Transport Phenomena in Materials
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Lecture -17
Energy Transport

Welcome to the session on energy transport as part of NPTEL MOOC on transport phenomena materials. This topic is also referred to as heat transfer in most of the courses.

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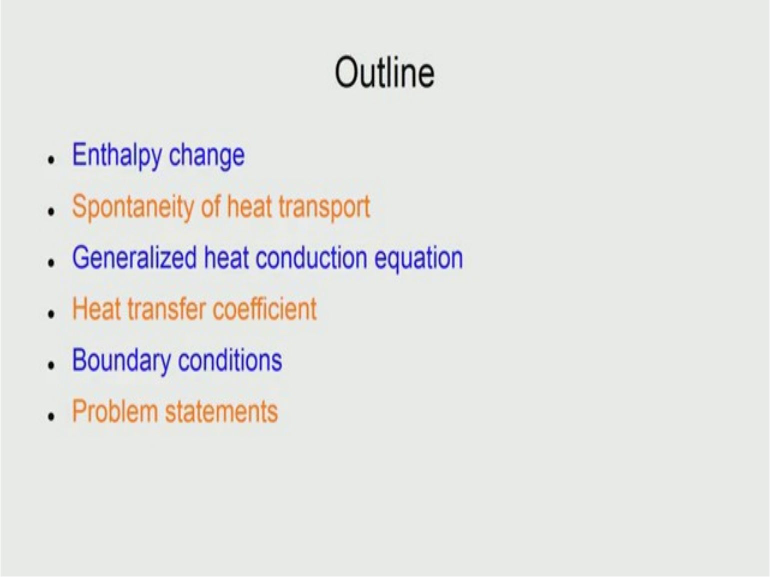

Outline

- Enthalpy change
- Spontaneity of heat transport
- Generalized heat conduction equation
- Heat transfer coefficient
- Boundary conditions
- Problem statements

So, in this course we are going to take up heat transfer after the fluid flow sessions mainly, because there is a strong linkage between these 2 phenomena. And in this session we are going to look at the enthalpy change from the thermodynamic principles and argue the spontaneity of heat transport and then motivate the linear constitutive relationship which we will be using here. Then derive the generalized heat conduction equation, which will be the governing equation for heat transfer. Then we will introduce the concept of heat transfer coefficient boundary conditions in problem statements that would basically be the starting point for us, to solve some of the problems on heat transfer that we encounter in metallurgical phenomena.

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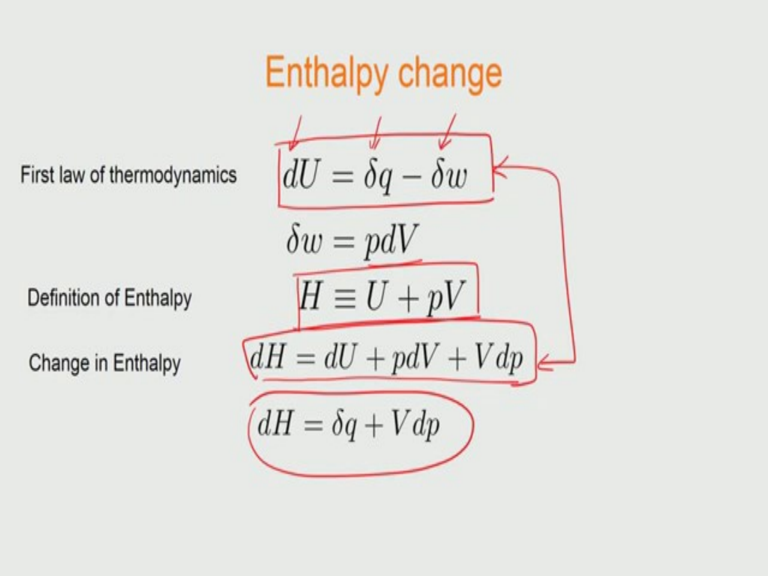

Enthalpy change

First law of thermodynamics $dU = \delta q - \delta w$

$\delta w = pdV$

Definition of Enthalpy $H \equiv U + pV$

Change in Enthalpy $dH = dU + pdV + V dp$

$dH = \delta q + V dp$

So, the enthalpy change is going to be discussed, motivated from the first law of thermodynamics, where we write the change in the internal energy as written here, which is basically the difference in the amount of heat that has come in and the work that is done by the system. So, we write for example, the differences with a different symbol here Δ, because these 2 are path functions.

Whereas, here you write with d, indicating that its a state function and we restrict the work to be the pdV work. So, that we will later on look at the constant volume and constant pressure situations, and see whether these terms will survive or not. We can also take the chemical work, but at this moment we are not going to take those into account.

The enthalpy as a thermodynamic function is defined here. So, we write this with a triple equal to sign, because this is actually the definition. And once we write, then we can actually look at the change in the enthalpy, as related to the change in the internal energy and the rest of the parameters. So, we write by straightaway taking the differentiation by parts for the p and b.

So, once we write this, we basically combine this equation with the first law and realize that we can express the change in the enthalpy to the amount of heat that has come into the system, and at constant pressure we see that the second term will drop. So, this is exactly the idea. Most of the engineering situations, the pressure is actually kept at one bar and ray. Often we also are using the condensed systems that is liquids and solids where the pressure effects are very minimal. So, therefore, most of our discussion will be with the heat transfer at constant pressure. So, once we write, then we can also see how the change in enthalpy can be expressed further on.

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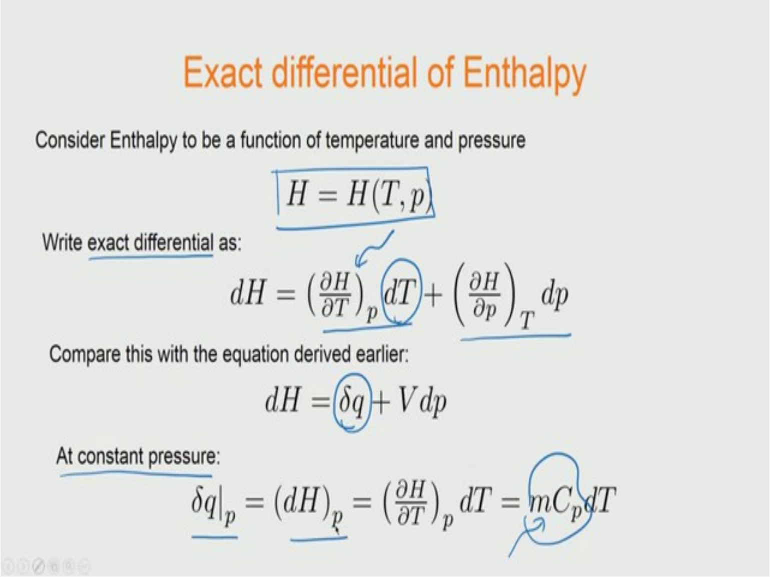

Exact differential of Enthalpy

Consider Enthalpy to be a function of temperature and pressure

$$\boxed{H = H(T,p)}$$

Write exact differential as:

$$dH = \left(\frac{\partial H}{\partial T}\right)_p dT + \left(\frac{\partial H}{\partial p}\right)_T dp$$

Compare this with the equation derived earlier:

$$dH = \delta q + V\,dp$$

At constant pressure:

$$\delta q|_p = (dH)_p = \left(\frac{\partial H}{\partial T}\right)_p dT = mC_p dT$$

So, here we are proposing that the enthalpy can be written as a function of temperature and pressure. So, for any function; that is a function of 2 variables, you can actually write what would be called as an exact differential. So, this is actually a mathematical concept, saying that the differential of enthalpy, if it is written as a function of temperature and pressure, can be expressed in terms of the partial derivatives and the differences of respective variables.

Now if this is so then what we can do is, that compare this equation with the one that we have derived. So, this equation when we compare, then we can already see that this term would be here comparable, and then we could also see how we can express the amount of heat that has come into the system to raise the temperature by a small amount.

So, then we can give some name for this particular quantity and there so we are actually introducing. So, we basically call that as the heat capacity and per unit weight or per unit volume or molar heat capacity etcetera, is depending upon the variable that is in front of it. So, this is how we define, but remember that it is all valid, when we are taking the pressure to be constant. So, basically motivated from the enthalpy, we basically say that the amount of heat that has come into the system, is actually leading to directly a change in the enthalpy, and that can be related to the temperature change via the heat capacity.

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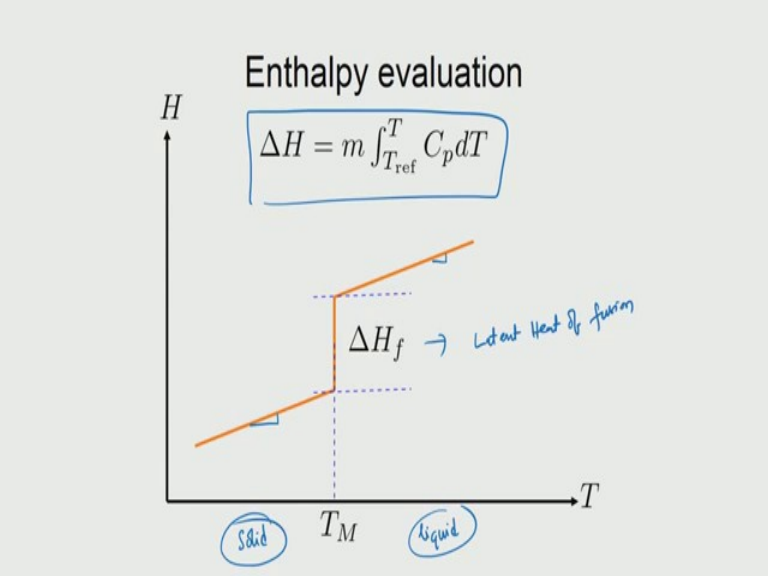

Enthalpy evaluation

$$\boxed{\Delta H = m\int_{T_{\text{ref}}}^{T} C_p dT}$$

So, once this is there, then we can see how this actually applicable for most of our systems. So, if you take many of the materials, pure materials for example. If you take pure iron and then the melting point is indicated here. So, as you increase the temperature at the melting point, there is a certain amount of heat that has to be given to break all the bonds between the atoms in the solid state, to make them into a liquid state. So, here we have got the solid state here and this is the liquid state and this is also called as a latent heat of fusion or enthalpy of melting and so on.

And you could see that this actually makes the expression for enthalpy a little complicated, because there are changes of slopes, but as long as you are not changing the phase; that is if you are sticking into only the solids are the liquids, then you can have a direct proportionality relationship between the enthalpy and the temperature, and the enthalpy change as you go along. This curve is given by the integration; that is written here. So, you could apply this, either in the solid state or in the liquid state, but when you do a cross, then you will also need to look at the latent heat of fusion without forgetting, because that is an addition that will come, because of the change of the phase.

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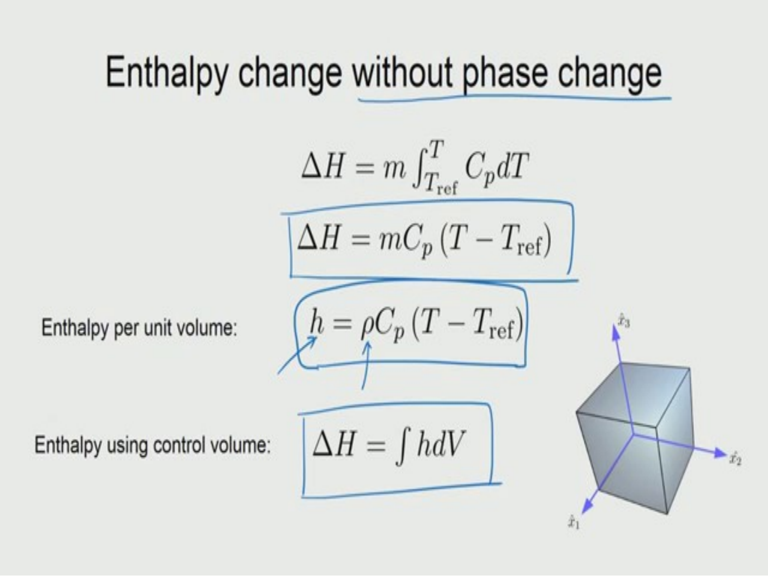

Enthalpy change without phase change

$$\Delta H = m\int_{T_{\text{ref}}}^{T} C_p dT$$

$$\boxed{\Delta H = mC_p(T - T_{\text{ref}})}$$

Enthalpy per unit volume: $\boxed{h = \rho C_p (T - T_{\text{ref}})}$

Enthalpy using control volume: $\boxed{\Delta H = \int h dV}$

So, assuming that there is no phase change, assuming that you do not have a phase change, then you can straight away use this straight line approximation and write that the enthalpy is related to the differences in the temperature, where the slope is nothing, but mC_p, and if you were to, then also apply it for a given domain that we know that the mass of the entire domain depends upon the geometry, and in transport phenomena we have been generally using the control volume approach..

