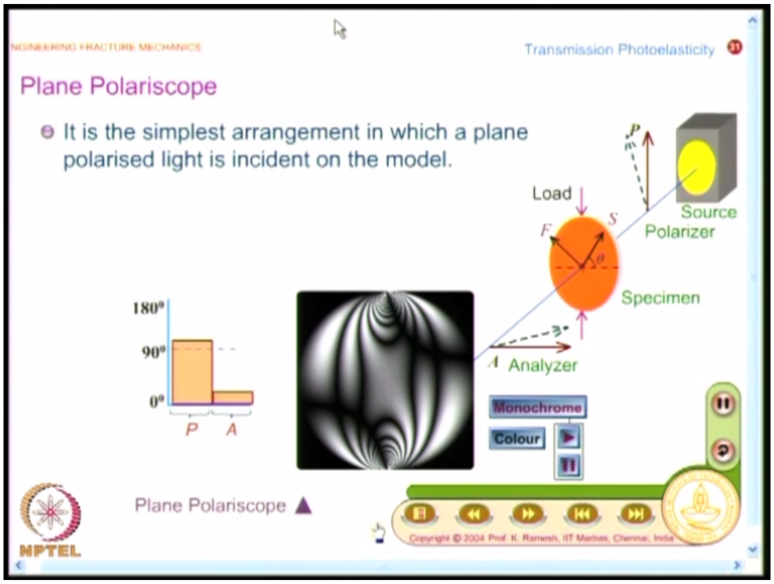

We had a light source, then we put a plane polarizer. Then you had a model which is loaded and you analyzed whatever the light that comes out of the model by an element similar to polarizer but the axis is different, because you do the analysis of exit light ellipse you call this as analyzer and what we saw in general in a plane polariscope you would see 2 fringe contours. One fringe contours move as the polarizer.

And analyzer crossed positions are moved and you have another contour which is stationary and we have also looked at. Suppose I increase the load, the contours that are stationary will move because they are effect of the load applied and we have also looked at from principals of solid mechanics in linear elasticity. The principal stress direction does not change only the magnitudes you have tan 2 theta = 2 tau xy/sigma x - sigma y.

When the loads are increased the individual magnitudes of shear stress and normal stress vary, but theta as such will remain constant and this we also understood this could be used advantageously even in a plane polariscope with monochromatic light source, how to distinguish a contour being isochromatic or isoclinic. You can rotate the polarizer analyzer crossed and if the contour moves then you call it as isoclinic.

Suppose I increase the load if the contours move then I find those are isochromatics. Subtle difference comes when you are dealing with 0th fringe order and what we did in the last class, we had analyzed what the light as it passes through the model how it comes out after the analyzer and we said intuitively because I see 2 fringe contours, the intensity equation should be a function of 2 parameters one is the relative retardation delta.

Another is principal stress orientation theta and what we did was we did a simple trigonometric analysis and what does it we did? We had this polarizer and after the polarizer you get plane polarized light which hits on the model. When it hits the model we look at the entry plane and exit plane. On the exit plane the model introduces a retardation which is indicative of the stresses developed at the point of interest.

Then whatever the light that comes out you analyze it using the analyzer and we found out what is the light transmitted by the analyzer which was a function of delta as well as theta and in this case, you had only simple optical element, a polarizer, model, and an analyzer. So trigonometric resolution was sufficient and convenient for you to do it and we also saw that we are going to look at circular polariscope. In a circular polariscope I need to have 2 more optical elements.

I need to have a polarizer, a quarter wave plate, model, another quarter wave plate, and an analyzer. So I have more number of optical elements. When I have more number of optical elements employing trigonometric resolution would be cumbersome. You will still get the result that is not an issue. You can always get the result by trigonometric resolution, but it is better to develop an appropriate mathematics which helps you to analyze when I have more optical elements in my experimental set up and this is known as Jones calculus and what is Jones calculus?

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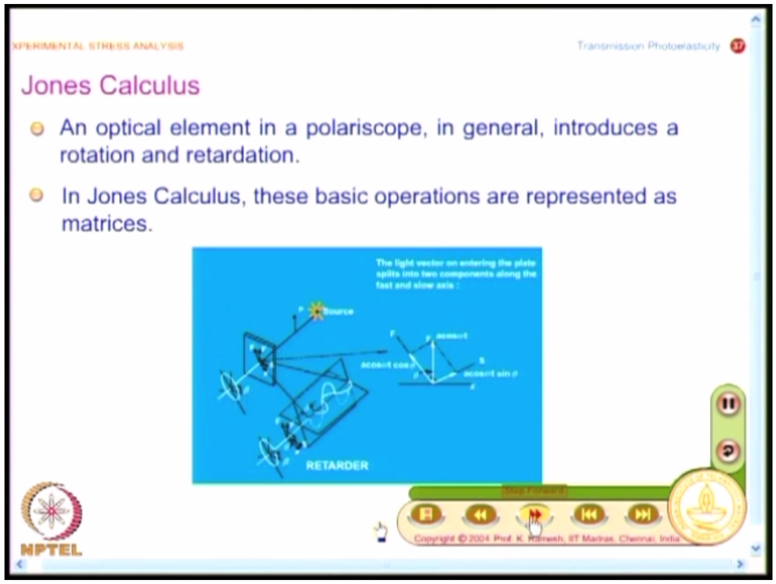

We will have to understand first of all that an optical element in a polariscope in general introduces a rotation and retardation and our interest is you know you want to have this as matrix operators with that the analysis becomes lot more simpler. So this is what we want to look at. The basic operations are represented as matrices and I said in one of the earlier classes when we introduced this.

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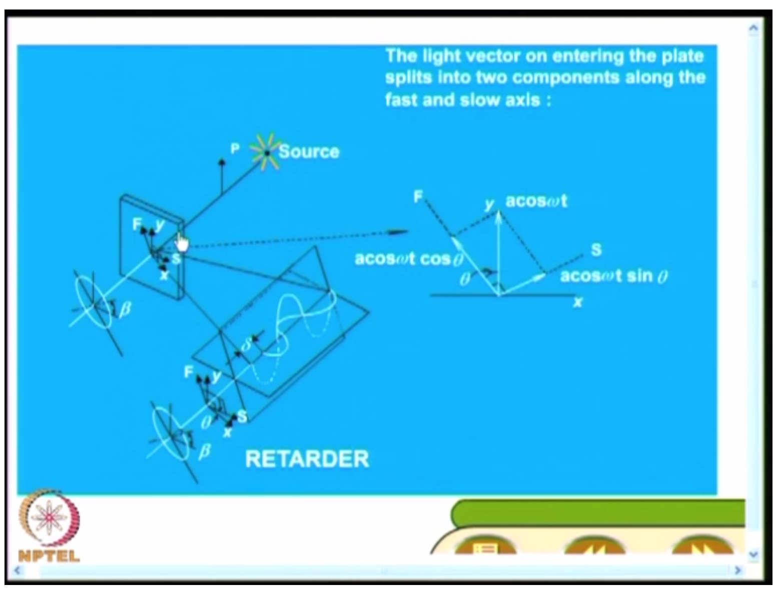

We are going to see this animation again and again in the course and we have actually analyzed a crystal plate and what we want to look at is when light impinges on this what happens within the crystal we have schematically shown with a larger sketch what happens on the plane on which it hits the light hits the front surface of the crystal plate and what happens within the crystal is shown here and if you look at what happens on the front surface.

