Transport Phenomena in Materials
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Lecture – 15
Simple cases in fluid flow - Spherical coordinate system

Welcome to the session on simple cases in fluid flow as part of the NPTEL MOOC on transport phenomena in materials. In this session, we will be taking up a problem in spherical coordinate system.

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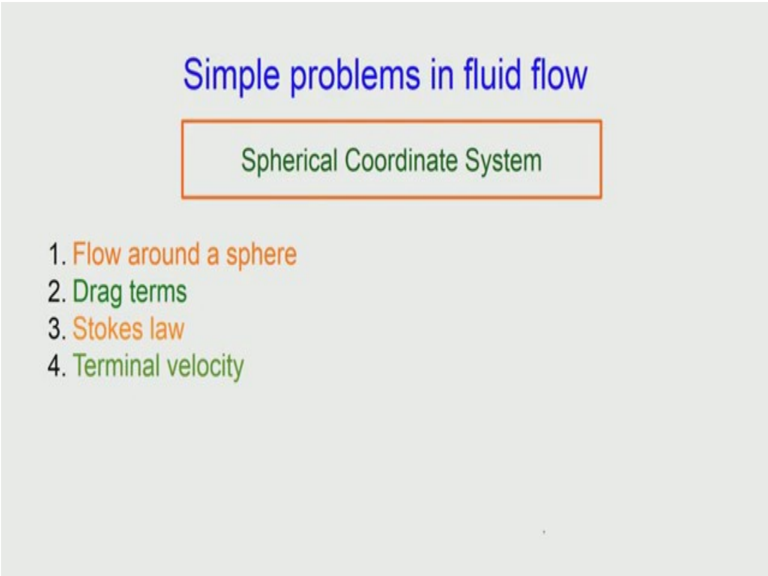

So, problem we are taking up is flow around a sphere; now in a spherical coordinate system even simple problems tend to be quite tedious because of the algebra involved. So, you may pause this lecture time to time workout some of these steps so that it will be clear at the end how we went about these derivations. So, we will take up the problem of flow around a sphere and once a solution of the velocity components are available; you will derive what are called the drag terms namely the force acting on the sphere because of the velocity gradients.

And then we will see how we can arrive at the Stokes law which is very important in the metallurgical industrial problems where the terminal velocity of particles is used in extractive metallurgy problems.

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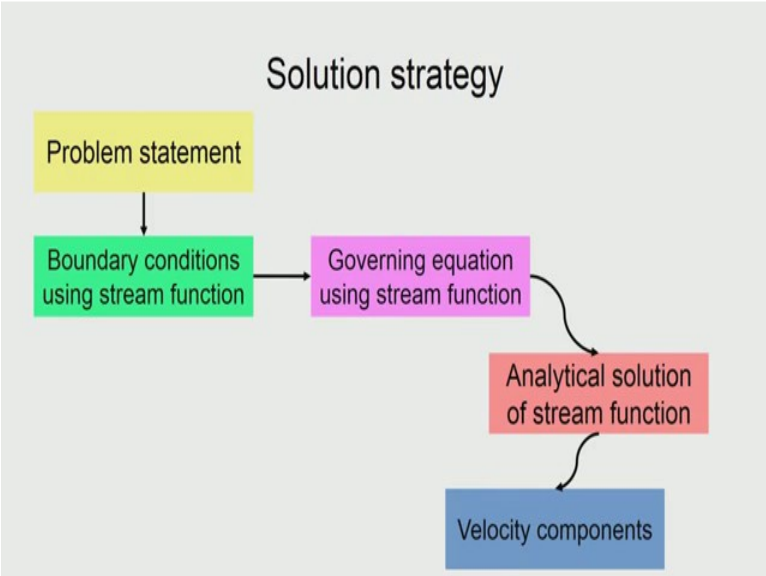

So, this is the strategy that we are adopting for this problem. So, we are actually going to start off with a problem statement as we normally do in most of the transport phenomena problems. And then we will be writing the boundary conditions, but here we make a deviation from the other problems we have done in previous sessions. We will be writing the boundary conditions using the stream function.

So, this is because the solution it will be accessible if we use the stream function for this particular problem. And then we will write the governing equation also in the stream function form which we have looked at already in an earlier session. And the analytical solution will be sought and once the solution is available as a stream function then we can again get back the velocity components because we have the definition of stream function already available with us.

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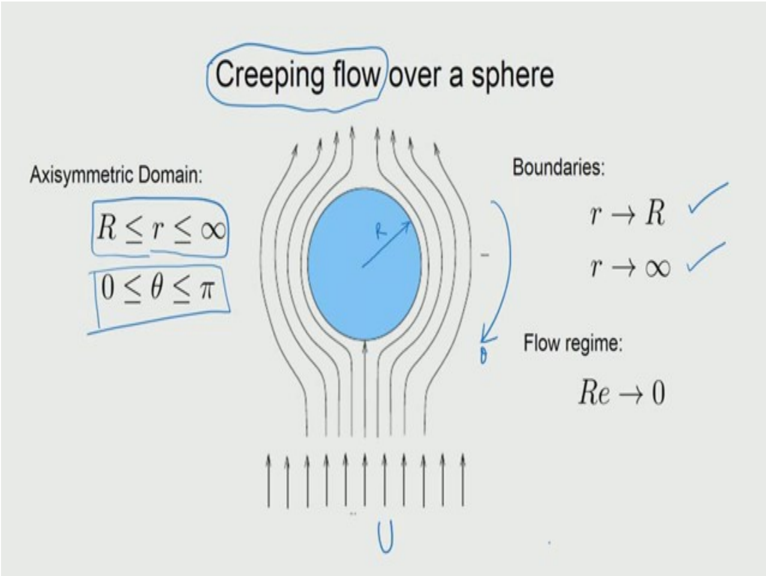

So, here is the problem; the problem is basically it is a solid or sphere which is smooth and rigid and the liquid is flowing around that. So, the direction of the liquid flow is shown vertically upwards and you may want to call the velocity vector as U which is in the upward direction.

So, what this problem will render is that the spherical symmetry is now lost because the intersection between the spherical, symmetry and the vertical velocity will be cylindrical. So, we will actually use single point system, but for an axisymmetric case. So, that is what the makes the problem a little tedious with respect to the algebra and we will see how best we can come about this limitation.

Now for the domain has to be defined; so, as we have discussed in an earlier session about the domains, the domain in our case will be the entire space around the sphere which means that in terms of the R radius if you take the radius of the sphere to be R, then the r small r above capital R; that is anything that is above the sphere is a part of our domain. And the θ is of course, varying in the complete range that is 0 to π which means that θ is going in this way and r is going in that way. So, the entire range of θ is a part of the whole domain.

