

Experimental Stress Analysis
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Lecture - 11
Retardation Plate, Stress Optic Law

The main focus of discussion in the last class was when the relative retardation is changed you are able to get light of different ellipticity, azimuth and also handedness. So that gives you a hope by measuring the characteristics of the light it is possible to find out what is delta and in fact there is a whole body of optic literature what you call as ellipsometry which tries to find out azimuth which finds out the handedness and also the ellipticity.

Fortunately, in photoelasticity particularly 2 dimensional photoelasticity we do not have to go to that much detail we can simply use a plane polariscope or a circular polariscope and analysis the exit light characteristics. So it is a lot more simpler only when we go in for 3-dimensional photoelastic analysis we invoke certain aspects of ellipsometry in more detail and what we looked at next was for all our photoelastic analysis it is desirable that we understand what is a light impinges on the model.

And we said that the simplest light that you can impinge on the model is plane polarized light and for getting a plane polarized light what we did we said that we are using a sheet polarizer and a sheet polarizer is like this that is what we saw in the last few classes earlier you have a sheet polarizer which is very convenient for you to rotate and it is also easy to have a larger field and it is desirable how this acts like a filter.

Because I said when you have a natural light this acts like a filter and you get only plane polarized light that comes after this and for you to understand this you should know some aspects of crystal optics only then you will be able to appreciate even the physics behind a sheet polarizer.

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EXPERIMENTAL STRESS ANALYSIS

Transmission Photoelasticity

Polarizers in Sheet Form

- Most useful polarizers, which come in sheet form in large sizes, are thin, light weight, cost less compared to prism type polarizers and can be easily rotated, employ dichroism.
- A dichroic material is one which absorbs light polarised in one direction more strongly than light polarised at right angles to that direction.

Incident natural light

Horizontal component fully absorbed

Vertical vibrator partially absorb

Linearly polarised transmitted light

Linearly Polarised light transmitted by a dichroic crystal

Back to main

And that is what we have looked at in the last class polarizers in sheet form and what we learnt was within the polarizer sheet the horizontal component is absorbed by the polarizer and for illustration this is shown which large thickness in reality this is very thin and this phenomenon is called dichroism. So a dichroic material is one which absorbs light polarized in one direction more strongly than light polarized at right angle to that direction.

So what do you find is from the natural light it allows the vertical component, vertical vibration is only partially absorbed and you have this horizontal component is fully absorbed. The net result is from the natural light source you get only a plane polarized light. So a linear polarized light is transmitted by a dichroic crystal.

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EXPERIMENTAL STRESS ANALYSIS

Transmission Photoelasticity

Polarizers in Sheet Form

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- Various materials are dichroic, either in their natural state or in a stretched conditions. Tourmaline is one example of a crystal being dichroic in its natural state.
- The most common dichroic polarizers are made of stretched polyvinyl alcohol sheets treated with absorbing dyes or polymeric iodine.
 - ★ Stretching of the sheet, orients the molecules parallel to the direction of strains and renders the material doubly refracting.
 - ★ The material becomes dichroic when stained with iodine.

Back to main

And if you look at the nature you know you also have certain materials which are dichroic.

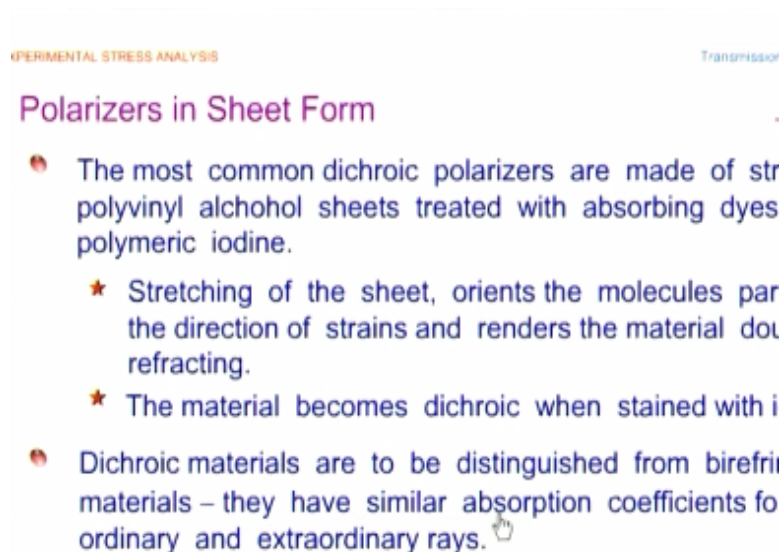
So what you find is you have Tourmaline is one example of a crystal being dichroic in its natural state like we have seen natural crystals which is birefringent. And you also have Tourmaline is one example of a crystal being dichroic in its natural state and what we have seen while making the polarized sheets was you had polyvinyl alcohol stretched.

So the other possibility is the most common dichroic polarizers are made of stretched polyvinyl alcohol. In general, they are stretched polyvinyl alcohol sheets treated with absorbing dyes or polymeric iodine. So what you have is various materials are dichroic either in the natural state or in a stretched condition. Tourmaline is one example of a crystal being dichroic in its natural state.