I have the reference axis as X and Y and the crystal we have reference axis as slow and fast which is rotated at an angle theta. So what you have is whatever the light that is represented with respect to XY axis they have to be represented with respect to the slow and fast axis of the crystal plate. When you are looking at the model it is the slow and fast axis at the point of interest which coincides with the principal stress directions.

And whatever the retardation introduce within the model is shown here schematically and this retardation is a function of the stresses developed in an actual model. In a retardation plate, it will be a function of the thickness of the plate and also the refractive indices of the ordinary and extraordinary rays that is what we had seen. So what we need to do is if I want to represent this crystal plate or the model which is loaded behaves like a crystal we want to replace this by one matrix which represents the rotation, another matrix which represents the retardation introduced.

So, if I understand these 2 in matrix notation it is possible for me to give a mathematical representation of how a retarder can be represented. So the moment I do this if I have several optical elements, it just put those matrices in the order in which they are arranged multiply them and you will be able to comment on what is the intensity of light transmitted. So what we are looking now is we have to look at what the retarder as done the function. Now we are looking it.

We have looked at by impinging a plane polarized light that is a very specific situation. In a generic scenario what you are going to look at is you are going to have light vector represented with respect to X and Y axis and the model as such has reference axis as slow and fast which is oriented in general at an angle theta so I do a rotation when it enters the model within the model it acquires the retardation so these 2 operations I would like to first get the matrix representation. If I do that it will make my life lot more simpler. So that is the goal. We will look it.

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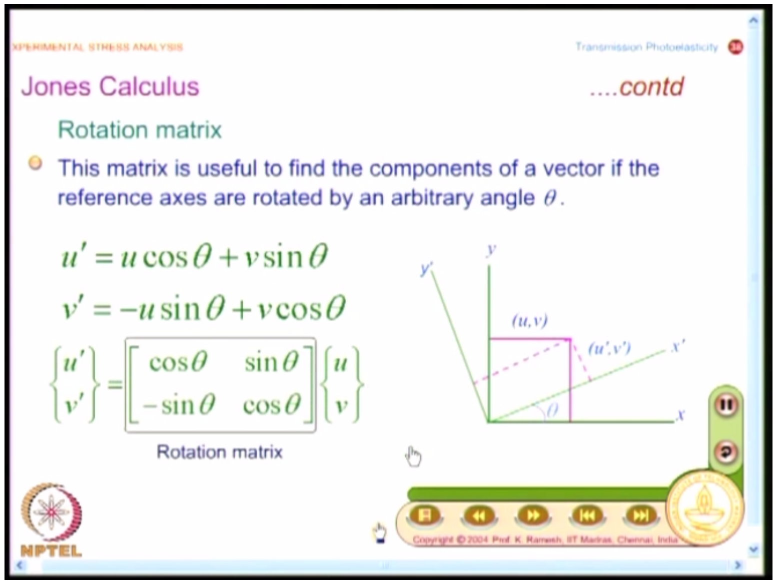

So what I have is the rotation matrix and rotation matrix you would have done in many of your other engineering courses whenever you come across transformation one of the simplest transformation is rotation. To find the components of a vector the reference axes are rotated by an angle theta and this is fairly simple and straight forward. It is only remembering what you have done in some of your earlier courses. So what I have here is.

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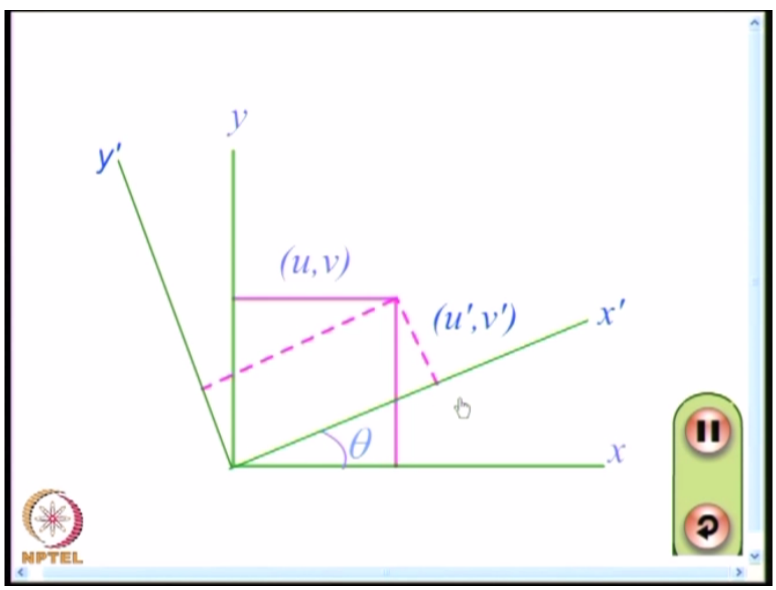

I have the reference axes X and Y. I have a point, I have a point labeled as u, v so I can find out u, I can find out v and what I want is I have another coordinate system at angle theta. I have this as x prime axis and I have the y prime axis which is perpendicular to this. So the transformation is simply obtained by finding out the expression for u prime and v prime from simple geometry that is all you need.

This you have done in your earlier courses. It is not something new. I think you can brush up your old memories and write on your own whether you can write the rotation matrix or in other words find out what is u prime, what is v prime in terms of u, v, and theta and you need to simply apply geometry here and then get the relevant components. It is not difficult. If you start writing all this in the class, then revising your notes become lot more simple.

It is only remembering what is that you have done in your earlier courses and fairly straight forward so I am going to have u prime as u cos theta = v sin theta and v prime as - u sin theta + v cos theta and this you would have done in your earlier course. It is only recalling what you have learnt and this could be represented as a matrix operator. What we are going to do is? I know u, v, I want to find out u prime v prime and this can be obtained by premultiplying u, v by a matrix cos theta - sin theta cos theta.