So, that we write the expressions for a given control volume and then we just integrate for the whole volume, depending upon the geometry than the integration can give you the results. So, we do not want to write the quantities for a entire body, but actually where a control volume. So, we write the enthalpy per unit volume here divided by the volume, and you can see I am using a small h to say that it is enthalpy per unit volume. So, you have $\rho C_p \times \Delta T$.

Now, the T reference is basically some reference point to indicate the scale of the enthalpy to start at zero, and usually you take it as a 298 K as a reference. So, you could also take it as any other reference, but remember that, its always differences of enthalpies that we are interested in very often. And from here you can already motivate that for any given body, because if you integrate the density over the entire volume you get the mass. So, you can already motivate that, the enthalpy change is nothing, but integral of the enthalpy per unit volume over the entire volume.

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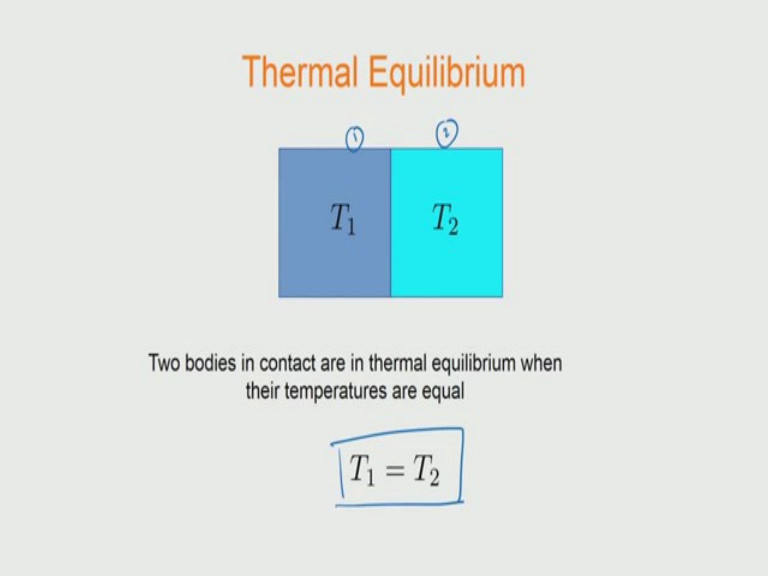

Thermal Equilibrium

Two bodies in contact are in thermal equilibrium when their temperatures are equal

$$\boxed{T_1 = T_2}$$

So, once that is there then we will actually use that concept later on to derive some of the expressions. So, here we bring about what we mean by thermal equilibrium, because heat transfer is basically a tendency of the system to go towards thermal equilibrium, and we say that there are 2 bodies here; one and 2 and these are at 2 different temperatures T_1 and T_2, and if they are actually at thermal equilibrium what we want to then conclude is the T_1 should be equal to T_2. So, which means that the measure of the equilibrium is basically given by the temperatures. So, that is how we want to define the thermal equilibrium as.

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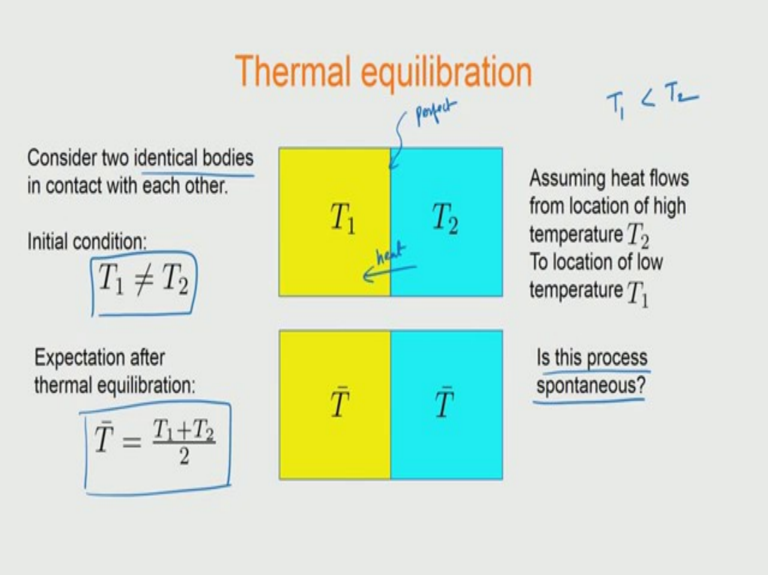

Thermal equilibration

$T_1 < T_2$

Consider two identical bodies in contact with each other.

Initial condition:

$$\boxed{T_1 \neq T_2}$$

Expectation after thermal equilibration:

$$\boxed{\bar{T} = \frac{T_1+T_2}{2}}$$

Assuming heat flows from location of high temperature T_2 To location of low temperature T_1

Is this process spontaneous?

And if you now take 2 bodies, that are at 2 temperatures T_1 and T_2 and if they are different and if they were to reach the equilibrium over a period of time. So, initially T_1 is not equal to T_2, and later on they reach in equilibrium, and if these 2 bodies are identical. There are 2 identical bodies, they have the same amount of mass and their contact is perfect contact. So, we say that the contact is perfect. So, that there is a good amount of heat that can go across to make the equilibrium be achieved.

And if that is for the (Refer Time: 08:27). So, then we can already see that the average temperature that will be seen in both of these bodies, is given by $(T_1 + T_2)/2$. This actually makes us that the intuition, that the heat actually is going in this case for example, if you were to tell that T_1 is a low temperature and T_2 is a high temperature. So, we say that the heat is going this way. So, that the T_2 is reduced to $\acute{T}$ and T_1 is actually increased to $\acute{T}$, where $\acute{T}$ is basically the average of T_1 and T_2.

Now this actually makes us come to a conclusion that heat is actually going from a location of high temperature to low temperature, and I want to just quickly remind you why would like to call this spontaneous process. So, here is where we remember something from thermodynamics; any process that increases the entropy would be spontaneous. So, if in this process of temperatures being equilibrated if the entropy were to become positive; that means, it is a spontaneous process. So, that we basically discover the cause and effect relationship, which we then use for the linear constitutive equation that we need for heat transfer.

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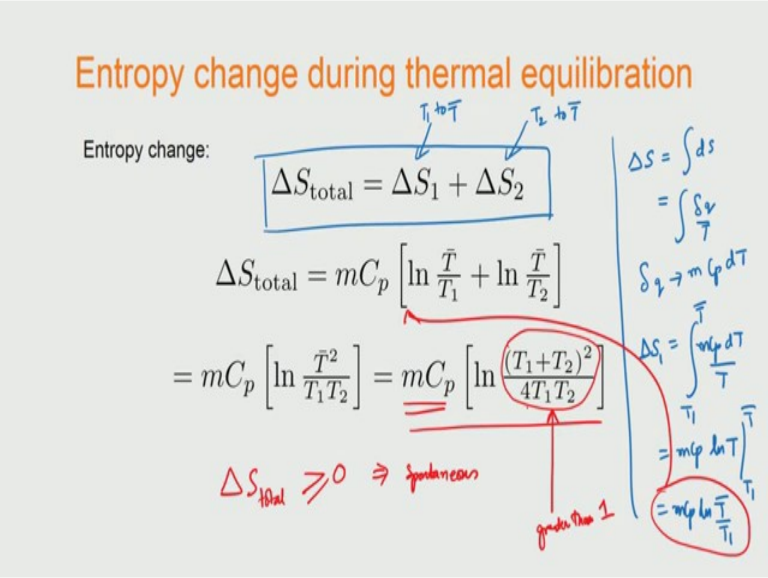

Entropy change during thermal equilibration

Entropy change:

T_1 to $\bar{T}$ T_2 to $\bar{T}$

$$\boxed{\Delta S_{\text{total}} = \Delta S_1 + \Delta S_2}$$

$$\Delta S_{\text{total}} = mC_p \left[\ln \frac{\bar{T}}{T_1} + \ln \frac{\bar{T}}{T_2} \right]$$

$$= mC_p \left[\ln \frac{\bar{T}^2}{T_1 T_2} \right] = mC_p \left[\ln \frac{(T_1+T_2)^2}{4T_1T_2} \right]$$

$\Delta S_{\text{total}} \geq 0 \Rightarrow$ spontaneous

greater than 1

$$\Delta S = \int dS = \int \frac{\delta q}{T}$$

$$\delta q \to m C_p \, dT$$

$$\Delta S_1 = \int_{T_1}^{\bar{T}} \frac{m C_p \, dT}{T} = m C_p \ln T \Big|_{T_1}^{\bar{T}} = m C_p \ln \frac{\bar{T}}{T_1}$$

So, you write the entropy change for the entire body in this manner. So, you say that the entropy change for the first body is ΔS_1 and for the second body is ΔS_2. This is going from T_1 to $\acute{T}$, this is going from T_2 to $\acute{T}$and you could write the ΔS is basically $\int dS$ and that we write as δ q /T and we already know that δq is given by for example, $mC_p \times dT$.

So, therefore, we can write here the ΔS as mC_p dT/T and if you want to go from T_1 to $\acute{T}$, then you could write it as mC_p ln T evaluated between the limits T and $\acute{T}$ and that would be $mC_p\ln(\acute{T}/T_1)$. Similarly you can then see how you could write it for the other expression also. So, you then see that it is coming here. Similarly it is also evaluated for the second term and then you can get the sum of the 2 entropies like this.

Now when is this process spontaneous? The process is spontaneous when the total entropy change is greater than or equal to 0. So, that is when it is spontaneous. So, we inspect whether it is spontaneous or not, by looking at only this term, because these are actually anyway multiplicative factors, and if this term that I am highlighting, if that is greater than one then what happens the logarithm of a quantity more than one will give you positive number and therefore, the entropy change will be positive. If this number happens to be less than one; that means it will give you a negative number when you take logarithm and then the entropy change will be negative, which means our process is not spontaneous. So, we will only look at this term, and to evaluate whether that term is positive or not.