Now, the boundaries are actually defined at two locations the first is quite straight forward, it is on the surface of the sphere. And then we have also a boundary condition available at the far field; that means, far enough from the sphere which is basically the limit of the radius tending towards infinity. And the problem we are going to look at is in the regime of fluid flow where the Reynolds number is very small; so, this is also referred to as a creeping flow. So, that is the liquid is going to follow the curve of the entire the sphere very closely and it does not actually separate out. So, that makes the problem a little easier to handle and we will give us a analytical solution in the end.

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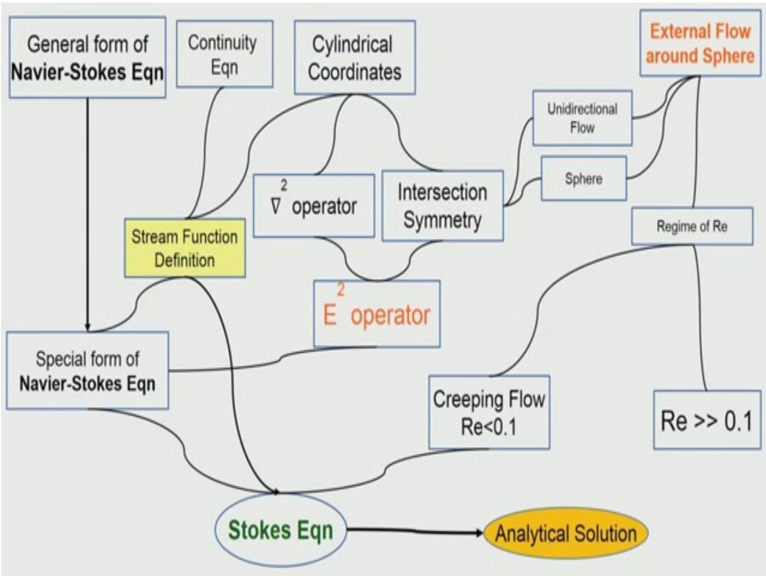

So, here is the concept map of how this derivation is done; so, each of these boxes contain some aspect that was discussed either earlier or will be discussed as part of this lecture. So, this is how we go about we have already seen the general form of the Navier Stokes equation and we can actually make a special form for the specific case of the Newtonian fluid, incompressible fluid and in a coordinate system that we are interested etcetera.

So, we do have special forms of Navier Stokes equation available and we already have seen what is the continuity equation that is the mass conservation and using the continuity equation and the idea that when you have two velocity components you can perhaps to create a function that will capture these two into a single unknown variable. So, stream function for example, so that idea is also going to be used

And we can realize that the Laplacian operator is different in different coordinate systems. So, you then see that for the coordinate system that you choose; operators are different and combining with the axis symmetry that we have then we see that some of the terms of the ∇^2 operator will be different and therefore, we create the E^2 operator. And we see that this problem is actually for an external flow around a sphere and we make the velocity to be unidirectional.

And the domain to be around a sphere and the regime to be applicable for creating flow. And then we put all these things together to arrive at the governing equation which is called the Stokes equation and then we seek the analytical solution; so this is how we go about. So, here we take up one at a time and one thing that we take up is generally highlighted in the yellow background. So, we take up the stream function first; the definition that will be refreshed for us to the use at this moment.

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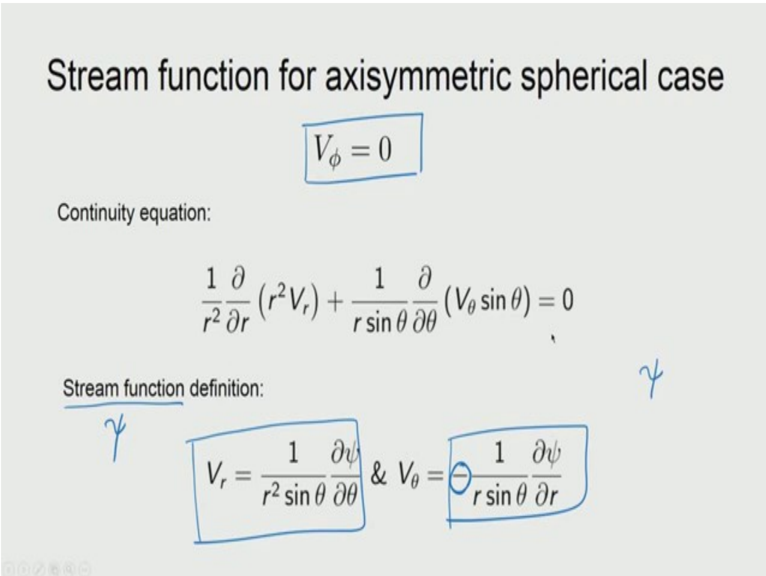

So, the axisymmetry is being imposed which actually implies that the ϕ component of the velocity is not there. So, there is nothing happening in the direction and which means that you can reduce the continuity equation to only two terms.

And whenever there are equations of two terms then you could use that as a constraint to create a function which actually will satisfy this equation and also give you the components of the velocity. So, that is a function that we are referring to; so, there is a stream function ψ. So, according to then this equation you could see what kind of a function would satisfy.

So, there is a sin θ there; so, we watch out and we have already seen which component of the velocity should be the positive and negative. So, this is actually from a convention that we have already seen earlier. So, this is the definition that we have and we are going to use this; so, we write our equations inside.

But we always look up this component definition whenever we want to translate from ψ to V_r and V_θ and vice versa. So, I would leave it as homework to check that when you substitute to these 2 velocity components into the continuity equation then the equation would satisfy.

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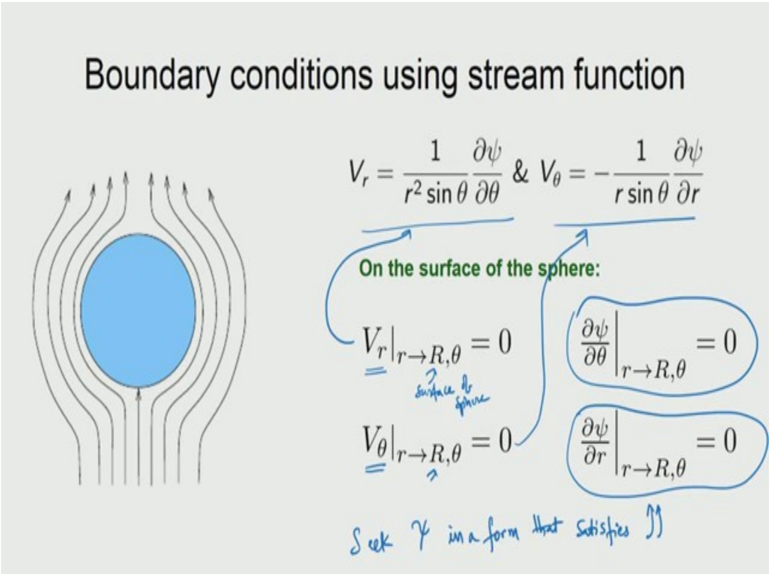

So, now we look at the boundary conditions. So, so the boundary conditions are to be written as follows; so, the solid sphere is impenetrable and it is smooth with the no slip condition on top. So which means that the velocity components are 0; so, the V_r that is the radial velocity on the surface. So, this means that on the surface of the sphere; so, that is 0.