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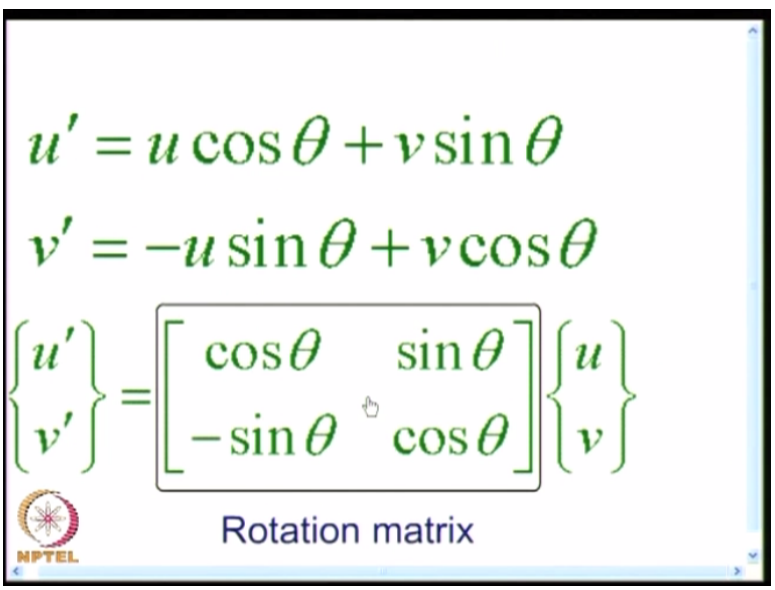

So I have this rotation matrix. So what I have is when the light enters the retarder I would put this rotation matrix to find out the components of light that passes through the slow and fast axes so I will have a rotation matrix and you have to very careful about what is the notation that you are using theta how it is defined. See in our development we define theta as the orientation of the slow axis with respect to the horizontal.

One can also develop with respect to fast axis, one can also develop with respect to slow axis. We will consistently find out theta as the orientation of the slow axis with the horizontal we need to have a sort of a convention. We use this type of convection for all our mathematical development. Now you have the rotation matrix. We will also try to find out how to represent the retardation introduced within the model by a retardation matrix.

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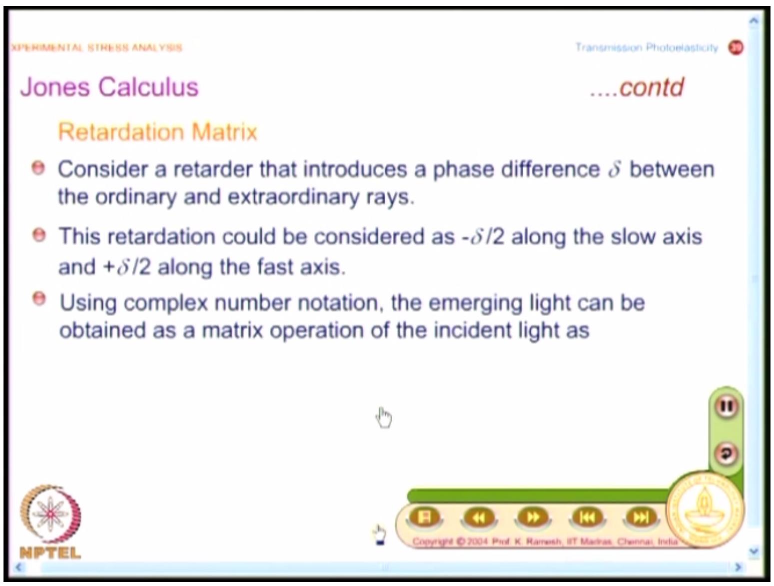

And what we want to do we will consider a retarder that introduces a phase difference delta between the ordinary and extraordinary rays and how we introduce this delta this can be written down in many different ways and I said that retardation could be considered as - delta/2 along the slow axis and + delta/2 along the fast axis. So the idea here is we would like to have a nice matrix representation and we will follow some convention to facilitate.

How we analyzed different optical arrangement if we develop some equations to start with the same set of equations could be used if you follow the same convention that is the idea behind it and we would like to have slow axis as oriented at angle theta and what we will do is we will use complex number notation and we will represent the emerging light. This makes our life lot more simpler and that is what we will do and when I do this I can represent the retardation introduced within the model comfortably as like this.

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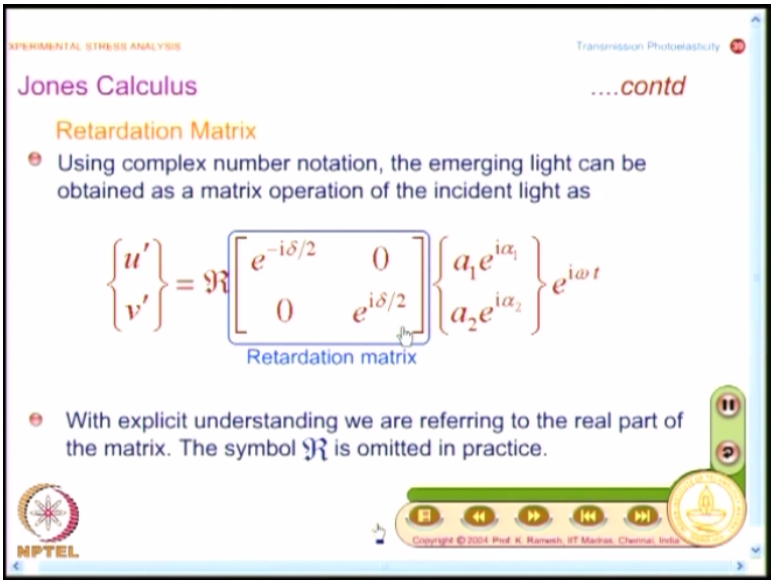

So what I have here is I have u prime, v prime which is = we take the real part of e power - I delta/2 0, 0 and e power I delta/2 and why I have e power - I delta/2 here because I understand that this is a slow axis and this is the fast axis. So because I write my retardation matrix with the first element as e power - I delta/2 I should always represent theta referring to slow axis with respect to the horizontal and this is the general light that is impinging on the retarder.

This what happens inside the retarder within the model or within the crystal plate you have ordinary and extraordinary beams acquire a phase difference of delta which is represented as one half split to the slow axis another half split to the fast axis and what we do in all our mathematical development is we will remove this all we will not write that explicitly for convenience. We understand that we deal only with the real part and we will simply have the retardation matrix written as such.

So what we have now seen is we understood a retarder in general introduces a rotation and retardation and rotation matrix you have seen in many of your earlier courses. it is nothing new when you are having a vector you want to represent the vector with respect to another axis which is rotated you use the rotation matrix. The same rotation matrix comes here also and within the retarder because of the if it is the model it is because of the loads that is applied.

