Transport Phenomena in Materials
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Lecture – 16
Friction factors

Welcome to the session on friction factors as part of the NPTEL MOOC on Transport Phenomena in Materials.

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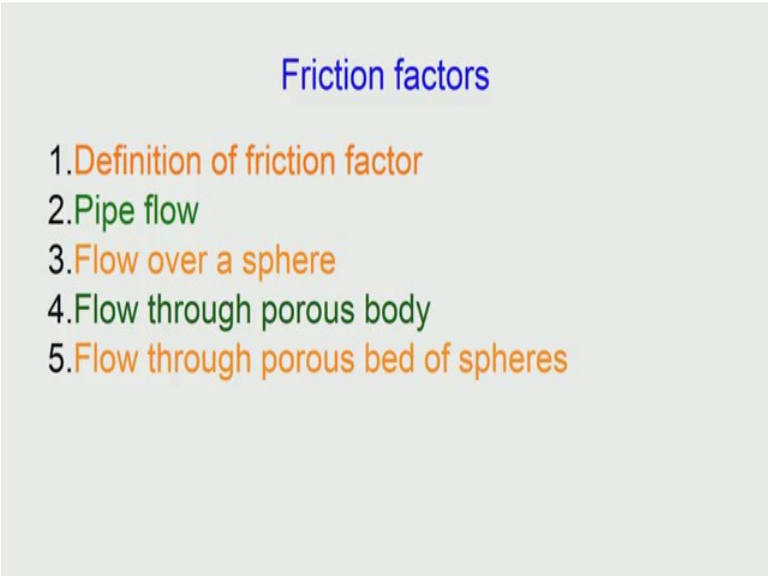

So, in this session we will be looking at the definition of friction factor; the friction factor is go by names of different ways of modelling them. And we will be taking up what is generally default one handled in many of the metallurgical textbooks. We will take four problems and then derive the friction factor expression for these; the analytical solution for the velocity for these four problems has already been covered in this MOOC.

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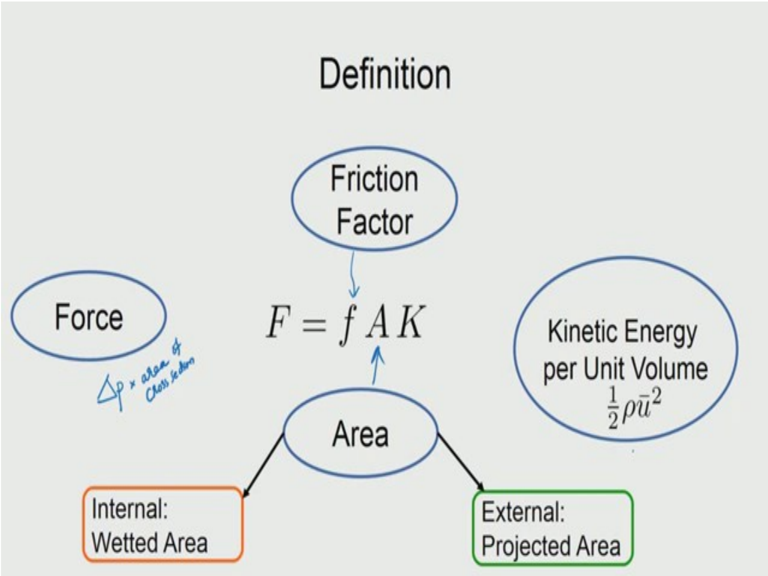

See the definition is as follows; friction factor is this quantity f that we have written here and it is defined as the expression here F capital F which is a force is equal to small f × area A × K; which is the kinetic energy per unit volume. So, if you see this K and if you multiply with the volume then you will get the kinetic energy half mv^2.

Now from this definition, we can already see that there is a meaning for this f and we will come to that interpretation in a moment. The force is generally to be modelled as follows; it is to be taken as the pressure drop that is across the domain that is causing the flow into the area over which it is acting. So, which means that it is normally the cross sectional area and this is not the same area as a that is written here because that is modelled in two different ways for two types of problems.

So, what are called as the internal flow for example, the pipe flow is an internal flow where the wall is in the surroundings and so, such problems we take the wetted area. So, here the wetted area in the case of pipe flow for would be for example, the perimeter × the length that is πdl. Now in the case of external flow that is the flow over objects like for example, flow around a sphere; then we take the projected area..

So, projected area in the case of a flow over a sphere would be for example, the cross sectional area of the sphere itself which basically is πr^2. So, you do have for example, different ways of defining A and so we need to watch out which type of flow we are looking at and accordingly take that expression. And the u is again the velocity that is taking representative; so, very often it is an average velocity that we are considering.

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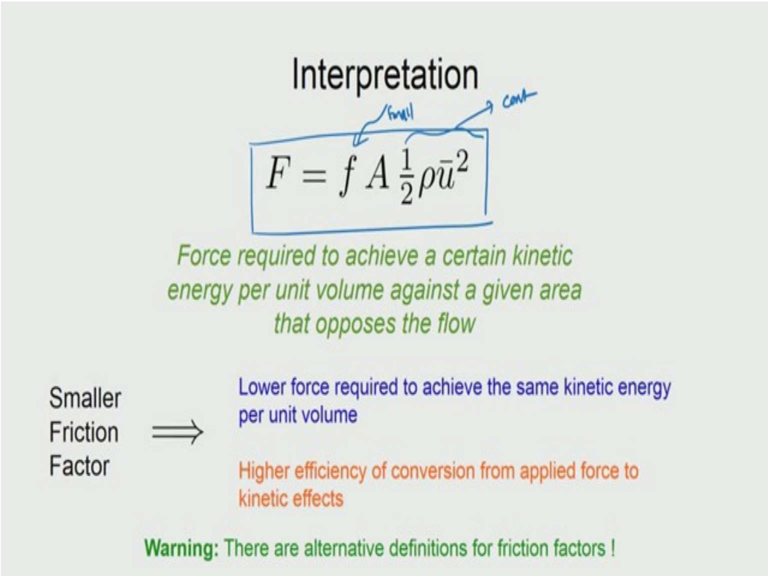

So, here the interpretation the meaning of this friction factor F. So, here if you see the way we have written the expression it is basically seeing that how we are able to convert the pressure that is applied over the cross section into the kinetic energy of the liquid that is flowing through. So, it is basically the efficiency by which we are able to convert the driving force for flow into the actual flow itself. And when a friction factor is taking a very small value, it actually implies if you can see this expression; this is small it implies that if this is remaining constant then you would require less force to achieve the same amount of kinetic energy.