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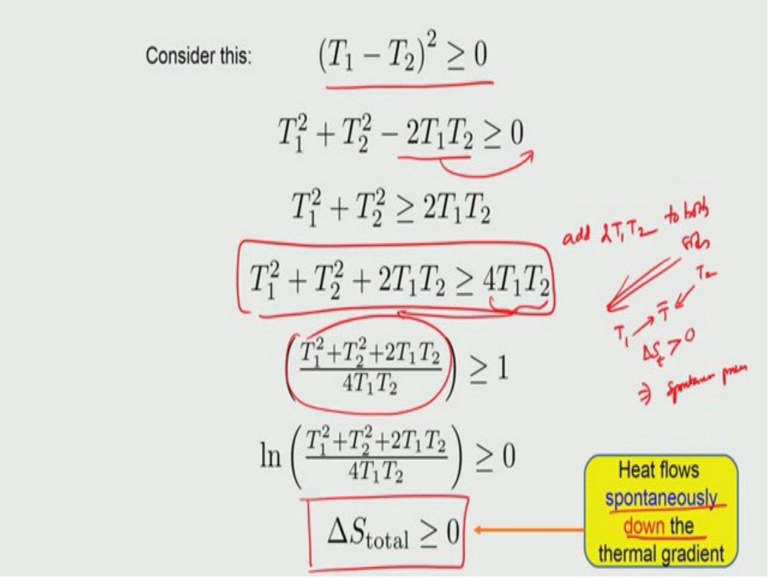

Consider this: $\underline{(T_1 - T_2)^2 \geq 0}$

$$T_1^2 + T_2^2 - 2T_1T_2 \geq 0$$

$$T_1^2 + T_2^2 \geq 2T_1T_2$$

$$\boxed{T_1^2 + T_2^2 + 2T_1T_2 \geq 4T_1T_2}$$

$$\left(\frac{T_1^2+T_2^2+2T_1T_2}{4T_1T_2}\right) \geq 1$$

$$\ln\left(\frac{T_1^2+T_2^2+2T_1T_2}{4T_1T_2}\right) \geq 0$$

$$\boxed{\Delta S_{\text{total}} \geq 0}$$

Heat flows spontaneously down the thermal gradient

add $2T_1T_2$ to both sides

$T_1 \rightarrow \bar{T} \leftarrow T_2$

$\Delta S_t > 0$

$\Rightarrow$ spontaneous process

You look at the expression here. So, as long as the T_1 and T_2 are 2 temperatures, if they are different you know that whether the T_2 is more than T_1 or less than T_1 the difference square is always positive, and therefore, we say that it is greater than or equal to 0, because T_1 and T_2 will be equal then you get 0 there. And you expand $(T_1 - T_2)^2$ and then take the 2 T_1T_2 to the other side and then add 2 T_1T_2 to both sides, then you see that you can write this expression. Once you write this expression you to take this $4T_1T_2$ to the denominator and then you can write that the entire expression is greater than or equal to 1, which means that the logarithm of that is greater than or equal to 0, which actually means that the entropy change for this entire system is given as greater than or equal to 0.

So, which also means that in the process that T_1 went up to $\acute{T}$ and in the process that T_2 went down to $\acute{T}$, it is actually that entropy change total is actually greater than zero, which implies this is actually a spontaneous process. And in this process what is happening is that the heat is actually going down this direction. So, heat going down the temperature gradient is a spontaneous process; that is exactly what is our conclusion from here, which means that we are now able to propose a constitutive relationship, where we can talk about the cause and effect relationship if the cause is the temperature gradient then the result will be the heat flow, and the sense of it is that the heat will be flowing down the temperature gradient, and that is exactly what we are writing here.

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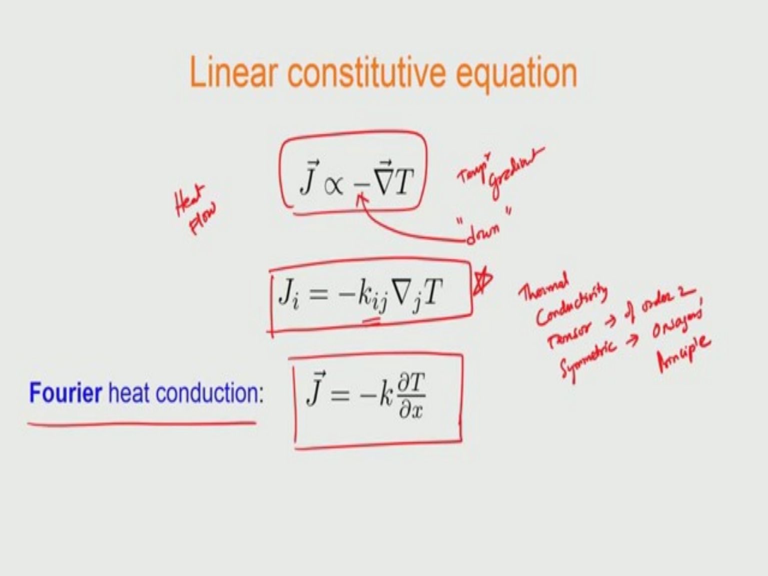

Linear constitutive equation

$$\boxed{\vec{J} \propto -\vec{\nabla} T}$$

Heat Flow

Temp^r gradient

"down"

$$\boxed{J_i = -k_{ij} \nabla_j T}$$

Thermal Conductivity Tensor → of order 2

Symmetric → Onsagar's Principle

Fourier heat conduction: $$\boxed{\vec{J} = -k \frac{\partial T}{\partial x}}$$

We are saying that here, J is the heat flow. So, in per unit area heat flux and this is the temperature gradient. So, we want to use this minus symbol we should say, down the gradient ok. So, we are saying that the heat flux is down the temperature gradient, and then if you want to look at these 2 as vectors and they should be related by a property and from the tensor relationships that we have discussed earlier in the beginning of this course, we know that the highest tensorial order for the property that can relate, these 2 vectors will be a tensor of order 2 and that is exactly what we also want to call as thermal conductivity tensor, which is of order 2.

So, that is what we can conclude here. And we have seen this linear constitutive relationship actually come out with a name Fourier heat conduction, where the heat flux is given as proportional to the temperature gradient where the proportionality constant is k. Now normally k is taken as a constant, but we know that the most general form of writing this relationship is, k is actually a tensor of order two, k is actually symmetric one also as we have discussed earlier, this is because of the Onsagar's principle. So, k_{ij} is a symmetric tensor. So, we will look at problems, where k is actually a tensor of order 2 with 9 components reducing to 6, because of symmetry and for different process symmetries how different values of k are required.

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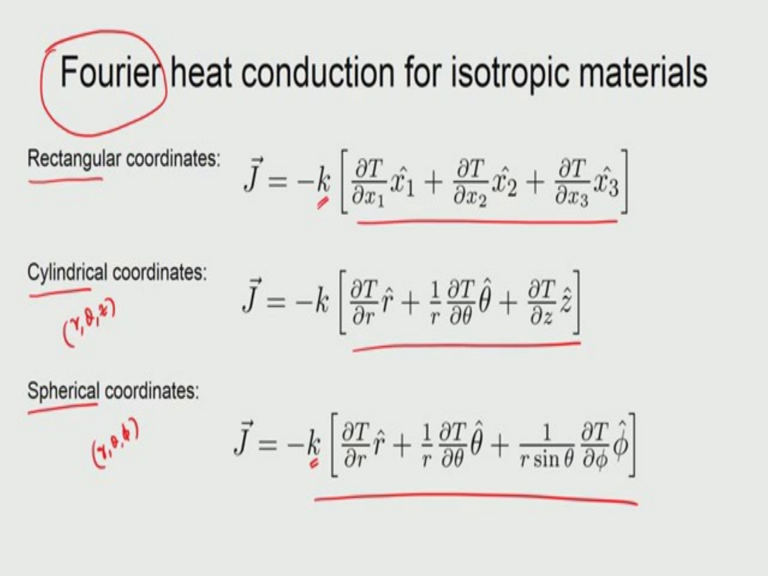

Fourier heat conduction for isotropic materials

Rectangular coordinates:

$$\vec{J} = -k\left[\frac{\partial T}{\partial x_1}\hat{x}_1 + \frac{\partial T}{\partial x_2}\hat{x}_2 + \frac{\partial T}{\partial x_3}\hat{x}_3\right]$$

Cylindrical coordinates:

(r,θ,z)

$$\vec{J} = -k\left[\frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{\partial T}{\partial z}\hat{z}\right]$$

Spherical coordinates:

(r,θ,ϕ)

$$\vec{J} = -k\left[\frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\phi}\right]$$

So, we will take that discussion separately and see how we can actually write this expression for isotropic material. So, for isotropic materials k is just one number and that we have written here, and in the rectangular coordinate system you could write the heat flux related to the thermal conductivity in this manner where we have written the gradient. So, the gradient is written here in the rectangle coordinate system, and if it is cylindrical you can write it with respect to the r θ and z components of the gradient and you can see that it is written here.

And for spherical coordinates of course, you write it for r θ and ϕ directions, the variation of temperature in those directions and we can write the (Refer Time: 15:45) like this. So, (Refer Time: 15:47) into the thermal conductivity with a minus sign is giving you the heat flux and that is what is basically called as a Fourier heat conduction ok.

So, in the realm of most of the engineering scenarios we normally encounter only Fourier heat conduction, because the non Fourier heat conduction is out of the purview for our course. It normally happens when we have laser pulsing at time skills that are very small like pico second or so, but most of engineering problems where the heating is done at the expected rates and pulsing is done in milliseconds or so, or even more then Fourier conduction equation is valid and that is what is written here.

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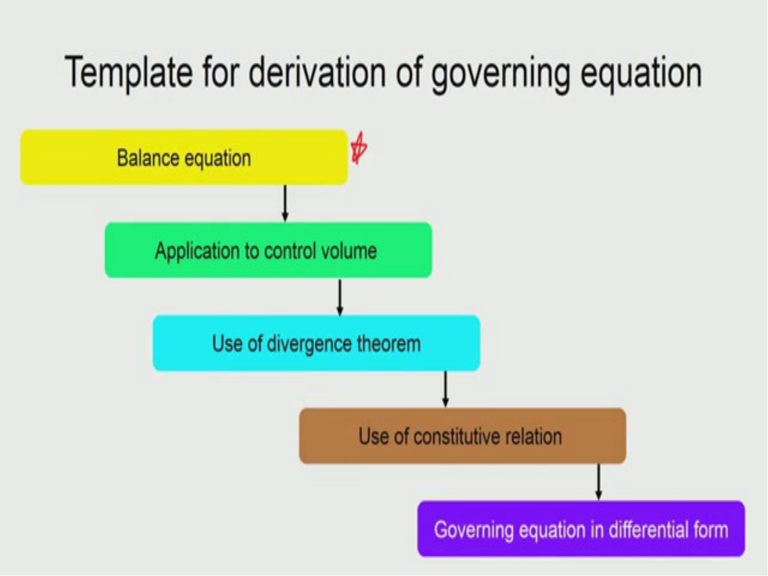

Template for derivation of governing equation

Balance equation

Application to control volume

Use of divergence theorem

Use of constitutive relation

Governing equation in differential form

Now we need to have a governing equation derived. So, the template for the governing equation is exactly the same as what we have done for the Navier stokes equation and that is given as follows. So, we start off with a balance equation. So, for Navier stokes equation we start by the balance of the linear momentum. So, in this case for heat transfer we start with the balance of heat or balance of energy. So, we started a balance equation, and then we apply this equation for a control volume. So, we write mathematically for a control volume, and then normally when you write it for a volume you would have the bulk terms, we volumetric terms as well as the surface terms.

So, the surface terms are going to be then converted to the bulk terms by using the divergence theorem. And then once you have that returned then you would actually look at only the integrand and then change it by using a constitutive relationship so, that the cause and effect relationship is used, and finally, we have the governing equation. So, this template will be applicable for all the transport phenomena, and usually if you follow this we are also very conscious of what is our starting point. So, it is a balance of enthalpy that is the basis for heat transfer governing equations. So, that is something that we have to be very clear, because there is a starting point for us.

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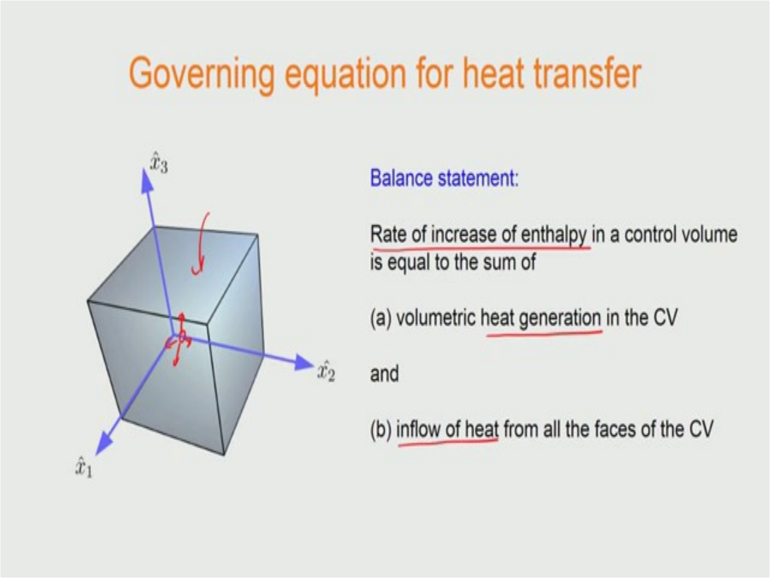

Governing equation for heat transfer

Balance statement:

Rate of increase of enthalpy in a control volume is equal to the sum of

(a) volumetric heat generation in the CV

and

(b) inflow of heat from all the faces of the CV

So, what we do is first we state the balance. So, balance is stated as follows. So, the rate of increase of enthalpy in a control volume is because of heat generation; that is done volumetrically in the control volume and the influx of heat from all the phases. So, if the heat is coming in and also there is a heat that is generated. So, therefore, then it should lead to the increase in the enthalpy of that particular control volume. Enthalpy being a thermodynamic function is any way related to temperature, so we would see that it eventually leads to increase in temperature, but the balance is written for enthalpy. So, we would write that ok.