The waves acquire a retardation and if it is a crystal plate you may have a quarterwave plate, you may have a half wave plate, or you may have a full wave plate to fill in your optical arrangement so it will give a particular value of retardation for a given wavelength. You understand that we do all this development for monochromatic light source. We have already seen retardation is a function of wavelength and our mathematics will become simpler.

If I confine my attention first monochromatic light source later on you can develop if time permits, we will also develop see what happens when I have a multiple wavelength. Now what we will look at is these 2 are elements only. Now we will have to understand what happens in a retarder how do I mathematically represent a retarder that is what we will see now.

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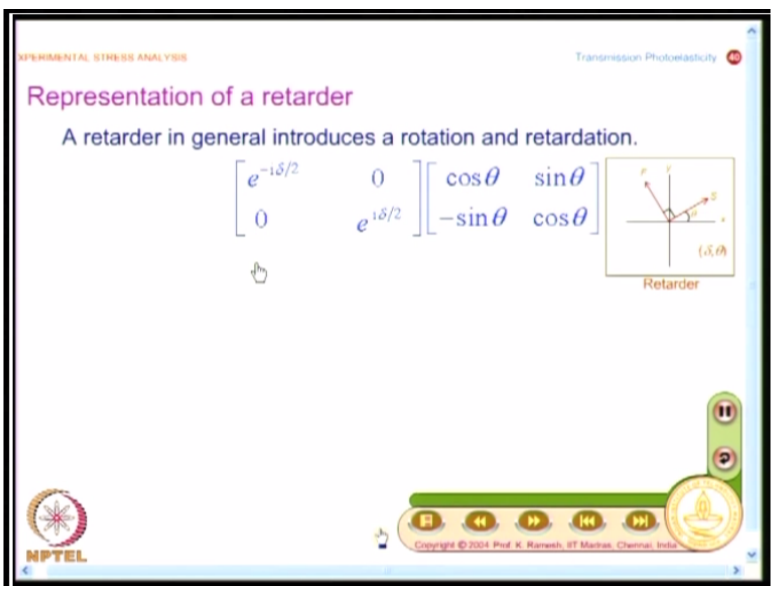

So what we are going to do is we are going to have representation of a retarder and here I have given.

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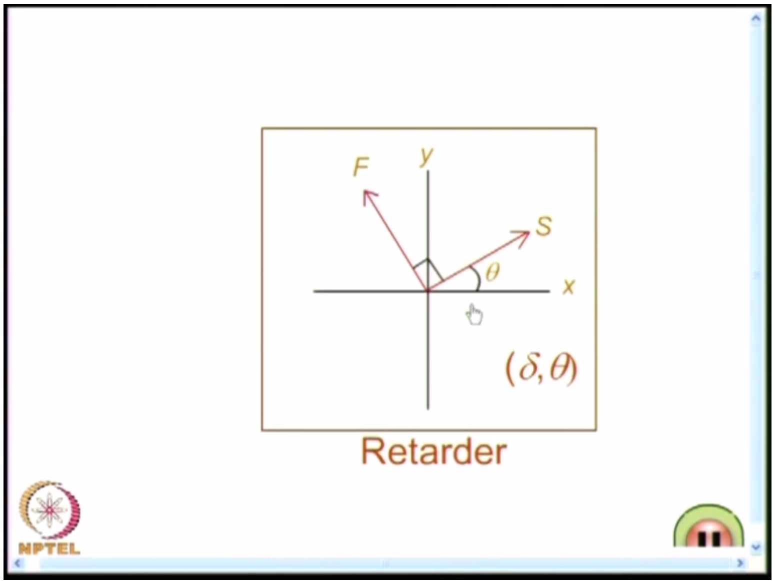

For clarity when I take a retarder I will recognize a retarder having 2 axes I will label one of them as slow and one of them as fast and I will find out the angle theta of the slow axis with respect to the horizontal. My reference axis is horizontal x and y and I will always represent the retarder by 2 parameters what is the retardation introduced and theta. So delta is the retardation introduced by the retarder and theta is the orientation of the slow axis with respect to the horizontal.

This you should never forget because when we develop mathematics we need to follow some convention. We are only representing a convention here and this convention we will consistently follow for all our analysis of other optical arrangements as well and what we will have to know I have this as the reference axis, the model has slow and fast as the reference axis, but I would finally like to have reference axes are only x and y and within the model.

I have a retardation introduced but this retardation also I will introduce only along the slow and fast axes. These are all subtle points. See if you understand this now when we go and develop compensation techniques why we do the steps involved in a compensation technique you will immediately understand carefully listen to what I say. When you take a retarder I have a reference axis slow and fast.

We introduce - delta/2 along the slow axis, + delta/2 along the fast axis and it comes out as a light ellipse. When it comes out of the light ellipse I will still want to have this represented with respected to the x and y direction. So retardation within the model is introduced only along the slow and fast axis only then I can do that. That is what the retardation matrix explicitly gives you.

So whatever the matrix operation I do I must rotate it back to reference axis so if I want to mathematically represent a retarder I need to have 3 matrices to represent a retarder with xy as the reference is the idea clear? I will repeat again and you will also do the mathematics behind it then it will become crystal clear that this what will fully represent the action of a retarder if you have xy axis as the reference because I do not want to keep changing my reference axis from one element to another element because element has a reference axis.

That reference axis we said one you can label it as a slow axis, one you can label it as a fast axis. though you do it arbitrarily when you are actually making a polariscope you can match your mathematical development with the polariscope that you have that is called calibration that is required in digital photoelasticity that may not be required in conventional photoelasticity because fast.

And slow axes do not play a major role in conventional photoelasticity but when you learnt the photoelasticity subject in 2010 you will have to know aspects of digital photoelasticity as well. so it is better that you learn the knowledge which will fit into digital photoelasticity comfortably so what we are now looking at is I have a retarder, I have a reference axis x, y. The moment light hits the retarder I will have a rotation matrix.

Then I will have a retardation matrix, then a reveres rotation matrix so that whatever the exit light ellipse is referred back to x and y reference so this is what I am going to do. So what I am going to have is I am going to have a rotation matrix so we will write one after the other also look at the animation you will have this appearing in a particular sequence that is how light gets modified in a retarder when it hits the model you have this rotation matrix cos theta sin theta - sin theta cos theta.

Then within the model it acquires a retardation, and this retardation matrix and rotational matrix are interrelated because I say theta refers to slow axis because it refers to slow axis I put retardation matrix in this fashion. I put e power - I delta/2. If I had referred theta with respect to fast axis this would change to what this would become e power I delta/2 because I refer theta with respect to slow axis I put this as e power - I delta/2.