And similarly, the θ component also will be 0 and this is basically no slip condition and in impenetrable wall condition. And if you look at the definition then this; when you inspect then it should also imply a condition on ψ. And that condition as you can see that if V_r work to 0; at r is going to capital R; that means, $\frac{\partial\psi}{\partial\theta}$ also should be 0 at the same condition.

So, this is the one boundary condition we get from the first condition. And using this definition for example, you could then combine here and then we will get the boundary condition with respect to the R and θ. So, you do have these two coming up and which means that we now seek; so, our problem will become seek ψ in a form that satisfies these things.

So, we must have a ψ which will actually obey this kind of a form; so, that it makes sense for our problem.

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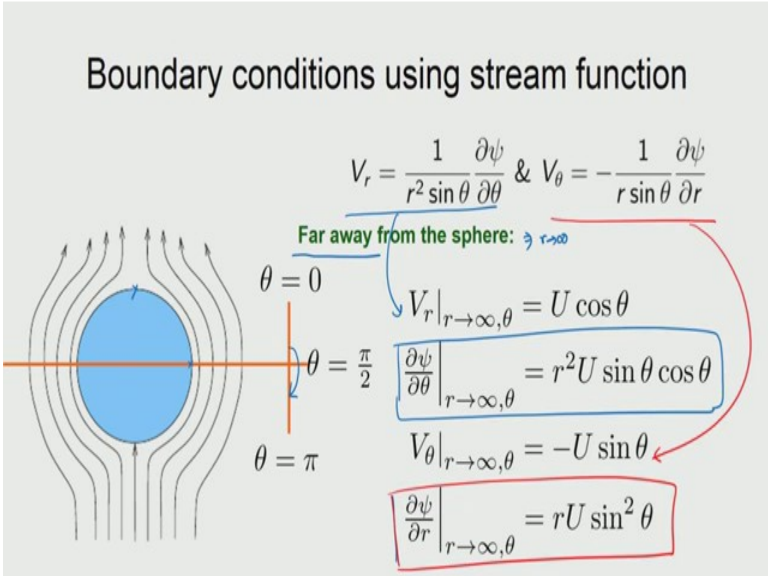

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So, we can then go further and see what we can also look at as a boundary condition far away. So, by far away we say that it is basically it implies basically when r tends to infinity and of course, applicable for all θs.

So, again we look at the definition and look at what kind of a; boundary conditions are coming out. So, if you see V_r definition and combine with this then you would get the condition that would be coming up here. So, how did we write the V_r as Ucos θ? You could actually see that the θ is varying in this manner and we are actually looking at the vertical velocity.

And which means that at this point θ is 0. So, velocity is U that will be coming up matching here and below it should be - U. So, cos π = -1 and that will be matching; at this position the velocity is 0 in the vertical direction which also will come as U cos (π/2) is 0. So, you could see that this way simply taking a component variable to write the V_r which is far away. And this will be actually all 0 on the surface, but at these respect to θ positions far away; this is how the velocities are the written. So, similarly you could also see how it has been used; θ is used so that you could actually combine with the boundary condition. And then, we would arrive at the far field condition for ψ ; so, we now have two sets of condition on ψ , the differentials when r is going to the capital R on the surface of the sphere and then r goes to infinity that is far away from the sphere. So, we have got four conditions on ψ.

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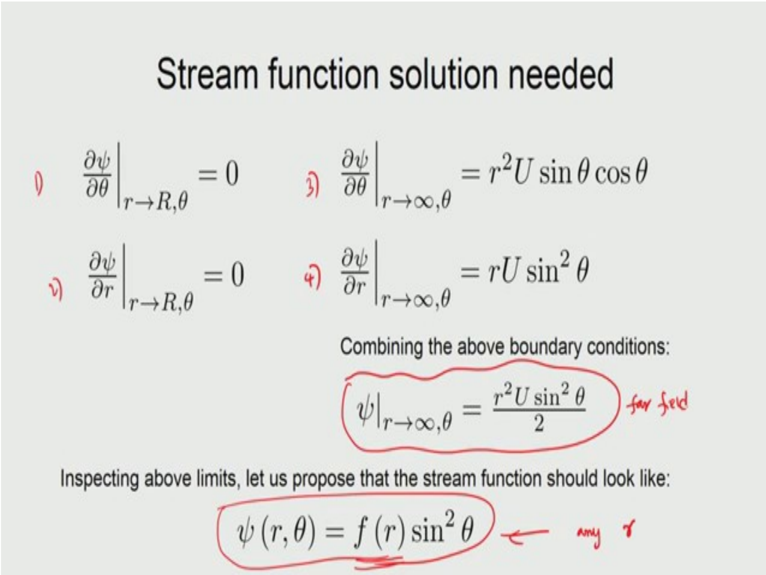

So, these are 4 are listed here and then we can see that when we combine these 4; that is; so you have got the 1, 2, 3 and 4. So, when we combine these then we can suspect what kind of a form of the ψ could satisfy. So, you could see that perhaps we can actually look at a form of this nature so that when we take a derivative with respect to θ; then you still have r square and then in r goes, you can actually see that it would actually give you the satisfaction.

So, you have this functional form that would be reasonable and we want to then see what is a generalized form that would actually solve the equation and so, you could actually seek that; perhaps it should be function of r. And then see what kind of functions will satisfy. So, because this is only at r is equal to; this is far field, but this we see at any r. So, what kind of a function f(r) is suitable such that at far field, it will give you this kind of a value; so that is the solution that we are now requiring.

So, what we normally do in all the integration and differentiation that we do in engineering problems is that whenever we want to propose a solution, then we insert that solution into the differential equation that should be satisfied and then see what form of the function will be reasonable. So, that is what we are going to do here also.

