Transport Phenomena in Materials
Prof. Gandham Phanikumar
Department of Metallurgical and Materials Engineering
Indian Institute of Technology, Madras

Lecture – 20
Heat transfer with advection

Welcome to the session on heat transfer with advection as part of the NPTEL MOOC on transport phenomena in materials. This is also called as a convective heat transfer.

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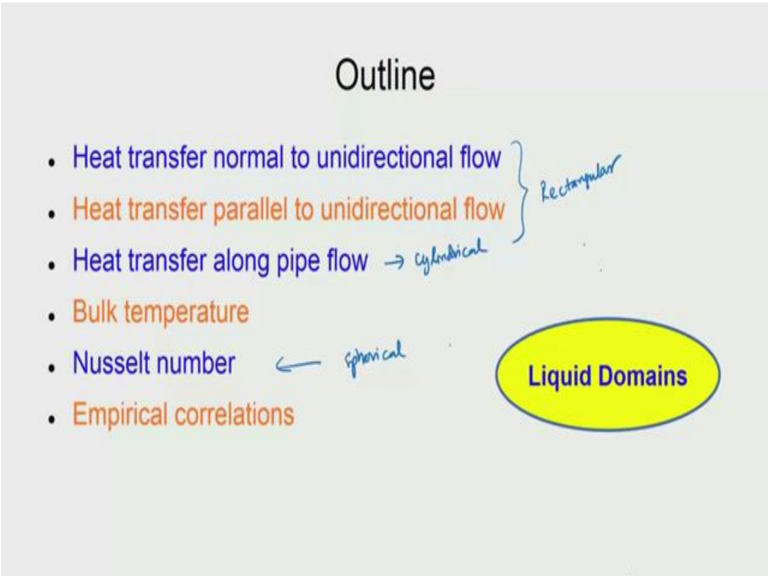

See outline for this session is as follows: We will take up a heat transfer along with a fluid flow in a rectangular coordinate system. The first two cases are in rectangular system and we will then move on to the cylindrical case also and in a very simple problem, while looking at the concept of Nusselt number, you will also look at a spherical coordinate system and through these sessions we will also be introducing new concepts like what is a bulk temperature, what is a Nussle number and so on..

And we will end the session with empirical correlations of Nusselt number, which basically will give us the heat transfer coefficient and once, the heat transfer coefficient is available, then we can use it as a boundary condition in the problems that we normally encounter in metallurgy. Now the domains for this session will always be liquid domains. So, which means that this fluid flow is taking place inside the domain and we want to understand how that flow will couple with heat transfer. So, let us take a trivial situation where we can rule out the effect of a flow on heat transfer.

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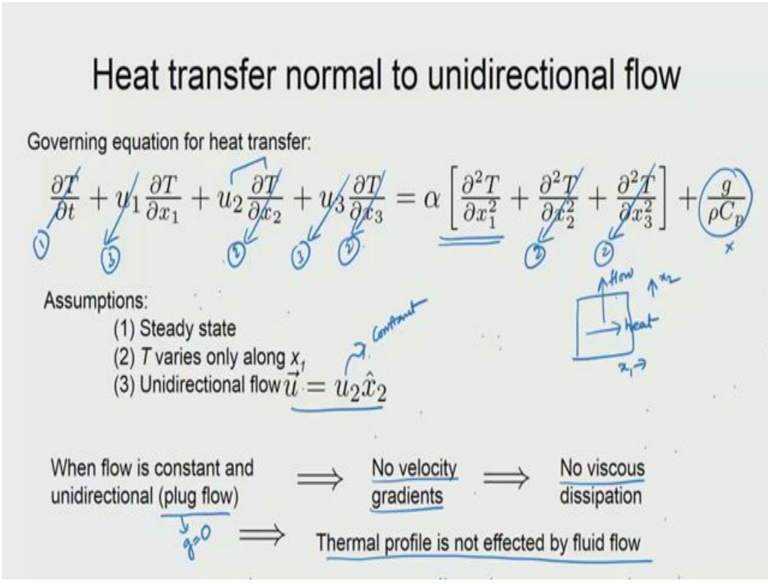

So, we are looking at heat transfer normal to the unidirectional flow. So, you could then pose a problem with the following assumptions. So, let us look at the equation which is generalized fourier heat conduction equation in rectangular coordinate system and say that, the situation is where, you have a flow taking place in one direction then the heat flow is in the other direction. So, this is the situation that we want to look at. So, if you take the state assumption, then you do not need this term and then, if you say that the temperature is varying along only x_1 direction, then you would not need the terms like this.

So, all these are gone because of the assumption 2 and then, we say that the directional unidirectional flow is along the u_2 direction. So, this is a x_2 direction, u_2 is in the x_2 direction, which means that u_1 is not there and u_3 is also not there. Now, you could see that though u_2 is present, it is multiplying with this term here, which is again dropped. So, this means that the flow is not affecting the temperature gradient at all on the advective term. So, this is the reason why, for example, it is necessary to look at which phase a flow compared to the unidirectional heat transfer.

However, the flow can also be affected in another manner. So, it can affect through the source term that is present here. So, if you drop all the terms on the left hand side, you have got only this term and then, the source term. Now, sometimes the unidirectional flow is termed as plug flow. So, which means that this u_2 is a constant and it means it is not a function of the other two variables x_1 and x_3, which means that there are no velocity gradients that are there.

Then if you look at viscous dissipation, which is basically caused because of velocity gradients, then g will be 0. So, for a plug flow, you can say that g is 0 and in such situations, then you could also drop this term, which means that your generalized Fourier heat conduction equation in case of a steady state heat transfer in one direction and flow in the normal direction. Then the thermal profile is actually not affected by the fluid flow.

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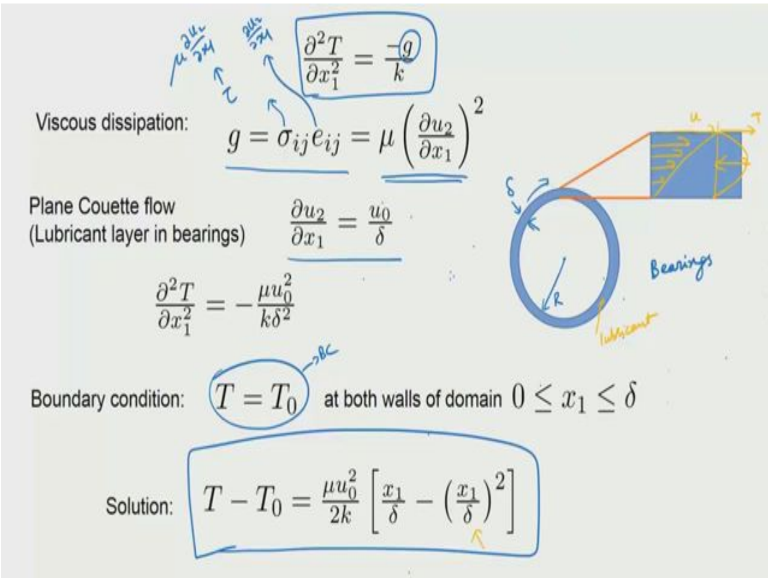

So, sometimes these are the situations where apparently the flow is there, but actually it does not affect. In situations where the plug flow is not the case, but unidirectional flow is actually the case with variation of velocity in the other two directions. Then, you do have a viscous dissipation coming in. So, viscous dissipation is written in this form and if you then recollect the Newton's Newtonian viscosity approximation, where we have already seen then this basically is tau, which is then given by μ times $\frac{\partial u_2}{\partial x_1}$ and then, this itself is again $\frac{\partial u_2}{\partial x_1}$. So, you could then write the g as this. So, which is basically W/m^3 and if such a viscous dissipation is taking place, then it goes into the g term here and affecting the temperature profile.

And then you could actually look at this equation and then solve it by integrating twice and you can then look at the solution that will come in this form and in the process of obtaining it we have made assumptions like the boundary condition saying that the domain on both ends has temperature T_0 and when is this kind of a situation possible in situations like bearings. So, in situations where you have got for example, outer wall moving and inner wall stationary there is relative motion between these two and then this thickness which is δ is very small compared to the radius.

So, the radius is R. So, compared to R if δ is very small, then we can have the situation and what kind of a flow are we then talking about we see that we are having a flow of this nature. So, clearly there is a gradient and that gradient is what is causing a viscous dissipation which is causing the heating now if the temperature variation is across this distance and you can see this parabolic. So, which naturally means that you would have the maximum heat at the center and therefore, if this was the velocity profile then the temperature profile would have something of that nature ok.

Now, the maximum heat that is actually obtained here, this δ is a reason why for example, what is in this region is getting hot. So, this means that, this usually is the lubricant. So, which means that lubricants do get hot because of viscous dissipation when δ is very small and the velocity gradients are steep and there is one reason why for example, lubricants have to be designed. So, that their viscosity does not fall significantly with increasing temperature. Now, that part.

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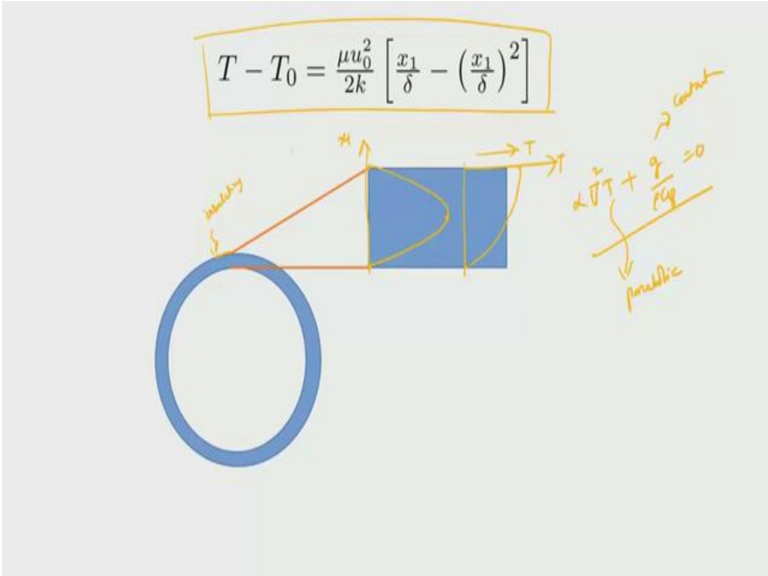

Let us look at how this solution is going to look like. You can already see the solution is written and then, you could then plot it. So, you could see from the profile that the temperature this way would be T_0. If you look at the functional form and there will be something maximum here and this distance is x_1 direction ok.

And this parabolic form is something very common to us. We have already seen this coming many times and you could already see that, whenever you have got the laplacian term playing a role along with a source term, then for example, you would have. So, whenever you have this kind of situation then usually this will be parabolic if this is constant term. So, that is something that we already saw and here, we can already see that in the problem, similar kind of a thing is coming. So, without even solving one can already guess what is going to happen.

Now, you could already see that this problem can be solved with the different boundary conditions, such as there is no heat flow across one of the walls, for example, what would when happen is the parabola will get shifted up? So, if you say that one of the walls is insulating for example, then, it would mean that you may have a temperature profile which is still parabolic, but then, having it is maximum on the surface because then, the slope of the temperature profile is zero and that would accommodate. So, like this you could actually look at the solutions for various boundary conditions. So, this is one way by which the fluid flow is affecting the temperature profile. Now, let us see what happens when the fluid flow is actually not normal, but parallel to the unidirectional flow.

