Transport Phenomena in Materials
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Lecture - 18
Heat Conduction Cases – Steady State

Welcome to the session on Heat Conduction Cases. We take up the steady state first. This is part of the NPTEL MOOC on transport phenomena in materials.

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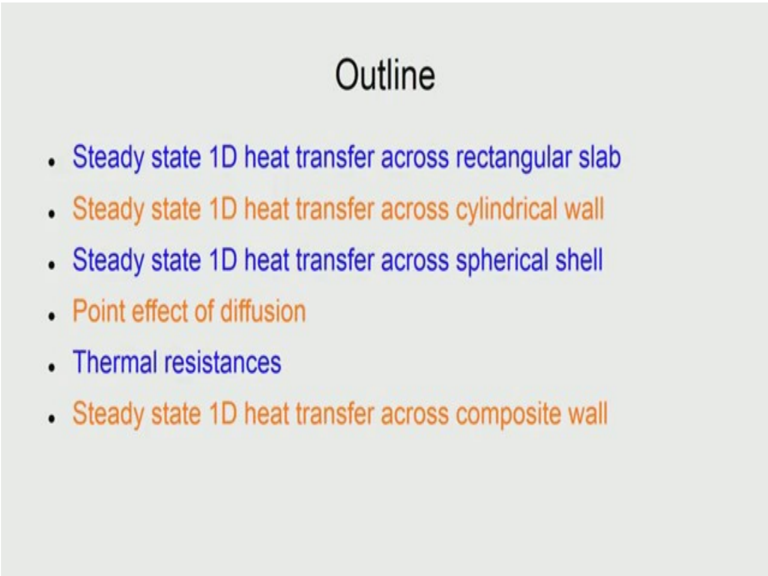

So, the outline is as follows; we first look at 1D heat transfer across a rectangular slab. Then we look at the cylindrical wall and spherical shell also, and see how the functional forms will be differing. We will take up the 2D and transient cases later on.

And through the process of looking at these 1D steady state heat transfers, we will come across what is called the resistance to heat flow or the thermal resistance, and that is where we bring the concept of point effect of diffusion. And then we will also apply this two heat transfer across a composite wall which will be very useful for us, because in metallurgy we come across the furnaces where heat treatment is done and all furnaces walls are basically composite walls.

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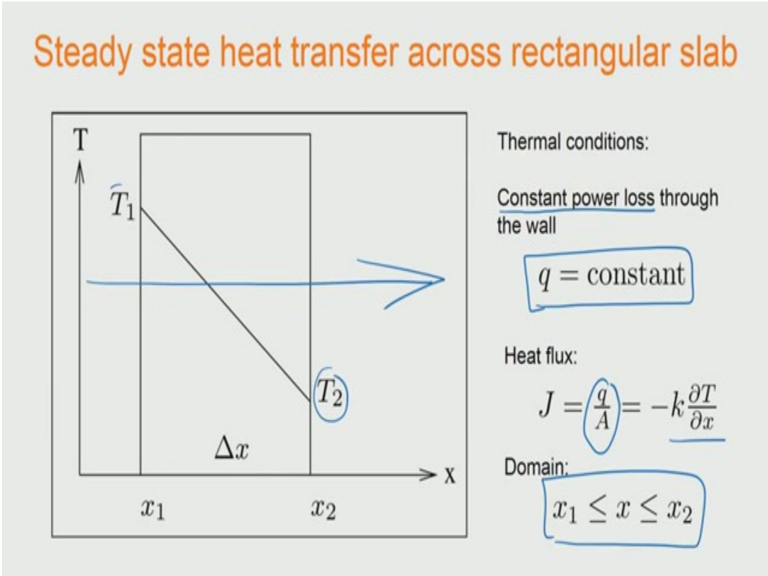

So, here is the first case, where we are looking at this steady state heat transfer across a rectangular slab. So, here the rectangular slab is shown here, and the temperature on either ends is given as a T_1 and T_2, and we have attentively given the temperature profile across this slab as a straight line. We will see that it is of course, valid and we look at the condition where the power loss is constant. So, constant power loss is looked at here and we say its power loss or constant heat flow. In the case of rectangular slab both are equivalent, because the cross sectional area A is same, but in cylindrical and spherical cases so there is a big difference between calling a flux being constant or the power loss being constant.

So, we say that this is basically constant across which means that the heat loss in this direction is constant, you know across the slab. The domain is identified, because we already defined that our distance along the x direction is between x_1 and x_2 and what happens beyond, is only coming through the boundary conditions and the boundary conditions are that at x_1 the temperature is T_1 and at x_2 the temperature is T_2.

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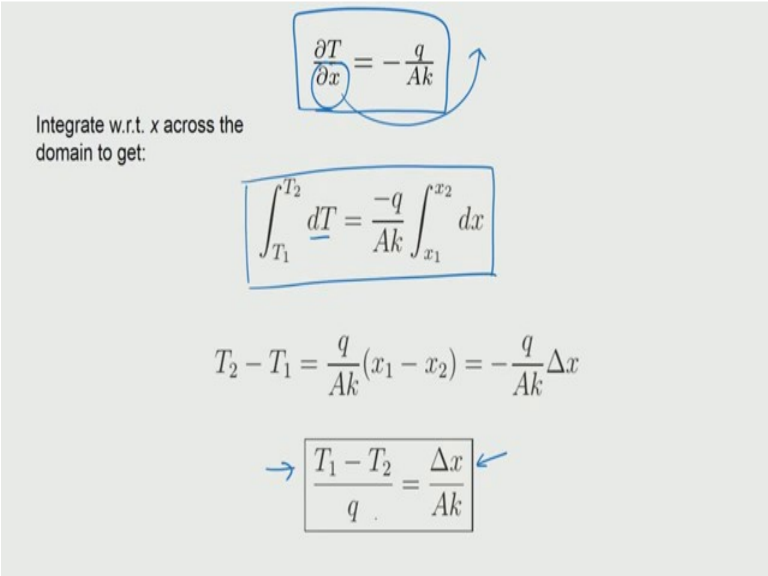

So, what we do basically is to write this equation which is basically the Fourier heat conduction equation and then take the terms on (Refer Time: 02:31). So, that you could write it with dT/dx on the left hand side. So, when we, what we do is, then we take this value to the other side so that we can write dT = - q dx /(A k) and then we do the integration across the domain and we saw that the domain has temperatures T_1 to T_2 on the temperature side and then x_1 to x_2 for the distance

So, we do this integration and this will give us the solution, and the final solution is looking here, basically it is giving you that the temperature differences are related to the distances; that is basically the straight line. So, this expression straightaway gives you the solution as a straight line which is already drawn for us in the schematic plot here.