Which means that getting to flow regimes in which friction factor is small would be good because that would be more efficient. And we have to also look at to the definition of a friction factor that is different in the different models, but by and large; we can say that when nothing is told then it is this way that we are considering the friction factor.

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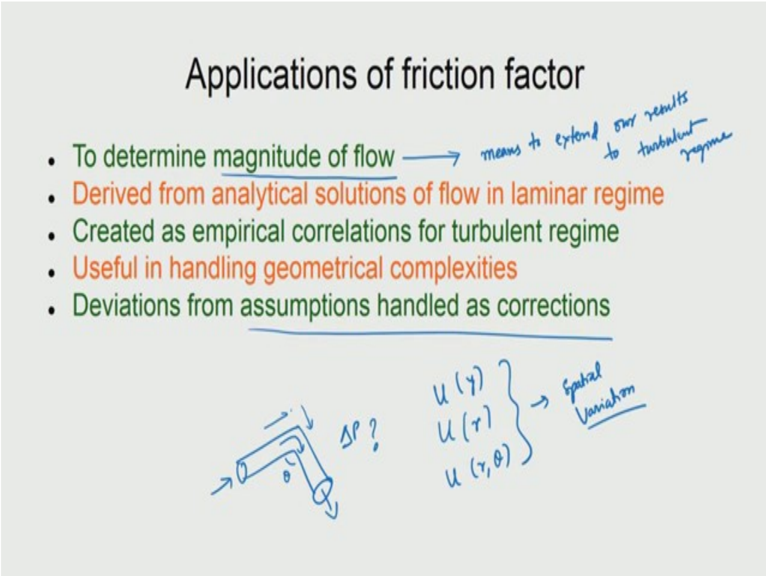

So some of the applications of friction factor why do we need this concept at all? So, we have already seen that we have the analytical solution for the velocity. So, if you have u and you already have u as a function of r, u as a function of r and θ, u as a function of y and so, on.

So, usually these are basically spatial variations that are available and very often the spatial variation is not relevant. For example, in situations like in turbulent regime the spatial variation is actually a function of time. And there is no point in for example, describing elaborately what would be only a transient in nature. So, it would be good for example, if we can take what would be time averaged velocity over the entire domain and see how that magnitude of the flow can be determined. So, the correlation for such magnitudes of flows can be derived for the analytical flows that we have already. So, we will be doing that, but we can use the same model to extend such correlations into turbulent regime.

So, in other words you can say that we actually use friction factor where, we are interested in the magnitude of flow, but not necessarily the variation of the flow. So, you cannot for example, get the spatial variation of flow; if you take the approach of friction factor, the way we are describing here .And we are also then conscious that it provides a means to extend what we have learnt our expressions, extend our results from the analytical solutions to turbulent regime that is because then we actually have in some way by which we can use those expressions for some industrially useful problems.

And we do have for example, situations where there is geometrical complexities for example, there is a pipe flow problem the pipe is actually not of exactly circular in cross section. So, then what do we do? So, we need some way by which we can handle that deviation as how it is affecting the pressure drop in our problem. So, we need a means and here again we are having that possible because we can handle that. For example, the pipes may not be smooth; it may be rough inside because of corrosion and how does that affect the pressure drop..

So, we will have those things handled because once we have a way to have the correlation of friction factor then we can add more terms to it as the industrial situation demands. And we also have situations where assumptions can be also deviated and given as a correction factor. For example, the pipe is actually straight line and we are actually saying strictly axial flow in the quasi flow problem we have discussed earlier..

Now for example, the pipe is exactly not straight, but it has some bends. So, each of the bends will involve some amount of change in the efficiency of the flow that will be taking place. Because there are these gradients that are taking place; so, the horizontal velocity has to come to a stop and the vertical velocity has to start.

So, those gradients are all going to dissipate the energy and therefore, some way by which each of these bends it because of the θ angle bend should relate to some pressure drops. Now how are we going to get these? So, these also can be modelled using the concept of friction factor. So, that way this concept is very useful in handling the real life problems and we will then see how these expressions look like.

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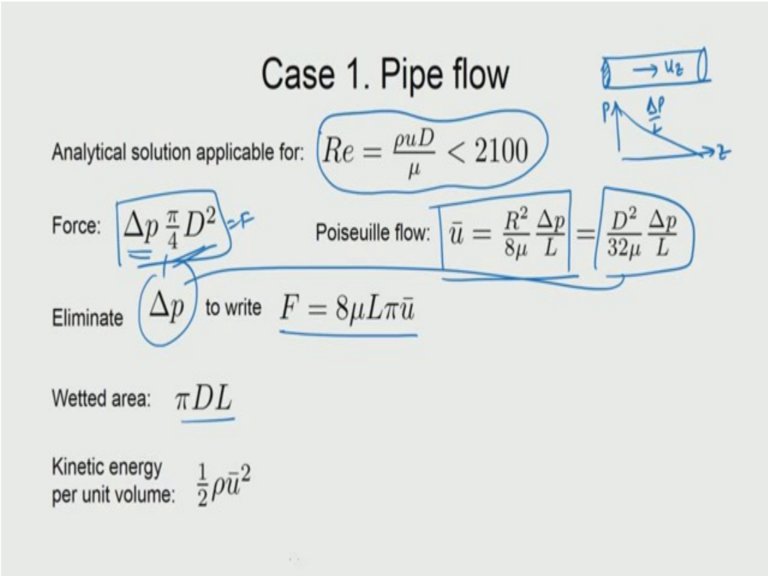

So, the first problem is reuse of the pipe flow problem that we have done earlier. And we have already seen that the pipe flow problem where we are looking at the axial velocity because of a pressure drop per unit length that is taking place.