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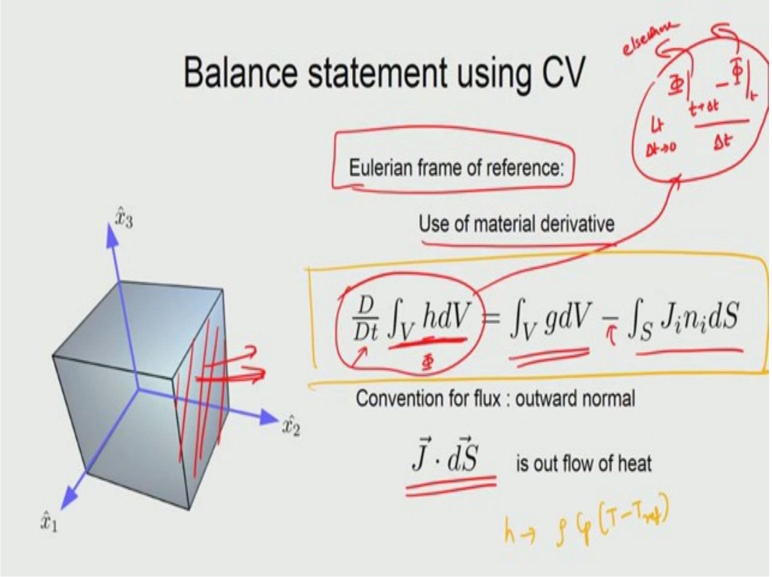

Balance statement using CV

Eulerian frame of reference:

Use of material derivative

$$\frac{D}{Dt}\int_V h dV = \int_V g dV - \int_S J_i n_i dS$$

Convention for flux : outward normal

$\vec{J}\cdot\vec{dS}$ is out flow of heat

elsewhere

$$\underset{\Delta t\to 0}{Lt}\ \frac{\Phi|_{t+\Delta t} - \Phi|_t}{\Delta t}$$

Φ

$h \to \rho c_p (T - T_{ref})$

And we write it now in mathematical form for a control volume, and we are then very consciously using the Eulerian frame of reference which we have discussed in detail in an earlier session, where we use a material derivative to look at the rate of change. So, rate of change with respect to the time, is used with capital D which is a material derivative; the reason being that this is for a control volume, which could be in a fluid also and in a fluid when you go whatever is the function that you want to write.

So, this entire thing for example, is a quantity let us say capital Φ, then capital phi at any time limit Δ T goes to 0. So, this is what we actually mean when you look at this quantity. Now here we see that at T + Δ T is the control volume elsewhere compared to here. So, the reason being that there is a velocity that is actually present and the control volume will actually move around. So, then if you use the fact that you could actually use the chain rule of differentiation, then you would actually get the velocities also, and that is what we discussed when we came about the material derivative.

So, please refresh that concept before we continue further, and then we will be able to write the rate of change of enthalpy as material derivative of this integral, and this integral what is there is basically the enthalpy of the entire control volume, and that is equal to the volumetric heat generation. So, g is the rate of generation of heat. So, rate of heat generation per unit volume that is written as g, and that would be basically watt per meter cube and then the influx of heat.

So, the influx of heat we always write with a minus sign. The reason is as follows; the J is actually by our convention is an outward vector, and when it is dotted with the surface normal of any phase that is also outward. So, when you dot it you get a quantity that is positive if the flux of any quantity j that is represented is going out of the control volume. So, you must actually then put a minus sign to indicate that we actually saw about the inward flux. So, that is why we say that, if this is the amount of heat that is going out of the phase that is indicated here then with a minus sign that actually then represents the influx.

So, what we have written in this particular equation is that is a mathematical form of whatever we have said here. So, this is actually this. So, these 2 are identical; one is in English and other is in the mathematical expression. Now, what we do is that we then go on to manipulate this expression further on to arrive at the governing equation.

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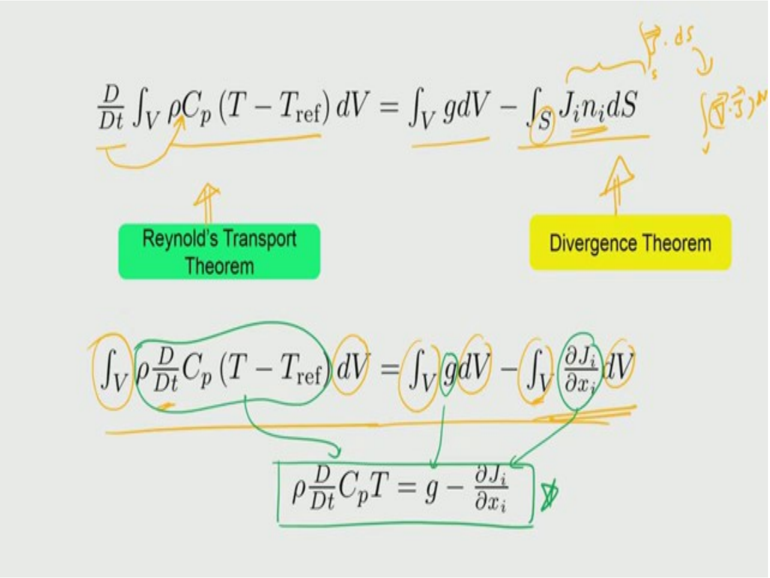

$$\frac{D}{Dt}\int_V \rho C_p (T - T_{\text{ref}})\, dV = \int_V g\, dV - \int_S J_i n_i\, dS$$

Reynold's Transport Theorem

Divergence Theorem

$$\int_V \rho \frac{D}{Dt} C_p (T - T_{\text{ref}})\, dV = \int_V g\, dV - \int_V \frac{\partial J_i}{\partial x_i} dV$$

$$\boxed{\rho \frac{D}{Dt} C_p T = g - \frac{\partial J_i}{\partial x_i}}$$

So, that we do as follows. So, we write the same expression here h, but h is then written with respect to the expression that we have already seen, it is $\rho \times C_p \times (T - T_{ref})$ So, that is what we write here, and integrated over dV that will give you the enthalpy and then the rate of change of enthalpy is indicated by this, and that is equal to the integral of the rate of heat generation volumetrically and the influx. So, we wrote that again neatly by expanding the small h.

And then we see that is bit awkward, because there is a differential here outside of the integral and the rest of them are actually integrals. So, we must take that in, and what enables that possible is the Reynolds transport theorem, which again has been discussed earlier and we know that it can be taken in at that position by this Reynolds transport theorem for any arbitrary scalar function, and luckily for as, temperature is a scalar function and therefore, we can directly do that.

And here we have basically integration which is over the entire surface of the control volume and if you want to then change it to the volumetric integral, then we must apply the divergence theorem. So, divergence theorem is over this index i. So, this basically whatever you have written is J dot dS and we know that that can be converted to del dot J dV integral over the volume and this is what actually we apply.

So, we have then taken the D/D t material derivative inside here it has come in and in here actually converting the, we are converting the surface integral into a volume integral here by taking the divergence. Once we write then we recognize that all these three terms are actually having the integration over the same domain, namely the volume of the control volume, and therefore, if this equation were to be true then it also means that the integrands are also equal. So, whatever I am highlighting that is all the integrands and they must also be following the same expression. So, that we then indicate here.

So, which means that we now have expressed the balance of enthalpy without using the integrations by writing it for the control volume and then adjusting, the term such that everything is over the same integration. So, we now have basically a governing equation for the enthalpy balance, but we still need to do some more, because here we write J which is the heat flux, which needs to be converted into the temperature differences, and that we do basically by using the linear constitutive relationship.

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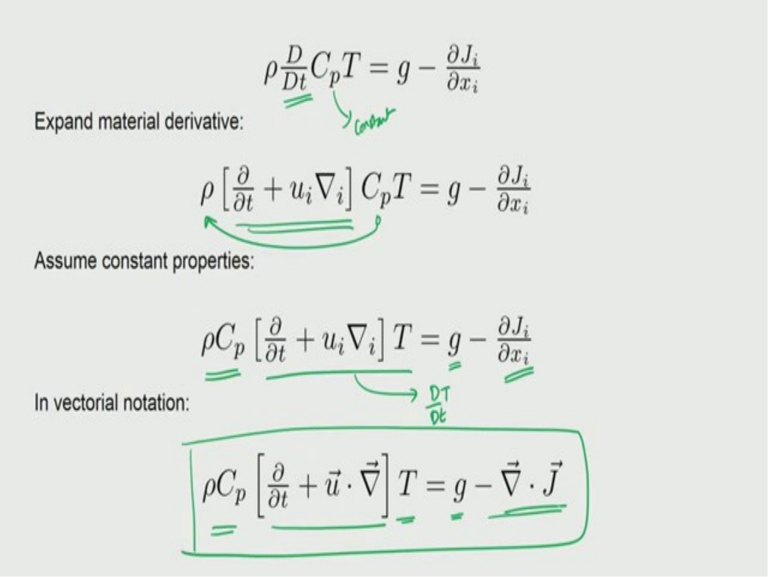

$$\rho \frac{D}{Dt} C_p T = g - \frac{\partial J_i}{\partial x_i}$$

constant

Expand material derivative:

$$\rho \left[\frac{\partial}{\partial t} + u_i \nabla_i\right] C_p T = g - \frac{\partial J_i}{\partial x_i}$$

Assume constant properties:

$$\rho C_p \left[\frac{\partial}{\partial t} + u_i \nabla_i\right] T = g - \frac{\partial J_i}{\partial x_i}$$

$\frac{DT}{Dt}$

In vectorial notation:

$$\boxed{\rho C_p \left[\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right] T = g - \vec{\nabla} \cdot \vec{J}}$$

But before we proceed we just let us look at the operator D/Dt material derivative and that we have already come across; that is expanded here. So, we expand the material derivative here, and if you then want to consider that this is constant, which means that it is not a location dependent quantity then you can take it out. What it means is that in problems where you have location dependent properties, then do not do this manipulation you can use the equation as it is, but in case of homogeneous media then you can take the location dependent properties, if they are not relevant then you can take this guy out, and then you can collect to the properties aside and then look at that the material derivative is acting only on the temperature..

So, this entire thing is basically d D/Dt of the temperature, material derivative the temperature that is all, and that is equal to basically g which is the rate of heat generation volumetrically and the divergence of the flux. So we then write this expression in vectorial form saying that material derivative of temperature with a property is equal to the rate of heat generation, and the total influx which is given by the divergence of the flux.