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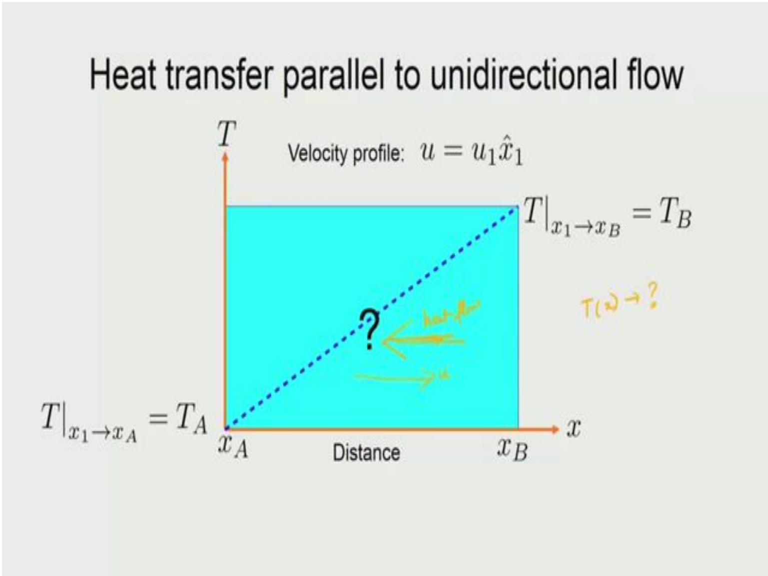

So, here is a situation. You have the flow along the x_1 direction. So, u is this way. So, heat transfer also is in the same direction. So, which means that basically anti - parallel or parallel it is not normal definitely. So, this means that, they will have some coupling that will take place. What we want to inspect is how the temperature profile would be. So, T as a function of x and we want to see whether this is going to be a straight line or not. So, our guess would be that it may not be straight line.

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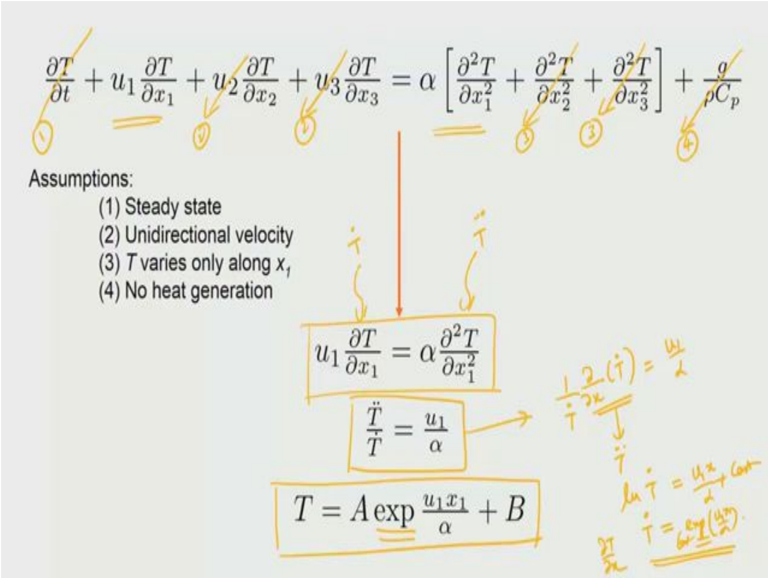

So, the way we solve is as follows. We again write the governing equation and see what would turn out to be. So, we see that steady state is to be assumed and then, the unidirectional velocity which is along the x_1 direction. So, u_2 is not there, u_3 is not there and the T is varying only along the x_1 direction, which means that along the x_2 and x_3 directions it is not there and of course, these ones are anyway 0 and then, there is no heat generation is an approximation. We want to say that we do not want to look at the viscous dissipation at this moment. So, which means that the governing equation will have only these two terms, which then is written here and you could then see that this and this can be written as $\dot{T}$ and $\ddot{T}$ which is then converting the equation to the simpler form.

So, you could then write this as $\frac{1}{\dot{T}}\frac{\partial}{\partial x}$. It is dot, because that is nothing, but this is $\dot{T}$, $\ddot{T} = \frac{u_1}{\alpha}$. So, when you integrate, you would get logarithm and then, it means its $ln\,\dot{T} = \frac{u_1 x}{\alpha}$ plus constant and then, when you exponent, you will get $\dot{T} = exp(\frac{u_1 x}{\alpha})$ into some constant term. So, like that you can actually look at the solution and then, when you integrate this, you get another constant and that constant is called B. Here, in our situation. So, you could then finally, see that the solution should be exponential form ok.

Because here you are writing $A\frac{\partial T}{\partial x}$ is exponential. So, solution also will be exponential. Now, the A and B constants can be determined using the boundary conditions, which we said that at x_A, it is T_A and x_B it is T_B.

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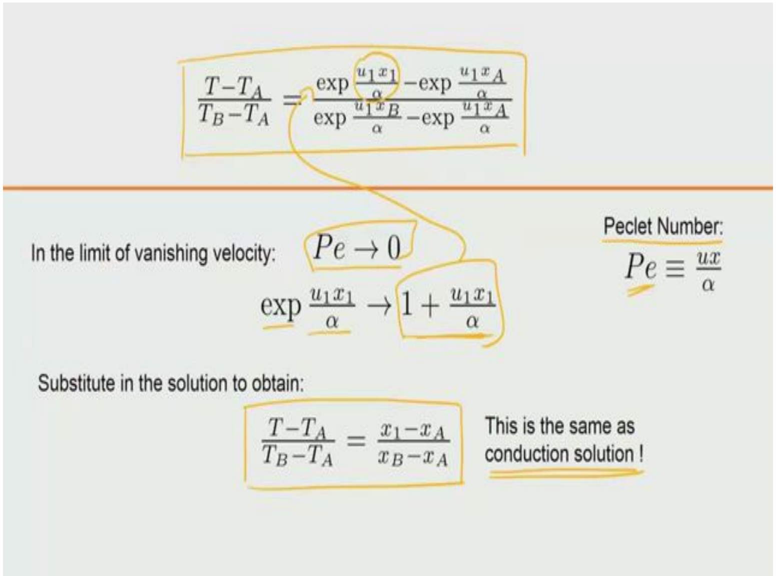

So, you could do that and then, solve the constants and then, you will get them as a very symmetric expression with exponentials and at this juncture, how the profile would look like is something where we can introduce a new concept called Peclet number. So, that is basically to look at this quantity that is appearing in the exponential. It is does not have units. So, you can then think of a number for that Peclet number.

So, Peclet number is defined as velocity into distance over the diffusivity. Now, in the limit of the Peclet number being very small, then what happens if the velocity is very small. In such situations, we already know that exponential of a small argument is approximately one plus that argument. So, with that approximation what happens is that, when you substitute this into here and you do the same thing for all the four exponentials, then you would see that the expression will come out to be very simple. It would just come to be the same differences in distances as temperatures. In other words, it is going to be just a straight line approximation, which is basically conduction solution.

So, we have retrieved the conduction solution though convection, which is a starting point in the limit of a convection being negligible so; that means the solution is valid. Now, how does this solution look like when we plot. So, for some typical values of alpha and u, you could plot and you could also vary the u to be either positive or negative.

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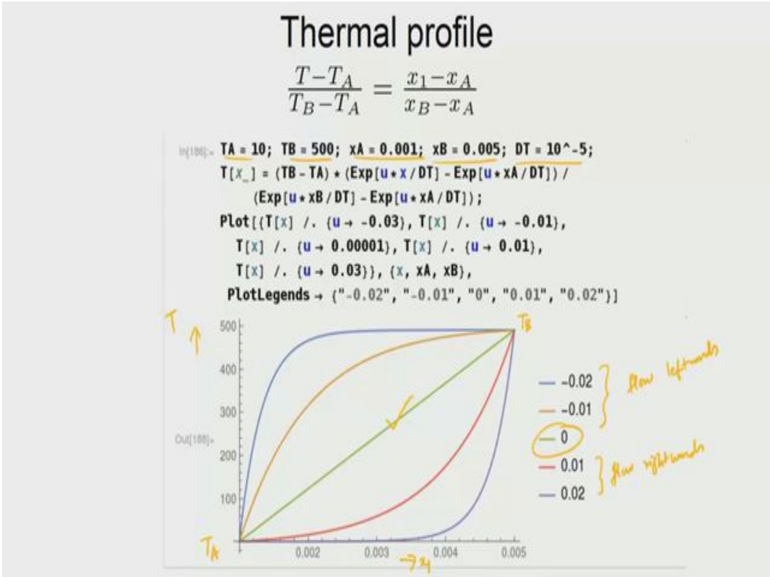

And that we do a simple mathematical script here, two line script and you substitute some values as you like and then, you can make a plot. So, you can say that on the left hand side is T_A, the right hand side is T_B and then, the distance is here x_1 and then, I am plotting the temperature here. So, you could then see that, you can have the flow to the leftwards and a flow rightwards and for both these situations, the temperature profile is actually deviating from the straight line. So, straight line is actually the conduction solution, which is actually available here, which we already know this, because a straight line is a solution for the conduction equation, when we drop all the terms except the diffusion now.

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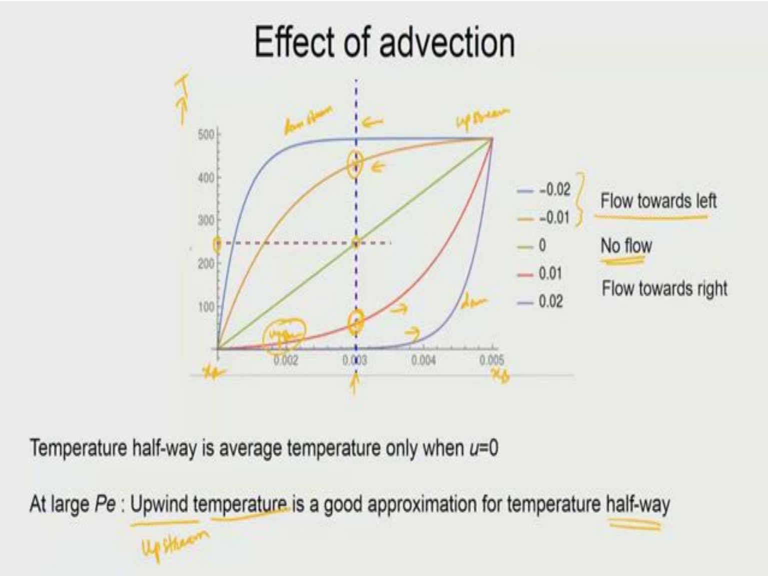

Now, just look at this profile a bit carefully. A midway between the two walls A and B, x_A and x_B midway you see here, if you go up then, where we cross over that, will be the temperature average temperature between the two walls and this is only true when there is no flow and when there is a flow, what would happen is that the temperatures are different depending on the orientation of the flow. So, which means that, basically you would not be reasonable to approximate the temperature halfway, when there is a flow along the direction of the heat transfer and you must always see that the temperatures are closer to the upwind or upstream temperature.

So, you can see that upstream this is for example, these two are where the flow going this way. So, this is upstream and this is downstream here, these two cases are for the flow in the positive direction. So, this is upstream and this is downstream. So, which means that the temperatures are approximated to the upstream, whether it is positive or negative and so, you should always say upstream or upwind temperature is a good approximation for temperature halfway and this is the case, when the fluid flow is present then, fluid flow is not present. You can actually take the average temperature to be the temperature halfway. So, there is an important effect of the flow on the heat transfer, when the flow is along the direction of heat flow ok.