Then fast axis will give e power I delta/2 and this would be referred with respect to what I have already rotated with respect to the reference axis of the retarder. I introduced retardation only along those reference axis, so this definition will have only the crystal plate or model reference axis, but we want to have if I have multiple optical elements it is better each optical element output is referred back to the xy axis then I can repeat the same thing for every optical element. Then you have a simple matrix manipulation that will tell you, what is the exit light?

In trigonometric resolution you have to find out it enters the model, it enters the retardation within the model and then rotate back all those steps you have to do it individually. Now you do not have to do that. If I have once I multiply all these matrices and keep it as a ready reckoner, you just plug in the value of delta and theta into this provided you define delta and theta correctly. Theta refers to slow axis with respect to the horizontal is the convention we follow.

You can follow any convention, but the convention we follow is that. So now I will also have a reverse rotation matrix.

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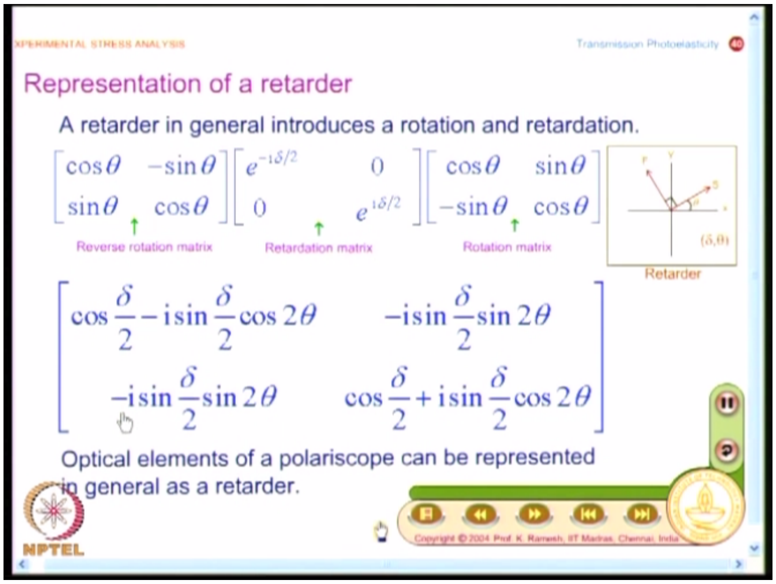

So I have a rotation matrix as light hits the retarder within the retarder it introduces as a retardation and whatever the modification that has been done by the model I want to refer it back to the original reference axis as x, y so I put a reverse rotation matrix, but what I want is I want you to do the matrix multiplication right in the class and when you do this you expand this e power - I delta/2 as cos delta/2 - I sin delta/2 and do the simplification.

So that is what I have been saying in photoelasticity you need to have matrix manipulation you also need to know trigonometric identities. If you know these 2 development of mathematical aspect of photoelasticity is lot more simpler and once you do this matrix multiplication and have this result as a ready reckoner then finding out what happens in a quarterwave plate or what happens in a half wave plate or what happens in an actual model it becomes very simple for you to write.

In fact, I want you to develop it right in the class so that the understanding becomes complete and when you refer back to the notes you do not have to feel that this is something different you have already derived it in the class and it is fairly simple and straight forward. All of you know how to do matrix multiplication and how to simplify complex quantities take a little bit of your time and then do this.

I think some of you are half way through and in matrix multiplication even the sequence matters. If I have put the matrices in a particular order multiply them in the same fashion and when I do the final simplification, the final matrix takes a form like this. I think some of you can verify. So I have this as cos delta/2 - I sin delta/2 cos 2 theta and I have this as cos delta/2 + I sin delta/2 cos 2 theta.

And if you look at closely there is only one small sign change between the diagonal elements and I have this as - I sin delta/2 sin 2 theta and this is - I sin delta/2 sin 2 theta. So this represents completely what the retarder influences the light that impinges on it. It accommodates what is the orientation theta. It also accommodates what is the retardation introduced by the retarder. So the representation of retarder clearly shows that I can represent it as a matrix like this and if I know this matrix and it is also very easy to remember they were not very complicated terms.

I have cos delta/2 cos delta/2 here and there is only a sign change I sin delta/2 cos 2 theta instead of cos 2 theta I have this as I sin delta/2 sin 2 theta. So it is also very easy to remember and even if you forget you can always go back and write this basic multiplication sequence and get the result even at the examination that is not difficult but what you will have to keep in mind here is we have taken theta as the slow axis with respect to the horizontal.

So now what we will do is we will look at what are all the optical elements that we may think off in a polariscope we may have a polarizer we will have a quarterwave plate so I need to know how do I represent a polarizer, how do I represent a quarterwave plate and a model will be represented as a retarder because model I do not know what is a delta introduces and what is the theta introduces.

We do not know we want to find out as part of the experiment the value of theta as well as value of delta. Delta you indirectly find out by finding what out the fringe order and theta you find out from the isoclinic angle. So model can always be represented by a matrix like this complex matrix like this and this is for a retarder. When I say retarder you have to keep in mind I have this axis in an actual model corresponds to principal stress direction.

And I implicitly look at only a single plane in practice you have a finite thickness because there is 2 dimensional model surface are free. You also make an assumption that principal stress direction does not change within the model thickness. Suppose I take a 3 dimensional model and take a slice out of it in general the principal stress direction will change from plane to plane you should never forget that.

So that is why 2 dimensional photoelasticity is fairly simple I could correlate whatever the result I get from optics to physical parameters comfortably. The moment I got to 3 dimensional photoelasticity what do I do? Mathematics becomes very complex. In order to simplify the mathematics, I said there is a stress freezing and slicing process I have a 3 dimensional model I lock in all the stresses I take out the slice.

When I take out the slice I will say that I will use a very thin slice bring in engineering approximation principal stress direction remains constant so this is how the approximations come. See you should know the procedure you should also know what are the approximations involved. You should not conclude you have taken a model of 6 mm thickness and put a diametrical disc compression you have analyzed it.

We have analyzed it because it is a 2 dimensional model and thickness is also sufficiently small and because on the way I apply the stresses principal stress direction remain constant is a reasonable approximation over the thickness. Only then interpretation is possibly because what is the interpretation that we finally come to we essentially find out what is the slow and fast axes and this axes should remain constant over the thickness of the specimen.

So, thinner the model it is better, but if you have a thin model when I apply compression it will buckle so you need to have thickness so that buckling is avoided so these are all practical considerations. So you have to understand the practical consideration and also appreciate the approximations involved and what we have seen in a retarder is retarder introduces a rotation and within the retarder.

You are adding or subtracting retardation only along the fast and slow axes that is only along the principal stress direction remember this then we do a reverse rotation. First we will take a very simple optical element like a polarizer where in there is no great mathematics involve, it is only a representation for convenience.