And when I come to the common polarize sheet they are made of stretched polyvinyl alcohol sheets treated with absorbing dyes or polymeric iodine and let us look at what each of these steps really influence. So what do you find is stretching of the sheet orients the molecules parallel to the direction of strains and renders the materials doubly refracting. So the first step is you stretch it and because of the stretching process the sheet becomes doubly refracting.

So it behaves like a crystal. You have 2 refractory indices you have ordinary and extraordinary travel through it. And what happens is when the material becomes dichroic when stained with iodine. So dichroic means it absorbs one component of light vector it allows the other component so essentially you get plane polarized light.

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EXPERIMENTAL STRESS ANALYSIS Transmissio

Polarizers in Sheet Form

- The most common dichroic polarizers are made of stretched polyvinyl alcohol sheets treated with absorbing dyes or polymeric iodine.
 - ★ Stretching of the sheet, orients the molecules parallel to the direction of strains and renders the material doubly refracting.
 - ★ The material becomes dichroic when stained with iodine.
- Dichroic materials are to be distinguished from birefringent materials – they have similar absorption coefficients for ordinary and extraordinary rays.

And you should also make a distinction and this is very important. The dichroic materials are

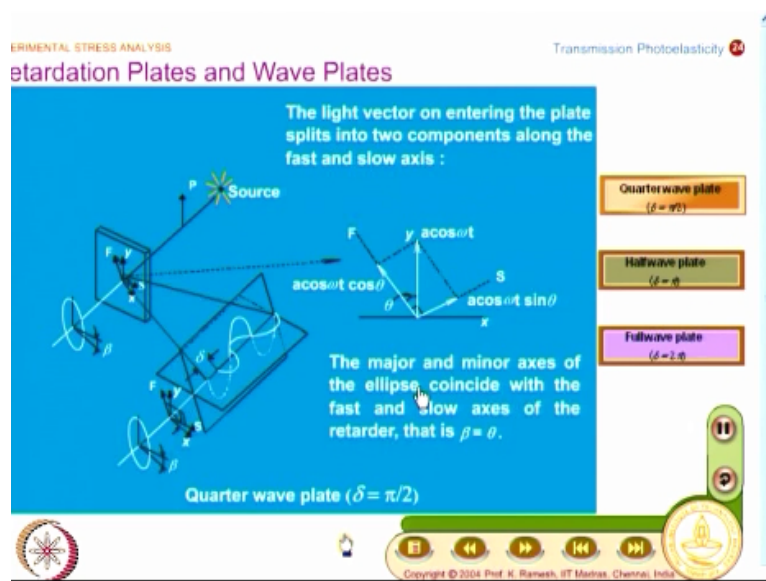
to be distinguished from birefringent materials. Birefringent materials have similar absorption coefficient for ordinary and extraordinary rays. See for you to do photoelasticity you need birefringent material and birefringent materials the absorption coefficient is same for ordinary and extraordinary rays.

But for polarized sheet you have a dichroic material it absorbs one of the rays completely and it allows the other ray and hindered and you are able to see and you should also see a very subtle point here. See I said engineering is approximation and when you are looking at polarization optics. I said after the polarizer and till the analyzer we do not assume any absorption of light intensity.

So in reality there may be a small absorption of ordinary and extraordinary rays which could be neglected. So we make that kind of approximation from practical standpoint. So what is important is we need to know what is a characteristic behind sheet polarizer because that have really advance photoelastic analysis because you have to work only with Nicol prisms then you had only a very small area for you to analyze the amount of polarized light availability is small the region of interest could be small.

Once we have sheet polarizer I can have a large sheet and large models can be looked at comfortably and what is fundamental to all of this photoelastic analysis is understanding what are retardation plates and wave plates.

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And this animation we will be looking at it again and again in this course and this is the cuts

of photoelasticity we will again have a look at it. I have a natural light source becomes polarized when it hits the front surface of the model you have this split into 2 light components. They travel within the model acquire a retardation and in general you get elliptically polarized light.

And we have seen by changing δ the characteristics of ellipse can change and there are few important cases which are of relevant to photoelastic analysis and we keep using a optical element call quarter wave plate. The name signifies it introduce us a retardation of $\delta = \pi/2$ and this is essentially a crystal plate. The properties are same at every point on this body of the crystal plate and what we have learned as the process of this animation laws.