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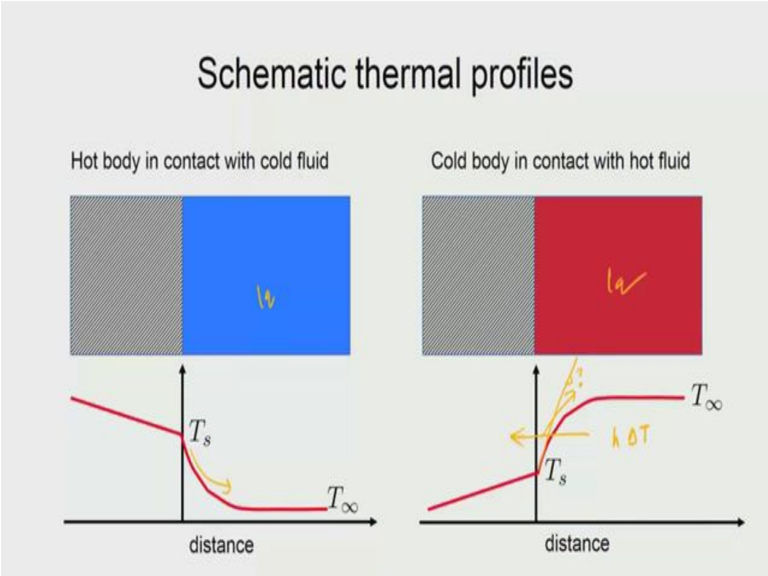

So, we have already seen these temperature profiles, when we were drawing the steady state heat transfer profile in an earlier session and we have been always drawing, for example, the temperature distribution in a liquid domain, whenever the heat transfer is happening, in the case of hot body in contact with cold body, we are actually drawing this way. So, that heat is going that way and in the case of cold body, with the hot fluid across and we will look at the heat going that way and the profile is drawn. So, you could see that in both the situations, we are drawing profile in an asymptotic manner and the slope here, is what we say, we are not really sure and there is a reason, why we want to write the heat flux in this direction. For example, e is given by $h\Delta T$ because we do not know the slope.

You can now see the reason for it, the slope here for example, is a function of the velocity magnitude, which we do not know in a general situation. So naturally, we then cannot use the fourier heat conduction equation here. We need something different. So, we use a dump factor heat transfer coefficient. So, this is the reason and these exponential profiles are drawn earlier itself, but we now know the meaning while we are drawing that way.

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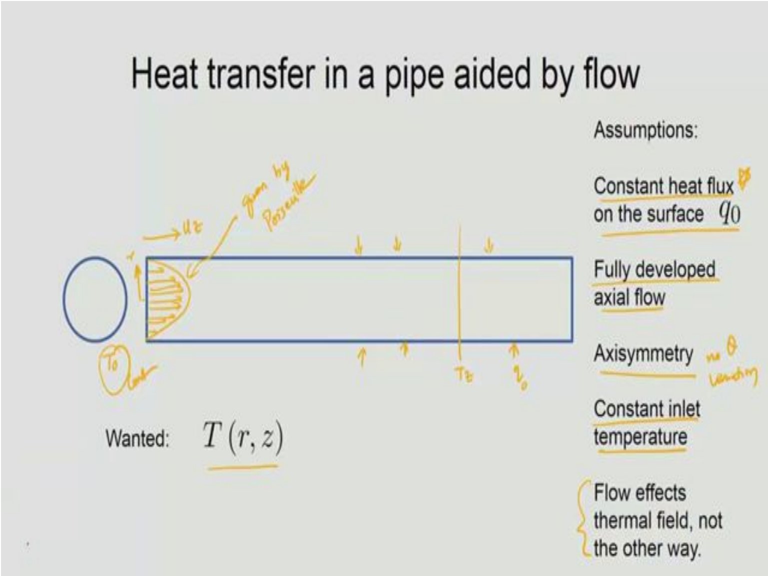

Now, we take a slightly long problem to solve, but at the end of it, we have a very elegant solution that is coming up. So, this solution is basically what we are seeking for a temperature profile in a cylindrical coil system or pipe flow. So, what we are saying is that, a cross section of the pipe flow is a circular and we say that, the velocity profile u_z as a function of r, should be exactly how we have decide derived at earlier, that is basically the so called flow. So, this profile is going to affect the heat transfer and we say that, the liquid that is entering is at a temperature T_0 and what is coming out at any z is a T_z and T_z is then a function of r and; that means, the temperature is a function of r and z and the temperature is changing because there is a heat, that is coming in through the periphery ok.

So, through the circumference we are getting the heat that is coming in and that is given as a constant heat flux q_0. So, remember it is a constant heat fluxes on the surface q_0 problems. If the problem is changed to constrict surface temperature, then that becomes a separate problem. So here, we are talking about constant heat flux and that helps us make some of the assumptions. We again make some assumptions like fully developed flow. So, this means that, this is given by the Porsecille expression then ax symmetry so, no θ variation and constant inlet temperature. So, the T_0 is constant.

And we say that, flow is effort in the thermal field and not the other way, which means that we decide the fluid flow and then, directly use it in the temperature equation. Then how the temperature is changing, will not change the flow by way of affecting the properties. So, we will not look at the cross coupling. We will look at only one way coupling. So, look at this problem and let us see how to go about.

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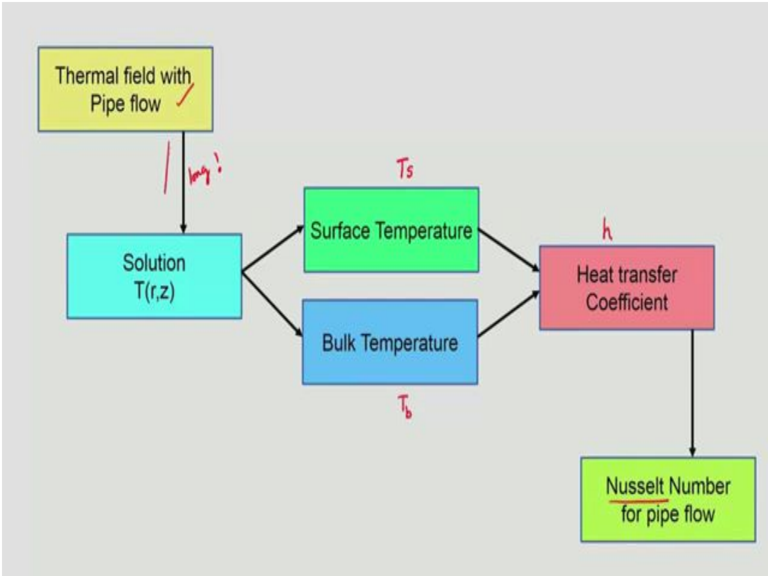

So, here is the overall map of these concepts because it is a long derivation. I want to just keep the bird's eye view. So, we want to derive the thermal fluid for a pipe flow and we will first seek the solution. So, this is a slightly long and once the solution is available, then we want to look at what with the surface temperature and what is called the bulk temperature. We will define that later on and from these two, we want to define the heat transfer coefficient and then from there, we want to then come to the concept of Nusselt number, which comes out quite an elegant expression in this problem. So, let us now embark on this particular process.

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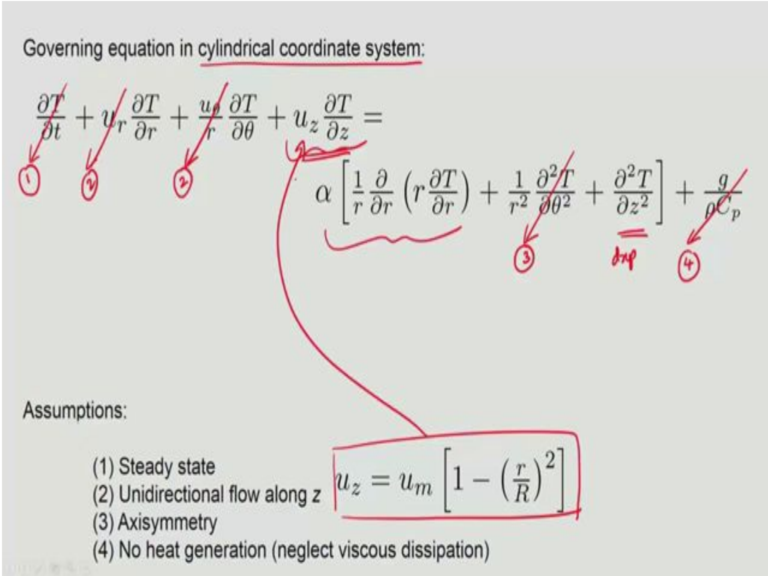

So, the equation to start is basically generalized Fourier heat conduction equation for a cylindrical coordinate system and that is what is written here. So, you would see that, this has many terms and we are going to drop many of them. So, steady state implies that we can drop this term and the unidirectional flow is along the Z direction, which means that u_r and u_θ are not there. So, which means that you could then drop this term and this term and there is axisymmetry in the problem. We say then T is not varying as a function of θ. So, we will drop this term and there is no heat generation term and which means that, you could actually see that viscous dissipation is neglected.

So, which means this is a fourth term. Now, we need to have a decision here and we say that here the $\frac{\partial T}{\partial z}$ is not a function of z, which means that the maximum way by which temperature is varying along the length, is only first order slope and not. So, we would like to drop this off. So this means that, we can actually simplify the solution a bit. So, which then leaves us with these terms and we have the flow decided already, the functional form which is what we would actually introduce into this ok.

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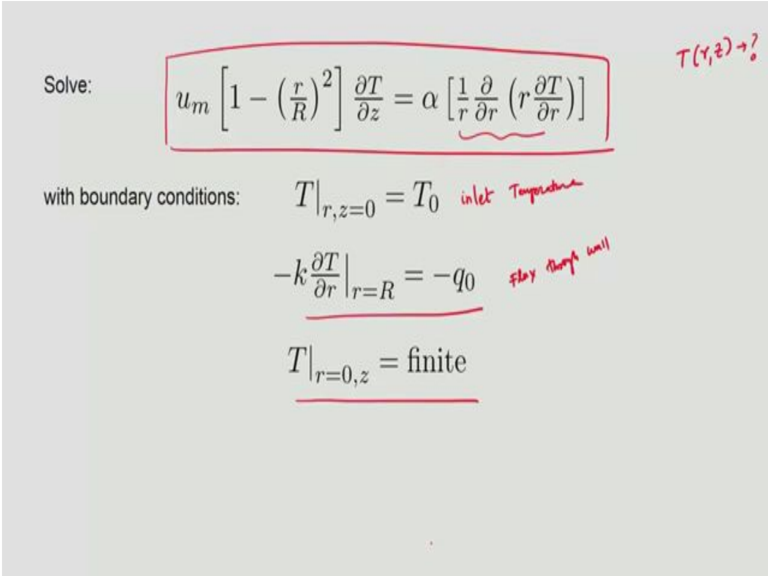

So then, that governing equation would come out o be here, in this manner. So, you would see that there is a $\frac{\partial T}{\partial z}$ term and then, $\frac{\partial T}{\partial r}$ term. So that, we actually seek temperature as a function of r and z. So, that is what I am wanting.

Now, the boundary conditions are to be given the boundary conditions, are such that at z = 0, that is at the beginning of the domain. This is the inlet temperature and the condition here, is basically the flux through the walls and then, we also make one more assumption or a limitation to the solution. We say that the at the axial direction, the temperature is finite which means that in case, when we are integrating, you only see the notorious nature of this term. We may have a logarithmic term and which will blow up at r = 0. So, we do not want that.

So, we want to say that, the temperature at the center of the circular tube is finite, which means that the logarithmic term should be dropped. So, these are the assumptions, the subject to which the solution has to be sort for this particular the equation. So, as it appears, this equation looks quite painful to solve, but we will do non - dimensionalization. So, that it can be solved very easily.