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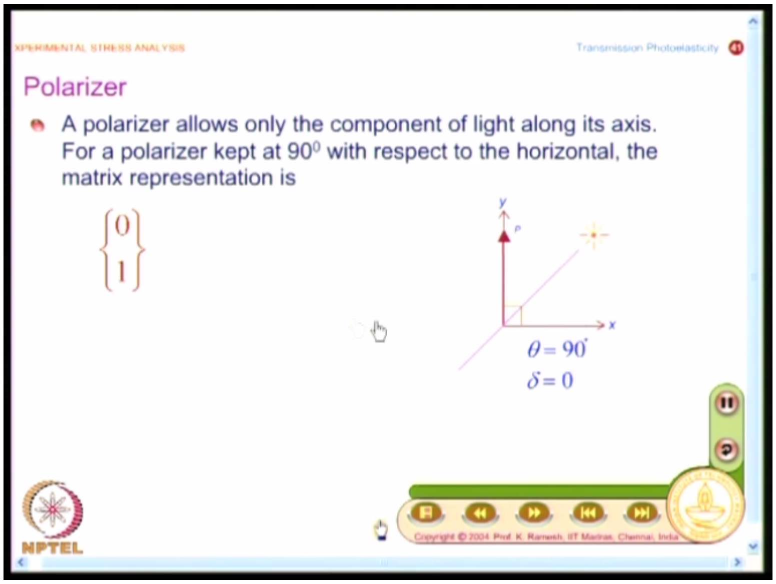

And what I have here is I have a simple polarizer.

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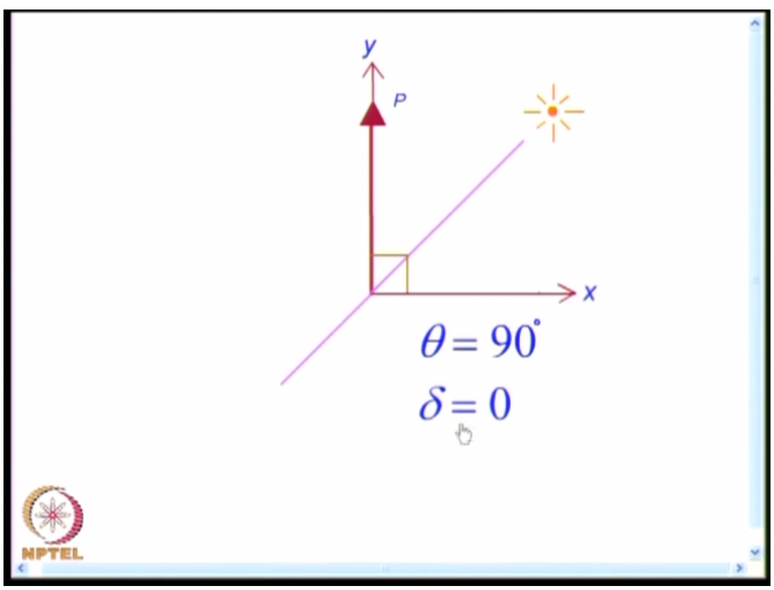

I can say that theta = 90 degrees and delta = 0. I do not even have to go for a matrix representation. I can simply represent this as a vector. I send only a component of light along the y axis is what my vector should say and that is easily represented by 0, 1 a simple vector like this is sufficient to represent polarizer nothing more is required only when I come to a quarterwave plate I need to have a look at how it is oriented and it is a retarder, but the retardation is known. A quarterwave plate gives a retardation of pi/2 radiance.

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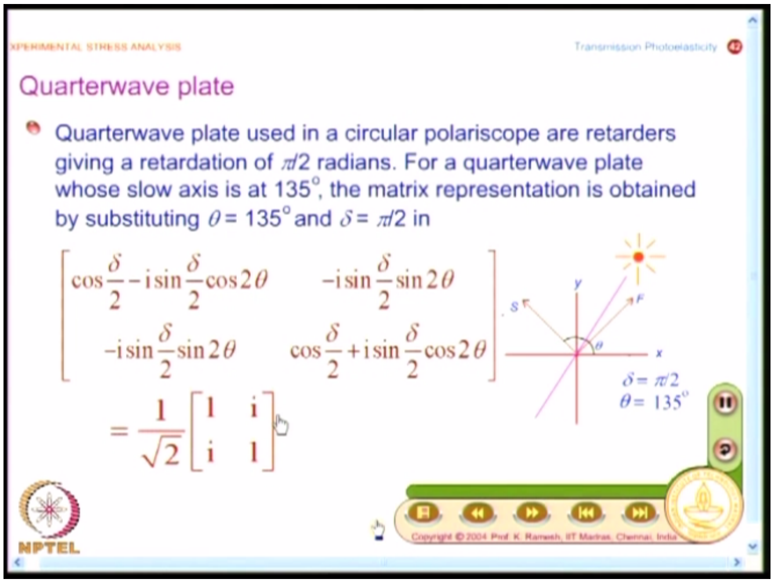

So that is what we will have a look at it and then we have this as quarterwave plate and quaterwave plate used in a circular polariscope are retarders giving a retardation of pi/2 radians. So we know the retardation and we keep the quarterwave plate at appropriate angles. One such angle is 135 degrees. The other angle is you can also keep it at 45 degrees and look at how the retarder is represented.

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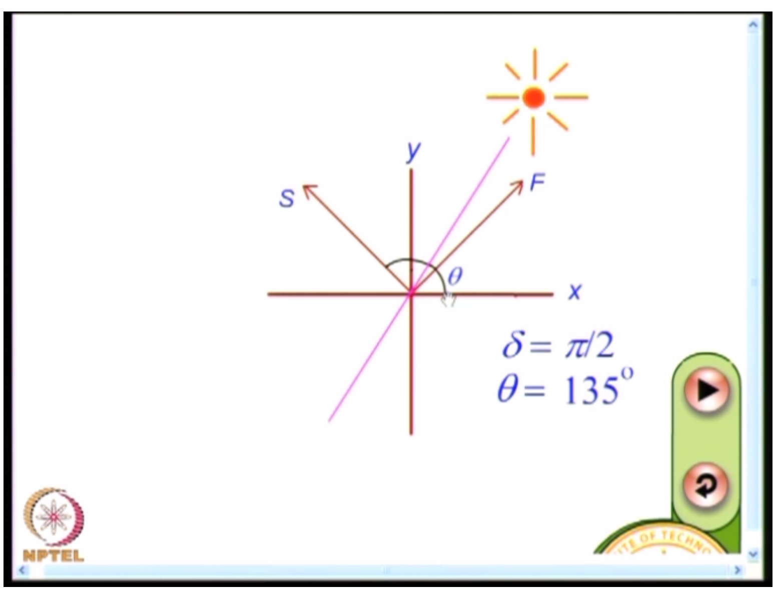

So what I have here is I have the reference axis as x and y. I have a quarterwave plate with slow axis adn when I show theta I show the reference from the horizontal to the slow axis. Theta is the angle of the slow axis and fast axis represented here. So when I have a quarterwave plate oriented at 135 degrees I write delta = pi/2, theta = 135 degrees. On the other hand, if I put theta = 45 I would get a different answer.