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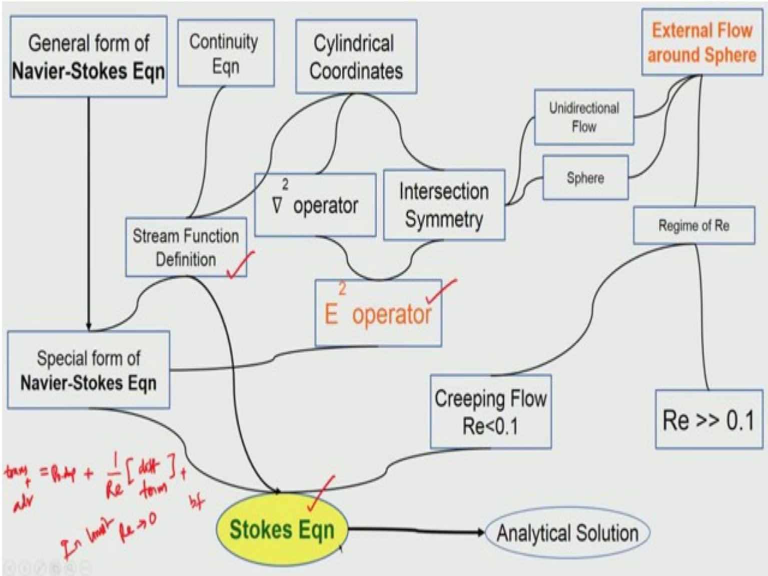

So, here we have then come up to the usage of the stream function. So, we now move on to introduce the E^2 operator.

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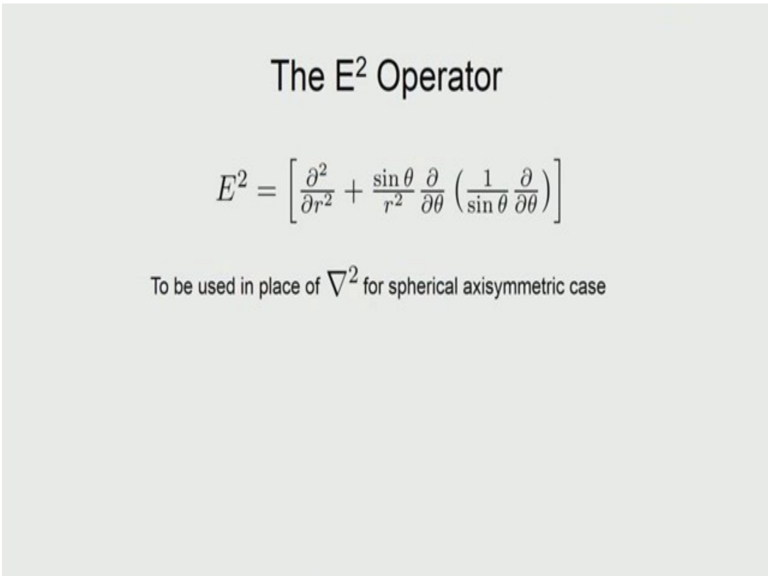

And the E^2 operator is basically in place of ∇^2 or del square operator that is the reason is basically because we are actually using the axisymmetric case. So, directly by looking up the ∇^2 operator; laplacian operator for single coordinate system would not help.

Because we have already constrained it to the axisymmetric; so, sometimes we will drop out and therefore, this is the operator we are going to use. So which means that the equation will have to be then looked at; now so, this is done, this is done. And when we see the Navier Stokes equation, we have we seen that you have got the terms that would have the Reynolds number with the diffusive term. And here you have got the transient and the advective terms and we have got the pressure drop term and then we have got the body force term.

So, which means that in the limit Re; tends to 0 then you could see that when Re is taken to the left hand side and right hand side on the numerator, then you would see that only the diffusion term will survive. So, we will see that we can actually take a special form of the Navier Stokes equation which is also called as a Stokes equation and use that as a governing equation for us.

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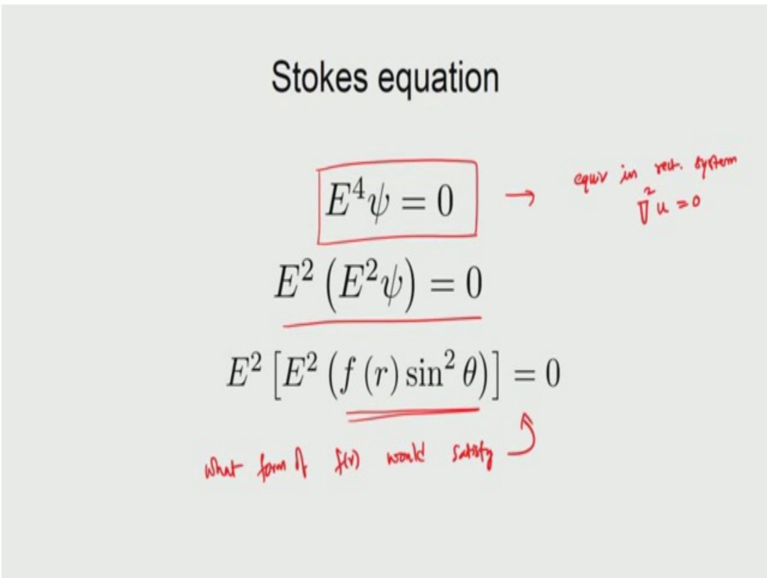

So, the form of that is here. So, this we have already introduced in an earlier session. So, this is basically its equivalent in the rectangular system would be for example $\nabla^2 U = 0$; so, that is the equivalent. But we already have used the stream function; so, when you use that it would then give you a fourth power and this is how the equations looking. And we have already seen that the E^4 is nothing, but E^2 acting on itself. So, it is actually nothing, but to the E^2 operator coming twice and that itself here actually already seen what is the form. So, we have seen the form here E^2.

Now, the functional form we are proposing the solution to be is here. So, what we see is that when we insert it into the governing equation, it should satisfy. So, here we are asking what form of f(r) would satisfy this equation; so, that is what we are asking basically. So, once we have it then we have the solution available.

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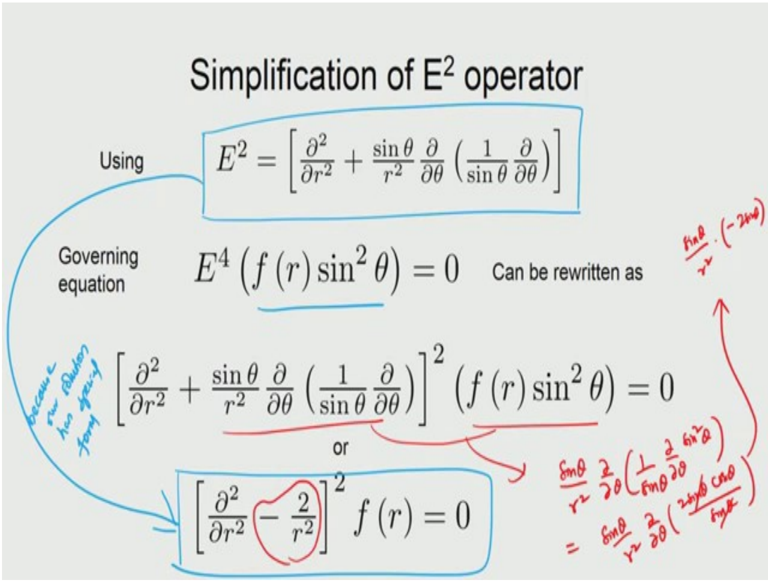

So, what we do is that we now go ahead and substitute the form that we have and see if you can simplify. The reason being that the form we have chosen is already separated in terms of the variables. So, f(r) and the $\sin^2\theta$ are separate; so, therefore, we could actually see if some simplification is possible. So, what we do is that we substitute and act the operator on this function and then we will see that if you look at how this part is acting on here.