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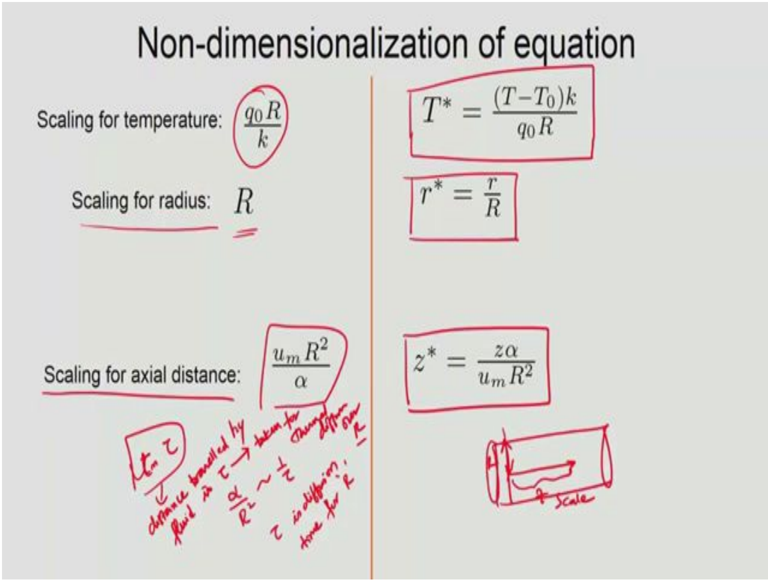

So, the way we do non - dimensionalization is, as follows we basically want to avoid these constants from appearing. So, we do not want these things to keep sticking around. So, we want to then go about in this following manner. Look at the units of q which is the flux, then if you take these units, then it comes to the temperature units and you could then use that as a scaling factor.

The reason, why we are using the $\frac{q_0 R}{k}$ as a scaling factor is, because the boundary condition comes out to be very elegant. So, we define the non - dimensional temperature in this manner. So, that we remove the T_0 from the temperature. So, that you only get the relative increase in the temperature and then, scale it with $\frac{q_0 R}{k}$. So, that the T^* is actually non – dimensional. Now, the scaling for a radius R is quite obvious. It should be the radius of the cylindrical tube that we are talking about.

So, r* scaling is quite a straight forward now. The scaling for that z distance is a little different. We could use the R itself as a scaling factor, but that would not help the equations turn out to be quite messy. So, what we do here is a scale that is actually interesting. So, look at the units of $\frac{\alpha}{R^2}$. So, $\frac{\alpha}{R^2}$ is basically having the units, like $\frac{1}{\tau}$, where τ is the diffusion scale. Diffusion time for a distance R, which means what you have written here is $u_m\tau$ into u_m. So which means that, this is the distance traveled by the fluid in time τ, which is taken for thermal diffusion over a distance R. So, this means that by the time. So, the analysis is shown here.

So, by the time the heat actually diffuses over this distance, how far did the fluid move? So, this is then the z scale and this is R. So, that way we actually have a very intuitive kind of a scaling here and as it turns out, that this kind of a scaling is going to give you a lot of a simplification of the equation. So, the z scale is $\frac{u_m R^2}{\alpha}$. So, you put in the reverse and then, you could see that Z^* is now, non - dimensional. So now, we see these three and then, go ahead and use it to simplify the equation ok.

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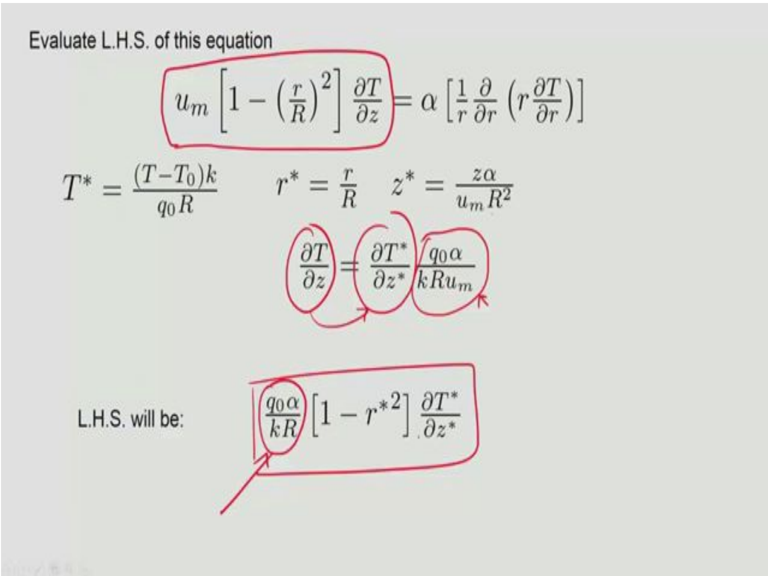

So, what we do is, first really take the left hand side term only. So, we take the LHS and we see that, what we need is $\frac{\partial T}{\partial z}$. So, $\frac{\partial T}{\partial z}$ is nothing, but $\frac{\partial T^*}{\partial z^*}$ and then, you could see that $T^*\frac{q_0 R}{k}$ is what should be there.

So, $\frac{q_0 R}{k}$ is there and z^* has a R^2. So, the R and R^2 will cancel and then, you will get the multiplicative factor here and this becomes the fact that comes, because we have done the non - dimensionalization and when we substitute, will get cancelled and therefore, the LHS will come out to be in this fashion, where for example, $\frac{q_0\alpha}{kR}$ is only present on the left hand side. So, make note of this. So, this is only the present on the left hand side and $\frac{R}{R}$ is already, readily there. So, that becomes R side itself.

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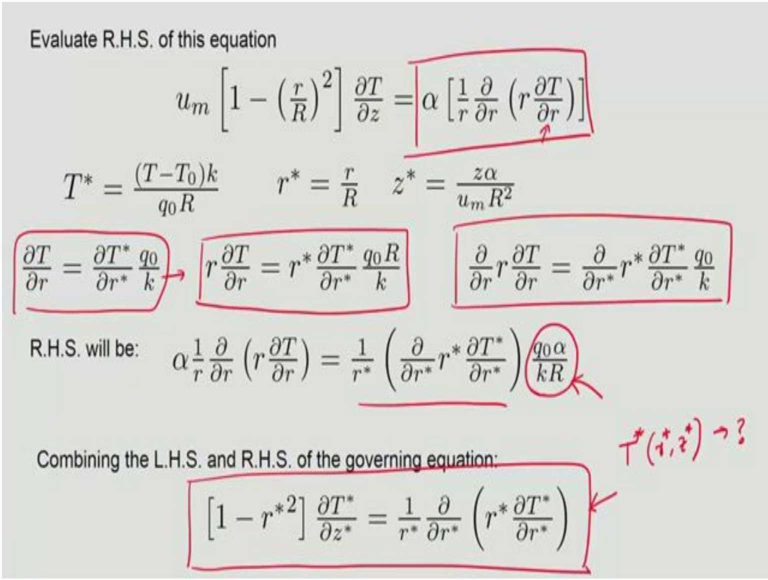

So, left hand side is now fully non – dimensional, accept of this term on the right hand side term, for example, RHS. So, RHS term is going to be non - dimensionalization using same expression. So, what we do is at first, we look at only this term. So, $\frac{\partial T}{\partial r}$ and that straightaway gives a $\frac{q_0}{k}$, because R is already gone with the denominator. So, $\frac{\partial T^*}{\partial r^*}$ will give you $\frac{q_0}{k}$. Now, we multiply with r, and then you get the second one. Then, you would get r present, because r is coming in the front and then, you look at the derivative of that. Then, we see that again, r has gone because you gone to the derivative ok.

So, again you get q naught by k. Now, the RHS will have 1 by r in the front. So, again r will come in the denominator. So, you could see that you are getting the same term which is multiplication on the right hand side also. So look at here, $\frac{q_0\alpha}{k}$ $\frac{q_0\alpha}{k}$. So, this means that when we substitute these two non - dimensional expressions the coefficients are getting cancelled. So, there is a beauty of the non – dimensionalization, which we have done, which means that finally, the equation is going to come like this, which is quite elegant, because it has no term except what we look at.

So, what we now say, that I want to evaluate T^* as a function of r_* and z^* and then, this is actually quite nice, because in this equation, there is no other quantity that is sitting around. Now, even the boundary conditions need to be then interpreted with respect to the non - dimensional way we have done, so that we will do.

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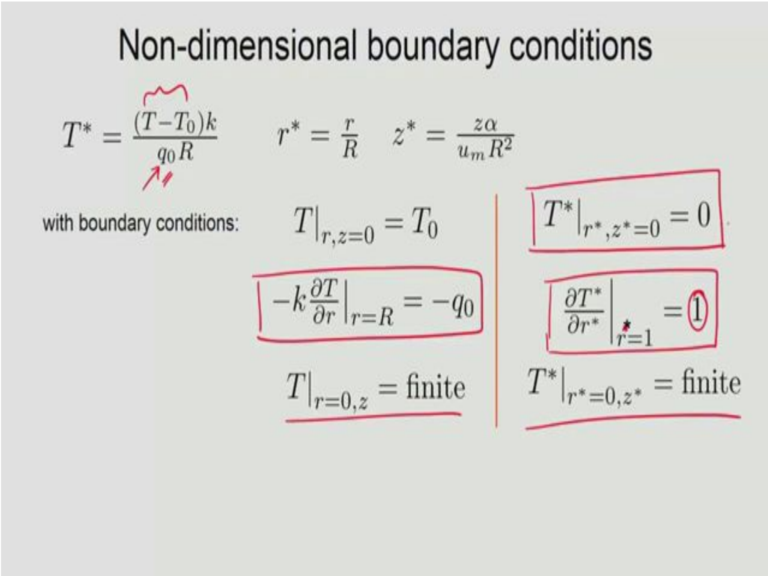

So, when we say the inlet temperature is T_0. So, because we have already seen that temperature scaling is with respect to T_0. So, the boundary condition for inlet condition is that, T^* is zero and the flux condition on the wall is giving as - q_0, which you already saw that we have chosen q_0 here. So, that actually gives you a very elegant boundary condition, for the wall as $\frac{\partial T^*}{\partial r^*}\,|_{r*=1}$, as just one. So, this one is what actually makes life very simple.

So, when you have zeros and 1's in the equation, the integrations become very easy and that is the hindsight, that which we have actually introduced q_0 into the T^* definition and then, when you look at the finite condition, it is the same. We just leave it as such. So, we now have the boundary conditions that are available. So, we can now pose the problem as follows.

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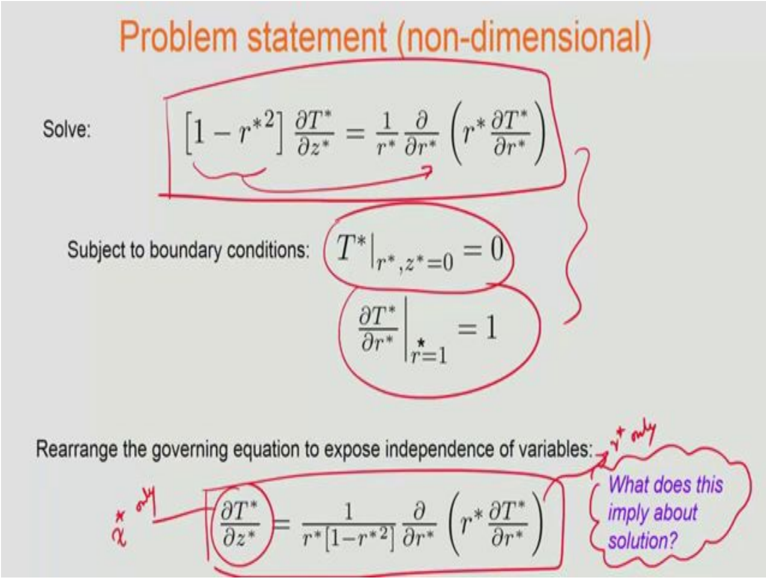

So, the problem is now a purely mathematical problem, it no longer looks like any heat transfer problem. We say that solve this equation subject to the boundary conditions that are given here ok. So, this again, you can directly look up any handbook and look at the solutions. We will actually see how we want to put the solution here itself. Now, look at the equation. We see that, when you take this to the other side, the equation looks rather simple. You see that on the left hand side, you have got only the z^* only on the left hand side, on the right hand side you got r^* only. So, this means that the temperature T^* could be thought of as a function, which is a sum of two functions that is, a function of r^* and function of z^*.