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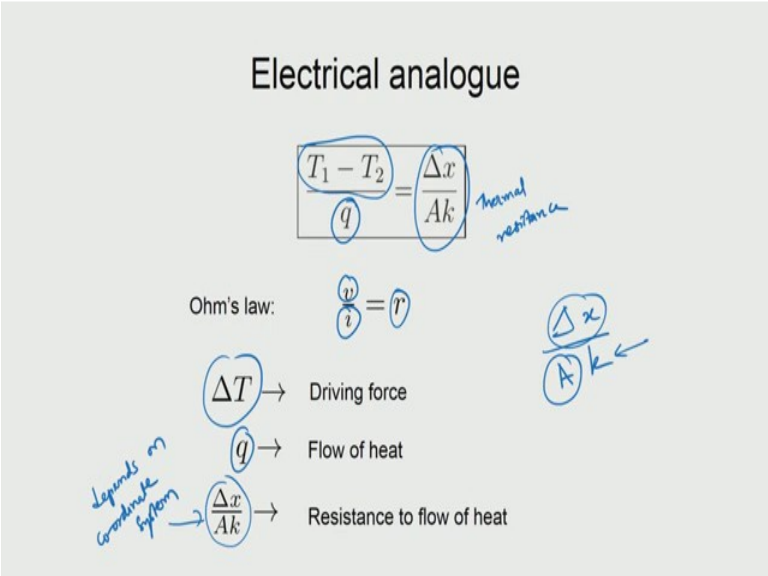

And what we see is that we do this analogy with respect to the electrical analogy, where we look at the Ohm's law, and what we see is that the difference in the temperatures which is basically the driving force for the heat flow and that can be looked at as a voltage, which is basically the driving force for the current flow and the heat flow itself is in the denominator, which is somewhat like the current flow and the ratio of these is a resistance, which basically is for us in the case of a thermal problem, it is basically a thermal resistance ok

So, that way we are able to then compare the ΔT the role of Δ T the role of q the heat flow, and the role of thermal resistance Δx/(A k). So, this form depends on the coordinate system. So, we must watch out, and in the case of rectangle coordinate system it is basically Δ x/(A k) which then we can already look at what is the implication.

So, what it implies is that, if the thermal conductivity is increased; that means, it is able to send the heat across very easily. So, therefore, the resistance should come down if there is a cross sectional area; that is large; that means, the heat can go across larger area. So, therefore, the resistance is less and if it has to go over a longer distance then that also means that the resistance is more. So, the proportionalities are, as we can only imagine from our daily life experience, but then we now have a mathematically defined way to look at what is called as a thermal resistance.

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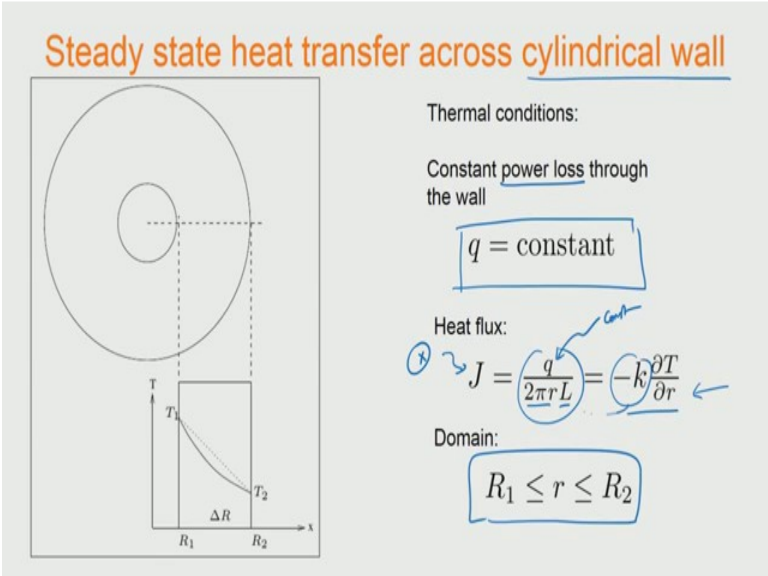

So, we can then look at the same thing in other coordinate systems as well. So, we take up this cylindrical wall now. And in the case of cylindrical wall we again look at the same problem, it is the power loss which is given as a constant and then we already saw that the power loss is q, and the flux is then given by q/area.

So, at any r the area is a cross sectional area over which the heat flux is happening, is given by the perimeter 2 πr × L (height) of the cylindrical surface through which the heat is flowing and that is equal to - k dT/d r, and it is a q which is constant this is constant, but not this, this is not constant. So, this is where the difference between the cylindrical and rectangular coordinate systems will come, and the domain of course, is defined between R_1 and R_2 where R_1 is the inner radius and R_2 is the outer radius of the cylindrical wall. So, now what we do is that we look at this expression and see how the temperature profile should look like and what we do is, we take this to the other side and right.

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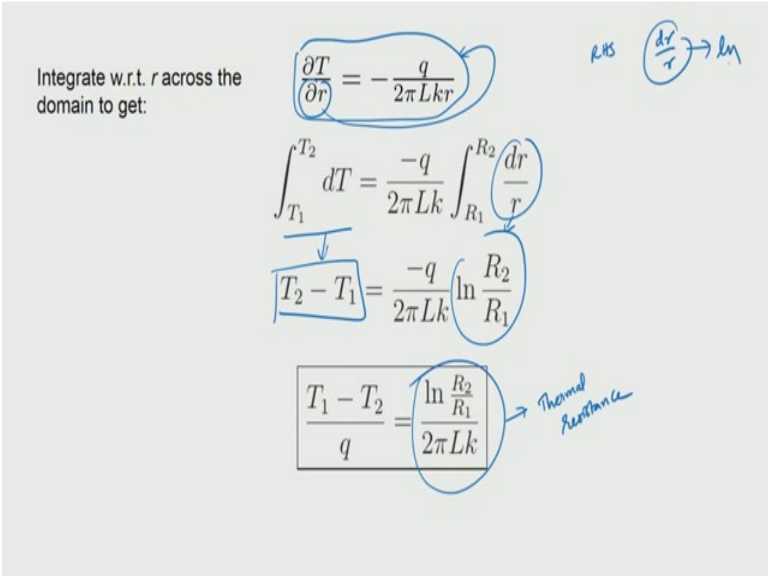

So, this is how it is written, and the moment we do this then what we can do is integration. So, we bring the d r to the other side. So, we can see that on the right hand side you have d r/r and this can integration you should give logarithm. So, that is what we will have.

So, the Δ T is coming out straight away from here, and here you could see that this is going to give you logarithm and rest of it, is basically directly applying the integration within the limits of the domain, and which means again we get the similar expression ΔT/q is equal to an expression, which has logarithm and this entire thing is basically again the thermal resistance in the case of a cylindrical coordinate system, and you can see that the denominator does not have area, it has 2 π L only; that is only the length scale which means that it definitely needs to be differentiated from the rectangle coordinate system. Now, what we do is that, we will see what happens when the inner radius of the cylindrical wall is very large.