And you see that such a flow is already modelled by us and here is the result for the average velocity. The velocity expression itself is given as parabolic that we have already come across. And we have seen that this kind of a result is applicable when the Reynolds number is less than 2100. So, the whole idea is one when what do we do about that number.

So, that is where we are actually headed and let us see for the range that is valid how does the friction factor appear using the method that we have seen. So, what we do basically is look at the terms that we have taken up. So, the pressure is Δp over which the domain entire domain is actually experiencing the driving force of flow into the area over which cross sectional area over which the Δp is acting.

So, that becomes the force term and the solution of the average velocity is already given here and that if you simplify then what we can do is eliminate. So, you can eliminate Δp between these two expressions. So, when you eliminate then what happens is that you can get what is the F. So, F is this is F, but you can eliminate Δp using this expression; so, when you substitute that you will get the expression for F and then the wetted area.

So, we know that the wetted area is nothing, but the perimeter into a length and that is given here and the kinetic energy per unit volume is here.

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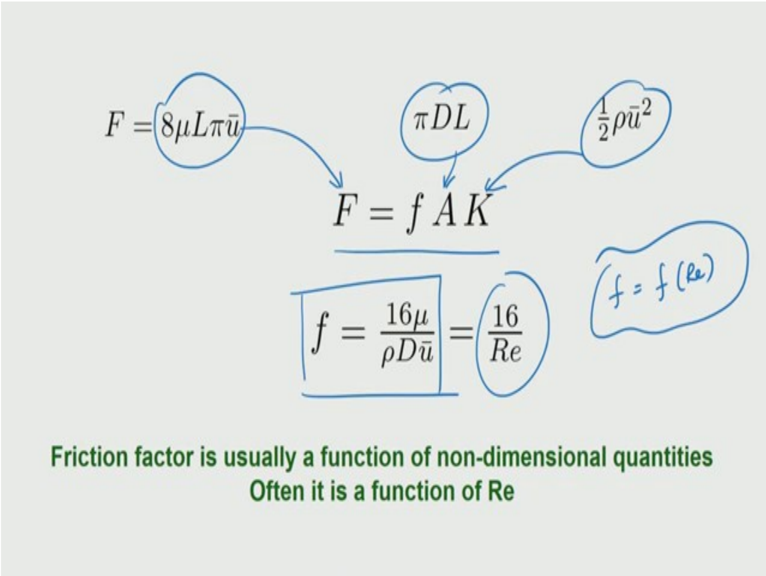

So, once we have all these expressions then we substitute these. So, we put that term here and the wetted area here and the kinetic energy per unit volume here. So, immediately we will get what the friction factor; interestingly once you evaluate you would get the friction factor to be like

$f = \frac{16\mu}{\rho D\acute{u}}$ and immediately you can see that in the denominator this is Reynolds number that is sticking.

So, which means that you actually have the friction factor is coming as a function of Reynolds number. And this was actually the design behind the formula that we have actually decided. And once we have this then we would see how very interestingly similar kind of forms will come in other problems also. Usually we can see from the units that the friction factor should not have any units at all; it is a non dimensional quantity. So, it must be also a function of only non dimensional quantities you normally do not encounter a friction factors as a function of for example, length or diameter etcetera.

They must always be normalized with quantities of similar dimensions so, that you basically have non dimensional quantities and very often in many of the problems it is a function of only the Reynolds number.

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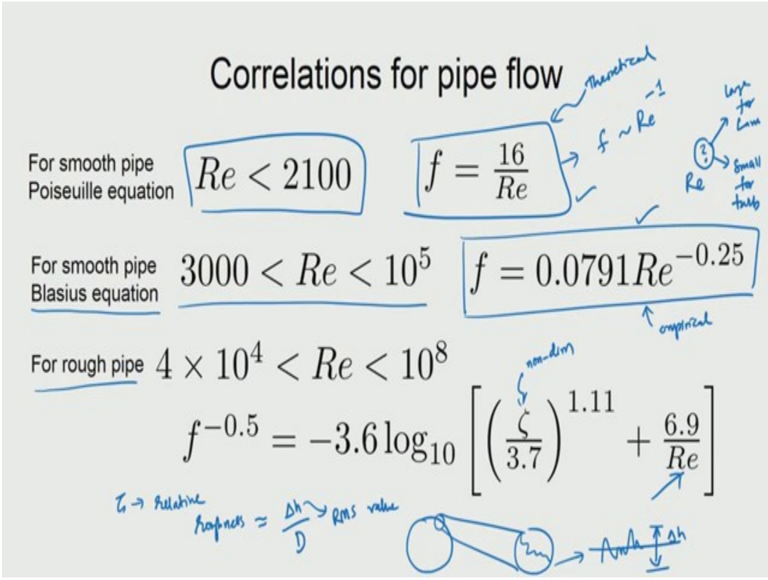

So, for the Reynolds number less than 2100; the friction factor is given. Now we can then use the same approach that is in a given experimental problem, we can actually measure what pressure drop is causing what kind of a velocity and make a correlation and then get F and they make their correlation with Reynolds number as the regression variable and then see how the friction factor should evaluate. And that actually is given in many open literature is that are available. And there are some results that are actually for a very wide range of Reynolds numbers and therefore, they are also very popular.

So, this relationship for example, is applicable for turbulent regime and it goes by the name Blasius equation and the friction factor is then given as a power of Reynolds number to raised to -0.25. Normally you would see that here for example, friction factor is going as a Reynolds number estimate of -1. So, you would see that the power here this quantity generally is small; if you look at the magnitude small for the turbulent regime and it is a large for a laminar regime which means basically that once you have these expressions, you can already make a guess which expression could be for what kind of a regime.

But nevertheless you must always look up the exact range of Reynolds number over which such a correlation has been fitted. So, please remember that this is empirical which means that it has been derived from experiments and this is basically theoretical because it has been derived from analytical expressions; it turns out that of course, this is also valid for experimentally you know similar problems over the wide range of Reynolds number up to

2100, but nevertheless you must know the origin. So, there are things that are from theory and there are things that are from experiments.