So, this kind of an approach will meet the hint. So normally, when we look at any differential equation, where you could actually separate the two variables on both sides of the = sign, which means there is a sum of two functions of those two variables could actually work out. So, we will then use that concept and propose a solution. So, the way we normally go about solving these equations is, we propose a solution, then we substitute in it and then see how the solution will come out.

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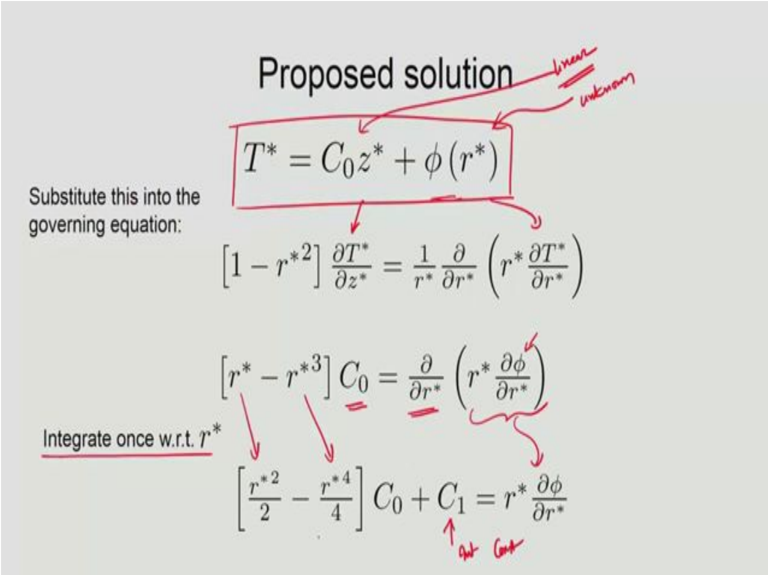

So, the proposal solution which will look like this, that is a sum of two functions. First function is just a linear function of z^* . So, we have already made the decision. So, remember this is linear. We have made a decision, because we dropped $\frac{\partial^2 T}{\partial z^2}$ term. So, this means that we only wanted the z variation to be linear. So, that is put and this is unknown and we want to know what that function ϕ is. So, this means that, the temperature variation along the tube, due to the heat flux from the surface is going linearly, is function of z^*, but there is a correction term that is coming, because of the ϕ, which will give you the temperature variation along the r direction. So, this is quite natural and it is also intuitive, the way we know how the pipe flow will be affected by heat flux from the surface.

So, which then we will substitute. So, we substitute this solution into the equation governing equation. So, when you substitute, then you would see on the $\frac{\partial T}{\partial z^*}$. We have only C_0 and on the right hand side, you have got only ϕ. So then, we have got simpler solution now. What we do is that, now that this is there, you could then integrate the left hand side with respect to r^*. So that you could get this thing off. So when we do that, r^* star goes to $\frac{r^{*2}}{2r^{*3}}$ goes to $\frac{r^{*4}}{4}$. So, this we do by integrating once.

But on the right hand side, when you integrate once, only this will remain and that is what is actually coming here and this is the integration constant. So, now, you can see that slowly we want to integrate. So, that on the right hand side, we have only ϕ. In the left hand side, we have got an expression. That way we got the ϕ figured out. So, we now see that this expression is available. So, we take r^* to the left hand side in the denominator and then, integrate once more. So, that is what we do.

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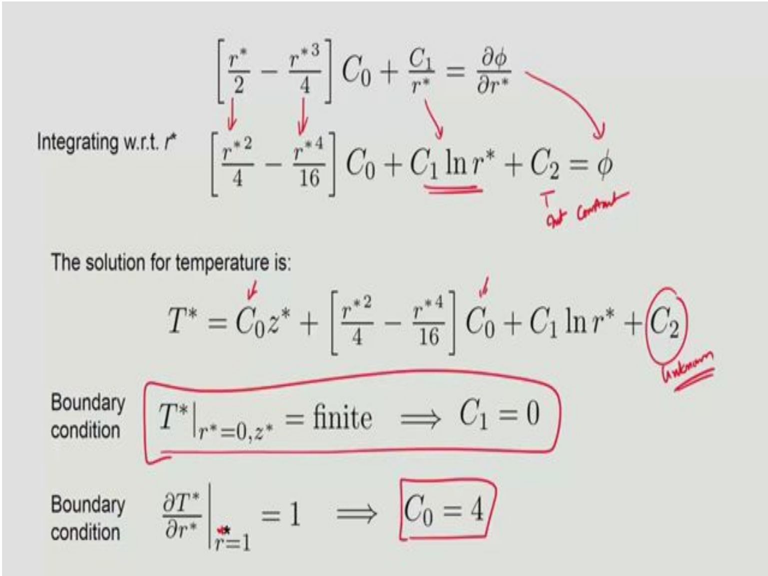

So, we have got $(\frac{r^{*2}}{2} - \frac{r^{*4}}{4})/r$, will give you $\frac{r^*}{2} - \frac{r^{*3}}{4} + \frac{C_1}{r^*} = \frac{\partial \phi}{\partial r^*}$.

Then, we again integrate with respect to r^*, then you will see that this goes as $\frac{r^{*2}}{2}\frac{1}{2\delta}\frac{r^2}{4}$ and this goes as $\frac{r^{*4}}{4}\frac{1}{4}$, that is $\frac{r^{*4}}{16}$ and this goes to the logarithmic term and then, this when you integrated will become just the ϕ and then, this is the constant term. So, we can see that now, we have got this guy, which we wanted to eliminate and then, we say that the boundary condition, which we have already decided that we do not want at $r^*=0$ any problem in the solution. So, we say to be drop C_1 ok.

And then once you do that, then the solution looks a bit simpler and the boundary condition that $\frac{\partial T^*}{\partial r^*}|_{r*=1}$ will make $C_0 = 4$. So, you have got these two in one go available, because when you substitute $r^*=1$, straight away you can see how that C_0 will come. So, this is the only thing unknown. So, keeping this unknown, as it is we will look at the solution now.

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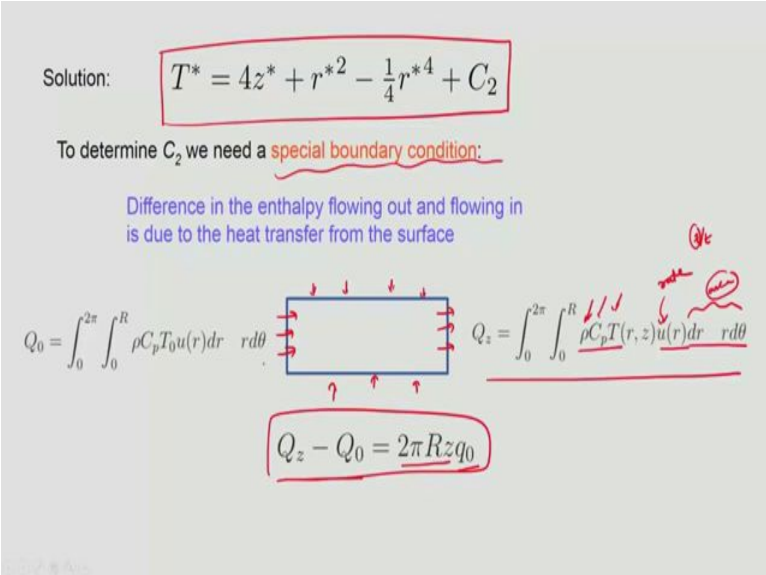

The solution looks quite simple except for one constant, that is there and if you suspect that constant should be just a number, but we want the number to be proper ok. So, we see that there is no boundary condition that we have, which we can use to get the value of C_2. So, what we do is, we introduce a very special boundary condition which basically talks about the enthalpy balance over the entire domain. So, what we want to say is that, whatever enthalpy is entering the domain and leaving the difference should have come only from the surface. This is quite intuitive, this is what it is actually so, but we want to then express it in the integral form. So, that the C_2 is available in the integral form. So that we can evaluate C_2. So, because most of the boundary conditions happen to be in definition form and that is not useful, because this C_2 will then get knocked off ok.

So, we say that the difference in enthalpy flowing out, minus flowing in that is $\frac{q_z}{q_0}$ is because of the heat flow coming from the surface and that is basically $2\pi rz$, which is the circumference into q_0 which will be the flux, that is coming in from the circumference. So now, what we do is at this, we then write it as a boundary condition and then see what will happen ok.

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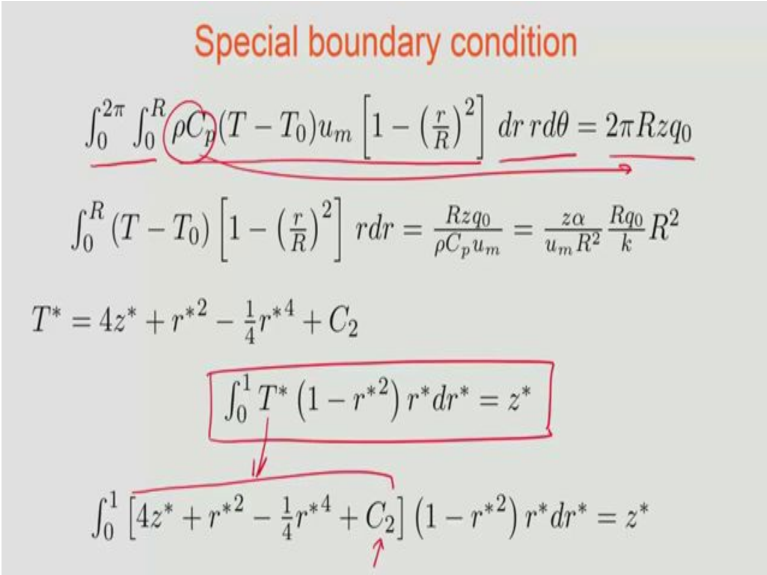

So, that boundary condition if you see, q_z is written in this form. q_z is the enthalpy that is flowing out. So, enthalpy is given as $\rho C_p T$ into the area element and into the velocity ok. So, that is basically giving you what is the enthalpy, that is coming out the u will tell you what is the rate, that is because in the z direction you have got. You will have because z over T essentially and you have got this is the area element. So, you have got area and then z there, is a volume x ρ is mass into C_p into T that is enthalpy and by T means a rate of enthalpy that is going out. So, similarly what is coming in is written and then, the difference of them is basically $T - T_0$ into this entire expression and that basically is written, in this form (ρCp $T-T_0$) x velocity x area element integrated over the entire circumferential area is going to give you the total amount of heat that has come in ok.

And this is basically the boundary condition that we want to use and luckily for us, we have already a non - dimensionalized $T - T_0$. So, we have those expressions that are coming quite nicely. So when, we then look at how would this expression look like, when we non – dimensionalize, then it would simplify in the following manner. We take the ρCp to the other side. So, you put in the denominator and then, we would see that the boundary condition we have written comes out to be a very simple one, which is looking like this. Now, this is what we do, is that we put the solution of T star in. So, that is what is put in here.

So, solution for T^* is available and rest of it is as it is and then, we will let see how we can go about integrating. Now when we integrate, we see that the C_2 is still kept inside and that is what is giving the value of C_2. So, the boundary condition which is an integral boundary condition, is then very useful ok.