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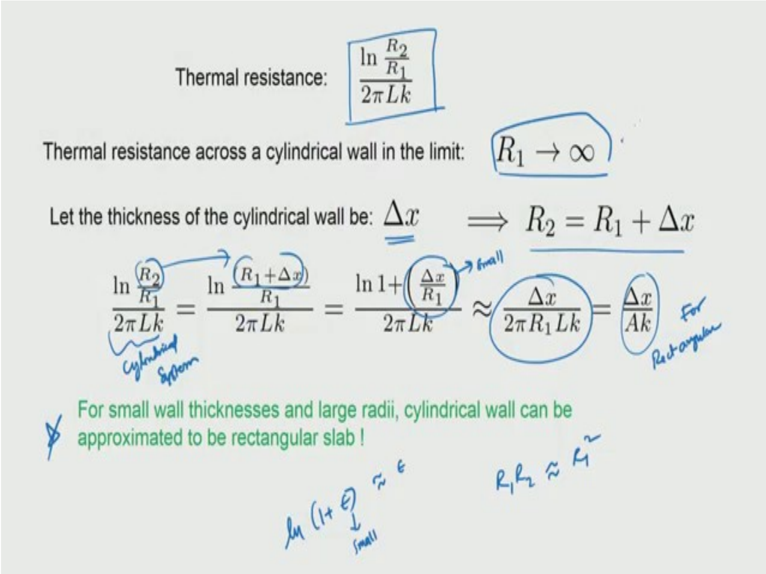

So, we take up the thermal resistance expression and we look at what happens in the limit of R_1 tends to infinity, which means that we take the Δ x to be very small compared to R_1, and if it is very small then it also means that $R_2 = R_1 + \Delta x$ and also, we can also approximate R_1 R_2 as approximately R^2_1 ok. So, if you do these approximations and then look at the thermal resistance expression. So, you can expand R_2 as $R_1 + \Delta x$, and then you can divide a numerator and denominator here with the R_1 and therefore, you get 1 plus this, and this is very small, and if you already know that $\ln(1 + \epsilon)$, when ϵ is small can be approximated as ϵ itself. So, then what happens is we write $\Delta x/R_1$ here and then immediately we can see that we retrieve the expression which is for actually rectangular coordinate system.

So, which means that we start off, which is for cylindrical system and with the limit it, we are able to arrive at the expression for rectangular coordinate system, what it implies is summarized here, it says that for small wall thicknesses and very large radii of a cylindrical wall, then we can approximate the situation to be a rectangular slab. Now, this is going to be useful, because in situations in metallurgy, where these things may be common, then we do not have to complicate the problem by going to cylindrical coordinate system. We may approximate it to be rectangular coordinate system also and get very reasonable answers; of course, subject to this being true.

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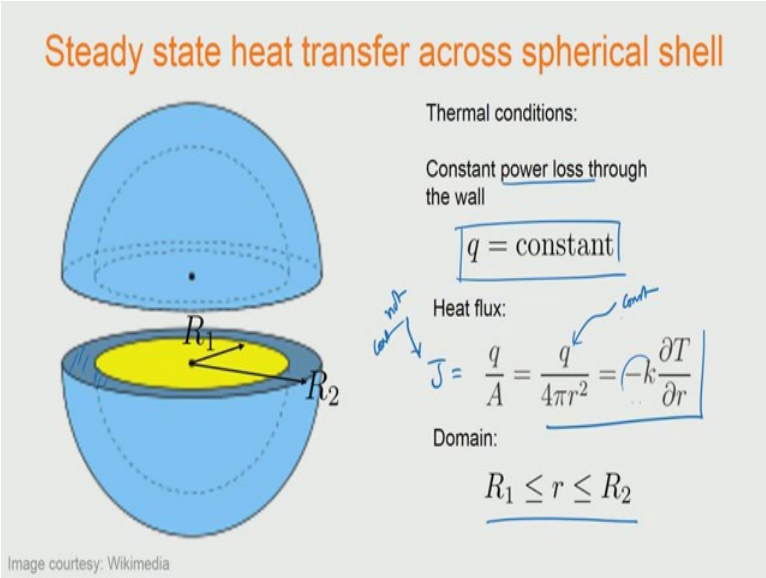

Now, let us look at the steady state heat transfer across a spherical shell, and here the spherical shell is shown here. So, this is the wall and the inside is showing as yellow, and outside is this blue, and the condition for which we are looking at, is again the same thing namely the constant power loss. Now, we will look at the flux expression. So, J and we write J = q/A and that is written as $q/(4\pi r^2)$which is then equal to - k $\partial T/\partial r$, which means that we are taking only radial heat transfer. There is a one day heat transfer and here it is the q, which is constant, and this is not constant; the reason being that the area is changing as you change the radius, and therefore, the flux actually is not constant when you go from R_1 to R_2, but the q is constant. There is a problem that we are having at hand and the domain is limited from R_1 to R_2 ok. So, what we do is that, we take this expression and this equation, we take the q to the other side.

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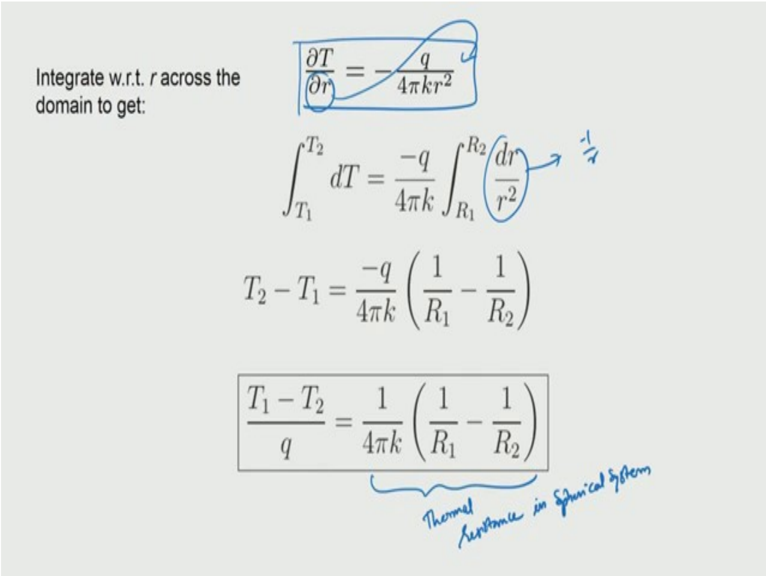

And then we integrate and so we have this expression and here the d r is taken to the other side, and we can immediately integrate to see this. Now, when we integrate, we get 1/r, because you have d r/r^2. So, this should give us 1/r with a minus sign and therefore, you can see that expression is given as ΔT/q is equal to an expression which gives you 1/4πk into the difference of the inverses of the radii. So, this means this is the thermal resistance in the case of a spherical coordinate system. Now, we can see that in each coordinate system the thermal resistance has a different expression for the length scales. Now again; like we did in the cylindrical coordinate system, let us inspect what happens when the inner radius of the spherical shell is very large.