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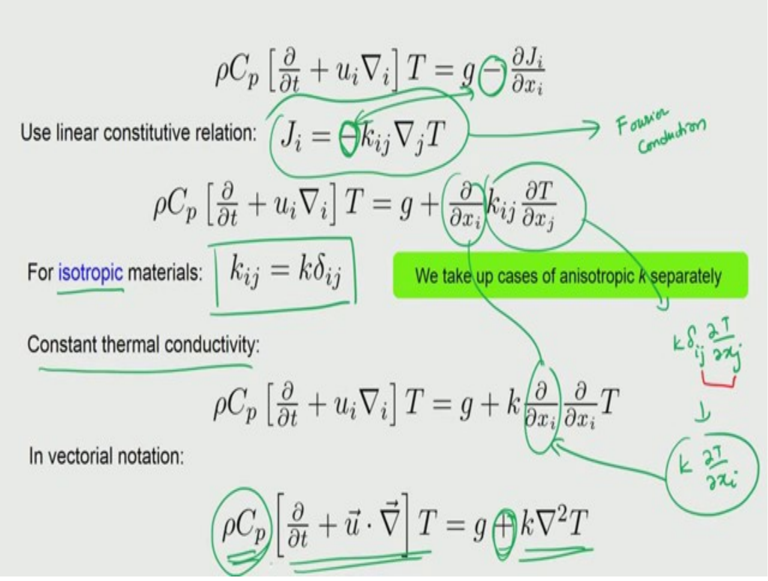

$$\rho C_p \left[\frac{\partial}{\partial t} + u_i \nabla_i\right] T = g - \frac{\partial J_i}{\partial x_i}$$

Use linear constitutive relation: $J_i = -k_{ij} \nabla_j T$ → Fourier Conduction

$$\rho C_p \left[\frac{\partial}{\partial t} + u_i \nabla_i\right] T = g + \frac{\partial}{\partial x_i} k_{ij} \frac{\partial T}{\partial x_j}$$

For isotropic materials: $\boxed{k_{ij} = k\delta_{ij}}$

We take up cases of anisotropic k separately

Constant thermal conductivity:

$k\delta_{ij}\frac{\partial T}{\partial x_j}$

$k\frac{\partial T}{\partial x_i}$

$$\rho C_p \left[\frac{\partial}{\partial t} + u_i \nabla_i\right] T = g + k \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} T$$

In vectorial notation:

$$\rho C_p \left[\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right] T = g + k \nabla^2 T$$

Now, the flux is then expanded, and we expand the flux in this manner, where we basically already have talked about it, it is basically the Fourier conduction, Fourier heat conduction equation, the flux is equal to -k × ΔT, and the k for example, here is a tensor of order two, then we want to make a simplification that for isotropic materials we want to consider it as an isotropic tensor and therefore, we can write it as $k \times \Delta_{ij}$.

. So, look at this expression. So, you look at only this expression, if you write $k\Delta_{ij}$ and then $\partial T/\partial x_i$, then we know the principle that whenever the kronecker Δ is coming for the match matching index you can then manipulate and then this can be written as $k \times \partial T/\partial x_i$. So, this is what we have written and that goes here. So, this comes straight away here. So, rest of it is here. So, which means that we can actually look at the last term, and recognize that it can be written as$k \times \nabla^2 T$. So, which is basically Laplacian that is coming in, and on the left hand side we have basically the ρC_p and then this is the material derivative.

So, we can see that upon introducing the Fourier heat conduction equation, the minus sign is actually taking care and you have got a plus sign here, because you have got 2 minuses here, these 2 getting canceled out. So, we now have the governing equation for heat transfer. So, we write it in a different manner. So, we already see that the properties are being assumed constant. So, constant thermal conductivity etcetera.

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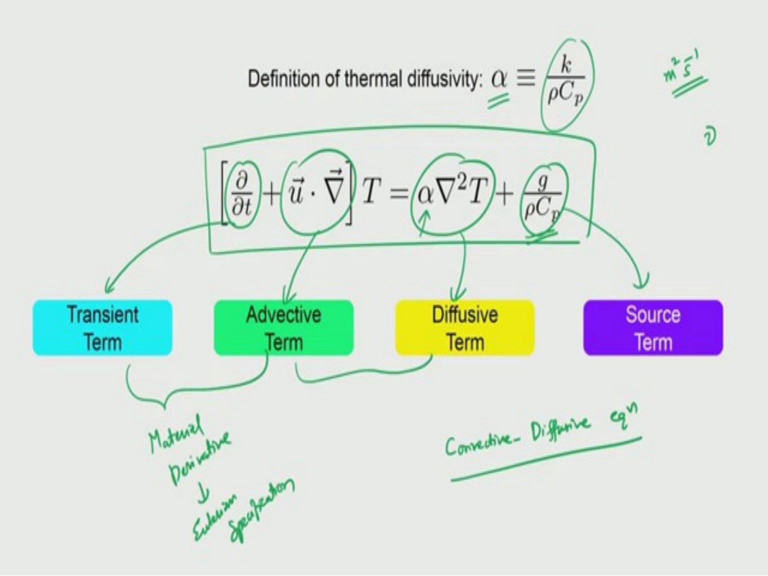

Definition of thermal diffusivity: $\alpha \equiv \frac{k}{\rho C_p}$

$$\left[\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right] T = \alpha \nabla^2 T + \frac{g}{\rho C_p}$$

Transient Term

Advective Term

Diffusive Term

Source Term

So, we can then take it to the right hand side and write it as follows. So, when you take to the right hand side k/ρC_p comes, and k/ρC_p can then be called as the thermal diffusivity, which has the units m^2/s, and we already saw some quantity of this nature and ν in mass transfer also, earlier the momentum transfer we look at what is called as the momentum diffusivity and that is basically ν kinematic viscosity and that has also the same units ok.

So, we then substitute that here and g/ρC_p comes about because ρC_p is taken to the right hand side. So, now, this is basically the governing equation for the heat transfer for us, with the advective term and the partial derivative at the time, all of it coming because of the metal derivative on the left hand side.

So, this first term goes by the name transient term and the second term goes by the name advective term. These 2 together are coming, because of the material derivative, and that is because we have used Eulerian specification, and this term is called the diffusive term, because it basically tells how the temperature variations are getting evened out. If we will see the result of that shortly when we solve some problems, and g is basically volumetric heat generation term and therefore, that is called as a source term.

So, we can see that the terms are very similar to that of the Navier stokes equation, you do have a transient term, you have advective term, you have a diffusive term, you have also source term. The only thing that was missing is in the Navier stokes equation you have the pressure gradient term, which is not relevant in heat transfer, but otherwise the four terms are there. And the way the terms actually interact on the equation to get you the solutions of different types is also similar and so, some of the solutions can already be guessed by looking at what kind of solutions are possible, if the equation looks like this.

This equation has another name, you could also call it as the so called convective diffusive equation, and this basically means that these 2 terms are being taken into account and so, in mathematical literature sometimes what people refer to as convey to diffuse equation is nothing, but this generalized equation that we have written, which falls into a template into which the Navier stokes equation or which analyzed heat transfer equation also can be cast. So, its a very broad template that is widely used ok.

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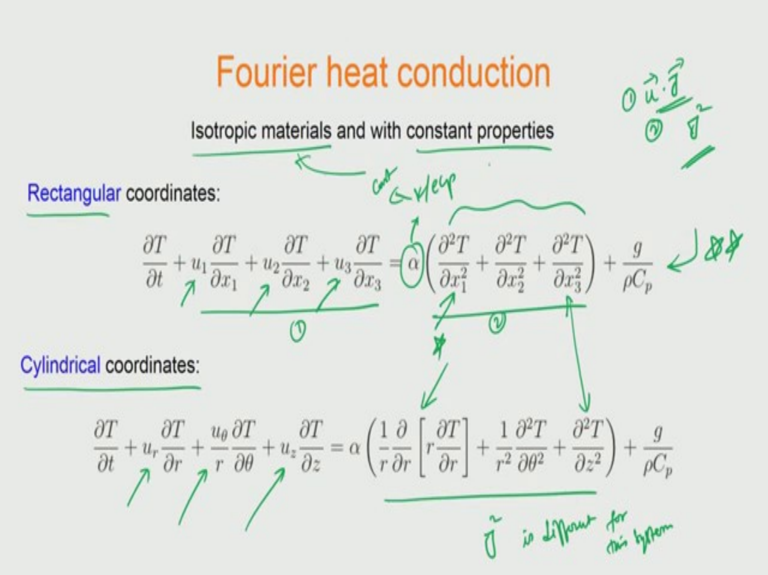

Fourier heat conduction

Isotropic materials and with constant properties

Rectangular coordinates:

$$\underbrace{\frac{\partial T}{\partial t}+u_1\frac{\partial T}{\partial x_1}+u_2\frac{\partial T}{\partial x_2}+u_3\frac{\partial T}{\partial x_3}}_{①}=\boxed{\alpha}\underbrace{\left(\frac{\partial^2 T}{\partial x_1^2}+\frac{\partial^2 T}{\partial x_2^2}+\frac{\partial^2 T}{\partial x_3^2}\right)}_{②}+\frac{g}{\rho C_p}$$

Cylindrical coordinates:

$$\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}=\alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{g}{\rho C_p}$$

① $\vec{u}\cdot\vec{\nabla}$

② ∇^2

$\tilde{\nabla}$ is different for this system

So, the Fourier heat conduction we now then expand. So, what we do is this equation we expand. So, we expand these terms in different coordinate system. So, for rectangle coordinate system we expand the advective term, we can immediately see that this can then be expanded here, and ∇^2T can be expanded here. So, this is the first and this is the second, first operator and second operator.

. So, we can see that the most general form of the Fourier heat conduction equation in rectangular coordinate system will have for example, these many terms. And you would see that the velocities are directly coming here and you have got the diffusion term on the right hand side. And the same thing can be also written for cylindrical coordinate system where the velocities are actually u_r u_θ and u_z and you would also have the Laplacian looking a bit different. So, watch out the Laplacian term different for this system. So, you can see first of all here ok.

So, the $r \times \partial T/\partial r$, 1/r here. So, this is a very important difference that you need to note down, but in the z direction by enlarge it looks the same. So, we can look up these operators, for different coordinate systems and substitute, which means that if you knew how this has come about and that is adequate. So, from here to here you could basically look up the operators and substitute and figure it out. So, that is adequate.

and we have written this equation for isotropic materials why do we say that, because here we are taking it as a constant and this is nothing, but $k/\rho C_p$, and k is constant and that is because you are taking about for isotropic materials, and why do we say its for constant properties; that is, because we have already correlated these constants k and ρ C_p you have taken from the left hand side and taken it under this. So, which means that it is only applicable, when the properties are not spatial coordinate dependent, that is also an assumption that we have written ok.

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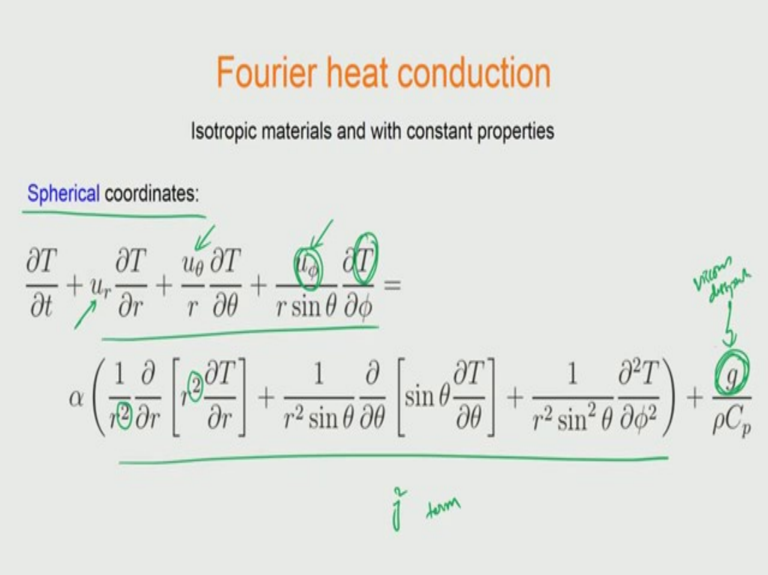

Fourier heat conduction

Isotropic materials and with constant properties

Spherical coordinates:

$$\underline{\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{\boxed{u_\phi}}{r\sin\theta}\frac{\partial \boxed{T}}{\partial \phi}} =$$

$$\alpha\underline{\left(\frac{1}{r^{\boxed{2}}}\frac{\partial}{\partial r}\left[r^{\boxed{2}}\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\frac{\partial T}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)} + \frac{\boxed{g}}{\rho C_p}$$

And for a spherical coordinate system. So, the equation looks like this. So, you have got u_r u_θ and u_ϕ coming in, and the Laplacian term for spherical coordinate system is looking like this. Notice that the power of r is different from the cylindrical coordinate system in cylindrical it is r and here for a spherical it is r^2 that is coming in ok. So, other than that you can actually look up these from handbook and substitute directly ok.