Because if you see $\partial^2/\partial r^2$would not act on $\sin^2\theta$; so, it is only on the f(r). So, the second differential is only $\frac{\partial}{\partial\theta}$; so, it should act only on the $\sin^2\theta$. So, if you now look at these two; so, what would happen is basically you have got; $\frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta\right)$. So, that would actually give you $\frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}$ and you have got 2 sin θ cos θ/sin θ.

So, then you would actually ruled out these and the $\frac{\partial}{\partial\theta}\cos\theta$ that would actually. Give you sin θ/r^2 and this will give you -2 sin θ. So, it will actually give you $-2\sin^2\theta/r^2$; so, $\sin^2$ is then taken as a common; so, $- 2 / r^2$ will come. So , $- 2 / r^2$ has come here; so which means that we have now a simpler form of the E^2 operator; so, the E^2 operator with the θ differential is here and E^2 operator without the θ differential is here and we were able to do this because our solution has a special form; namely of this form that is we are seeking a solution which is actually in two parts and that reason why we are able to make this simplification.

So, we now see that we have a simpler you know situation of only differentiation with respect to r and what kind of a solution we can seek for f (r)?

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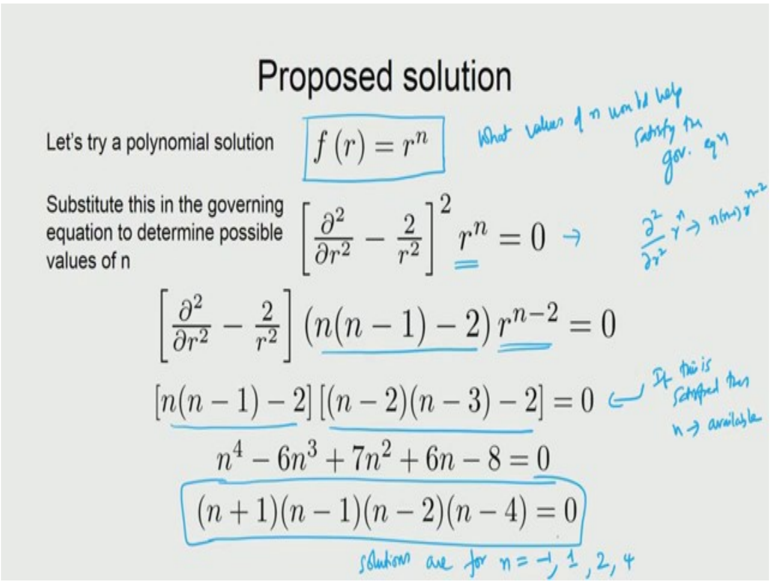

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So, we propose that we seek a solution of f (r) as a possibly a polynomial. So, r^n and see what values of n would help satisfy the governing equation. So, the governing equation is here; so, what kind of a values of n would satisfy. So, we go ahead and substitute that f(r) here as the r^n.

So, when you act first time; so, you have basically ∂^2. So, when you act first time, you have a $\partial^2 / \partial r^2$; r^n that would basically give you $n \times n- 1 \times r^{n-2}$. And then the second equation is nothing, but $2 r^2 \times r^n / r^2$.

So, you basically arrive at this; so, that is $n \times (n – 1) - 2$ that is coming here. Now when you act the same thing the further for the second differential on this function, then you would actually see that you got one more set of quantities that are involving n. And then we knock off the r because we want to see for what values of n this is satisfied.

So, if this is satisfied then we got n available. So, the lens can be used to then expand the r^n and then you have got the solution. So, we multiply and check the functional form; so, we have got a polynomial form and luckily for us this polynomial can be factorized and the factors actually look like this. So, straight forward from these factors you can already see that the solutions r for n = -1 then + 1 + 2 and + 4.

So, we can see that the solutions are readily available and we can note that the functional form that we have r^n can take any of these ns which any of these values which means that a combination of those that should also work as a solution; so, that is what we do.

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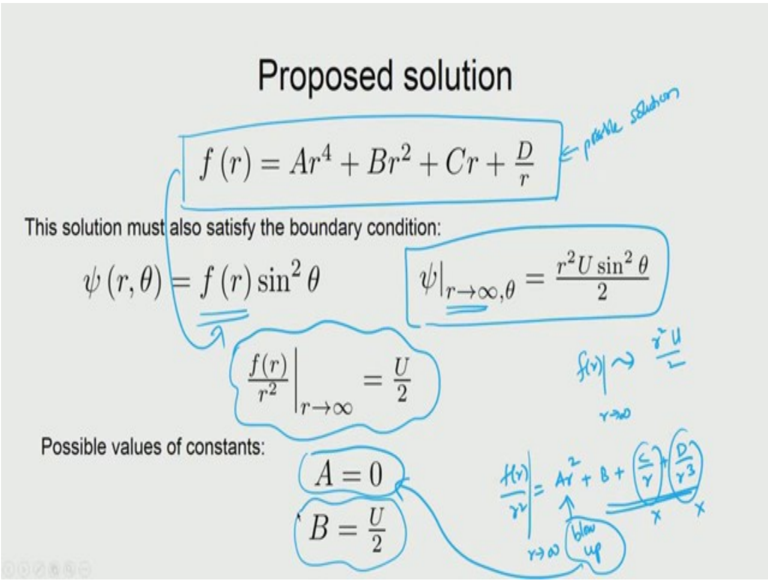

So, we take a combination of all those forms. So, the n = - 1 is giving the B term the = + 1 is giving the C term and so on. So, A, B, C, D are numbers or quantities which would actually be multiplied and then this summation actually is a possible solution; so, this is basically a possible solution.

Now it is only a possible solution, but we need to ensure that it actually satisfies the boundary condition and we already have a boundary condition requirement here; that is in the limit of far field velocity you must have this particular equation that has to be satisfied. So, which means that when if you want to use f(r) here, then it means that if you compare with this f(r) ; in the limit of r tends to infinity should give you $r^2U/2$, And which means that $1/r^2$ f(r) should give you U/2; so, we now have a constraint on the f(r).