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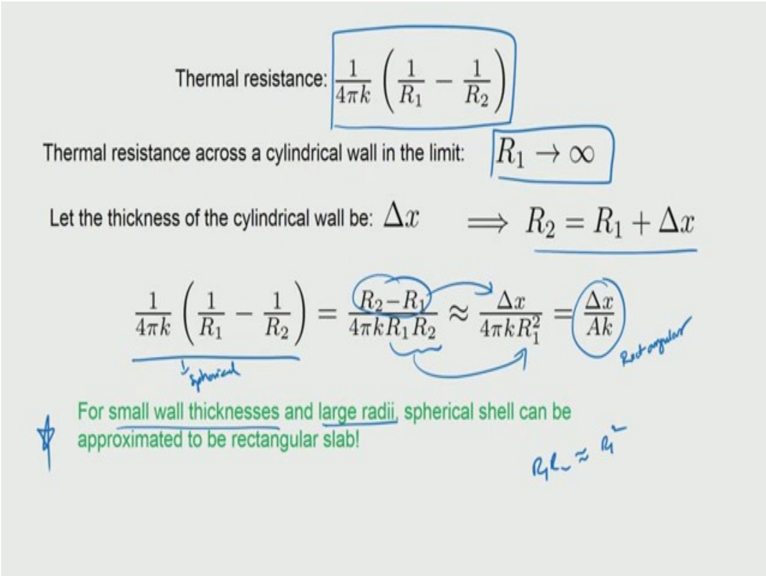

So, we take the thermal expand, the thermal resistance expression and we subject to the limit of R_1 tends to infinity; that is very large radius, which means that R_2 can be written as $R_1 + \Delta x$ and $R_1 R_2$ can be approximated as R^2_1. So, we look at the thermal resistance and see that it is here, the difference is Δx and the denominator we just simply approximated as R^2_1 and immediately we could see that $4 \pi R^2_1$ is nothing, but the area at the radius is equal to R_1, and therefore, again we see that the answer we get is similar to the rectangular coordinate system.

And so we start with spherical and we arrive at the rectangular expression in the limit that the inner radius of the spherical shell is very large compared to; for example, the thickness of the shell. So, what is also implies is just like we did that in the cylindrical coordinate system. We can compare here also saying that for small wall thicknesses and large radii the spherical shell can be approximated to be a rectangular slab. So, we can also simplify problems, sometimes to get the order of magnitude, estimates using this kind of a simplification.

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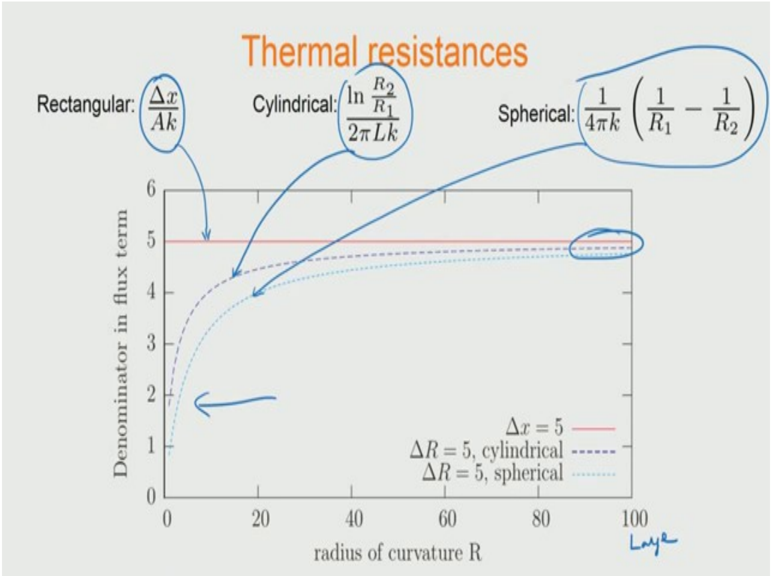

So, here we now write and summarize what are the thermal resistance that we came across in the case of rectangular system. We got the thermal resistances Δx/Ak in the case of cylindrical, we got it as ln (R_2/ R_1) /2 πLk, and in the case of spherical system we see it as 1/4 πk into the difference of the inverses of the radii.

Now, it implies that at large, at large radii, then we see that all three expressions should give you the same value as a rectangular case, which I again see that all the three curves for these three functions I will sort of merge at very large radius, but as you come towards the small radii, you can see start a divergence between these values. So, you could see that the rectangular case is flat, the cylindrical one is drooping down and the spherical one, it droops the much more. So, what this implies is that, the resistance actually is changing and it is very small for the spherical case, intermediate for this cylindrical case, and it is constant for the rectangular case and the resistance is going into the denominator of the flux term, which means that we can say that the flux itself is changing in the inverse manner.

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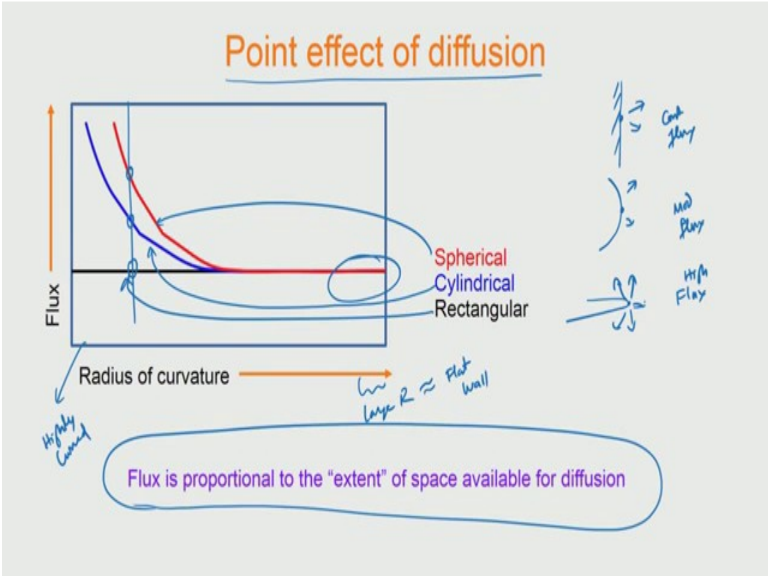

So, what this implies is that at large, this is actually large radius of curvature which is approximately basically a flat wall, and when you take a flat walls, all three coordinate systems will give you basically the same flux across the shell, but when you look at highly curved walls, which means at low radius of curvature, then you see that the three fluxes are different, and for any such value, if you see that you would see the rectangular case will be flat. Here cylindrical one will be here and in this spherical one will be here. So, which means that the three values are going to be different and the spherical one will have a higher flux.

Now, this effect that, because of the curvature the flux is changing for the same ΔT is called as a point effect of diffusion, what it implies is that when you have situations like this; a slab versus a cylindrical surface versus a very spherical one; so as the curvature is increasing then you have a flux; that is large and here it is moderate flux and this is basically constant flux. So, what happens is that, as this a location from where the heat transfer has to take place. So, it is actually accessing more and more space into which the heat has to go away and that extent of space; that is available for heat to go away is basically leading to the higher value of flux, and this is manifesting in the form of the mathematical expression which is different for the spherical versus rectangular coordinate system.

So, which means that we are, basically we come to conclusion that the flux is proportional to the extent of space available for diffusion. Now this point effect of diffusion has already been noticed in many other phenomena. And in the case of metallurgy for example, we notices as sharp objects are losing heat very fast and we say that the corners of a casting will solidify fast. So, that actually is nothing, but an application of point effect of diffusion, which basically says that amount of space available for diffusion dictates, how fast the diffusion can take place.