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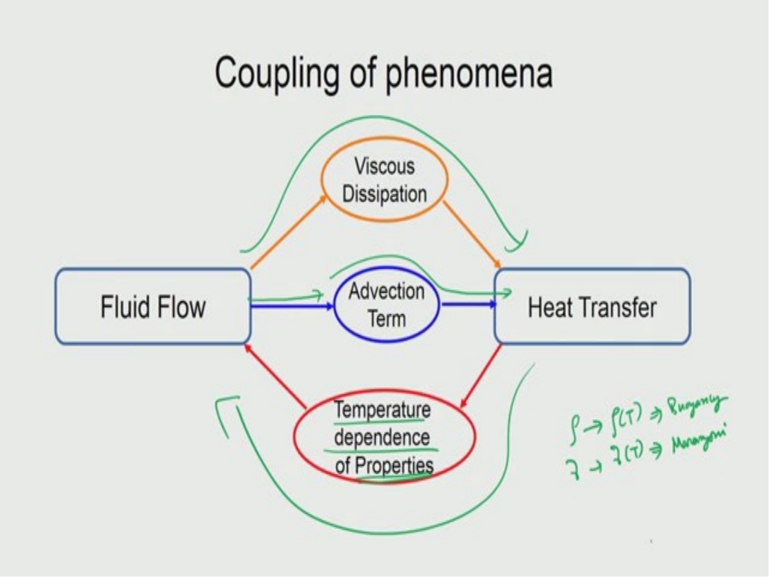

Coupling of phenomena

Viscous Dissipation

Fluid Flow

Advection Term

Heat Transfer

Temperature dependence of Properties

$\rho \rightarrow \rho(T) \Rightarrow$ Buoyancy

$\gamma \rightarrow \gamma(T) \Rightarrow$ Marangoni

And these are going to affect the heat transfer in the following manner. So, you could see that you have got the temperature and also you have got the velocity. So, how does the fluid flow affect the heat transfer and vice versa. So, that is the concept and we are not going to go in depth, because we would have coupled problems being solved later on, but at this point I would mention that the fluid flow is going to affect. We would say that the fluid flow is going to affect the heat transfer mainly via the advective term. So, this is how mainly it will happen. So, the advective term is already visible here. So, this is advective term. So, advective term is what is going to cause an effect of the fluid flow on the heat transfer.

, but there was another way by which it can affect and that is through viscous dissipation; that is because when the velocity gradients are very steep, and that would lead to basically release of heat and that would also be affected and that would come by the g term. So, that this term is coming from viscous dissipation is possible.

So, in situations where there is a very narrow gap of liquid. For example, in lubrication situations the velocity gradients are very steep and viscous dissipation can be quite high, and that also can lead to heat transfer. So, the way fluid flow effects heat transfer is by mainly by advection term, and sometimes when steep gradients are present also by this viscous dissipation.

Now, how does heat transfer affect the fluid flow that would actually affect via the properties. So, you could see that, it is actually affected via the temperature dependent properties. So, if you have for example, ρ that is dependent on temperatures. So, then it will lead to what is called buoyancy flow, and if you had for example, surface tension that is the dependent on the temperature, you would have what is called the marangoni fluid flow etcetera.

So, like that heat transfer can lead to fluid flow and fluid flow can lead to heat transfer, and in such a coupled situation how do we go about. So, we do it step by step, we take one at a time and at this juncture, we basically take heat transfer alone and avoiding the advection term and that is when we write the equation for solids.

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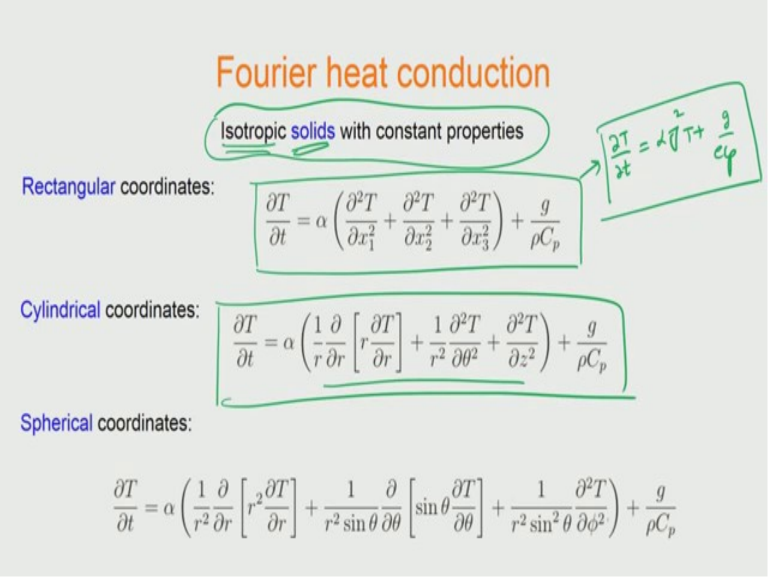

Fourier heat conduction

Isotropic solids with constant properties

Rectangular coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x_1^2} + \frac{\partial^2 T}{\partial x_2^2} + \frac{\partial^2 T}{\partial x_3^2}\right) + \frac{g}{\rho C_p}$$

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{g}{\rho C_p}$$

Cylindrical coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{g}{\rho C_p}$$

Spherical coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\frac{\partial T}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right) + \frac{g}{\rho C_p}$$

So, we write, for solids, isotropic solids how the equation should look like. So, on the left hand side you could see that we have dropped the adjective term, mainly because the velocities are zero. So, the Fourier heat conduction equation for solids is appearing to be very simple and this if you write it in the way that normal is seen in the books you could write it in this fashion. So, it is as small as this. So, you are actually expanding the Laplacian here in this equation, and in cylindrical coordinate system it is expanded here, and the Laplacian is actually here, and in spherical coordinate system, similarly it has been expanded. So, for

Fourier heat conduction in solids which are isotropic, then you can see that the equation is quite small and that can be solved out.

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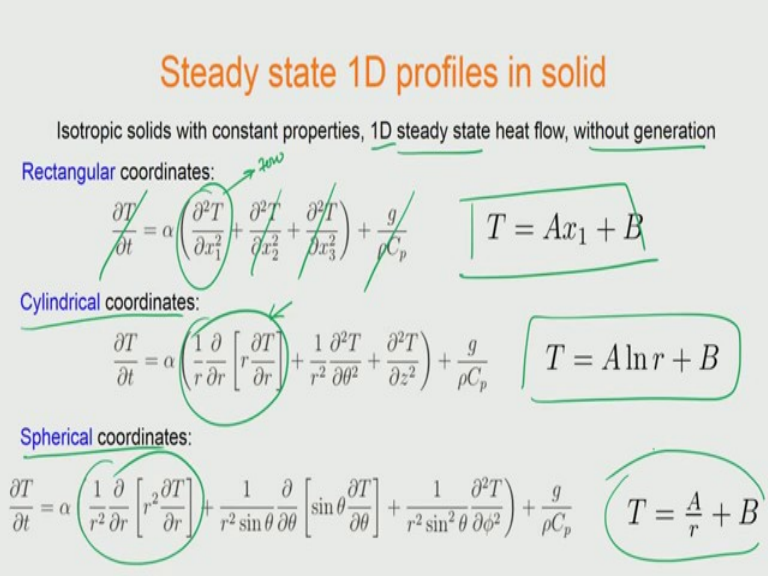

Steady state 1D profiles in solid

Isotropic solids with constant properties, 1D steady state heat flow, without generation

Rectangular coordinates:

flow

$$\cancel{\frac{\partial T}{\partial t}} = \alpha\left(\frac{\partial^2 T}{\partial x_1^2} + \cancel{\frac{\partial^2 T}{\partial x_2^2}} + \cancel{\frac{\partial^2 T}{\partial x_3^2}}\right) + \cancel{\frac{g}{\rho C_p}} \qquad \boxed{T = Ax_1 + B}$$

Cylindrical coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{g}{\rho C_p} \qquad \boxed{T = A\ln r + B}$$

Spherical coordinates:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\frac{\partial T}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right) + \frac{g}{\rho C_p} \qquad \boxed{T = \frac{A}{r} + B}$$

Now, what kind of simple solutions are possible? So, this is something that we need to inspect, because upfront we can actually already get some insight of how the heat transfer is taking place. So, consider these equations and look at the simplest situation possible, when for example, you are looking at steady state. So, you do not want to look at that and we say 1 D. So, you do not want to look at variation in other dimensions and without heat generation you do not want to have this term.

Which means that you have only one term; that is set as 0 and what kind of solutions are possible, and you can already see that this kind of a equation would give you a line as a solution. So, we could see immediately that straight lines are valid solutions of temperature under one dimensional steady state heat flow without heat generation. So, mx + b , Ax + b is definitely one of the solutions that is possible.

; however, when we go to cylindrical coordinate system, the same thing when you apply to for example, here, then it will be logarithmic variation and in spherical coordinate system you look at this term you would see 1/ r variation. So, you could already see that steady state heat flow in 1 D without heat generation can appear to be straight lines or logarithms are $1/r_s$ all of these are valid solutions. So, we can then imagine how the temperature profile would look like in a given problem, draw some schematics and thereby evaluate how the entire thermal profile should look like. So, that it will help us posing the problem properly.

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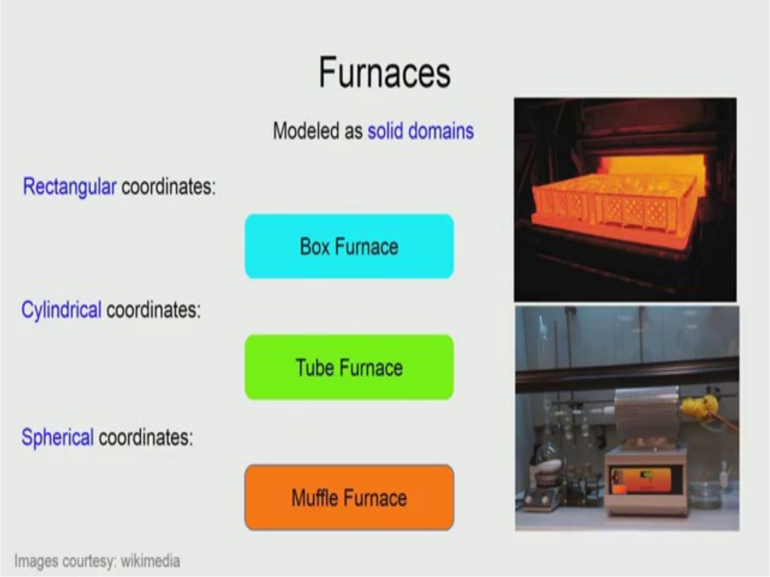

Furnaces

Modeled as solid domains

Rectangular coordinates:

Box Furnace

Cylindrical coordinates:

Tube Furnace

Spherical coordinates:

Muffle Furnace

Images courtesy: wikimedia

Now, here is where actually all these three coordinate systems are going to be useful metallurgical scenarios. So, in metallurgy we have heat treatment; that is happening for you know many components, and that is one problem where the heat transfer is directly used in a solid state, and so we have basically the furnaces where the heat treatment is being done.

So, we do have what are called box furnaces, cube furnaces and muffle furnaces or even pit furnaces. So, normally box furnaces can be modeled using rectangular coordinate system, tube furnaces can be modeled using cylindrical coordinate systems, muffle furnaces or pit furnaces where the sample is very small compared to the volume of the heating zone that is around it, then you can actually consider it as a spherical coordinate system. So, we have scenarios of all these three coming up in metallurgy, which means that ability to solve heat conduction equations in rectangular cylindrical and spherical coordinate system is something that all metallurgical engineering students should be able to possess ok.