Now if you look at f(r)/r^2 then you would see that it would be A r^2 + B + C/r + D/r^3 and now it in the limit of infinity.

You would see that these three will not give you a problem because C and D will vanish. So, these will drop, but this will blow up. So, which means with the only way we can actually have a meaningful solution is if A is 0. So, because of this condition we say that A is 0; so, that removes the problem for us. And B straight away we can see that f(r)/r^2 should be taking a value of U/2; so, B must be U/2. So, we have got a two of the constants determined in this manner.

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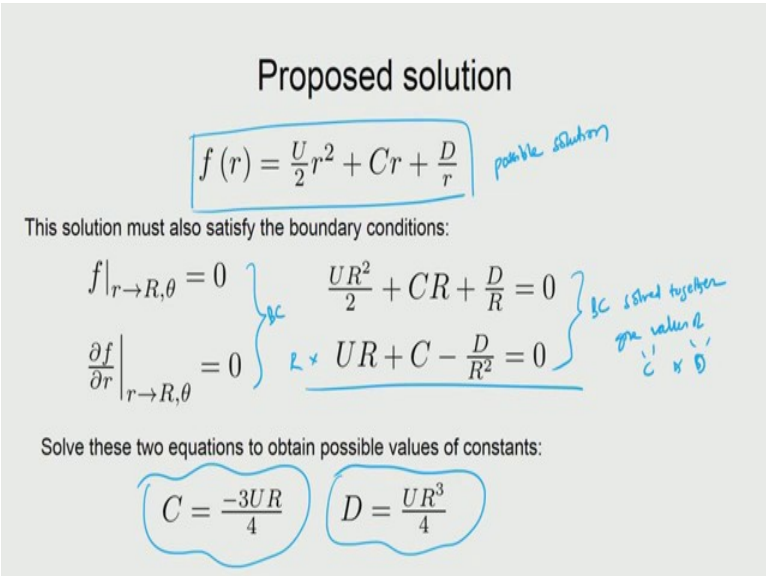

And now we see that when you substitute A and B values. So, your possible solution form is now simplified like this; we now have boundary conditions also for the velocity; on the surface of the sphere and that actually will also be useful. So, we have seen these boundary conditions already and this actually gives you when you substitute there are two equations and these two boundary conditions solved together; they give the values of C and D which are basically the integration constants or solutions of this particular equation that we have written.

So, which we do it; so, you can do it by multiplying the second equation with R and subtracting and then so on. So, if you do that then you arrive at the value surface C and D; so, please do that, so that you are getting these expressions the same way and we also when spend time on these expressions and then you can also commit them to your memory and that will be easier to remember the final form later on. So, now, we have got all the four values determined.

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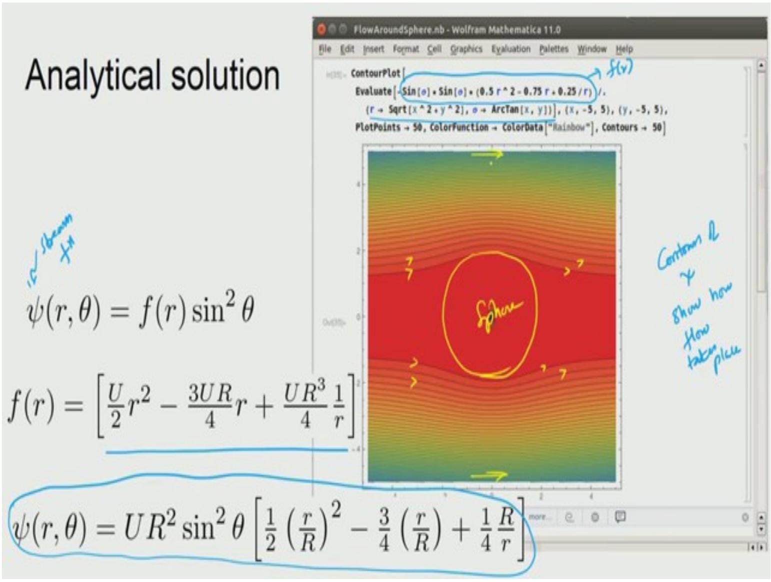

So, we have got the solution available, but solution is this; the solution is available to us and the ψ the stream function is f (r) × $\sin^2\theta$. f (r) is already available to us. So, we now have the entire ψ form that is available here.

Now, we already know that contours of stream function show how the flow takes place. So, what we have done here is a small one line MATLAB mathematica script to show you. So, you can see that we have actually seen this function that is nothing, but the f (r) and then we are actually converting the coordinate system from the polar to the x y here and when plotting them in a range. And you are you can immediately see that the contours look like how we have imagined. So, to just show you clearly the sphere is of radius 1 because we have taken the value of U to be 1.

So, this is the sphere and we can see that the velocity should look like that this is how we already imagined how it should be moving. So, it should go around the sphere and far away it should have the unidirectional velocity. So, you can already see that the far away the velocity is unidirectional. So, this is how we can actually see that the solution we obtain actually makes sense.

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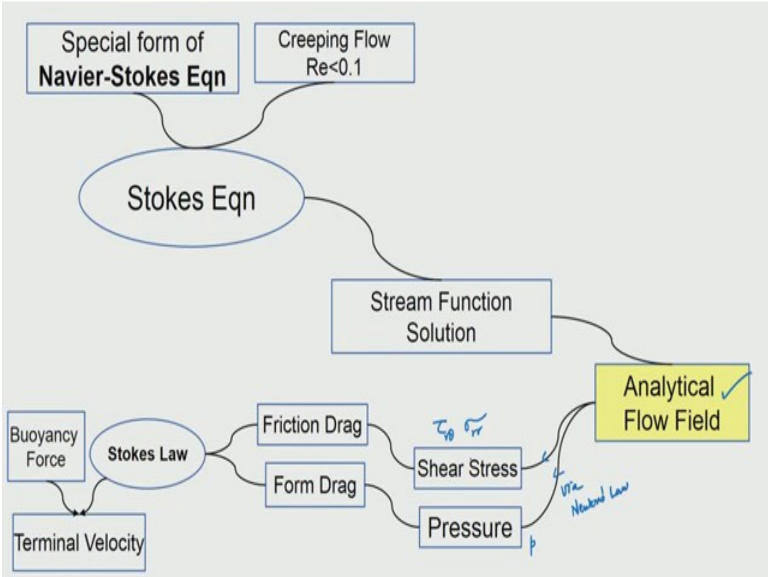

And once it is available. So, we basically now have the solution.