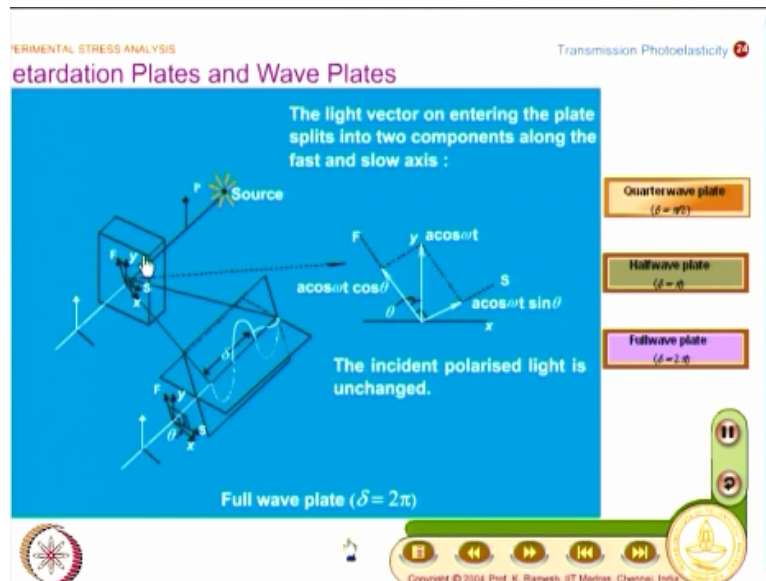
You find one of the rays travels faster and you have on this plane the ray travel faster and you call that axis as F axis and you have another plane which is perpendicular to this the ray trails behind it you call this as a slow axis. So once you go to your crystal plate you will always look for a fast and slow axis and what is the advantages when $\delta = \pi/2$ we have already seen from various states of polarization.

The major and minor axis of the ellipse coincide with the reference axis here it is labeled as fast and slow axis and essentially the azimuth of the ellipse is 0 if I have this as horizontal. If I have this axis as horizontal in a vertical axis the azimuth will coincide with the major and minor axis of the ellipse will coincide with the reference axis fast and slow axis. So that is an advantage.

And this understanding is very much important when we look at elaborate optical set up where how do these optical elements contribute to formation of different types of fringe pattern because we are going to look at the plane polariscope we are also going to look at a circular polariscope in conventional photoelasticity. If you go to digital photoelasticity people had no restriction on what kind of input light you should send.

People have experimented with various combinations and there you will know a physical appreciation of how these elements contribute to the complete experiment you will be able to understand it better.

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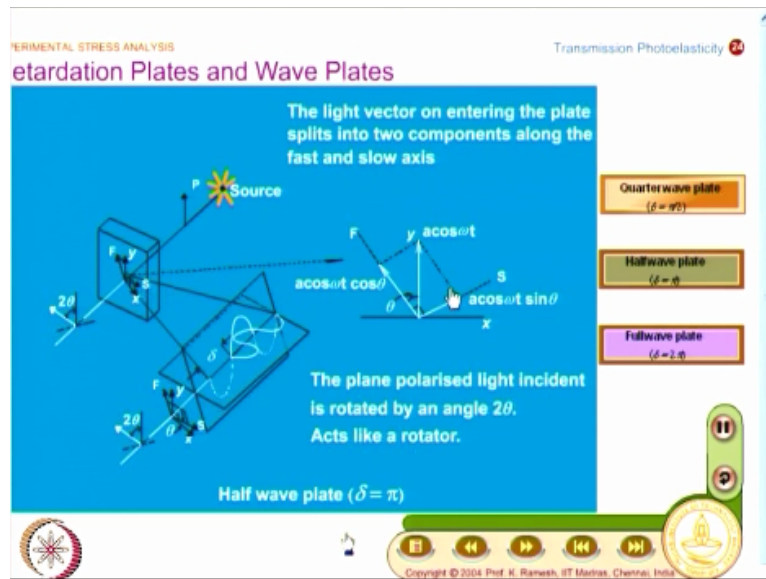


On one hand we have quarter wave plate on the other extreme I have a full wave plate. What is the definition of full wave plate? It introduces the retardation of one complete wavelength. So it is as good as the crystal plate is not there when I have a full wave plate it is as good as the crystal wave plate is not there. So this could happen at 2π , 4π , 6π the essential process is same.

So whatever the input light I send the same light will come out at the exit point and this is a very important aspect and this is what we will use it for investigating what happens in a plane polariscope. Though we developed various states of polarization by looking at elliptically polarized light that knowledge is essential for appreciation. For understanding plane polariscope you will have to just analyze whether the light coming out of the model is plane polarized or not.

And do you find here I have also shown schematically that the thickness of the plate is increased to provide you large value of retardation.

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And what happens when I go to a half wave plate. When I go to a half wave plate a simple argument is I will have instead of $a \cos \omega t$. Suppose I will give only in this there is a retardation this will become $a \cos \omega t + \pi$ so that is nothing, but $-a \cos \omega t$. So you will have a component here. So you will have the resulting will be in this direction. So plane polarized light which is incident on the model remains plane polarized, but rotated by angle 2θ very interesting.

So what do you find here is when you go to 3-dimensional photoelasticity we also learned what is a rotator that could be reasonably understood when you look at how the half wave plate behaves it is for a plane polarized light. So similarly people found there are ways that you can rotate the light ellipse. The elliptic characteristics will remain same only the azimuth will change its direction so that is what a concept of a rotator.