Now, we look at the composite wall problem and here the ability to draw the schematic temperature profile across a composite wall would already solve the problem half of it and later on when we plug in the numbers and do the simple analysis of the solving linear equations and we got the solution.

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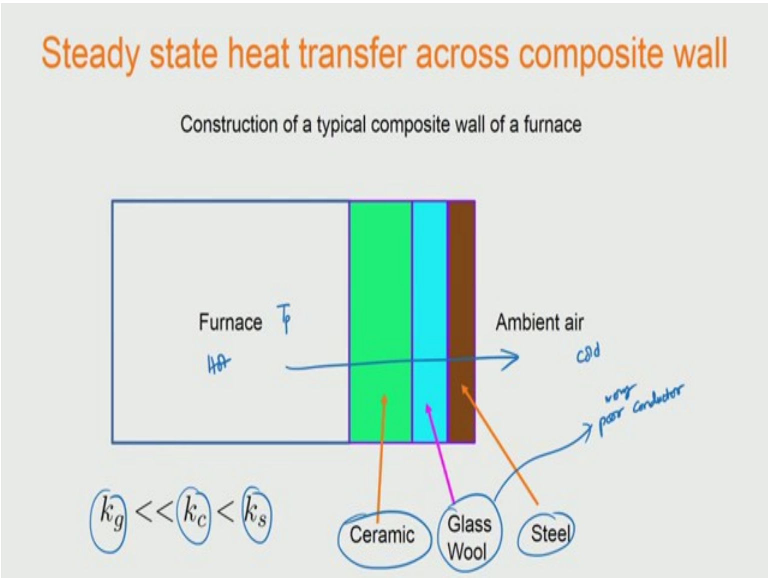

So, the wall is going to look like this, most of the furnaces in the metallurgical for metallurgical laboratories would look like, this is the furnace inside is hot and the ambient air is cold or at room temperature 25 ºC and the inner wall of the furnace, what is facing the hot air is usually ceramic and the outer shell which is containing this furnace is basically steel, and what separates between the ceramic wall and steel is glass wool. Now glass wool basically is a very poor conductor of heat. So, this must be very poor conductor of heat. So, that the heat does not go from the ceramic brick onto the steel container, because whatever heat comes to the steel container will go into the ambient air, and that will tell you how much of power loss is happening, because this furniture furnace is kept at a high temperature over a period of time.

So, how much power you draw from the socket, electrical socket, once the peak temperature has achieved. It depends upon how much heat is actually going out of the furnace through this composite wall, which means that we can design the wall to dictate the thicknesses of these different layers, as well as the properties of these three layers in such a way to minimize this heat loss, so that we draw less amount of power to maintain the furnace at the same temperature.

So, we normally see that the thermal conductivity of the glass wool is very small compared to the ceramic, which is again smaller compared to the thermal conductivity of steel. So, the steel will have the highest thermal conductivity for this kind of a configuration. So, we will look at how to go about drawing these schematic temperature profiles across the furnace.

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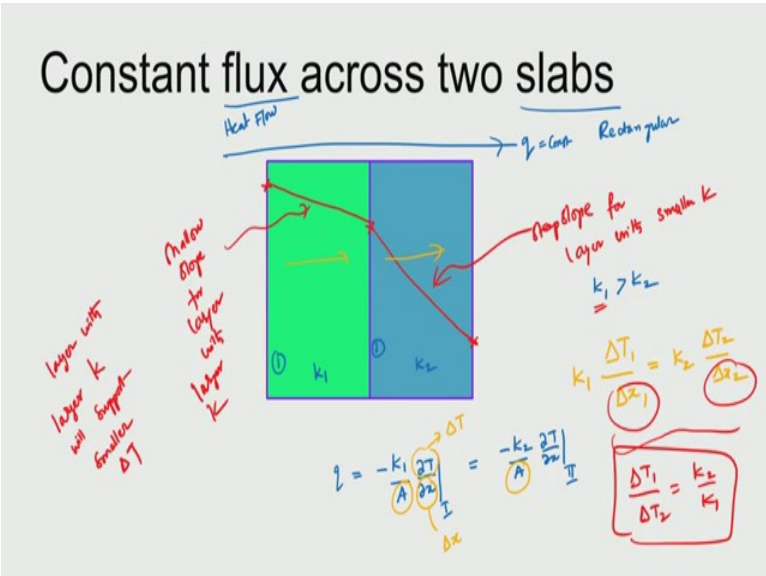

So, for that what we do is, first let us look at how to draw the situation when there is a constant flux across the slabs. So, we take rectangular case, which means that constant flux and constant heat flow are same, and we say that, that is basically q which is going out in this direction and that is constant ok. Now, what we do is that, let us assume that the first case k_1 is the thermal conductivity.

Second case k_2 is the thermal conductivity, and let us assume that k_1 is greater than k_2. So, if this was the heat that is going. So, we would write that $q=\frac{-k_1}{A}\frac{\partial T}{\partial x}$across the first wall and that is also equal to $\frac{-k_2}{A}\frac{\partial T}{\partial x}$ across the second slab. Now, we look at this expression and realize that you could; for example, simplify the situation, saying that the cross sectional area is same which is true here, and we approximate the dT as ΔT, the temperature difference across, because at steady state the temperature profile is anyway linear and this basically is Δ x which is the thickness of the slab.

So, which means that this expression will tell you that $\Delta T_1/\Delta x_1 \times k_1 = k_2 \times \Delta T_2/\Delta x_2$. Now we again look at the situation where you can think that these two walls for example, are at different thicknesses, then you can approximate and see what is the answer, but let us for moment ignore the Δ x thicknesses and see what would this conclude.

So, here we can see that $\Delta T_1/\Delta T_2 = k_2/k_1$. So, which means that if k_1 is larger, its going to the denominator. So, T_1 should be smaller, so which means that the layer with larger thermal conductivity will support smaller ΔT. So, that is what you can conclude from this expression. So, which means that here k_1 is larger. So, the thermal differences between the one end of the first layer to the other end should be smaller ΔT, because k_1 is larger and k_2 is smaller. So, it can support larger the ΔT.

So, what happens is that when we draw, you can draw with the two slopes; such that the shallow slope. So, we say shallow slope for layer with larger k, and we will draw steep slope for layer with smaller k. So, which means that the layer which has poor thermal conductivity, will be able to withstand high temperature differences across it. So, this is a straight forward, the conclusion that we can draw from this steady state heat flux balance, assuming that the heat that is arriving from the left. For example, the heat is arriving from the left; the same amount of heat is also going so there is no accumulation of heat at the interface.