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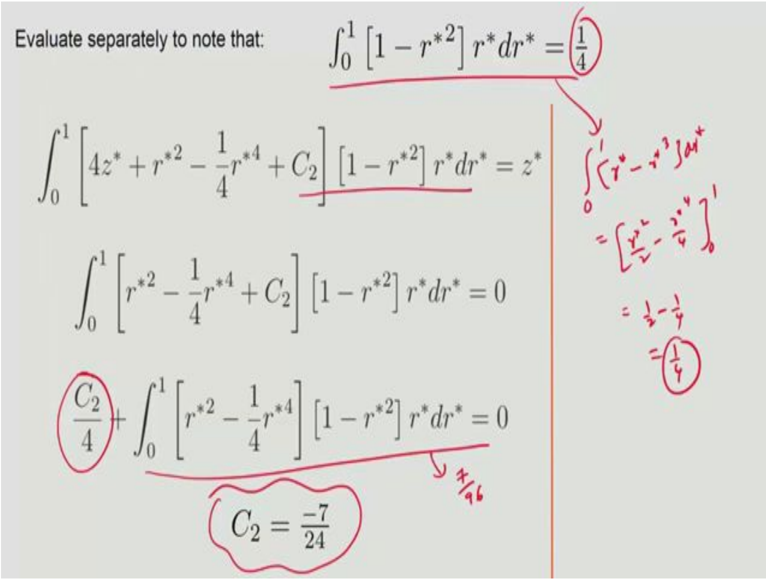

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Now, what we do is that we especially, separately evaluated this and then keep it. So, we can then see that, this is 0-1. 1 - r^2 , that is $[r^* - r^{*3}]dr^*$ and that would be like $\left[\frac{r^{*2}}{2} - \frac{r^{*4}}{4}\right]$ between 0 and 1, that is nothing, but $\left[\frac{1}{2} - \frac{1}{4}\right]$.

Now, this $\frac{1}{4}$ is very useful because, it comes in twice, while we integrate, become you can bring it twice. So, there is one that comes because of C_2 into this. So, that comes straight away $\frac{C_2}{4}$. So, we made the simplification and straight away we write that and then, the second expression is coming to the rest of them and then, that integration is going to give you $\frac{7}{96}$. Now, this is something evaluated in a similar way, as I have drawn and written here. So, it is just integration from 0 - 1. So, it is quite straightforward, only polynomial integration. So, that will give you, though this basically gives you that, the C_2 is - $\frac{7}{24}$ ok. So, once we have got the value of C_2, thanks to the boundary condition, then we can substitute in the solution..

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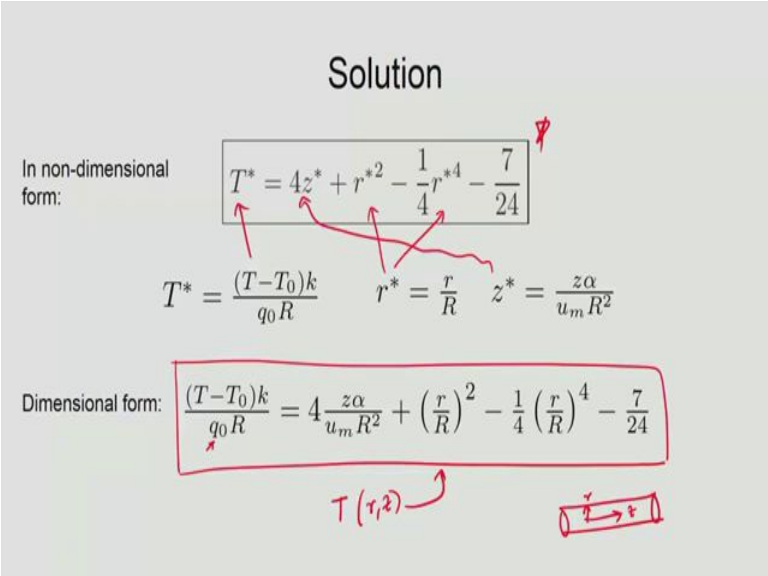

And then, give the solution here, in the form of T^* . The solution is $4z^* + r^{*^2} - \frac{1}{4}r^{*^4} - \frac{7}{24}$ and then, when we substitute these things in, then you could actually see how the solution would look like in the dimensional form of r,z is now available.

So, technically our problem is solved. We have done with the solution and for the pipe at any r and z, we can then find out the temperature. It is subject to the boundary condition, that the heat flux on the surface is constant which is q_0, it is also given here ok.

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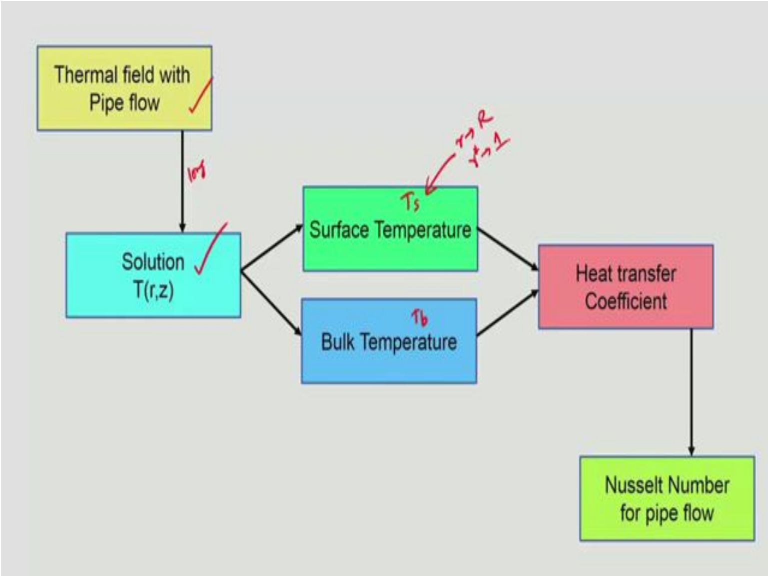

So, now what we do is that, we go ahead and use this solution to arrive at the concept of a bulk temperature and Nusselt number. So, where are we? So, we have set the problem and this long derivation is done and we have got the solution. Now, we only need to get the surface temperature and the bulk temperature concept. So, the surface temperature is nothing, but at R tends to capital R. So, that is nothing, but substituting that $r^* = 1$. So, that is quite straight forward we will do that.

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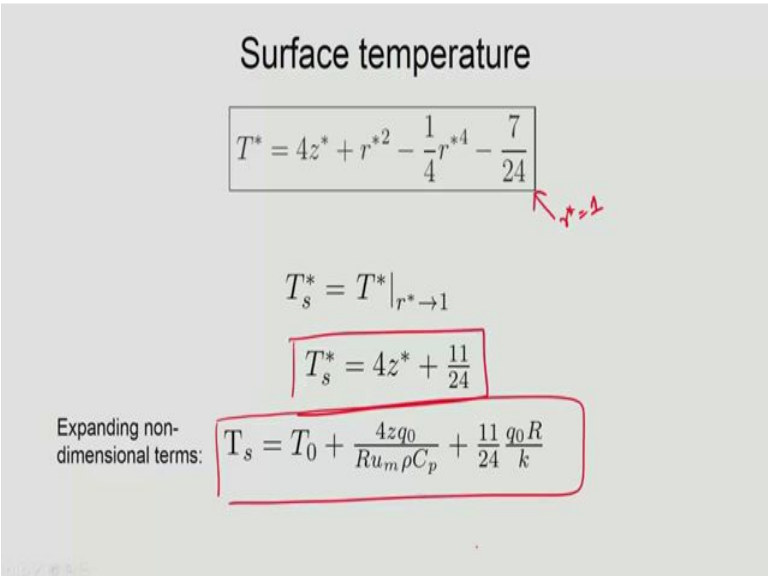

So, simply substitute here $r^* = 1$ and you get what is T^*. So, surface temperature is then given as 4 z^* and then, this is $1 - \frac{1}{4} - \frac{7}{24}$ that gives you straightaway this. Now, this when you expand using the non - dimensionalization expressions, then that gives you the surface temperature as a full expression here ok.

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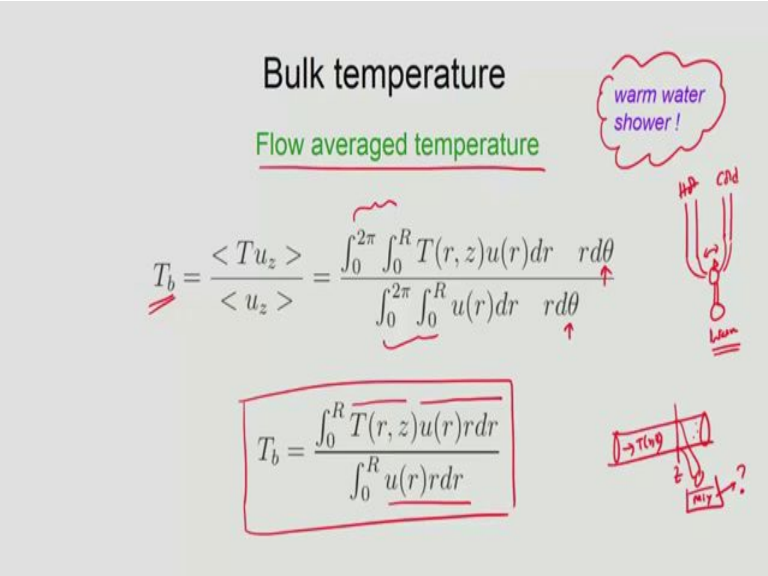

Now, the bulk temperature has to be introduced. So, the bulk temperature, I want you to think about, how you get a warm water shower. Normally, you have got the hot water coming through one pipe and the cold water coming through another pipe and then, you have a knob which allows you to mix the flow rate between the two and then, you have got the warm water here. So, what are you doing basically, what you are doing is, the heat that is coming from the hot water and the cold water is getting mixed by a ratio and then, you are getting the warm water.

The ratio depends upon the flow rate. So, as you can already see that, if the flow rate was very low for hot water, then you would not mix much of cold water. You lower the fraction so that you can get the warm water at the same temperature. So, these some common daily life experience, that we are actually using, what is called flow average temperature? So, the averaging is done with respect to the flow rate and that is exactly what is done with the bulk temperature also. So, bulk temperature is defined as flow average temperature $\frac{Tu}{u}$ and in case of a constant plug flow, you just simply multiply the velocity and that should do, but in other case, for example, the velocity is a function of the R and therefore, you can actually then substitute and then integrate over the entire cross sectional area.

So, this is the integration over the entire cross sectional area and then, you can get the bulk temperature evaluated. Now, the bulk temperature actually means that whenever you have the temperature that is varying as a function of r and z, we do not know how to put thermocouples and measure the temperature. So, what we do is that, at any point at any z, if you want to take that liquid out and to a container and then let it mix and then ask what the temperature of that mixture is and that will be the bulk temperature basically. So, this concept is very useful to know what the effective temperature that is coming out is and the way we evaluate is written here.

Now, we are actually dropping the θ integration on the denominator, numerator because there is no θ variation. So, you would get 2π in the numerator as well as denominator will drop it and you will simplify the bulk temperature definition with only the r variation here. So, we have got T and then, velocity rdr, that is the averaging that is done, now velocity is only available as an expression. So, we can substitute and get this.