So what you have here is in a half wave plate I have $\delta = \pi$. In a quarter wave plate $\delta = \pi/2$ and the full wave plate $\delta = 2\pi$. And let us summarize these concepts are also summarized in this slide.

(Refer Slide Time: 14:14)

EXPERIMENTAL STRESS ANALYSIS Transmission Photoelasticity 15

Retardation Plates and Wave Platescontd

- The emerging light is in general, elliptically polarised.
- If the thickness is such as to produce a phase difference of $\pi/2$ radian, then it is a quarterwave plate ($\lambda/4$).
- If it is π radian, it is a halfwave plate.
- If the retardation is 2π , then, one gets a fullwave plate and the incident light is unaltered.

So what you find the first observation is when I have a crystal plate even when I send a linearly polarized light impinging on the crystal plate. The emerging light is in general elliptically polarized. So that is the general observation number one. And if I so adjust the thickness of the plate to produce a phase difference of $\pi/2$ radian then I call that as a quarter wave plate. It is also labeled as $\lambda/4$ plates.

If the retardation is π radian, then it is a half wave plate. If the retardation is 2π radian one get a full wave plate and the incident light is unaltered. And this understanding is very important. Even before we go and find out the expression for δ a clue is given and the sketch was also drawn that thickness is one parameter which I could play with to get different values of retardation.

That is the simplest when you look at the expression when you look at it acquires retardation within the model. So it is easy to anticipate by changing the thickness. So what we will have to now look at it when I say a crystal plate I should look for 2 reference axis one is a fast axis and slow axis and I will also have to know what is its refractive indices n_1 and n_2 I should know what is its thickness.

So what I will now try to do is I will get an expression for δ which is a function of the optical properties of the crystal plate that should be our next goal because first thing is we said δ is very crucial any changes in δ is reflected in the nature of exit light ellipse and now we go back and find out whether δ could be evaluated from the parameters of the crystal whatever the crystal plate that we think of. So that we will do that.

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EXPERIMENTAL STRESS ANALYSIS Transmission Photoelasticity 20

Retardation Plates and Wave Platescontd

- The relative retardation (δ) is expressed in terms of the thickness and optical properties of the crystal plate.
- Since the velocities of propagation within the crystal is different for the two rays (ordinary and extraordinary), they will take respectively h/v_1 and h/v_2 seconds to traverse the plate.
- This time difference contributes to the phase difference. Let the frequency of light be f , then

$$\delta = 2\pi f \left(\frac{h}{v_1} - \frac{h}{v_2} \right) = 2\pi h \frac{c}{\lambda} \left(\frac{1}{v_1} - \frac{1}{v_2} \right)$$
$$= \frac{2\pi h}{\lambda} (n_1 - n_2)$$

So what we want to do is we want to express the relative retardation in terms of the thickness and optical properties of the crystal plate. So what we want to do is when I say optical property what are the properties I have refractive index n_1 and refractive index n_2 or you can also classify it as ordinary ray of refractive index and extraordinary ray refractive index that is fixed for a crystal.

The difference between the crystal and the model is every point in the crystal has same behavior, every point in the model in general will have different behavior depending on the stresses introduced. Suppose I take a tension (()) (17:32) apply uniform tension then it will behave like a crystal plate only because you are applying a uniform state of stress. So every point will become identical leaving that apart in a generic situation the local stress state dictates what would be the properties of the crystal at that point of interest.

So now what we want to do is we want to get an expression for delta as a function of the thickness and properties of the crystal plate. So how do you go about what is the clue? See we know one ray travels faster suppose I fix the thickness because in practice we will have a model of a particular thickness being analyzed. So thickness of the plate is fixed. So one ray will travel the same thickness faster than the other.

So looking at in other words the time taken to traverse the thickness by these 2 rays will be different and that is why we looked at when we learned (()) (18:45) law I said you have learnt it in your physics course at the school level there you read only about $\sin A/\sin R$ you never

bothered to look at as ratios of velocities and I said in photoelasticity we have a purpose we want to look at ratios or velocities.

Now what we used that knowledge and identify that the rays will take different time interval to traverse the thickness. Suppose I have v_1 as a velocity v_2 as the velocity I will have h/v_1 and h/v_2 is the time taken to traverse and then I have ωt as the phase and ω is nothing, but $2\pi f$. So using this input it is possible to write an expression for δ that is what we are going to do.

So refractive index whenever we want we will look at as ratios of velocities and we will look at as a tensor when I want to relate to stress. So I use it in a way that will help my theoretical development that is what I am going to do. And so what you find here is the velocities of propagation within the crystal is different for the 2 rays. They will take respectively h/v_1 and h/v_2 seconds to traverse the plate.

And we take h as the thickness of the plate, you know, t if I use it indicates time so we want to have a difference symbol for the thickness. And we take advantage of our understanding on refractive indices. So this time difference contributes to the phase difference. Suppose we have the frequency of light be f then I can write the expression for δ . Can you try out how will you write δ ? Make an attempt even it is wrong it is fine that is how you learn things.