Now the problem of a deviation from assumptions, we have assumed in the pipe flow that it is a smooth rigid straight pipe. And in case it is a not smooth; if there is a roughness that has been developed inside because of the corrosion etcetera then what do we do? So, we then define this ζ variable which is basically called as a relative roughness. So, a relative roughness is basically defined as the roughness itself which is basically Δh divided by the diameter. And Δh is basically usually RMS value that is a root mean square value. So, if you look at a pipe and inspect the region which is the inner surface then that region would appear to be a rough like that.

And this profile if you then what to see what is the root mean square value. So, this Δh is what is given as a roughness; in other words the roughness is nothing, but in length units and you can say roughness of 50 microns roughness of half an mm, roughness of 1 micron etcetera. And if you divided by the diameter, we get the relative roughness; so, this is a relative roughness, so it is basically non dimensional. So, we have basically the friction factor made as a correlation for not only the Reynolds number, but also the relative roughness which means that even for rough pipes we do have a way by which we can go ahead and evaluate what would be the pressure drop required if you want to push fluid through it at a particular velocity.

And for the velocity you evaluate the Reynolds number and then plug it in with the roughness and you get the friction factor that will give you what if the Δp look like etcetera. So, like this we can actually solve real life problems by using these kind of a correlations. And when we plot these three friction factors over a complete range of Reynolds number, we see something interesting we plot it to using log log plot.

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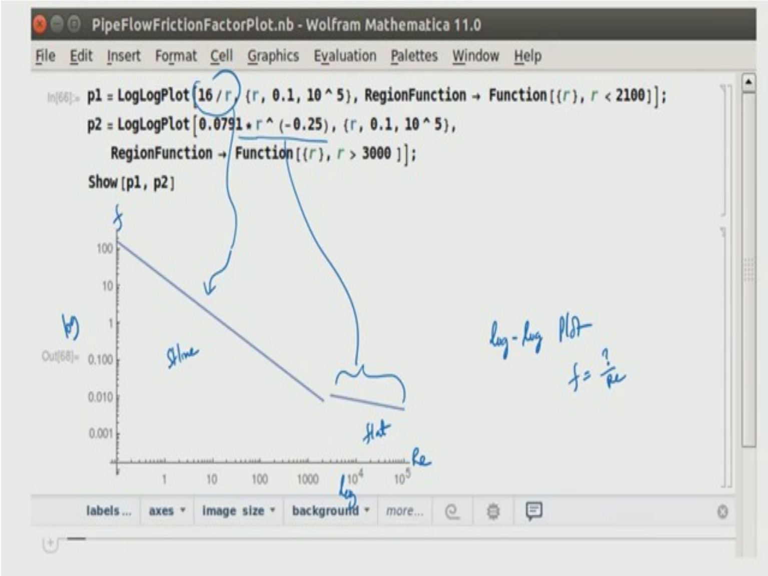

So, you notice here that it is log and the log plot. So, remember that it is always made a log log plot and the log log plot of a function that looks like f is equal to something by Re, you would be naturally have a negative slope of one which is 45 degrees in the negative way, which you can already see that is visible here.

So, at this region is coming from this expression and this region is coming from this expression. So, you could see that in the left hand side of the log log plot of a friction factor as a function of Reynolds number; you would have basically a straight line for the laminar or analytical derived the friction factor and you would have fairly flat region; so, you could actually see that straight line drooping downwards and fairly flat region for the laminar and turbulent regimes respectively..

So, we will see whether this kind of a trend is replicated in other kind of domains or so. So we will see that and that would be something as a learning as we go along. So, I am actually showing you how to plot multiple functions in different regimes; in a mathematical notebook by taking a screenshot here.

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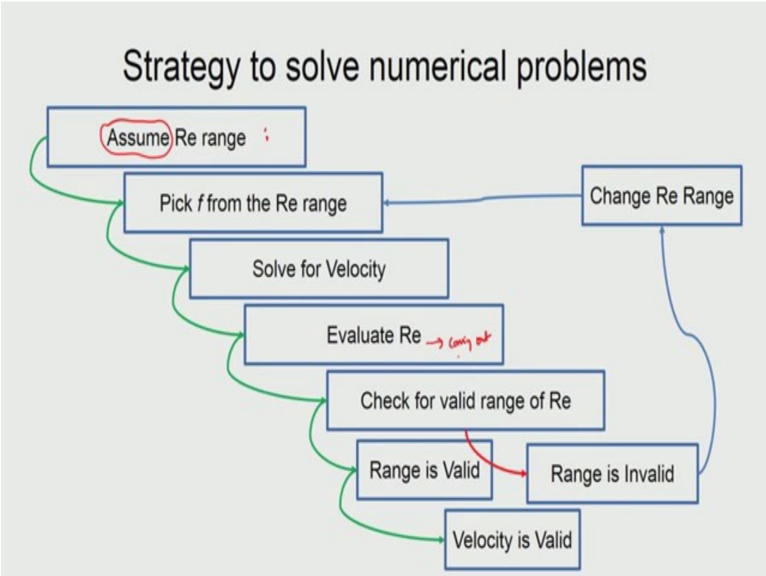

So, the strategy to solve numerical problems once you have these correlations is as follows. Basically up front once we have a problem where the pressure drop required is to be determined and then we do not know for example whether the flow is in laminar regime or turbulent regime. So, we do not know; so upfront what we normally do is look at the diameter of the pipe and if it is large if it is large in centimetres or more then maybe it would be turbulent and go ahead and start with the turbulent regime expression.

So, in other words we have to basically assume the regime of the Reynolds number and pick the friction factor from that. Once you pick the friction factor expression which means that then you could use it to solve for the velocity. So, the velocity solution would then require for example some iteration depending upon the complexity of the expression.

For example, if you take the rough pipe expression; you do have the logarithm term. So, there will be some iteration that is required and usually if you use a software like MATLAB or mathematica, you can directly solve for it; otherwise you could also use a calculator and use what is called single point iteration scheme and then get to this kind of solution. So, we would have some practice of such problems in a tutorial anyway, but once you solve for the velocity without forgetting; we should always evaluate what we do the Reynolds number?

And see that there whether Reynolds number that is coming out is in the range that we have picked. So, if it does not fit in the same range if it is not in the valid range of the Reynolds number; then what we have to do is that immediately change the expression to another range depending upon what range we have got.