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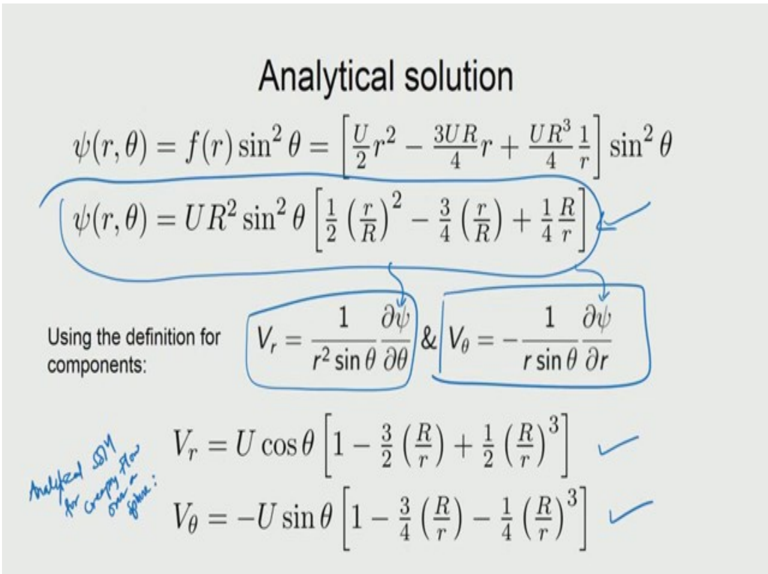

So, we now can get this is in the form of size. So, we can get see then how the flow field is in V_r and V_θ. So, that we will do now the ψ is available and so, then we use the definition of V_r and V_θ here and then substitute this ψ solution we have got and substitute that into these expressions and then we are able to get the components straightaway.

So, it is a simple differentiation that we are doing nothing and there is no step in between to wonder at all. So, substituting the expressions of course, for ψ you get V_r and V_θ

straightforward. So, we now have basically the velocity components that are available. So, this is basically the analytical solution the exact analytical solution for the Stokes equation for the limiting case of creeping flow.

So, for creeping flow over a sphere; so you can see that the number of steps that are taken to arrive at the velocity components is way more than what it had taken for problems in rectangle coordinate system, which is also the reason why we must try and reduce the dimensionality of our problems. So, to see whether some estimates can be made for a rectangular coordinate system already and in case the situation does not allow we can then go ahead and make it a simpler problem in a coordinate systems like cylindrical or spherical as the requirement makes us do.

Now we have achieved up to here; so analytical solution is available. So, technically the problem is over, but we now want to take this a little bit further up because of a particular metallurgical application that we normally or familiar. So, what we do is that we will use these flow fields to arrive at the pressure and shear stress distributions. So, both the τ and the σ; that is basically the r θ and r r and here pressure, so we want to look at these quantities also; and how do we get these? This is basically via the linear constitutive equation namely the Newton's law which is actually connecting the shear stress and the velocity gradients.

So, we will use and get those things and then from there we will get the drag terms and then from that we will get the Stokes law and finally, the terminal velocity concept which is very useful in many of the metallurgical problems.

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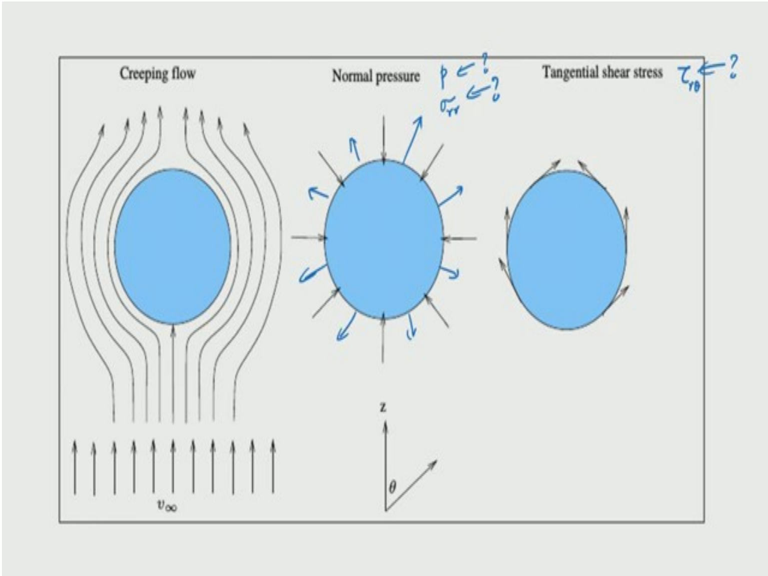

So, the solution is already available to us and the pressure and the stresses are then displayed here. So, we you can see that the pressure is actually in this direction and if you want to look at the rr then that would be in the opposite direction. So, because of the convention that pressure is in the direction of compression and therefore we have got those things like that.

So, we want to determine these; what are these expressions and once we have them then we can look at what would be the components of forces that are acting on the sphere in the Z direction and then look at the summation etcetera.

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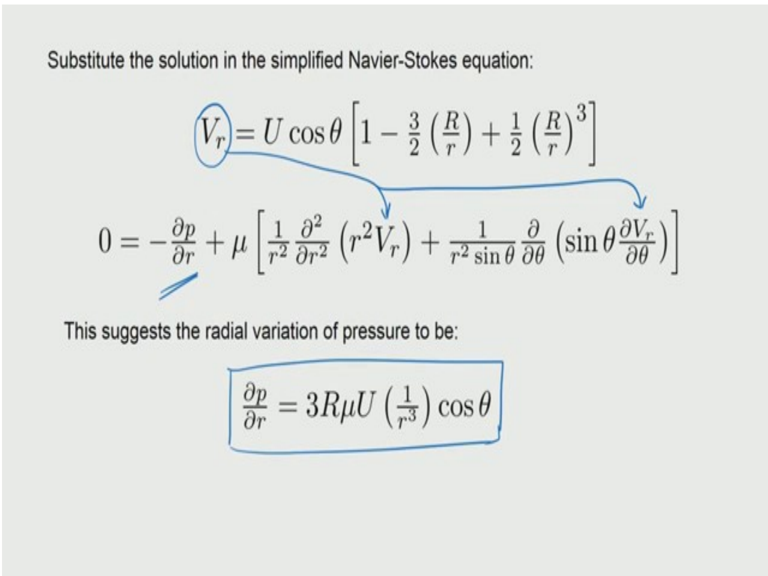

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So, for that what we need to do is go one step back and look at the Navier Stokes equation, substitute the velocities and get the pressure distributions. If you have noticed in the previous, session we have already done that; to see how the pressure variation will come once we have the velocity components available. So, you want to do that here also; so the velocity components are available. So, then we substitute that into the Navier Stokes equation.