So what I have here is I know that ordinary and extra ordinary ray travel with different velocities and I understand because it has to traverse the same thickness this will take different time intervals. Now the question is can I write in expression of δ in terms of the parameters that I know. You can go in stages first you take the time difference then look at how it can be converted into phase difference then bring in certain identities f can be written in different ways.

And finally write down that expression in terms of difference in refractive indices that is how we will go. We will look at difference in velocities first, difference in time taken then finally write it as $n_1 - n_2$ that is by requirement that is how I want the results to be reported. I can do that it is very simple. I think some of you must have got it and that is what you have here. So I have this as $2\pi f h/v_1 - h/v_2$.

I will again rewrite f as c/λ where c is the velocity of light and we have already seen if I write c/v_1 , c/v_2 I can write it as n_1 and n_2 these are absolute refractive indices and you have to note a very key important observation on this expression. I have expression for δ which is given as $2\pi h/\lambda * n_1 - n_2$. So what you find suppose I say that I want to have a $\delta = \pi/2$.

I want to have a quarter wave plate I can find out what is the thickness corresponding to that and what is hidden here h will become a function of wavelength it is very important. See I said from mathematical development of photoelastic analysis the mathematics becomes lot more simpler if I confine our attention to monochromatically light source where does this come.

Till now we have not looked at we wanted to see colors so we used white light we enjoyed seeing those bright colors, but when I come to mathematical analysis we find the crystal plate behaves like a quarter wave plate for a given wave length. The same is applicable for wave plates as well as the model behavior. So we would confine our attention our mathematics will become lot more simpler if I use monochromatic light source and do my photoelastic analysis.

Now you have also Achromatic quarter wave plates where the plate gives a phase difference of $\pi/2$ for different wave length for a range of wave length you have such plates available. You know when there is a problem you have opportunity for research and research has developed a way to overcome this. So that goes on parallelly. So what you have to understand is in photoelasticity why the wave length is important monochromatic wave length comes hidden in the expression.

Now what we will do is we will go back and then see what great scientist have contributed how this could be related to $\sigma_1 - \sigma_2$ more by induction rather than clear mathematical development.

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EXPERIMENTAL STRESS ANALYSIS Transmission Photoelasticity

Stress Optic Law

- Consider a transparent model made of high polymer subjected to a plane state of stress. Let the state of stress at a point be characterised by the principal stresses σ_1, σ_2 .
- Maxwell formulated in 1852 the relations between stresses and the indices of refraction as

$$n_1 - n = c_1 \sigma_1 - c_2 \sigma_2$$

$$n_2 - n = c_1 \sigma_2 - c_2 \sigma_1$$

n_1 and n_2 be the refractive indices for vibrations corresponding to two directions. n be the refractive index in the unstressed state, c_1 is called the direct stress optic coefficient and c_2 is the transverse stress optic coefficient.

So what you have here is we call this. This law relates stress and optics and I call this as stress optic law. It relates stress and optic so I call this as stress optic law and when I do this I consider we have transparent model material and this is made of a high polymer and we also take for simplicity subjected to a plane state of stress. And what you will note is each statement you have to qualify.

Let the state of stress and the point by characterized by the principle stress as sigma1 and sigma 2. I am looking at a 2 dimensional state of stress. I can have matrix involving sigma x, sigma y, tau xy and so on, but I can also represent the same stress sensor in terms of its principle stress values. So that is what is indicated here. Let the state of stress at a point be characterized by the principle stresses sigma 1 and sigma 2.

And Maxwell in 1852 formulated relations between stresses and the indices of refraction as he conducted a series of test and then found out that $n_1 - n$ is related to $\sigma_1 - \sigma_2$. It is a function of the material constant. So what he found was he found the direct stress optics coefficient and there is a transverse stress optic coefficient and what he found was $n_1 - n$ that is n_1 and n_2 or the refractive indices of the ordinary and extraordinary ray.

And n is the refractive index in the unstressed state. So based on a series of experiment he was able to establish a relationship $n_1 - n = c_1 \sigma_1 - c_2 \sigma_2$ and $n_2 - n = c_1 \sigma_2 - c_2 \sigma_1$. A similar exercise could be extended for 3 dimensions which I am not paying attention now. And what do you find here we are interested in $n_1 - n_2$. So I subtract these 2 equations then I can group the terms.

And mind you that c_1 and c_2 depends on what is the transparent model material that I am going to use. So there is a material parameter that comes in the formulation. Whatever I see is also function of the material that I use. The arithmetic is very, very simple if I want to relate it to $\sigma_1 - \sigma_2$ the arithmetic is very, very simple there is no great deal about it, but to understand the physics we had to look at how a crystal behaves.

Reinforce ourselves that for a 1 single incident ray there will be 2 refracted rays. The 2 refracted rays are plane polarized in mutually perpendicular direction. In general, they will be elliptically polarized when it comes out of the crystal all that knowledge is required to appreciate the link, but if you look at the mathematics it is very simple.