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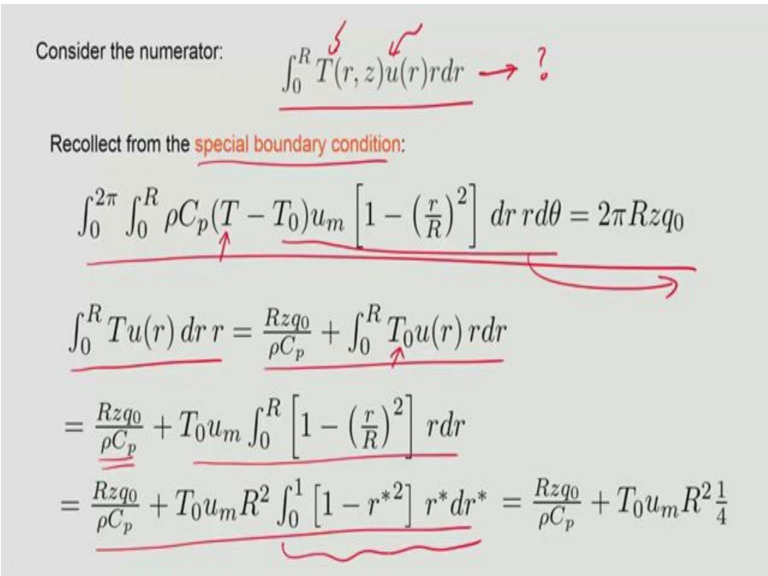

So, the special boundary condition can be used to evaluate this. So, when you substitute the expression for T and u. So, it will be quite formidable. So, instead of getting lost with respect to that particular algebra, what we do is that couple of sides back we have used the special boundary condition. So, we will recollect that. So, we have got this done right. So, use this. So, we write this expression, we bring it on to this slide and here, we write it and we see that the expression is written and what we wanted is only with respect to the temperature. So, take the rest of it onto the right hand side. So, which means that we can actually write $\int Tur\,dr$ is nothing but, then written in this form. So, to the right hand side.

Now once, you take it to the right hand side then it is constant here. So, you can then go ahead and evaluate. So, we are actually postponing the evaluation of the bulk temperature. We just using the expression as it is and then reusing the analysis that we have done earlier. So, now, when you take to the right hand side, then you could see that the first term is actually coming with a constant and this integration can be done and you have got the expression, the T_0 is available in the second term and then, you could then substitute the r by capital R to the r^* and simplify your mathematics. So, that you could see this integral and again, this integral is reused from what, we have done earlier.

So, we have done it here already. So, we reuse that result here and say that there is nothing, but 1 by 4 ok?

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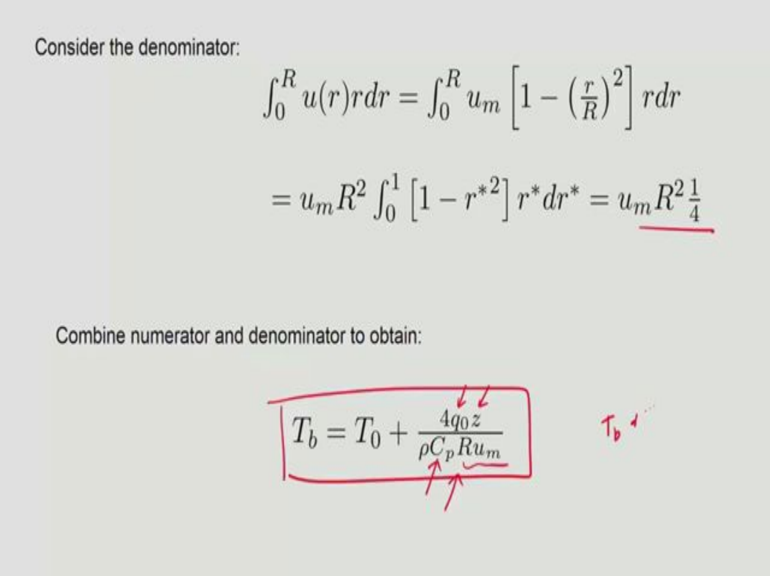

And the denominator is given here. So, when we look at these two terms, then we get T_b as T naught plus some quantity. Now, this means that this is the additional rise in temperature. As you go away in the distances z, because of the flux q_0 and naturally, how much of rise should come with respect to rho C_p and then, the time scales, etc are then taken into account by R and here.

So, this is how the bulk temperature is then derived and given as an expression. So, once you have got Tb and Ts available then we can see what happens to the difference between them. So, the difference is then evaluated to define the heat transfer coefficient as follows.

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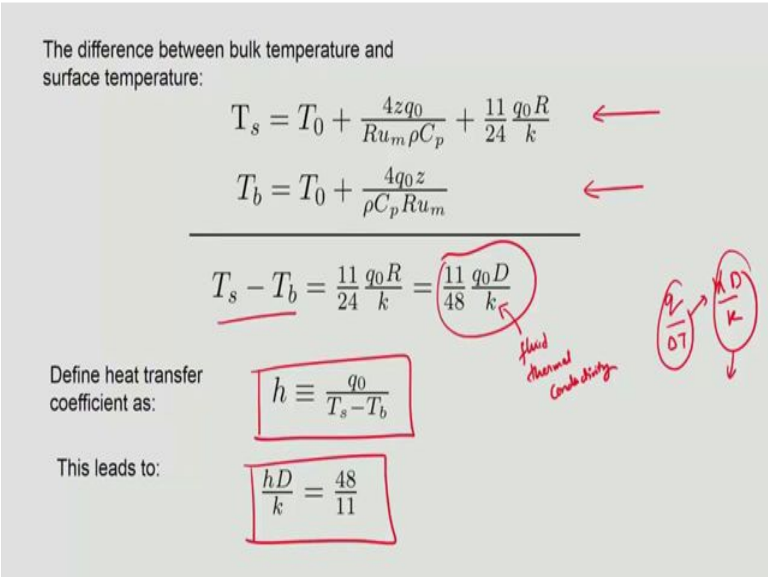

So, T_s expression we have already derived T_b expression. So, let us look at T_s minus T_b and that comes to be quite simple and it comes out to be also something that we already recognize. So, you see that temperature difference and q_0 are there. So, you have got q and then, there is a temperature difference, if you go to the denominator. So, this is somewhat like the heat transfer coefficient D_3 is a length scale and by k.

So, this we already see that it is a very useful kind of a non – dimensionalization, that is we have done, but please note the k is of the fluid thermal conductivity. So, it is not the biot number we are talking about, that is why we give a separate name 'Nusselt number.' So, what we do is that we define heat transfer coefficient in this manner that is for the pipe flow. We defined as a $\frac{q_0}{T_s - T_b}$ and if you define that, then you get this non - dimensional number to be very simple expression as just a number $\frac{48}{11}$ and this, we want to define as Nusselt number ok

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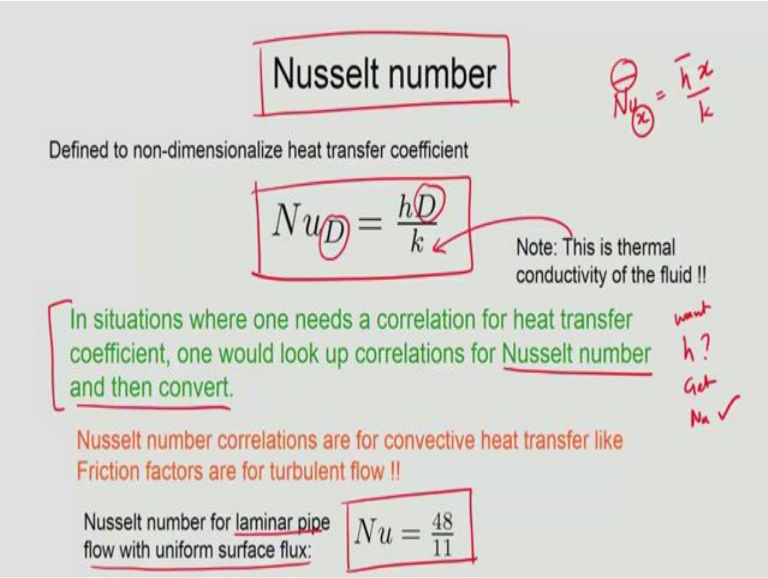

So, Nusselt numbers is defined in this fashion and remember that k is that of the fluid in the case of Biot number, k is that of the solid. So, that the subscript for the Nusselt number. We should always be looked at because, sometimes you may see an expression like this, which means that you are most probably talking about this and sometimes, you may have an averaging, which means that you may have an average to the way, the heat transfer coefficient was measured, etc. So, like Reynolds number you also watch the subscript and superscript for Nusselt number ok?

Now, why is this concept certainly being brought in the reason is that, these correlations for heat transfer coefficient are not available as the raw quantity, you would actually encapsulate with respect to the length scale and the thermal conductivity of the fluid and then, it is a Nusselt number correlations that are available. So, if you look up a handbook and you already have Nusselt numbers available for various situations, you can use them, look at the definition of a Nusselt number and then, get the heat transfer coefficient correlation. So for a given situation, if you want heat transfer coefficient so, you want the heat transfer coefficient first to get Nusselt number and then, you can go back and evaluate what would be the H and then, use it as a boundary condition.

So, it is very important in metallurgical scenario. You have situations where the fluid flow could be quite complex in various geometries and you do not have to go ahead and solve fluid flow in all the situations, very often, the Nusselt number for those situations already available as a correlation. So, we can use it and get the heat transfer coefficient and then, our problem will then become just a conduction problem and the reason why this Nusselt number derivation is important, is because it comes up in a very elegant manner. It says that Nusselt number is just 48 by 11 for laminar pipe flow along with the uniform a surface flux conditions.

So, you could see that even when the surface flux is not constant, but you would have a constant surface temperature then, Nusselt number will come to be constant, but be $\frac{48}{11}$ to be a some slightly lower number. So, you could actually look up those correlations. Now, there is another situation where Nusselt number turns out to be just a constant, just a number and we look at it and that is a very small problem and to expose the beauty of the Nusselt number correlations, let us go through that.

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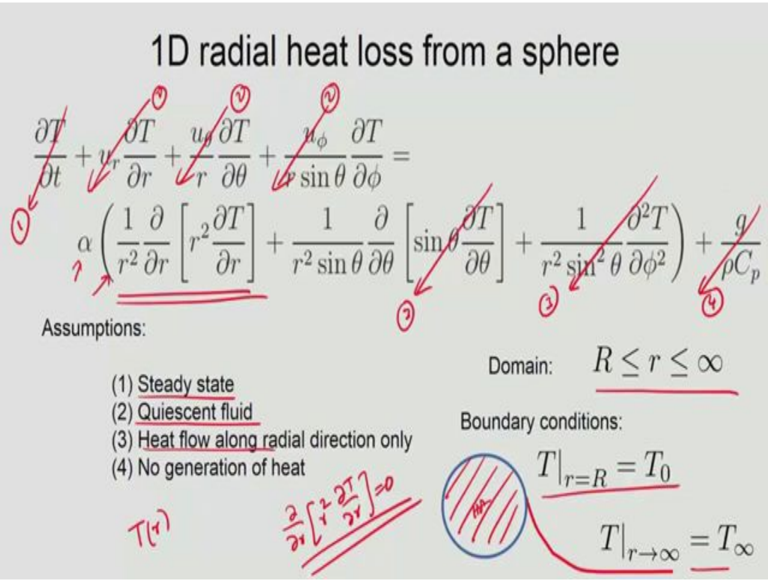

So, this is a situation of a sphere which is, let us say, hot sphere which was losing heat into fluid. Now, the fluid, we say is a quiescent fluid that is its not actually having an advection. So, that I will then use it to drop these guys out by either assumption 2 and we say that this heat loss is taking place at steady state. So, which means that I would like to drop the stuff and we want to say that, the heat flow is only in the radial direction which means that then, I would like to drop this and then, we say that the fluid flow is not happening. So, there is no heat generation at all. So therefore, I drop this.