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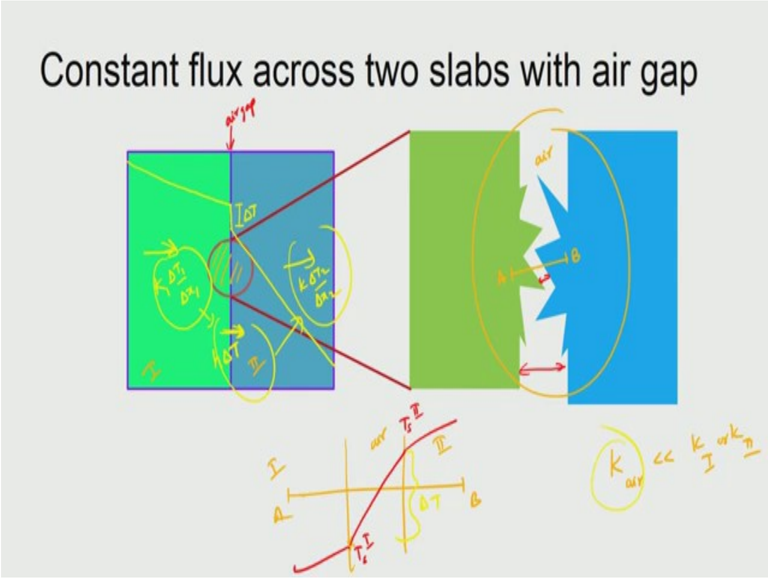

Now, what we do is that we will use this analysis and see what happens when there is an air gap. So, let us say the same situation is happening; only thing is at the contact between the two layers is not very good, and we zoom in this region and see that what happens here is, there is an air gap. So, this is air and assuming that these two blocks are basically metallic and so which means that thermal conductivity of air is definitely much less than either k_1 or k_2. So, what this implies is that, this gap now when you try to plot the temperature across a location like A B what happens is like this A B. So, if you look at the layer differences. So, this is the first layer and this is the second layer this is the air gap

Now, you would see that the temperature plot should actually be like this. So, that the surface temperature on the second wall surface, temperature on the first wall are different and they will be quite steep; that is because the air that is between them is able to withstand that large temperature difference, because its thermal conductivity is very poor.

So, basically this straightforward application of what we have discussed just a slide back. Only thing is that the difference the gap between the two walls is not constant, somewhere it is large and somewhere it is small, but definitely it would withstand a large temperature gap. So, what this implies is that, if you were to then drop the temperature profile across this wall, assuming there is an air gap. So, here let us say there is an air gap, then how do we draw the temperature profile. So, for that what we do is, draw it in this manner.

So, we draw it here for example, the k_1 is large. So, we put this way, and the k_2 is high. So, what we do is that, we make it in this manner. So, that there is a ΔT that is basically, because of these ΔT. Now which means that when you look at the heat that is going in this direction. In this direction it has to go through this gap, which actually should also be modeled. So, we then write an additional term here. So, this heat that is going through the bulk up to the interface would then be proportional to $k_1 \Delta T_1/\Delta x_1$, but the interface resistance is then modeled as $h \times \Delta T$, and then in the other wall again from here, it would be $k \times \Delta T_2/\Delta x_2$.

So, we can then see that the interfacial resistance can also be taken into account, only thing is that instead of directly matching these two quantities, it goes via this quantity. So, that the thermal resistance also is coming into a picture. So, for most of the situations in metallurgy, we have to ask, is there thermal resistance or not in the case of situations like continuous casting. This actually plays a very big role. We do actually have this air gap between the caster and the metallic material; that is solidifing and inside this air gap, we try to introduce ceramic materials to control the thermal conductivity and thereby achieve different cooling rates to avoid cracking etcetera. So, we must pay attention and in these model problems we may ignore, but we must know that we are making a very conscious decision that there is no air gap which actually is not reality in most of the situations.

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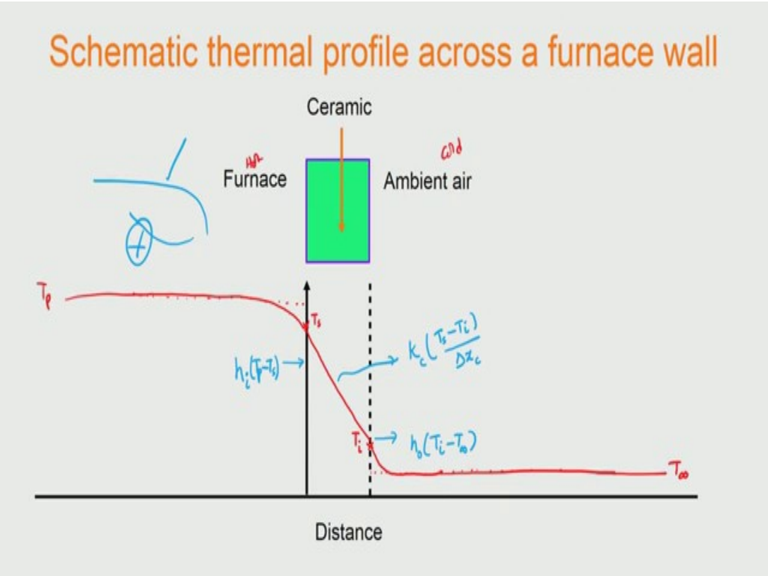

So, let us a for moment we ignore this particular aspect, we do not want to talk about the air gap at all, and want to only draw the schematic temperature profiles across the furnace wall.

So, between each of these walls that are in the composite wall, we say that there is no air gap with that assumption, let us see what happens. So, here I want to just show how we go about drawing this schematic profile, knowing whatever we have discussed till now. So, furnace is hot. So, which means that the temperature on the left hand side, very far away inside the furnace should be flat, because it does not know that heat is being lost through the wall and ambient air is cold. So, which means the temperature profile far away in the air is low here, and its not knowing that there is a furnace out there keeping heat. So, the temperature is flat.

So, these curves should come in and what we do is that we know that the temperature of the wall of the furnace is not the same as the temperature of the furnace. So, you few then were to look at what will the peak temperature. The peak temperature if it is here, the wall temperature is somewhat here and therefore, we should draw it is like that. So, T_s surface temperature would be something like that, and we asymptotically take it to the peak temperature in the furnace and this difference T_p - T_s is what is actually driving the heat flux into the inner wall of the furnace and outside also the same thing.

So, if you were to then note down what would be the T_∞, which is far away temperature and the outer wall would have a temperature slightly high, and you would have asymptotically, this is how the profile should look like and this is the difference T_i - T_∞ is a temperature difference over which the heat transfer is taking from the outer wall of the brick that ceramic brick, and between these two if you have assumed that the steady state heat flow is taking place in 1D then you could join them. So, this is how the temperature profile would look like.

So, you could see that what it implies is that the heat flux; that is actually going here is given by the heat transfer coefficient; that is for the inner wall into T_p - T_s × area, and the area we are assuming to be constant and what is actually going out from here is basically heat transfer coefficient for the outer wall into T_i - T_∞, and here what is actually coming from the one end of the ceramic wall to the other end would then be given by k of the ceramic × T_s - T_i divided by the thickness of the ceramic.