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EXPERIMENTAL STRESS ANALYSIS

Transmission Photoelasticity

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Stress Optic Law

$$N = \frac{\delta}{2\pi} = h \frac{C}{\lambda} (\sigma_1 - \sigma_2)$$

$$\sigma_1 - \sigma_2 = \frac{N F_\sigma}{h}$$

$$F_\sigma = \frac{\lambda}{C}$$

- F_σ is known as the material stress fringe value with the units N/mm/fringe.
- In photoelasticity fringe order N is always positive.

Now I am going to rewrite this delta in a form convenient for us to use and those steps are fairly straight forward there is no great mathematics involved here. So I have this as $2\pi h / \lambda (n_1 - n_2)$. Now we know $n_1 - n_2$ in a different form. So I put this as $c_1 + c_2 * \sigma_1$ and σ_2 and what I have is this is the material parameter and for convenience we replace it by another symbol in order to differentiate it from velocity of light we use the capital C.

I can recast this equation. So I have this as $2\pi h / \lambda * C * (\sigma_1 - \sigma_2)$. So it is a function of the material that I am going to use and it is also a function of the wave length that is very important and you know if you go to any of the optical techniques you would not tell the retardation in terms of radians. It is lot more convenient if I label it as fringe orders.

And I do not know how many of you have really looked at what is the fringe order. If you look at fringe order it is defined as $\delta/2\pi$. So if I say fringe order of 1 the relative retardation is what is relative retardation $\delta/2\pi$ I say so the retardation is 2π . If I say fringe order of 1 and fringe order of 1, fringe order of 2, fringe order of 3 you can go and I can also have partial fringe orders.

So what we will do is we will recast it we will segregate the term $\delta/2\pi$ then the expression becomes lot more simpler to look. So I will write this $n_1 - n_2$ as $\delta/2\pi$. So I can recast this equation as $h^*c/\lambda(\sigma_1 - \sigma_2)$ and I can write $\sigma_1 - \sigma_2$ finally as in this fashion Nf σ/h and we have introduced a new symbol for the term λ/c in this fashion.

And mind you this is a very famous relation in photoelasticity. $\sigma_1 - \sigma_2 = Nf \sigma/h$ a very famous relation and if you know only this expression it is not sufficient it is misleading that is why I put immediately $f \sigma = \lambda/c$ because if you look at the basic equation because you bundled some of those quantities by a new symbol there is a chance that you may misinterpret where is the wave length comes in the expression.

So there could be mistakes like this and you know when we ask question in the examination then we understand that you have not understood it until then it look as if it is crystal clear. Only when questions are asked you find that your understanding is not complete. So do not remember only this final expression always think $f \sigma$ is a function of wave length. Why we say function of wave length, why we emphasize this.

If you go to photoelastic benches some of the earliest benchers, they had a mercury arc lamp that was one of the easily available monochromatic light source. Then people had sodium vapor lamp. So these 2 wavelength you come across. Some of the old polariscope they have only mercury arc lamp, some of the recent polariscope they may have white light as well as the sodium vapor lamp and somehow the material property is stable may have been obtained for a particular wave length.

And you may have to use that property for you mathematical analysis then I have to convert from one wavelength to another wave length. So that is all that you can get by looking at $f \sigma$ is a function of wave length and this is considered as independent of wave length for

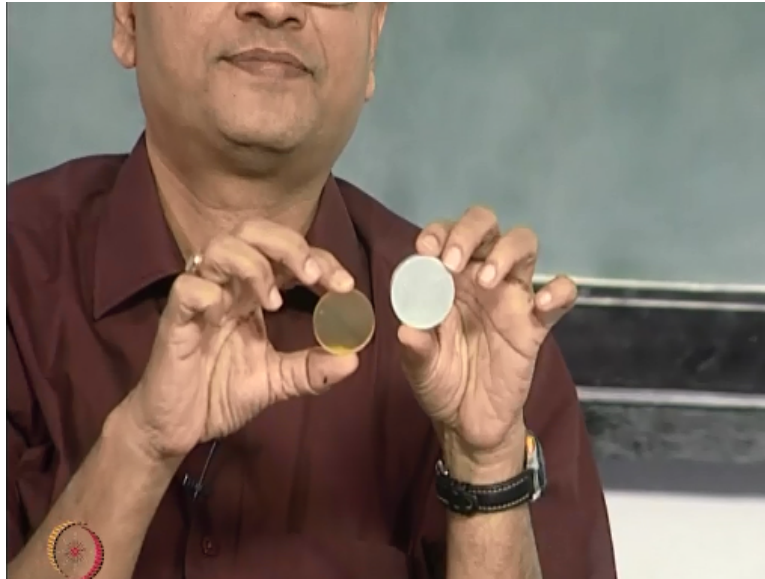
the most part of analysis and this also gives that this is a linear expression within limits. Suppose I apply loads which are very close to plastic region and I have very high stress gradient.

This relationship is no longer linear and you have to use it with caution. See sometimes you look at an expression whether the expression tells you or not you assume many things. So one of the first wrong assumption that is possible is it is independent of λ is a wrong conclusion you can arrive at because people have introduced f_{σ} for convenience and f_{σ} how it is defined this also has a units newton per millimeter per fringe and f_{σ} is known as the material stress fringe value.