So, which means that my problem is reduced to only this term is equal to 0, which means that the α and $\frac{1}{r^2}$ can also be dropped. So, my equation is quite simple it is nothing, but $\frac{\partial}{\partial r} r^2[\frac{\partial T}{\partial r}] = 0$. So, this is the equation, that I need to solve to get the temperature as a function of r away from the sphere and subject to the boundary condition, that on the sphere the temperature is T_0 and far away infinite distance away from the sphere fluid temperature is T_∞ ok. So, subject to do that in the domain between r and ∞. Let us inspect how the solution would look like.

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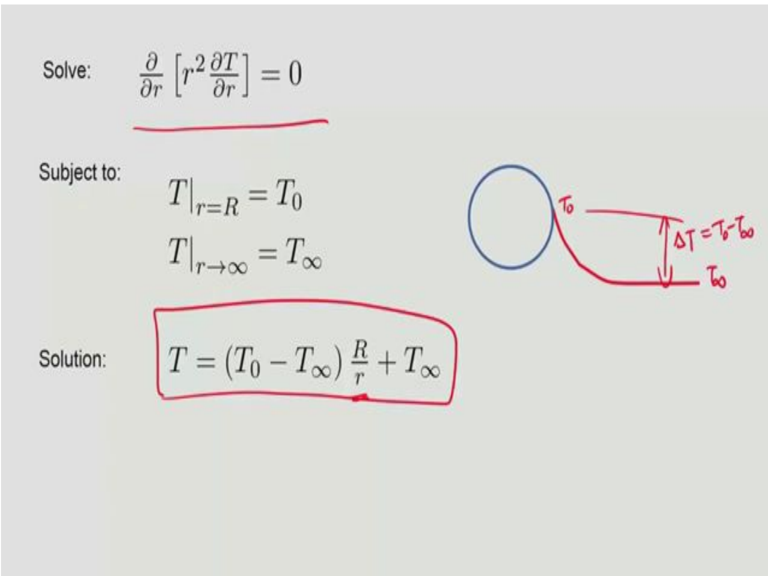

And this is already done by us earlier. So, you could see that this can be integrated twice that will give you a functional form of $\frac{1}{r}$. So, this is a solution you can already inspect that when r tends to capital R then you get T_0, when R tends to ∞, you get the T_∞. So T_0 and T_∞ ok.

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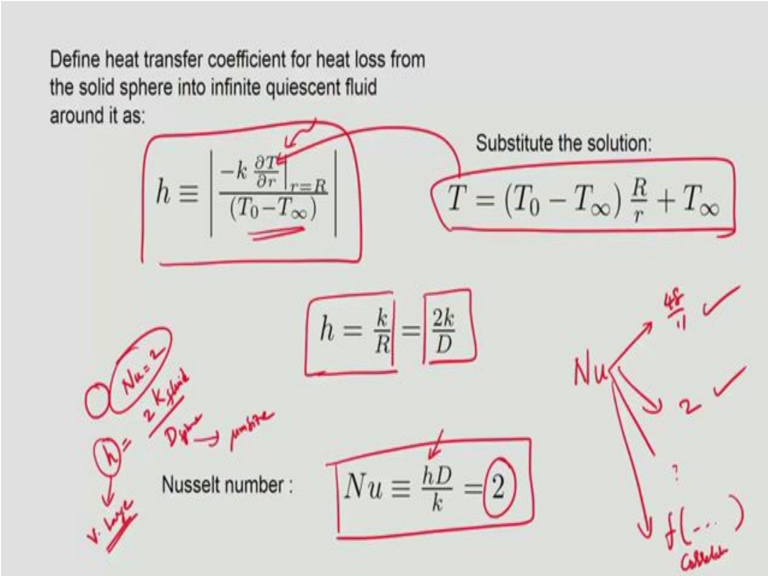

And this solution then can be used to define the heat transfer coefficient and then inspect what would be the Nusselt number. So, heat transfer coefficient is defined as at the conductive heat flux divided by the ΔT. Now, the ΔT here, is quite obvious is nothing, but the difference between these two. So, this is the ΔT, which is basically T_0 - T_∞. So, we use that in the denominator and the conductive heat flux away from the sphere is given that $-k\frac{\partial T}{\partial r}$ evaluated at r = R and this is the solution that we have for the temperature. So, we substitute that here and then, we will see that it straightaway gives you $h = \frac{k}{R}$, which means $\frac{2k}{D}$ because we want to then look at what is the expression that comes out with D, because Nusselt number has that.

So, we can straightaway see that Nusselt number for this problem comes to be very elegant just two. So, you could see that there are problems in which Nusselt number comes to be $\frac{48}{11}$ sometimes 2 and so on and very often; it will be a function of various quantities at the correlation. So, these two cases we know what are the situations for which it comes. So, like this, look out for a very elegant expression that can actually help us in get the getting the heat transfer coefficient. Now, straight away from here, we have some conclusions that can be drawn. Now, what does it mean when we say a sphere will have a Nusselt number = 2. It means that the heat transfer coefficient is given by $\frac{2k_{fluid}}{D_{sphere}}$, which means if the sphere is in micrometer size, then heat transfer coefficient is going to be very very large, which means that, when you want to solidify a liquid metal, without any partitioning, then if you can spray this liquid metal as a tiny droplets, then each droplet would have such high heat transfer coefficient, that it could freeze at almost a million kelvin per second.

And then, you will have partition the solidification, you get metallic powder that is very homogeneous. So, in the spray forming of a metal powder manufacturing, what we are actually using is that $N_u = 2$, idea to get the h to be as large as you can and thereby avoiding slow solidification in segregation, etc. So, you can already estimate how the heat transfer coefficient B for a tiny sphere, by just knowing that $N_u = 2$ and any other change, because of the fluid flow we are looking addition ok.

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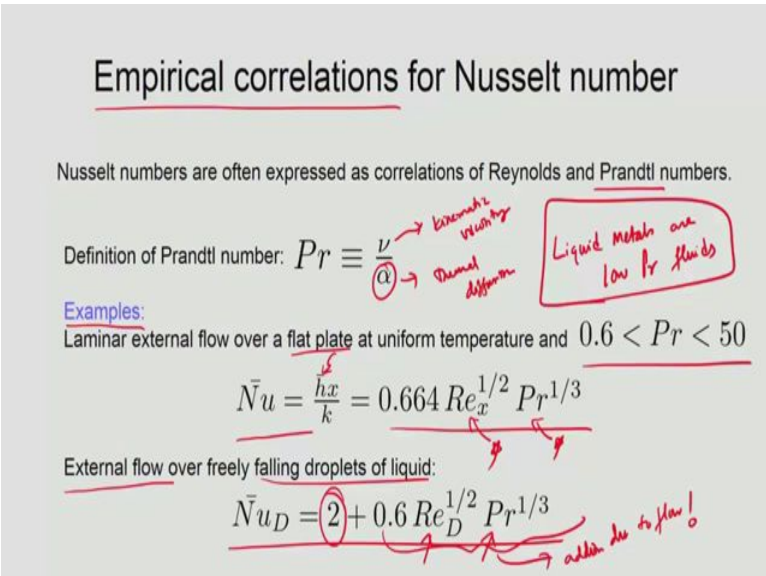

Now, there are empirical correlations that are then available for Nusselt number because we now know the functional form, how to go about some of these correlations are given as a function of the properties. So, the property is one of them of course, is Prandtl number, we call it as a property because this is basically kinematic viscosity and this is basically thermal diffusivity. So, the both are properties. So, basically Prandtl number is nothing, but it is a property of the material and we want to take a ratio of these two, because we are actually looking at coupling off with the fluid flow and the heat transfer and liquid metals. The liquid metals are low Prandtl number fluids ok.

So, this is very important for us to remember and what happens is that, in many - many correlations, when the Prandtl number is very small, we can make many approximations. So, you can say that the thermal fuel gets the set up very early and so on. So, like that we can actually make approximations and there are some examples that I am just listing here. So, the complete list of correlations that you would find useful will be available in the course website, but for some examples you can see that the Nusselt number will be given as a correlation like this for example.

It will be given as a function of Reynolds number and Prandtl number, which means that as the velocity changes and you can substitute and get the Reynolds number, Prandtl number for that particular fluid you are using and straight away you get the Nusselt number. The moment the Nusselt number is available, you can get the heat transfer coefficient. So, for example, if you say that I have changed the gas that is actually cooling the flat plate from argon to helium, then how much the heat transfer coefficient will change? So, you could see that it comes via this because of the Prandtl number, because of the Prandtl number you have got α and then, α is going to be different from argon to helium. Helium is a little bit more and straight away you can then see how the h will be changing.

And then, if you say that the flow rate of that gas is doubled, what would happen then? We say that it is a Reynolds number that is changed, then you can see how the heat transfer coefficient is changing like that and for all these correlations, always remember that there will be validity range, beyond which you should not be using those correlations, for example, that external flow over a fully dropping droplet of liquid, you could say the correlation is given as two plus something. We now know, where this two has come from it, is from the pure conduction solution and rest of it is because an addition due to flow.

So, we can already see, we can make meaning out of these expressions already and here also, the Reynolds number and Prandtl number will tell you how the heat transfer coefficient will be modified as processing conditions or the fluid is changed in a situation like, for example, external flow over a freely falling droplet. Now, like this you can actually look up the Nusselt number correlation. So, for many - many problems arrive at the values and then, get the heat transfer coefficient and use it in the boundary condition for the thermal problems.

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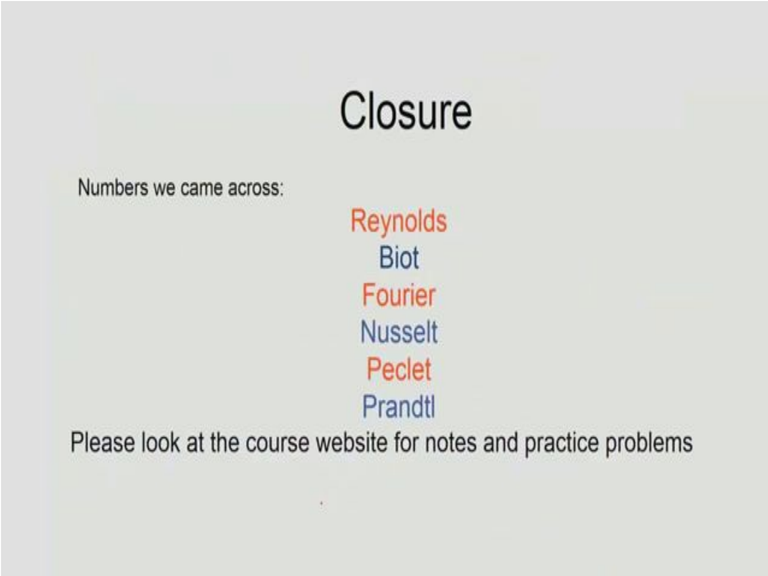

So, over these sessions in both fluid flow and in the heat transfer, we have come across a number of the non - dimensional quantities. We have come across the Reynolds number and we were looking at the scaling of the new, navier stokes equation. We came across the Biot number, when we wanted to see how we can choose the lamp heat capacitance method to be valid or not, we came across the Fourier number in that process and we come across the Nusselt number, because we then see that it comes out to be a simple number for many - many problems, where the fluid flow is directly affecting the heat transfer.

We came across Peclet number, when we saw situations where the flow is along the heat transfer direction and we could encapsulate the distance with respect to the velocity and the diffusivity and then, we also came across Prandtl number to find out the ratio of the diffusivities of the momentum and the thermal fluid. So, there are many more numbers that will be very important in this subject. We will come across them as we do some of these tutorial problems, but now I think this is adequate for us to look at the heat transfer with and without the fluid flow and look at the course website for notes and practice problems that are numerical in nature.