So, to just illustrate that point let me tell you here; you see that if you assume this problem and you get the range that is actually very small which is less than 4×10^4; then you go to the other expression here. So, like that you basically change the expression so, that you pick the correct friction factor and then we can again repeat the same scheme.

So, usually you would converge to one of the regimes as long as the problem is well defined. And once you converge then what happens is the Reynolds number you get would be in the same range as you have started for the friction factor and then you can just see that the velocity that you got is actually valid and then you can see that the problem is solved. So, one needs actually an intelligent guess to pick the correct range for a solution, but even if you get it wrong; it does not matter because as you keep going through this cycle, you would be actually coming closer and closer to the correct solution.

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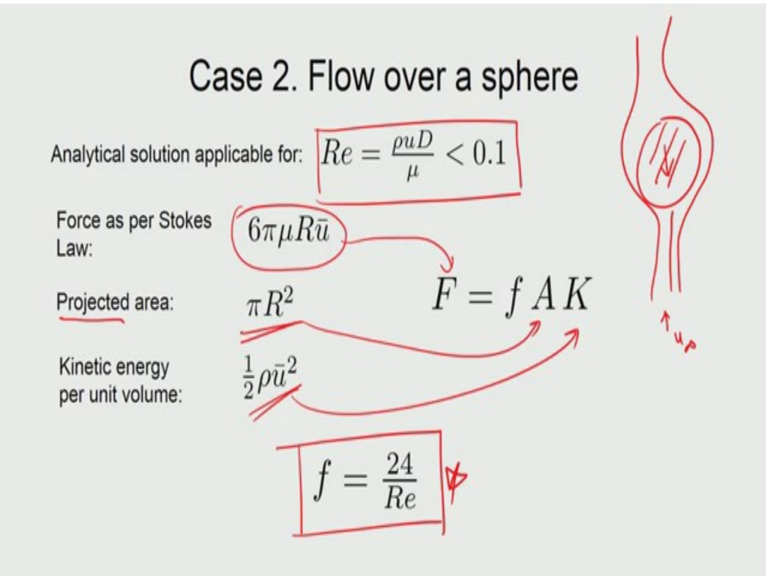

So, now let us move on to a problem which we have solved earlier the flow over a sphere. And as you have seen it is a very tedious a derivation, but in the end we will see how the final velocity; terminal velocity can then be used for correlations. So, here is a situation you have got a sphere and you have got the flow that is going around. And this far field velocity is u, which sometimes we write u_∞ also, terminal velocity and in case the solid is actually falling down and liquid is stationary then you call a terminal velocity, but the problem would still be same namely flow over a sphere.

And this is in the creeping regime; the analytical derivation we have done is for the creeping regime which means that the Reynolds number; critical Reynolds number is very small is 0.1. So, I have extremely small values of Reynolds number only you can use this. Now the way we have derived the Stokes law; we have already done the integration of the pressures and stresses that are acting on the sphere and arrived at the force. So, we directly can use a force we do not have to do any more manipulations there and because the flow is external we will be using the projected area so, that gives you πR^2 and then the kinetic per unit volume is already there kinetic energy per unit volume.

So, you then substitute those and then you would go over to the f. So, f is available here; so you can see immediately that f is coming very similar to the expression which we have derived in pipe flow; there it is 16/Re and here it is 24/Re. So, which means that the way we are going about derivations seems to be a fairly universal in its application.

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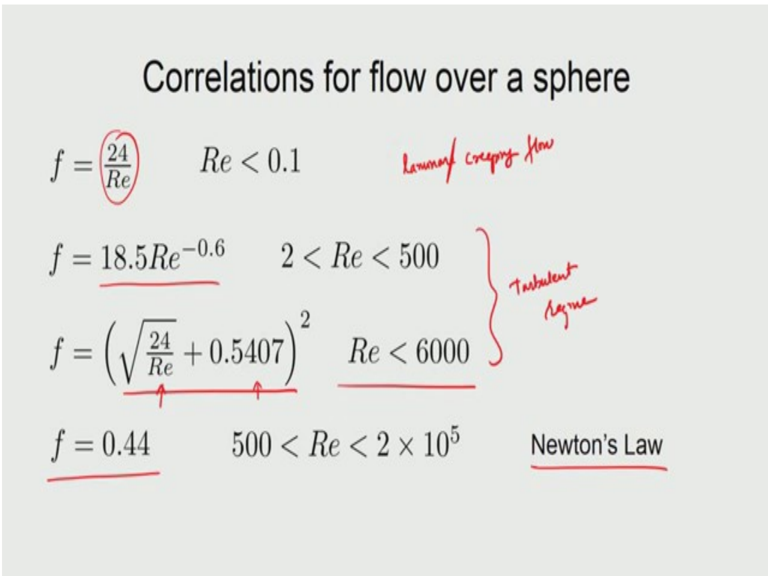

And we can then see how fiction factor was modelled in an experimental correlations for the rest of the Reynolds number regimes. So, this is a laminar or actually strictly speaking it is a creeping regime; creeping flow regime.

And these are the basically in turbulent regime; so, you could see that the friction factor expression is given in different manners in the different regimes. So, you could see that for intermediate range that is basically for the Reynolds number less than 6000, you are actually using an expression which seems to fit both the low Reynolds number regime as well as high..

Because at low you are actually having 24/Re which is already repeating here which means it is something like a hybrid function; to span across a large Reynolds number range. And for intermediate, you do have a more accurate function that is available which is made from the regression. And as only you can already see from the power that here is a power is -1; here is power is -0.6.

So, as you go into the turbulent regimes; the power of Reynolds number for the regression that is done in the empirical correlations is generally tends to be small. And interestingly, it has been observed that at very high Reynolds numbers; you do have the friction factor coming out independent of the Reynolds number, it comes out as a constant and that also will be Newton's law; there are a lot of things that go by Newton's name and we must not confuse between these.

So, friction factor being a constant is also observed in this domain.