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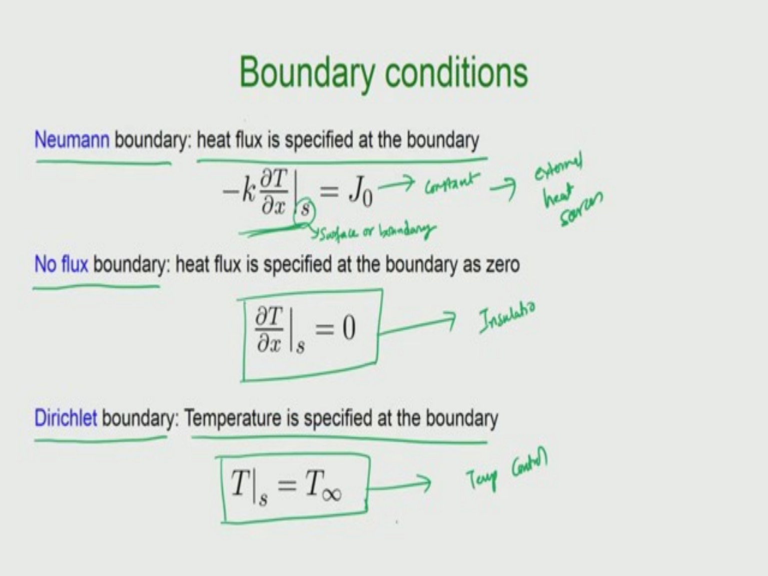

Boundary conditions

Neumann boundary: heat flux is specified at the boundary

$$-k\frac{\partial T}{\partial x}\Big|_{s} = J_0 \rightarrow \text{constant} \rightarrow \text{external heat sources}$$

s → surface or boundary

No flux boundary: heat flux is specified at the boundary as zero

$$\boxed{\frac{\partial T}{\partial x}\Big|_s = 0} \rightarrow \text{Insulation}$$

Dirichlet boundary: Temperature is specified at the boundary

$$\boxed{T|_s = T_\infty} \rightarrow \text{Temp Control}$$

Now, when we solve these problems we have to pose the boundary conditions, because that would only make the problem unique or a well posed problem. So, there are many boundary conditions that we can write. The first boundary condition we come across goes by a name Neumann boundary. So, Neumann boundary condition is where the heat fluxes specified at the boundary. So, the heat flux is given already by the Fourier's heat conduction equation here, and we are evaluating at s which means that it is evaluated at the surface or the boundary. So, the heat flux that is evaluated the boundary is given as a constant. So, J_o is basically a number. So, once you have the heat flux as a number then; that means, that you are actually using what is called as a Neumann boundary condition.

So, sometimes you may say that the surface is insulated. So, there is no heat that is a getting in are getting out. So, in such situations you can use what is called as a no flux boundary which is a special case of Neumann boundary condition where J_o is zero. So, when J_o is 0 you can drop k and you can say that the temperature profile would have a maximum or 0 slope at the surface, and that would be a no flux boundary condition.

And Dirichlet boundary condition is when the temperature is specified, the value of the parameter is specified not the slope. So, slope is specified is called Neumann boundary condition, the value is specified is called the Dirichlet boundary condition. So, the temperature at the boundary is specified it is called the Dirichlet boundary condition, which is also quite popular.

So, usually these are very popular, because you can achieve the insulation to give this. This actually requiring some task because you need temperature controls, and therefore, these are not very easy to achieve. These are actually also easy to achieve, because you actually have external heat sources that can give heat flux at a particular rate to the body. So, if you have metallurgical scenarios, usually it would be encountering any of these things coming in as boundary condition. So, we must inspect the problem and see which boundary condition is suitable at which location of our domain.

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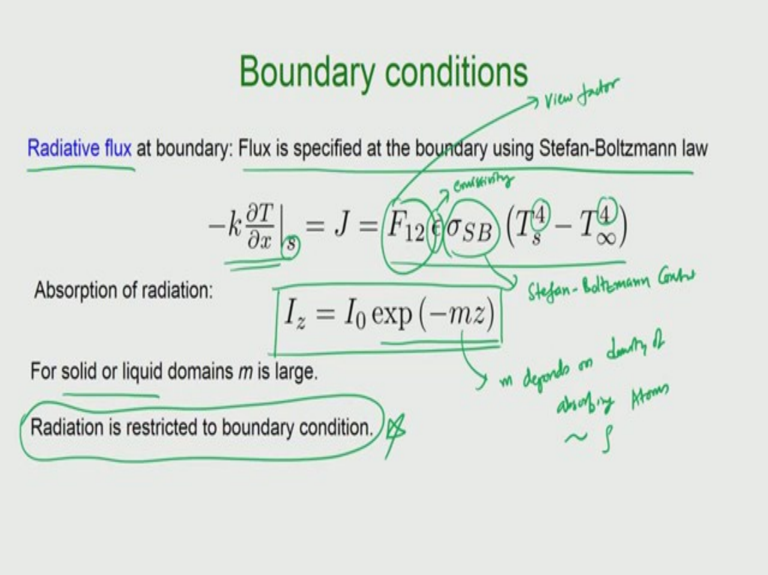

Boundary conditions

Radiative flux at boundary: Flux is specified at the boundary using Stefan-Boltzmann law

$$-k\frac{\partial T}{\partial x}\bigg|_{s} = J = F_{12}\,\epsilon\,\sigma_{SB}\left(T_s^4 - T_\infty^4\right)$$

view factor

emissivity

Stefan-Boltzmann Constant

Absorption of radiation:

$$\boxed{I_z = I_0 \exp\left(-mz\right)}$$

m depends on density of absorbing Atoms ~ ρ

For solid or liquid domains *m* is large.

Radiation is restricted to boundary condition.

And we also have, very often the flux condition not specified as a constant, but as a function of temperature and very common boundary condition like that is a radiative boundary condition. So, bodies that are exposed to radiation from outside or which are losing heat, because of radiation can be basically specified as a flux at the boundary losing heat by radiation given by Stefan Boltzmann law and; that means, that the flux at the boundary s indicates that the surface is specified by the heat loss by radiation. So, the fourth power should tell you that its a radiation equation. So, this is the Stefan Boltzmann constant and this.

So, σ_{SB} is Stefan- Boltzmann constant ,ϵ is emissivity and F_{12} is basically called as the view factor which is indicating, whether the radiation is actually able to go away or is it being shadowed etcetera. And normally radiation is treated not as a heat transfer mode within the domain, but only as a boundary condition. So, the reason is as follows; most of our metallurgical problems involve the domain to be either solid or liquid, and when you have solid or liquid then they will attenuate electromagnetic radiation given by the beer lamberts law, here that is given.

now that it is exponentially the intensity of the radiation going through the condensed matter, is going to down go down as a exp(-mz). This m depends on the density of absorbing particles or atoms, which means that it is proportional to the density of the material, which means that in gases you would have radiation going to the complete domain, but if the domain is made of not gas, but solid or liquid, then within a short distance, because m is proportional to density which is very high for the solid or liquid, m being very large means that the I_z will be 0 very soon which means is that the radiation is going to be absorbed only within a first few microns of the body, which means that mainly is a boundary condition.

So, that is a reason why radiation is treated as only a boundary condition in metallurgical problems. Of course, if I am going to model a furnace the interior of the furnace then its not true, but most of the time we are actually having the domain not as a furnace, but to the sample that is inside the furnace and therefore, this kind of a thing is applicable. So, radiation is only a boundary condition ok.

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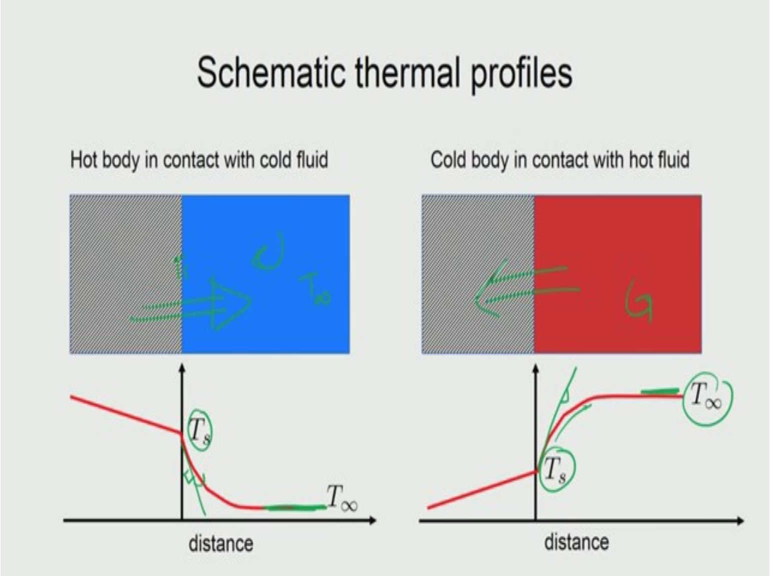

Schematic thermal profiles

Hot body in contact with cold fluid

Cold body in contact with hot fluid

So, here is where we want to then look at what happens if our boundaries where you have got a fluid in contact. So, what happens is that, when there is a fluid and if you are going to look at hot body in contact with a cold fluid, then you know that far away in the fluid the temperature is low. So, you can see here the temperature is low, and this is at T_∞, and the interface is at T_s that is given here. So, we see that the temperature may fall in this manner. The reason why we draw it parallel to the axis far away is, because the interface is far away and so, the heat loss is not felt very much away from the interface.

So, which means that the temperature profile has to be drawn in this manner, asymptotically going towards T_∞ when every hot bodies in contact with cold fluid. If it is a cold body in contact with the hot fluid we actually also make it as asymptotic, but going to a high value. So, asymptotic manner is always drawn, and this kind of a heat transfer would indicate for example, in this case the heat transfer is going in this direction, in this case heat transfer is going in this direction.

So, in these kind of situations it is very difficult to ascertain what will be the slope of these variations so, the reason being that they depend upon the fluid flow. So, if you have the liquid that is convicting, then it would change the slope and very difficult to find out what is the slope here. So, because of that what we do is that, instead of expressing the heat flux at this interface as slope what we do is, we just take the difference of the temperatures and then express and that is where we actually come across the definition of heat transfer coefficient.

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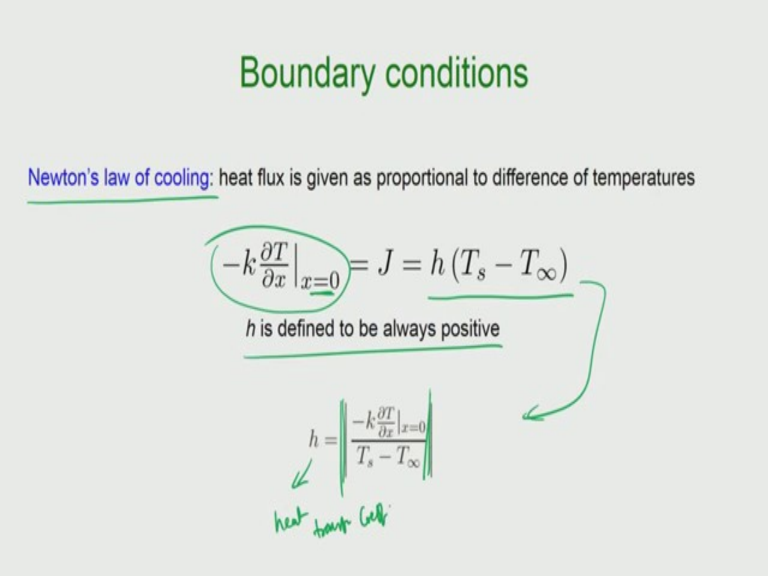

Boundary conditions

Newton's law of cooling: heat flux is given as proportional to difference of temperatures

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = J = \underline{h\left(T_s - T_\infty\right)}$$

h is defined to be always positive

$$h = \left|\frac{-k\frac{\partial T}{\partial x}\big|_{x=0}}{T_s - T_\infty}\right|$$

heat transf Coeff

So, we say that in boundary conditions you may also have a situation where you want to call it is a Newton's law of cooling and the heat flux at the boundary, which is given by x= 0 is written as h× (T_s - T_∞). So, difference of temperatures into h and we also need to remember that h is always defined to be positive, which means that from this expression you can immediately see that h is defined with an absolute magnitude of this particular quantity; that is flux you take it this is a positive or negative, whether it is going towards the left side or right side take the magnitude divided with the temperature difference and that would be giving you the heat transfer coefficient. So, h is what we now define as heat transfer coefficient, which is watt per meter square ok.