Unless I keep the quarterwave plate slow axis at 45 degrees I cannot put theta = 45. So you have to represent theta very carefully. Theta is always in our representation orientation of the slow axis with respect to the horizontal. Once I know delta and theta it is child's play. You have an expression what a retarder does, plug in the values of theta and delta and because these values are very nice even your sin and cosine terms reduce to known simple quantities.

It is very simple make an attempt and try to find out what the values are. You know the retarder is cos delta/2 - I sin delta/2 cos 2 theta. Then I have - I sin delta/2 sin 2 theta - I sin delta/2 sin 2 theta similar terms here this is symmetric actually. Cos delta/2 + I sin delta/2 cos 2 theta. This is the representation of a retarder. Now we want to find out for a quarterwave plate which is oriented at 135 degrees. The angle is also given.

Why do we have it as 135 degrees. We want to have a circular leap or a rays light because we want to have a circularly polarized light we keep it at 135 and 45 in conventional photoelasticity. In conventional photoelasticity these are the 2 orientation of the quarterwave plate popularly used. So when I substitute delta = pi/2 theta = 135 you will get a very simple expression. you will get a very simple expression and it really makes your life simple.

When you have to go and analyze a circular polariscope and I wanted to right now plug in the values of delta and theta and simply it and it will definitely make your life simple when you want to analyze circular polariscope in fact I would recommend all of you to try out trigonometric resolution for a circular polariscope then analyze the same problem by Jones calculus then you will find out how Jones calculus is very simple until then you will not find out.

Because you have to do matrix multiplication you may find I have to do matrix multiplication which I have studied long time back I am not able to do comfortably now, but if you do that it becomes lot more simpler to analyze any type of optical elements kept in a polariscope at any orientation you can comfortably analyze and in fact Jones calculus paved way for the development digital photoelasticity also.

You could analyze any given optical arrangement very quickly by employing Jones calculus. I think you must have done the simplification. Pleas verify the result. It is simply 1/root 2 1, i, i and 1. And if I change to theta = 45 you will have a very small change in this representation as simple as that. So now what we will do is we will go and analyze what happens in a plane polariscope.

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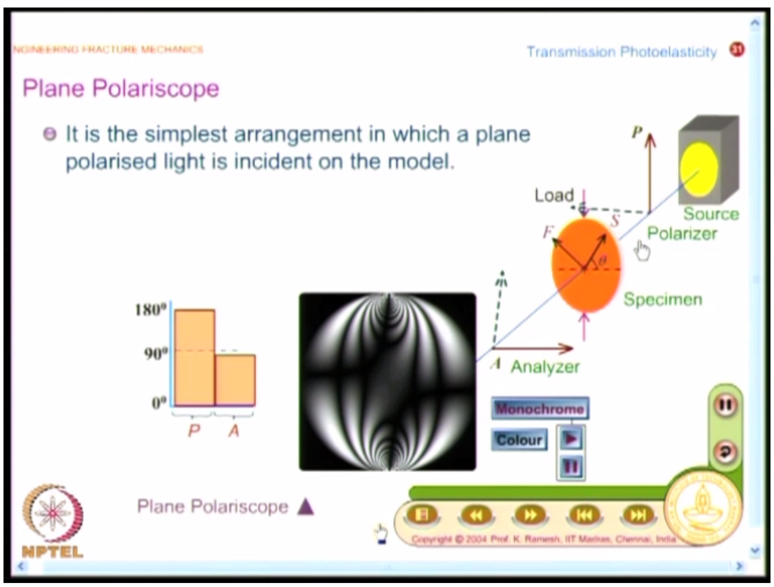

So what we are going to do is we have looked at plane polariscope where I have a polarizer, I have a model and I have an analyzer. So what I need to do, I need to put polarizer in its mathematic representation then just replace only the model by the matrix that you all saw that is all you have to do. I do not even have to worry about the analyzer. If I look at only the horizontal component of that I know what is the light transmitted by analyzer it is as simple as that it is too simple for a plane polariscope.

Trigonometric resolution is also needed because that gives you understanding what happens in each optical element that is needed. We did a logical explanation. We did a trigonometric resolution. We will also analyze the plane polariscope by Jones calculus. The reason is you understand all the 3 methodologies. A logical explanation may not be possible when I have many optical elements, many optical elements will complicate your logical explanation so we will see what happens in a plane polariscope.

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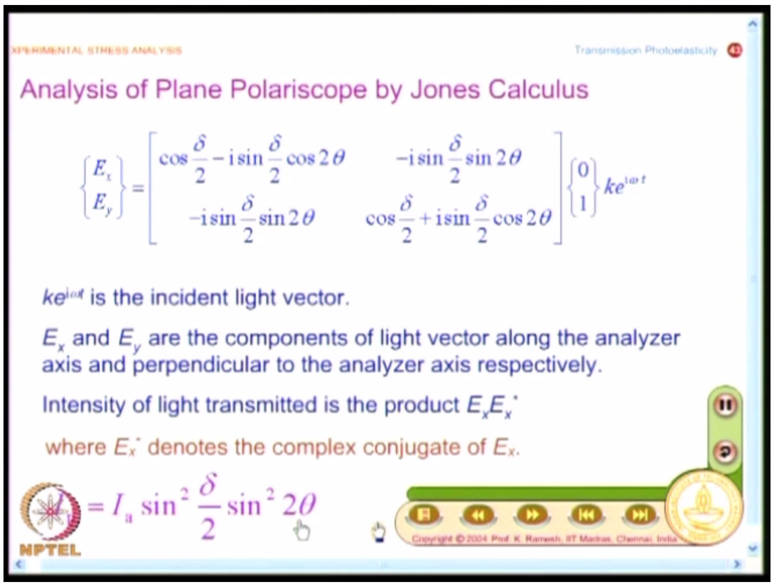

We are going to say analysis of plane polariscope by Jones Calculus and I will repeat the animation. You will see here my interest is to find out the light vector Ex and Ey and I would also write this starting from the right first I will say this is the light impinging on the model so I will put K e power I omega T first then I will look at what is the first optical element represent the optical element, first optical element here is polarizer.