And this has the units like this and when I plug in here I will get stress as (σ) (35:06) that is the purpose here and many things you can understand from this expression. See when I introduced fringe from photoelasticity even when you looked at famous problem of 4-point bending where you had tension and compression side the fringes were always labeled positive integers.

On the other hand, when we went to my ray you found that fringes are numbered both positive and negative whereas in photoelasticity you always number it as positive integers why is it so that comes from this expression because when I say σ_1 - σ_2 I always arranged the principle stress as in algebraically decreasing order that is σ_1 is algebraically is the greatest σ_2 is middle and σ_3 is the least. So this will always be positive.

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Okay let me go back to my other question that I raised about 3 classes back. I took an aluminium disk and also a Polyurethane disk. I said poly is one of the photoelastic model material and then imagine that I apply a same load. I said what is the nature of the stressors developed because Polyurethane is a plastic which has low modulus you can visually see the deformation.

Aluminium is so hard it has about 70 GPa and this is about 0.3 GPa. It is very, very small value. So the deformation is definitely different there is no 2 opinions about that. The question I asked was for the same load and for the same size how the stressors would be it is a plane problem. Have you brushed your solid mechanics and found out what do you anticipate stress will be same?

I am happy to hear that. That is a key point without which there cannot be any photoelasticity. Suppose I have 3-dimensional model story is different. This we will see towards the end of the discussion on photoelasticity. We look at the relevant mathematic equation and then show I said in all the experimental technique the Poisson's ratio is a nuisance value.

The Poisson's ratio will do the spoil sport when I go to a 3-dimensional problem. In planer problems the stressors are same. It is very, very advantageous. Okay now let us look at the expression. Now what I have see if I have to use this expression my interest is to find out σ_1 - σ_2 that is very clear. From the experiment I will have to find out what is the fringe order.

And depending on the material that I use you know if you look at photoelasticity for class demonstration we bring in Polyurethane and then you have polycarbonate, you have epoxy, you have perspex, you have even glass they are all photoelastic materials and even the recently introduced (()) (38:50) they are all photoelastically sensitive material and we use this for certain purposes.

In class it is easy for me to apply the load and then show the generation of fringes very conveniently and when I do one experiment I want certain amount of stability I do not want model to deform and introduce large deformation. When I introduce large deformation the whole analysis become different. So I want to minimize deformation so that is only reason why I chose different material.

There is also another reason availability and then you have what is called time and effect we will see all those issues later. So the essence here is there will be chances for you to use models of different materials for each of these material I need to find out the material stress range value. Now the question is by looking at this expression for a given problem I change the material what would happen to the fringe.

We have just now seen I take aluminum disk or Polyurethane disk for the same load applied stresses do not vary instead of aluminum disk I am going to have a heralded disk or a polycarbonate disk or Perspex disk and so on. So in such a scenario what happens $\sigma_1 - \sigma_2$ will not change at a point of interest. So this product will change appropriately. So if I have f σ is small I will have more fringes.

If I have f σ is high, I will have less fringes. This product will remain a constant and if you look at the kind of problem that can be coined the arithmetic is very, very simple. If you understand the physics behind it if you anticipate that this is how it has to chose the left hand slide. Here it is the left hand side $\sigma_1 - \sigma_2$ does not change and only the right hand slide changes so they will adjust.

So you will see more fringes less fringes more fringes less fringes is not the indication of the values. You need to know the material parameter only when I know that I can evaluate the stressors and also this is very important if I find out this parameter inaccurately then all my match between experiment and if I want to do the comparison whatever I do from the

experiment and analytical method they will not match if I measure this quantity carelessly.

I have to do sufficient care in finding out. So what we will do is we will have a detailed discussion on how to find out $f\sigma$ as accurately as possible and from photoelastic point of view we will have to find out how to get the fringe order. I cautioned you even several classes back that in all optical techniques finding out the fringe order is tricky you do not get it in the first go.

You have to use auxiliary methods, you have developed engineering equipment, you have to verify from various methods of finding out and then another question is you find fringe order and the fringes you do not find out in between fringes. So you have to use compensation techniques. So finding out fringe order is an issue. So if I have to find out the stressor I have to know the fringe order N and material parameter.

So what way we will proceed is we will first go and see how to get the fringe order then we will have a discussion on how to find out $f\sigma$, but even before we discuss on these issues let us have a look at what is photoelasticity can give. See I said photoelasticity can give you directly only fringe order and the principle stress direction. How it is going to give, how we have to do?

We have to look at the optical arrangement understand then go about it, but even before getting into the details we can now find out from our strength of material knowledge can I extract what information if I know these 2 quantities that would be of interest because even before we want to do an experiment on photoelasticity you can be assured what I can get as information from stress analysis point of view to the extent possible.

So that is what we will see in the next slide.