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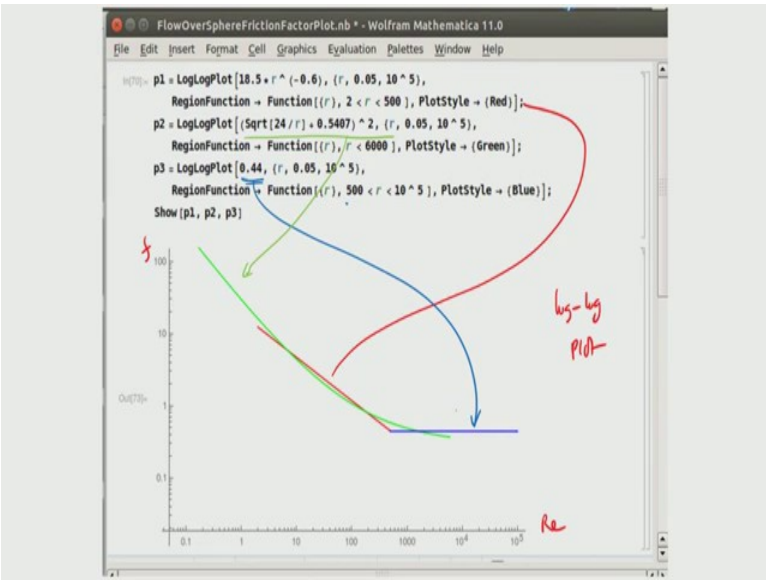

So, when we plot again all these things you could see that they also follow the same trend. That is you do have a straight line on the left hand side of the log log plot of friction factor as a function of Reynolds number. So, the left extreme at very small Reynolds number you have basically drooping down straight line plot and as you go to high Reynolds number you have a flattened profile that is coming out. So, it is very similar to what we have seen in the pipe flow and the functions we colour is for what is given here; so, the first step thing is red line.

So, I write here there and then you have got the blue guy. So, 0.44 is here and the green line is here the hybrid expression; so, hybrid expression is actually able to cover part to the flat region as well as the straight region. And you could see that the behaviour is a very similar which means that when we do not know anything; then we can perhaps make some assumption on how the friction factor could be varying, the exact numbers we may not get it right because we need their correlations for it.

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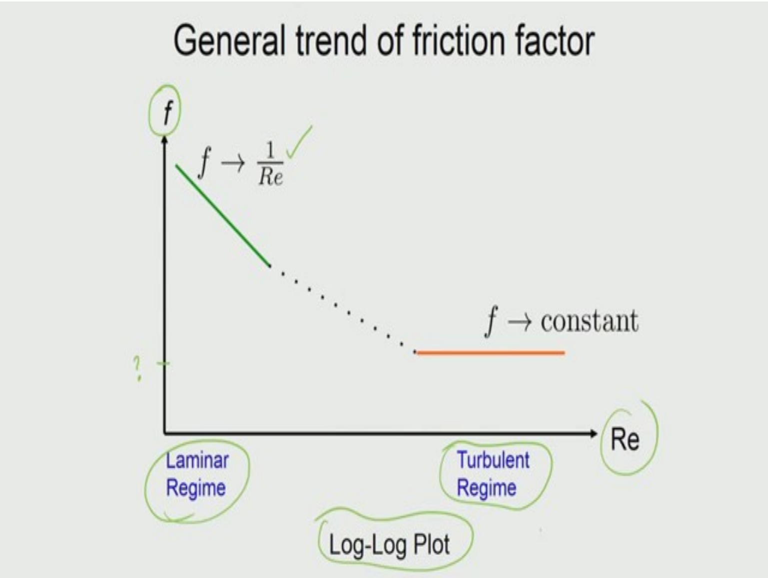

But we can get the trend and the trend is as follows; the trend is that when you take a log log plot of a friction factor as a function of Reynolds number. Then for very low Reynolds number that is in laminar regime, you see that it is a function of 1/Re that numerator can be different for different problems depending on the geometry, but it will be function of 1/Re which means that when you are going to take ratios and compare that the relative values then you do not need the constant in the numerator; so, you can go ahead and make this assumption and already come to some solutions of the problems.

And at a high Reynolds number regime which is turbulent you could actually make an approximation that the friction factor is constant. What value that would be is again something that we need from the experimental correlations, but we can make an assumption that is a constant and make already use of such things for some problems.

So, we will have few such problems already using these trends in the tutorial and its very good idea to have this particular image commit to our memory; that is friction factor as a function of Re will be sloping downwards as 1/Re and then becoming flat at very high a Reynolds number. So, this image if it is there in our mind then there are many problems that can be solved directly to get the trends.

Now, let us then see how we have modelled the porous medium flow. So, porous medium flow was modelled as a bundle of tubes.

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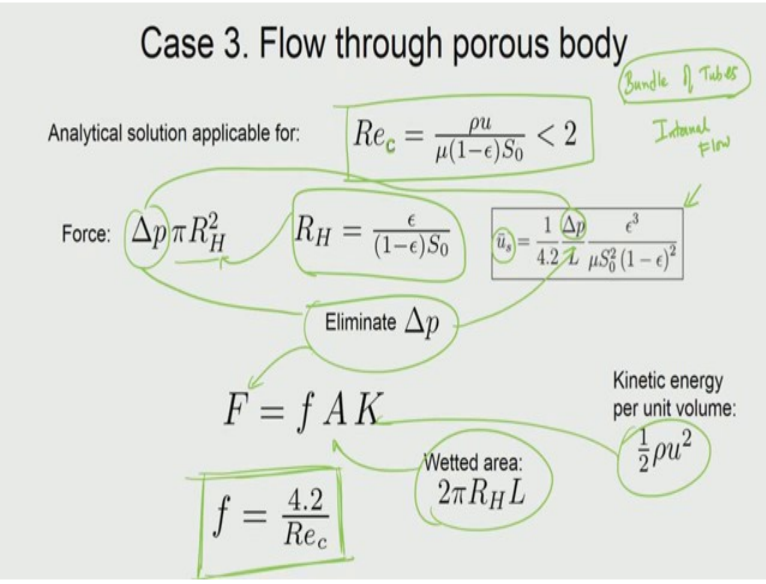

So, that is how you got the analytical solution here which we gave a name Blake Kozeny equation and the bundle of tubes would mean that it is actually internal flow; so, this is important. The model that we have chosen is that it is an internal flow which means that even though the porous body is made of spheres, where actually the liquid is going outside of the sphere; it is still not treated as an external flow, it is treated as an internal flow because of the void is being modelled as a pipe.