So, we can then see that these three expressions should then be matched if you want steady state, and that is how we can go about solving the wall temperatures for a given form level. So, now, we have drawn this schematic. So, I want to again inform you that this asymptotic should be such that far away into the furnace, the temperature profile should be flat. So, the curvature inside the hot should be like this and not like this. This is wrong, the reason is the temperature cannot keep on increasing, because far away into the furnace the temperature has to be equilibrated. So, this is correct and this is wrong. So, watch out the curvatures when we draw the schematic temperature profiles.

Now, that we have drawn this for one wall.

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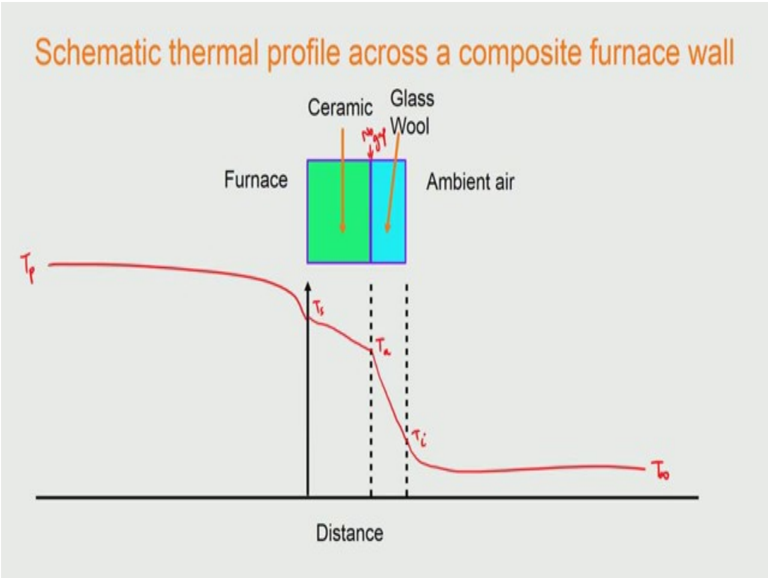

So, we increase the layers and see how this should be drawn when you have a composite wall, and then while drawing that we already know are practicing. So, we realize that the ceramic is having a better conductivity compared to the glass wool. So, we say that the glass wool should withstand a large gradient and then the ceramic should withstand is smaller amount of gradient, and then we see that this has to go something like that and this has to go something like that.

So, we now see that this is how we can draw it schematic temperature profile from the furnace interior into the ambient air, far away and we could then give symbols if you like and T_s and the T_i and you have some T_a. So, you can give symbols. So, here for example, the two temperatures are matching and that is, because there is no air gap here. So, here we say no air gap. So, with that assumption we can then draw the schematic.

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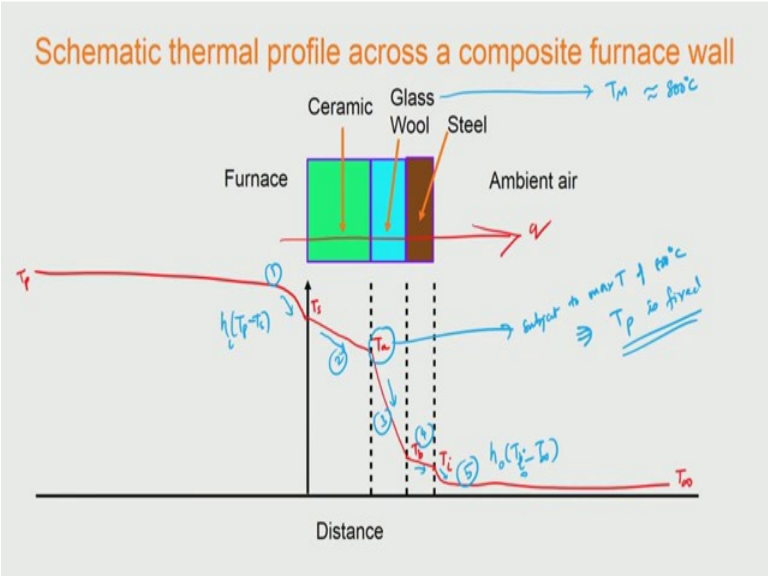

Now, when we have one more layer, the process of drawing is that similar except for that. Here we realize that this steel has the thermal conductivity, it is very high. So, it would not be able to withstand much of the temperature differences. So, practically that is very flat. So, you will draw it very flat like that, and then for the glass wool it can be started very high temperature gradient, and for ceramic there is somewhere in between. And therefore, now we could draw in this manner and flat temperature profile here and goes flat here. So, T_p here, T_∞ here, this will be interface temperature, this is surface temperature inside, if you like T_a and it is like T_b etcetera.

So, we can choose these symbols as we like, but the profile is like this, and we see that the steepest temperature gradient is maintained by the glass wool which is protecting the heat from being lost into the ambient air, and the overall heat flow is in this direction which you can see already that from the slopes you already know which way the heat is going.

So, you can see that the heat is going down this, because of the gradient down this, because of the gradient down here, because of the gradient down here and down here into the air. So, this is how the heat flow is happening, and that is if it is steady state, then the slopes have to be adjusted according to the thermal conductivities for the respective shells ok. So, when we now want to write expressions for each of these, we already know that for this first expression you write h × T_p - T_s here, the last one it will be h. So, this is inside this is for outside into T_i - T_∞ and so on. So, if you write those expressions they would look like that, so the first one.

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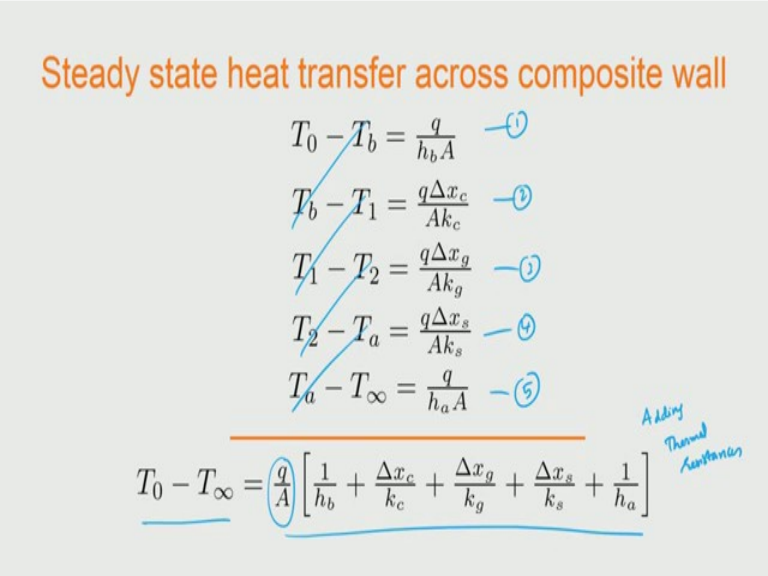

So, the first this expression would look like this, and this expression would look like this, this expression would look like this, this would look like this and here would look like this. So, you write expressions for all the 5 locations across which the heat flux is being written. and once you write you can then see that when you add, you would see that these will get cancelled, and you can write T_o - T_∞ is equal to this expression in which we have taken q/A as common out and you can see that $1/h_b$ + Δ x_c/k_c etcetera