(Refer Slide Time: 43:40)

PERIMENTAL STRESS ANALYSIS Transmission Photoelasticity

Stress Information Obtainable by Photoelasticity

- If the fringe order and the material stress fringe values are known, then, one can obtain the principal stress difference.

$$(\sigma_1 - \sigma_2) = \frac{NF_\sigma}{h}$$

$$(\sigma_x - \sigma_y) = (\sigma_1 - \sigma_2) \cos 2\theta$$

$$\tau_{xy} = \left(\frac{(\sigma_1 - \sigma_2)}{2} \right) \sin 2\theta$$

F_σ is material stress fringe value in N/mm/fringe.
 h is thickness of the model.

So what is stress information obtainable by photoelasticity. So here for this discussion we assume at a point of interest you know the fringe order we have not yet looked at the fringes how to find out the fringe order what is optical arrangement all that we will take it up later. Suppose I know the fringe order I know the material stress fringe value and I also find out theta what I can do.

So I will go to the more circle and look at what it is and take an advantage from your knowledge it is not new it is all we build on your understanding of solid mechanics, strengthen materials, the foundation has to be strong that is why we had a review of solid mechanics. You should know about more circle. You should know that stress is tensor and more circle represents this beautifully.

And what I have here I have sigma 1-sigma 2 is given as NF sigma/h and if I know the more circle I can easily write sigma x-sigma y=sigma1-sigma 2 * cos 2 theta. You all know more circle you draw in the sigma and tau plane I draw a circle and each points denotes a plane and here it is x plane and this is y plane and in more circle all this angles are twice the angles that is why they are at 90 it is shown at 180 degrees.

And when I have so many points on the boundary it shows all the possibly infinite planes you can find out what is the normal and shear stress absolutely no problem that is why it is a beautiful representation I do not whether you looked at more circle from this point of view. When I said all the possible state of stress in all the infinite planes when I have a point of interest that is what you understand as stress tensor.

And a beautiful graphical representation is Mohr's circle. So on the Mohr's circle every point on the circle denotes a particular plane and using this you can also find out what is the principal stress plane what is the magnitude of σ_1 and what is the magnitude of σ_2 and simple geometry will help you to find out what is difference in normal stresses and also the shear stress.

What is the value of shear stress you all know it? It is simply $\frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$. So what you find is from photoelastic analysis a simple normal incident can give you fringe order again and θ at a point of interest and if I know the material stress fringe value of the model material then I can go use Mohr's circle, find out normal stress difference as well as in plane shear stress.

So I can find out in plane shear stress very comfortably and this is what I said if you remember and recall one of the very important problems in engineering is I have a 3-point bend specimen. I want to find out what is the variation of shear over the depth. I said when I go closer to the point of loading though you have read in your simple strength of material course that shear varies parabolically over the depth.

This is no longer so when I go very close to the load application point and I also mention doing this analytically is possible however you have to represent this (τ_{xy}) (47:51). On the other hand, photoelasticity can give this information directly. So what you need to find out closer to the point of loading I need to find out τ_{xy} . τ_{xy} means I have to find out fringe order and θ .

And in fact you will do this as part of one of your laboratory experiment. You will find that so nice, so elegant. It is a very key point of strength of materials. Though you learn it varies parabolically near the lower application point, near the surface shear is maximum and even if you want to do it by numerical analysis you have to discretize the model very carefully and also model the concentrated load as accurately as possible.

And is there anything like a concentrated load. It is an obstruction. Is there anything like a rigid body it is again an obstruction. The concentrated load and rigid body goes together. In reality all bodies are deformable. So you live on approximations and same concepts are

required when we go and understand and interpret what is the result from photoelastic analysis.

So what we have looked at here is stress information obtainable by photoelasticity. Suppose I suppose $N \theta$ and $f \sigma$ I do not have to say I get only difference in principle stresses. I can also find out difference in normal stresses and in plane shear stress. So in this class, what we have looked was we started looking at how to understand the physics behind the simple polarize sheet.

We found that they display the behavior of dichroism very useful we have taken advantage of that. Then we moved on to find out an expression for relative retardation and we found it is a function of the thickness of the crystal plate and also depends on the wave length and we also reasoned out why mathematic has become simpler when we use monochromatic light source in photoelastic analysis.

Then we move on to establish what is stress optic law. Maxwell has conducted a series of experiments he found out there is a material parameter also comes in the equation. And I cautioned this material parameter is also function of wave length you should not forget that because if you look at the expression $\sigma_1 - \sigma_2 = NF \sigma / h$. It does not give an impression that it is a function of λ , but you have to keep in mind it is a function of λ .

So the problem could be $f \sigma$ is determine in one wavelength and I do it and experiment in another wave length so you may have to do the modifications. Then I use different materials then I have to do. If you look at arithmetic in photoelastic analysis they are very, very simple, but the physics behind it little involved that is what we are looking at it and once you know the physics you can easily solve the problem.

And the most challenging and crucial aspect its finding out the fringe order values that requires some type of training, understanding and that is where digital photoelasticity aims in minimizing your effort.