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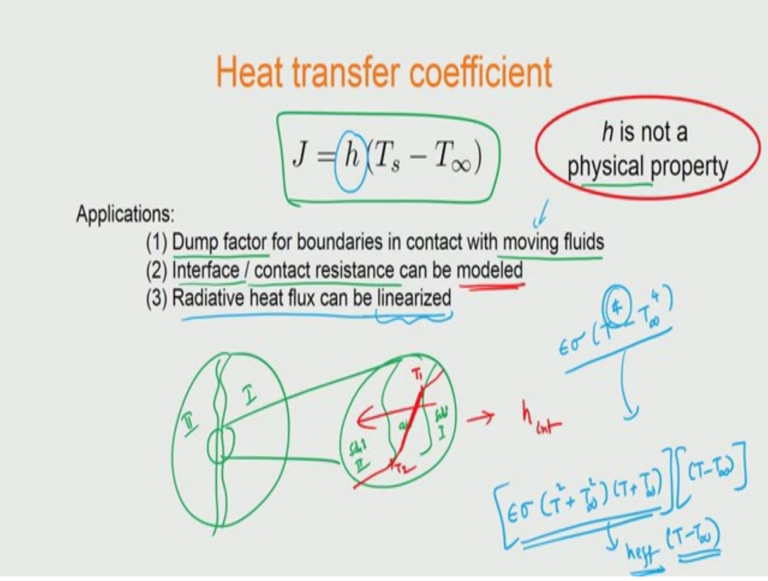

Heat transfer coefficient

$$J = h(T_s - T_\infty)$$

h is not a physical property

Applications:

(1) Dump factor for boundaries in contact with moving fluids

(2) Interface / contact resistance can be modeled

(3) Radiative heat flux can be linearized

So, heat transfer coefficient is then defined once here. So, we should not confuse that to be a physical property the reason being that, it depends not only on the thermal conductivity of the fluid that is in contact with the solid body, it also depend upon the velocity. So, that are there in the liquid, the geometrical structure of the interface and so on. So, therefore, is basically a dump factor. So, you can think of as a dump factor whenever there is a boundary in contact with a moving fluid. So, the fluid property like thermal conductivity plays a role. So, if you have a fluid that have a higher thermal conductivity then h will be higher and so on.

It is also very often used to model what is called contact resistance or interface resistance, what we mean by that is as follows. If you have 2 bodies, then if their contact is not perfect, if you were to then zoom in here. So, you see that between them there is an air gap, and this is a solid 2 and this is solid one.

So, which means that the temperature profile across this body for example, could be like this, which means that the temperatures at these 2 ends will be different, which means that the heat flow this way should be dependent also on this gap. And this air gap is what is causing the contact resistance, and very often this air gap is a function of this surface roughness etcetera which is very difficult for us to model. So, the entire thing can be dumped and say that this leads to a h which is at the interface. So, heat transfer coefficient can also be used at interfaces where there is a contact resistance that is present, and for situations like spot welding resistance spot welding, this plays a very big role.

You also can use this concept to do what is called linearization. So, in radiative heat flux you can also use it for linearization. So, you could look at this for example, $\epsilon \times \sigma \times T^4 - T^4_\infty$. Now this is a fourth power law, and if you want to then convert it into a linearized equation you could write it in this manner $(T^2 + T^2_\infty) \times (T + T_\infty) \times (T - T_\infty)$.

So, you could see that these both are equivalent, and then this entire thing can be written as $h_{eff} \times T - T_\infty$. So, you could actually linearize a fourth power equation into the heat transfer coefficient kind of a scenario, where the effective heat transfer coefficient can be given as $\epsilon\sigma \times (T^2 + T^2_\infty) \times (T + T_\infty)$. So, that way we can actually linearize using this. So, h can be used for multiple purposes, it can be used for linearization, it can be used to model the air gaps, contact resistance and it can also be used as a dump factor, whenever there is a fluid with unknown fluid flow that is taking place ok.

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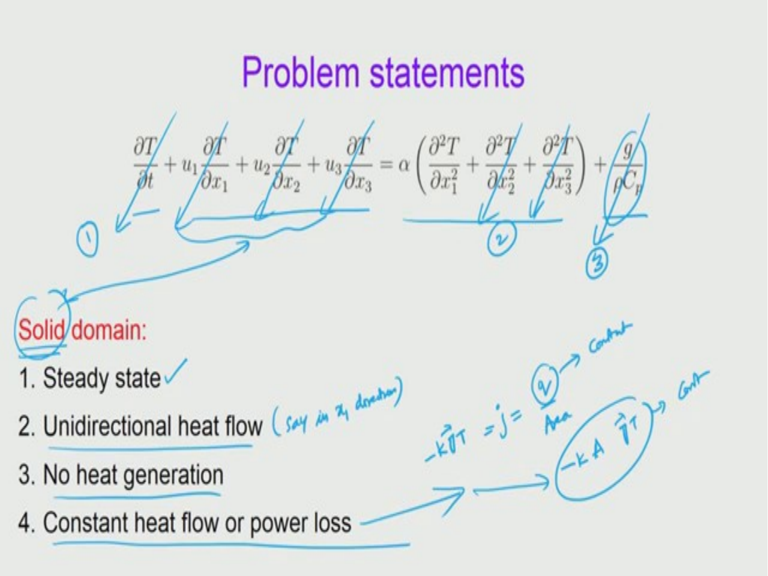

Problem statements

$$\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{①} + \underbrace{u_1 \cancel{\frac{\partial T}{\partial x_1}} + u_2 \cancel{\frac{\partial T}{\partial x_2}} + u_3 \cancel{\frac{\partial T}{\partial x_3}}} = \alpha \underbrace{\left(\frac{\partial^2 T}{\partial x_1^2} + \cancel{\frac{\partial^2 T}{\partial x_2^2}} + \cancel{\frac{\partial^2 T}{\partial x_3^2}} \right)}_{②} + \underbrace{\cancel{\frac{g}{\rho C_p}}}_{③}$$

Solid domain:

1. Steady state ✓
2. Unidirectional heat flow (say in x_1 direction)
3. No heat generation
4. Constant heat flow or power loss

$-k\vec{\nabla}T = \dot{j} = \frac{q}{Area} \rightarrow$ Constant

$-kA\vec{\nabla}T \rightarrow$ Const

And we need to then now move on to make a couple of problem statements. So, we will do it only for 2 domains and 2 situations and then the problem becomes well defined and therefore, we can start to solve the equations. So, this is the generalized heat conduction equation we have written, for the domain which will have solid and liquid etcetera. So, we have the advective term, we have the steady the transient term, the advective term or the diffusive term and the source term that are present.

So, we make statements about the problem to make it little more well defined. So, if you say that the problem is under steady state heat transfer. So, the moment we say steady state; that means, we drop this and make that, because of the assumption one and we say that, let us say it is for a solid domain. So, the moment we see a solid domain you know that u_1 and u_2 and u_3 are all zero. So, the entire thing can be dropped and this is, because its a solid domain.

Then we say unidirectional heat flow. So, we let us say in x_1 direction, which means that along the x_2 and x_3 directions there is no variation of temperature. So, we can then use that to drop these. So, as per the assumption 2 we can drop those. And if you say that no heat generation; that means, we can drop in this. So, like this we can actually drop terms, depending upon the problem it's not necessary that all these assumptions should be there in a given problem, but we should look out whenever these assumptions applicable and they accordingly reduce the number of terms in the equation, to help us integrate and arrive at the solution.

Sometimes we may also have situations like this constant power loss. So, which means that you may have situation like J can be thought of as q/area, and q is then given as a constant, and J is already written we know that k × grad T. So, you can think of that this, means that - k × area × area × grad T is constant. So, this can be an assumption, also that can be given which can also simplify the problems. So, we can actually reduce the number of terms depending upon any of these assumptions.

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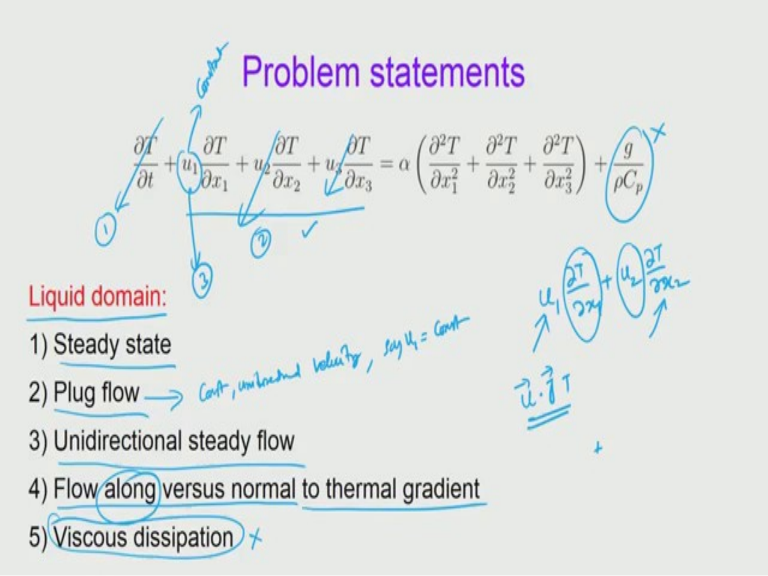

Problem statements

$$\cancel{\frac{\partial T}{\partial t}} + u_1\frac{\partial T}{\partial x_1} + \cancel{u_2\frac{\partial T}{\partial x_2}} + \cancel{u_3\frac{\partial T}{\partial x_3}} = \alpha\left(\frac{\partial^2 T}{\partial x_1^2} + \frac{\partial^2 T}{\partial x_2^2} + \frac{\partial^2 T}{\partial x_3^2}\right) + \frac{g}{\rho C_p}$$

Liquid domain:

1) Steady state
2) Plug flow → Const, unidirectional velocity, say u_1 = Const
3) Unidirectional steady flow
4) Flow along versus normal to thermal gradient
5) Viscous dissipation ×

We can also make these problem statements for liquid domain. So, four liquid domains you already see that these things should be present, and that is because advective term is present. Now steady state would mean that you could drop this. So, first term can go. Plug flow. By plug flow we mean constant and unidirectional velocity, which means let us say u_1 is equal to constant, which means that you can drop u_2 and u_3 if it is a plug flow. Plug flow along x_1 direction would mean this is a constant and u_2 and u_3 has zero. So, you could think of that.

In case if it is written as unidirectional steady flow, you could also think, if it is a third assumption then u_1 could be a function of x_2 for example, that is allowed, but u_2 and u_3 will be 0 in case the directionality of flow is along x_1 direction. So, you could actually simplify the problem by these kind of statements if the flow is along or normal to the thermal gradient. So, what we mean by is as follows; this equation that you have written. So, look at the terms $u_1\frac{\partial T}{\partial x_1}+u_2\frac{\partial T}{\partial x_2}$, let us look at only the first 2 terms.

Let us say that the temperature variation is allowed along the x_2 direction, but its only the u_1 that is actually present, which means that both the terms will get dropped, there is reason is that, this will not be present and this will not be present. So, like that sometimes depending upon the velocity and the direction of the temperature gradient, you may actually have some of the terms on the advective term being dropped ok. So, when do these terms is survive, when for example, the velocity that you have specified and the gradient, they are aligning with each other. So, only when such situations so along so, only then the terms survive; otherwise there will be known as survive.

Sometimes we may have viscous dissipation being present or not. So, if you say that the viscous dissipation is not present; that means that you can actually drop this term. Like this even in a liquid domain, you may have the governing equation for temperature having less terms, because of the assumptions that we make.

So, using these problem statements we start off, first of all with the generalized Fourier heat conduction equation, then using these assumptions, we reduce the number of terms and then we have a simplified partial differential equation which when we integrate, and then using the boundary conditions as the integration constants we then arrive at the solution of temperature as a function of distance. So, this is how we can arrive at the thermal profiles in a given problem. So, with this we close this session and then look at the practice problems for any numerical problems that you could work with in the course website.