So, we could see that we have essentially seen that when you do this addition, we are actually seeing that we are adding the thermal resistances. So, which means that we are actually taking the (Refer Time: 32:01) analogue and then simply adding them, as if they are in series and then finding out what with the temperature difference which means that when it comes to composite wall problems, we can go ahead and use the resistance analogue for the thermal field also and then solve the problems

Then when we have some situations; like for example, what would be the maximum temperature a furnace can withstand. So, normally the glass wool would have some melting point of the glass. So, let us say, it is given as let us say 800 ºC, which means that the hotter wall; that is in contact with glass wool shall not exceed 800 ºC and which means this is subject to a maximum temperature of 800 ºC. So, this implies that T_{peak} is fixed, once you fix the geometries and the materials automatically T_{peak} is fixed, which means that knowing what will be the melting point of the glass wool, we already see that what is the maximum temperature the furnace can be operated

If we operate at higher temperature than that then the glass wool would melt and then there is then no support between the ceramic and steel, and then immediately you can see that the heat loss will be not balanced the way we have planned and that would lead to; for example, accidents and fire in the furnace and so on. Now, you can see how this expression can be modified if the furnace happens to be different ok.

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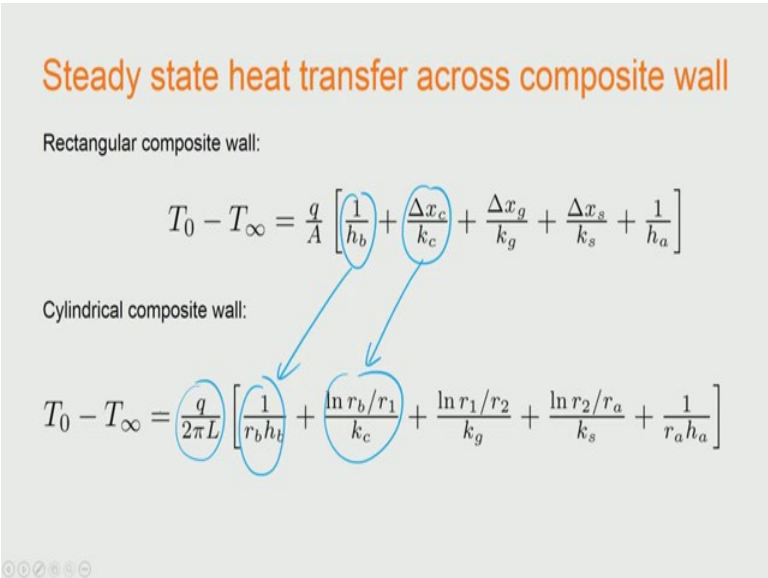

So, if it was actually a tubular furnace, then cylindrical coordinate system has to be used and you could see that the thermal resistance would then be different and you see that instead of Δx, you see that its logarithm that is coming etcetera. So, this we have already seen that each of the expressions would be modified, but the way you solve the problem would be the same, and therefore, you could also go ahead and do the same estimate for T_o - T_∞ which is basically sum of all the thermal resistances and with the q that is coming in front.

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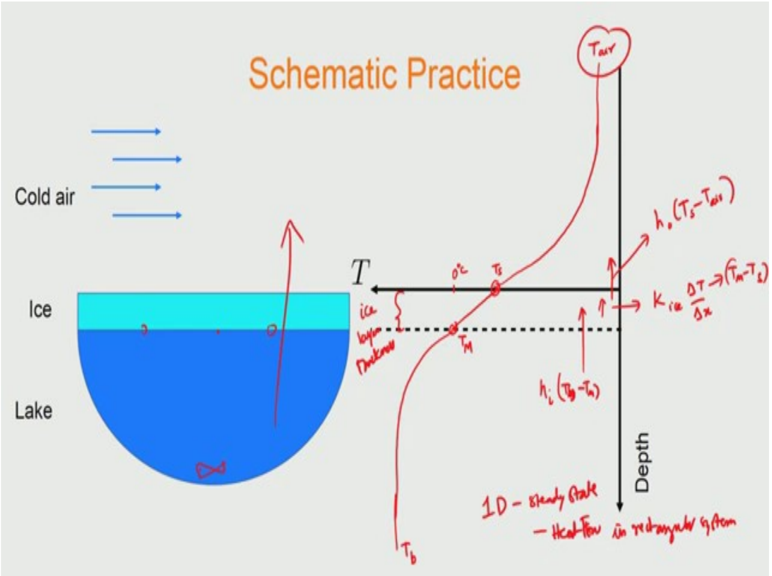

So, let us say that we have used this problem and let us say then we can see how this can be applied. So, here i am giving a practice problem. The practice problem is as follows. Let us say that there is a fish that is actually living under the lake and the lake is frozen on the surface, because of cold air and we want to drop a thermal profile across the entire system in the vertical direction, and prove that the fish can survive even in the cold winters, and this is actually really life experience in northern parts of Europe and so on.

Now, how do we go about drawing the profile here? Now the way you draw the profile here is identical to the way you draw in the furnace problem also, because you do have different layers of different thermal conductivities, and you could then start off by fixing tongue temperatures that we know. So, how do we go about?

We know that cold layer unless it is much lower than 0ºC , it will not actually lead to formation of ice. So, the temperature must be very low on the cold layer sink and along the depth if you see there must be no variation, very much high in the sky, because it does not know that there is a lake; that is absorbing all these chillness and therefore, it should be flat. And way below the bottom of the lake, you must have a slightly higher temperature for the fish to survive. So, you may have the temperature profile that is actually flat with respect to the depth.

Now, one temperature that we know is that the interface between the ice and the water is at 0ºC. So, which means that somewhere here. So, this entire layer which means that somewhere here you could draw the axis and we say that this is 0ºC , so this point we know. And then; that means that the surface has to be lower than that so you draw like that.

So, which means that immediately you can draw the profile in this manner, and then here, so which means that you can now estimate what would be the heat coming this way, what is the heat coming this way, what is heat (Refer Time: 35:53) way. So, here the first one would be for example, heat transfer coefficient I here the inner side into for example, T_{bulk} and then T this is 0 ºC, which is T_m. So, T_b - T_m and here it would be like for example, the thermal conductivity of ice into Δ T/Δ x and the Δ T itself is given as T_m - T_s, so this is T_s, and this is T of the air. So, this heat that is lost would be then h outer into T_s - T_{air}. So, you can already see from the values that T_{air} must be much less than T_s, so that the heat can go in this direction and then you can then get a balance.

So, which means that depending on the temperature of the air you can already estimate by the balance of these fluxes, what would be the thickness of the ice layer thickness, what would be the ice layer thickness, so that this balance can be there. Now that actually tells you whether the ice layer is thick enough for somebody to skate on the lake or something like that. So, you can actually take the same principles and apply two totally different scenarios, but in all these situations what we are doing is basically 1D steady state heat flow in rectangular systems. So, with that we can actually solve some of the numerical problem. So, we will do that as a part of the tutorial that will also be there in the course website.

So, with that we close the session and you can look up the course website for notes and practice problems.