So, that is exactly a mistake that many students make while applying these expressions. So, be watchful the model is very important at to arrive at the expression we thought of it is a tube and therefore, is an internal flow expression. And these expressions are valid for Reynolds number defined for the porous medium in this manner; that is the reason why there is a subscript that is given. So, Reynolds number less than 2 is where this particular expression is valid and this is the expression. So, we have got a Δp here connecting with the superficial velocity.

And we are going to use superficial velocity only for all these porous medium approach because we know that the actual velocity is of no relevance for us. Because difficult to measure and superficial velocity can be measured experimentally and therefore, that is what we are going to use. So, when we say u in porous medium; that means, it is superficial velocity. And the force term is nothing, but Δp into the cross sectional area over which the pressure is being acted on, which we extended the model as a πR^2 because it is a tube, cross sectional area is πR^2, but for R; we take the hydraulic radius. So, the expression for hydraulic radius is only available; so, we basically use this expression into that.

And the Δp actually we eliminate using this expression. So, then we will get what will be the value of f for the force. And once that force is available then we can then substitute it in here and the wetted area is taken here for the area term because it is actually internal flow. And the kinetic energy per unit volume; the u is basically same as u^s, which is superficial velocity. So, when you substitute all these things a very beautifully many of these things cancel out and you would get basically the friction factor as $4.2/Re_c$. And this actually also will elegant because the only constant that is sitting here is what is actually coming from the by expression from the Blake Kozeny equation .

And then the functional form is again very similar to the other problems namely 1/Re. However, the Reynolds number is redefined for this particular problem using hydraulic radius and the rest of the quantities as applicable.

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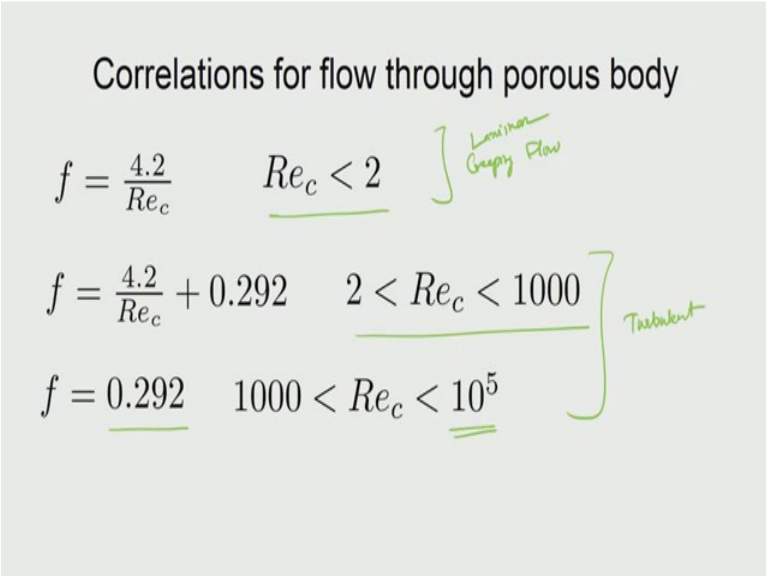

So, how does this expression then change when we go to other regimes? So, it is again seen here that friction factor is defined for less than 2 Reynolds number. But when you go to turbulent regimes these are all turbulent regimes; so, this is basically creeping flow regime, so for turbulent creeping flow or laminar regimes.

So, for turbulent regimes you do have correction and again like the Newton's laws that we have seen for flow over a sphere at very high Reynolds number, we see that the friction factor turns out to be constant. So, which again means that when we take a porous medium flow and it is told that the flow is turbulent; then we can just straightaway it take f as a constant and then go ahead and see how the pressure drops can be connected with the velocities.

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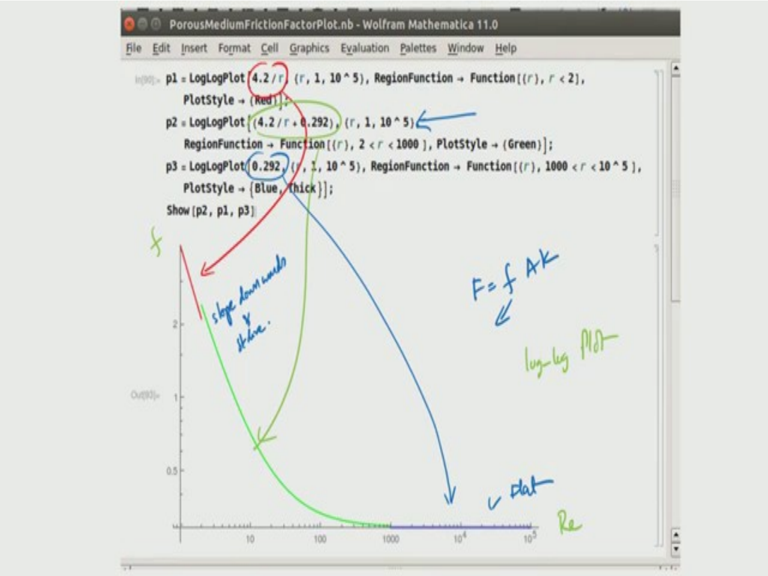

And when we plot these using the mathematical notebook, we will see that the three expressions are appearing to be the same. So, the log log plot has a the friction factor as a function of Reynolds number is appearing to be straight line drooping downwards on the left hand side for the low Reynolds number and a flat region on the right hand side for the turbulent regime.

So, you could see that the 0.292 is appearing here; so 0.292 is here. So, you have got the hybrid expression that is here; so, the first expression, the linear expression is here; the hybrid expression is given here in between and the expression for the turbulent regime is given here. So, you could see that the hybrid expression is a producing; the behaviour at laminar regime quite well on the lower side and the flat on the higher side quite well.

So, this is a very useful expression here 4.2/r + 0.292 and you could actually see that it behaves the same way as we have discussed earlier; which means that here it is basically slope downwards and a straight line and you could see that here it is basically flat. So, when we do not know any expression about these things; then we define the friction factor the way we have seen earlier. So, when we write like this then this behaviour is already known it is in a same fashion namely straight line on the left hand side and flat on the right hand side.