And we have already seen how to put polarizer. I will simply put 0 and 1 and what is the next element you have only the model and you have already written down how to represent a retarder. In fact, you can put the rotation matrix, retardation matrix and reverse rotation matrix and then finally arrive at this, but instead if you directly write it saves your time and it is very simple to remember.

It is not difficult which will be in a position to do and this would give me what is the component of light along the x axis and y axis and what I need to find out I need to find out the amplitude, because I said all the sensing elements only record amplitudes so that is what we will see now. So what I have here is k e power I omega d is the incident light vector. Ex and Ey are components of light vector along the analyzer axis and perpendicular to the analyzer axis respectively.

And intensity of light transmitted is Ex * Ex * where Ex* is the complex conjugate and once I know Ex I can easily find out Ex* and this matrix multiplication is very simple and I get the expression for light simply as Ia sin square delta/2, sin square 2 theta which is same as what you have got by trigonometric resolution absolutely no change only the mathematical procedure is different and here both trigonometric resolution.

And Jones calculus require the similar effort there is no change in the effort involved effort is almost similar. You really do not see the advantage of using Jones calculus for a plane polariscope, but definitely improves your understanding how to apply Jones calculus in analyzing optical elements of a polariscope. Now what we will do is we will look at what is a circular polariscope. I have the optical diagram here.

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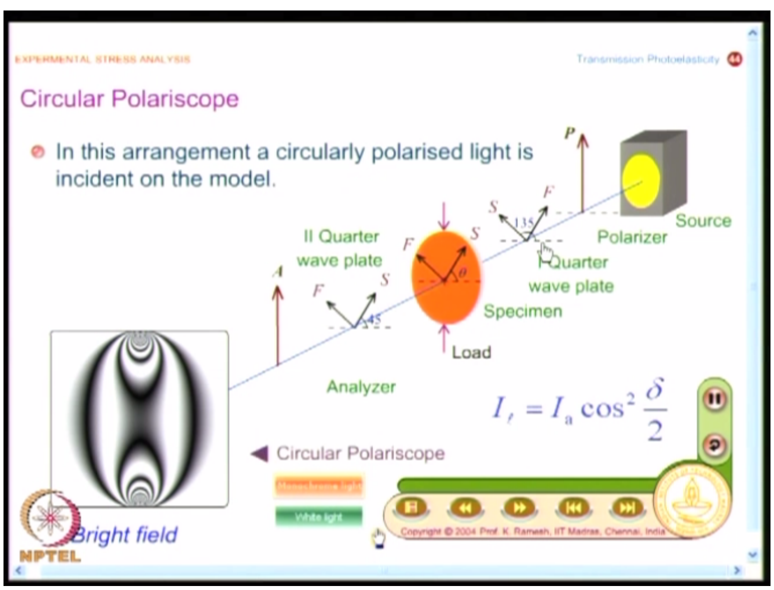

And let us what are all the elements that you have. I have the light source after the light source I put a polarizer. Then I put a quarterwave plate which is oriented at 135 degrees. Then I put a model which is loaded. After the model I put a second quarterwave plate adn this quarterwave plate is kept at 45 degrees then I put a analyzer. so in a circular polariscope I have 2 elements before the model and 2 elements after the model and if you watch it very carefully.

I have the fast adn slow axes and the slow and fast axis of the second quarterwave plate are crossed what I have in first quarterwave plate and what I have in second quarterwave plate are aligned such slow axis of second quartewave plate is perpendicular to the slow axis of the first quarterwave plate and I have the polarizer and what I see here is I keep the analyzer parallel to polarizer.

Then I have the background as bright. I will repeat the animation you will see that when the analyzer is horizontal in addition to quarterwave plates being crossed polarizer and analyzer also crossed. You will see the background as dark. All that we will develop equations later. First make the observation and I also bring in the expression which you can derive we will also derive and what you find here is the intensity of light transmitted is a function of delta alone.

It is no longer a function of theta so what I see in the screen I see only one set of fringe contours. So first knowledge in using a circular polariscope is instead of 2 fringe contours I see only one set of fringe contours and I repeat the animation you just have a look at it. So what I have here is I have the polarizer, first quarterwave plate, model, second quarterwave plane no changes done.

Instead of analyzer vertical I have kept analyzer horizontal. I see this as background as dark and I see fringes also in a particular fashion. See if you recall in a plane polariscope we had only the dark field we looked at the bright field, bright field was not giving any information and here I have the light intensity transmitted as Ia sin square delta/2 and when I keep it vertical what I have I have this as bright field and this equation also has changed.

Now you know the intensity of light transmitted so what I would suggest is do the analysis of light that passes through the circular polariscope purely by trigonometric resolution look at what happens in the polarizer, what happens in the first quaterwave plate, what happens in the model, what happens in the second quarterwave plate, then how to you look at in the analyzer and then which component you are looking at both horizontal and vertical component give you physical information.

So take this as exercise complete this exercise and come for the next class. Do it only by trigonometric resolution it will run into several pages and we will do the same thing with Jones calculus you will find how elegant and simple it is. So from now onwards we will use only Jones calculus for all our development, but please do the analysis by trigonometric resolution for a circular polariscope.

I have a polarizer, I have the first quarterwave plate, I have the model, I have the second quarterwave plate, and I have the analyzer and you can also have a look at this what happens in a white light when I put it in white light I see colored fringes beautifully and this flip between the dark field and white field and watch what happens you know whatever these 2 are complementary fringe patterns look at that the whatever the gaps that you see in one field it will be filled by the other field just look at the animation.

If you watch from that prospective you will find that there is a 90 degree shift so that is what we will see. In this class what we have looked at was initially we looked at what we do in a plane polariscope, what kind of fringe patterns you get and we recall that we did a trigonometric resolution to find out what happens to the light that passes through the model and how to perceive it at the analyzer we found that it is a function of delta as well as theta.

And I mention when you have multiple optical elements doing trigonometric resolution is lot more cumbersome and it is better that we develop a new mathematics called Jones calculus which will simply your analysis when you have multiple optical elements. So in Jones calculus you try to represent the modification introduced by each of the optical elements as a set of matrix operators.

So we found that a separate matrix for rotation, a separate matrix for retardation then we represented how to mathematically identify the role of a retarder. We had a mathematical representation of a retarder and I said that in this the angle theta refers to slow axis of the retarder that is how the final matrix was obtained and once you know the retardation matrix any set of optical elements in a polariscope can be comfortably analyzed.

If you know delta and theta finally we looked at what are the elements in a circular polariscope and I suggested that you try to do a trigonometric resolution and you I will find that it becomes quite cumbersome for the number of optical elements that you have. On the other hand, when we do by Jones calculus the mathematics become lot more simpler which we will see in the next class.