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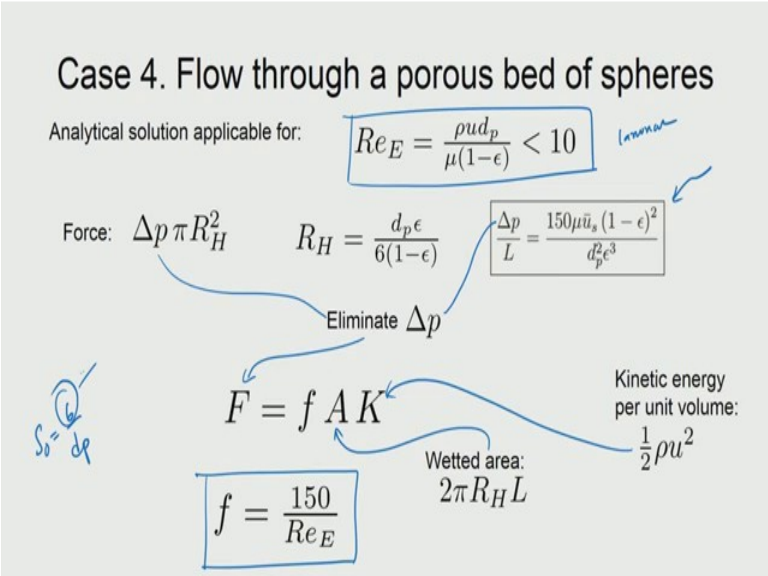

So, the expression for porous medium which is made of a bed of spheres is actually identical to the porous medium you have seen earlier; except further to the S_o is taken as $6/d_p$; though the reason being that when you are assume that the spheres are all touching at the point contact. And then you can directly get the wetted area per unit volume of the solid analytically and that comes as a $6/d_p$; where d_p is the diameter of the spherical particle that is consisting in the bed.

So, we have redefined; then the Reynolds number to accommodate this. So, S_o is replaced and then we have redefined the Reynolds number and that is where the subscript is changed now which is E and so which means that for a laminar regime that is within 10, then you have an expression that is available. So, we have got 4.2 in the Blake Kozeny equation; we multiply with 36 because the S_o is coming as a square. So, you see that 150 will come in there and we basically eliminate Δp between these two and the expression that comes in you substitute an f.

And then the wetted area is going there and the kinetic energy per unit volume is going there and when you substitute and cancel our terms then you get the friction factor for flow through a porous bed of spheres as 150/Re; which is a very elegant because you can see that it is also similar to the other expression except for the factor of 36 and we already know where that at 36 coming; it is coming from here.

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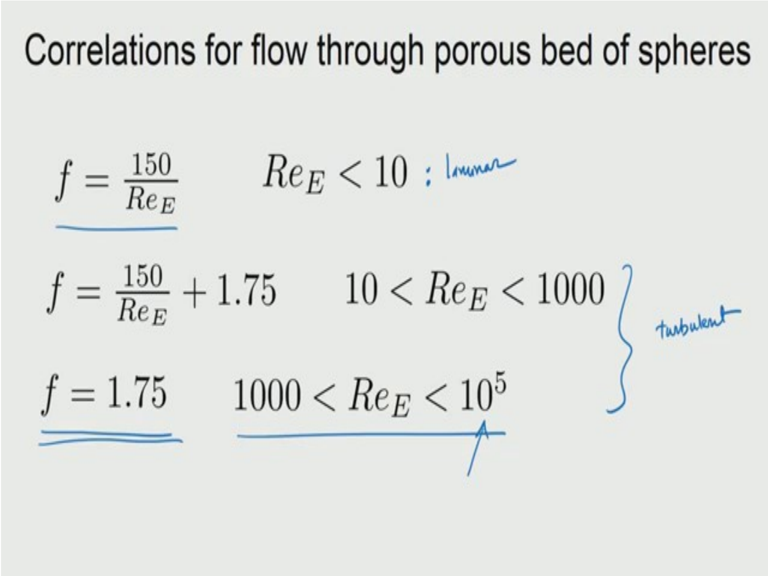

So, the friction factor expressions are given here for the laminar regime, you do have them analytically derived. And for the turbulent you have them as empirical correlations and we have got a hybrid expression and if a constant value. So, constant value is of course, for a very large Reynolds number which means that for a very varity of industrial problems; we can go ahead and use this assumption and solve some of the problems..

So, we have now got the strategy, we have got the expressions which means that we can now solve some numerical problems by using these expressions which we will do it in tutorials as part of this MOOC.

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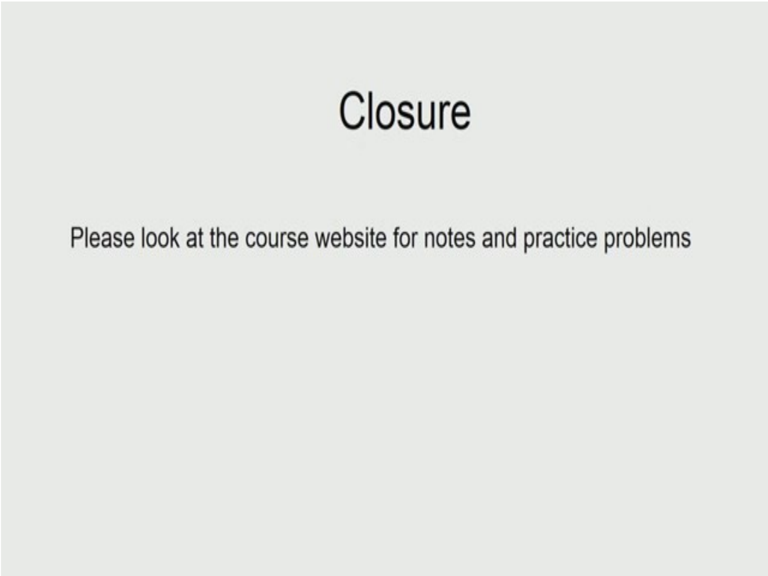

So, at this moment we will close this session and the course website will contain the notes about some more details about these and also some numerical problems for you to practice.