But keeping the pressure variations intact so that we can get some functional form a p,. So, we do that; we substitute and see that we get a condition which we need to integrate to get the p. So, we got the condition for pressure variation in the radial direction given that the radial velocity component is so and so; also so, we have got one term there.

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Substitute the solution in the simplified Navier-Stokes equation:

$$V_\theta = -U\sin\theta\left[1 - \frac{3}{4}\left(\frac{R}{r}\right) - \frac{1}{4}\left(\frac{R}{r}\right)^3\right]$$

$$V_r = U\cos\theta\left[1 - \frac{3}{2}\left(\frac{R}{r}\right) + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]$$

$$0 = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial (V_\theta \sin\theta)}{\partial \theta}\right) + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right]$$

This suggests the angular variation of pressure to be:

$$\boxed{\frac{\partial p}{\partial \theta} = \frac{3}{2}\frac{\mu U}{R}\left(\frac{R}{r}\right)^2 \sin\theta}$$

And similarly we substitute the θ component here and we substitute that also here and we have substitute the r component here and we can then get the variation in the θ directions. So, once these two are available we can then see what kind of a pressure form would let us combine both. So, do try out by substituting the velocity components in this equation and arriving at the pressure variation as that is written here. So, that is a couple of steps of algebra that quite straightforward.

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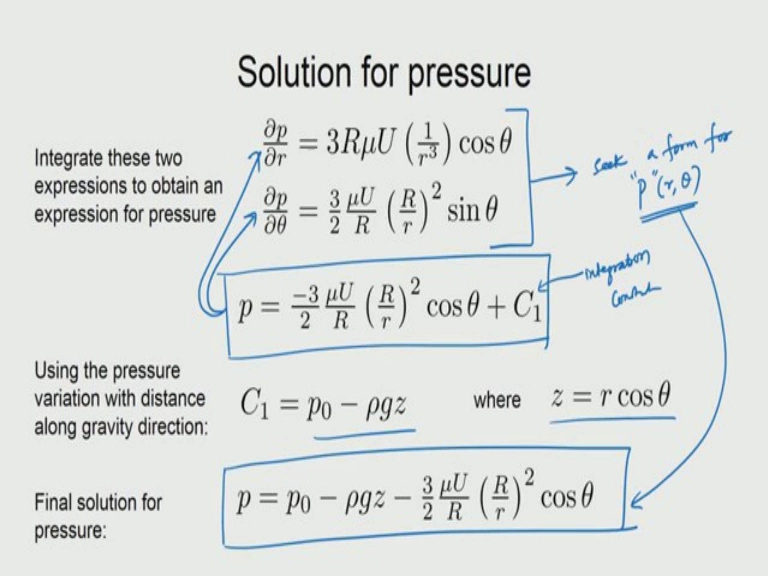

So, these are the two conditions and from here we seek a form for the pressure. So, we want it to be a function of r and θ and we only know that differentiation is respect to r and θ. So, by looking at them we have to guess what kind of the form would allow us to get this.

And this solution is then suggested as follows and C_1 is basically the integration constant. So, it is easy to see that once we have got the suggested form when we differentiate would you get these two variations that you can verify. And the integration constant is determined by looking at the pressure variation as a function of distance, we already know that from static problem. So, ρgh as the hydrostatic head is already known to us, p_o is the reference pressure atmospheric pressure; you can take for all practical purposes p_o to be 0 and then solve the problem that is not a problem at all; so, z direction that the distance is rcos θ and we are that here.

So, which gives us basically the pressure form function of r and θ is available; so, similarly we can get the; this is done.

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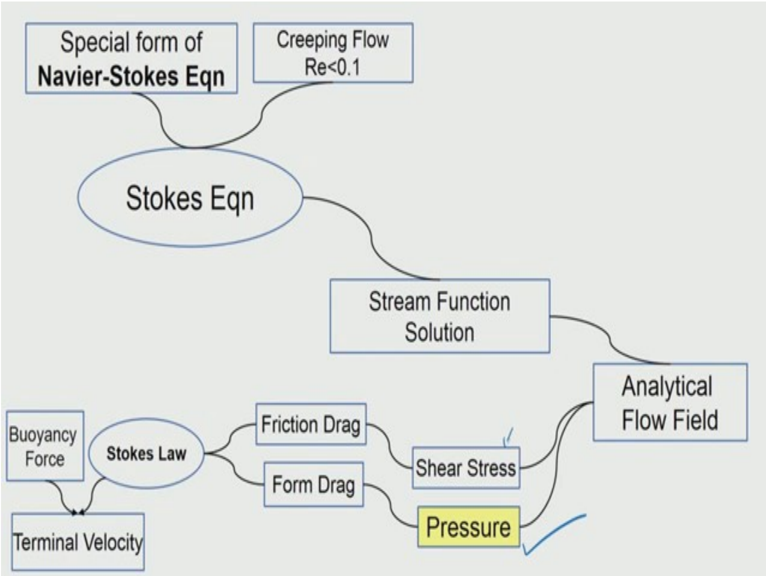

So, similarly we now seek what would be their shear stress distributions.

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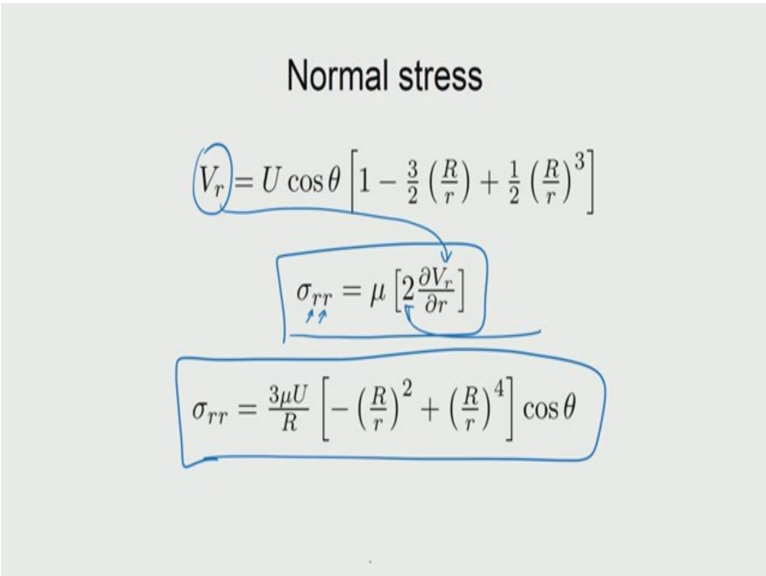

So, for that we again look at the just the connection between the stresses and the velocity gradients. So, that is available here for the spherical coordinate system and the velocity components are there. So, we can substitute and you have a 2 here; why is there? Because we see that the stress is actually a symmetric tensor.

So, when we have the same index if then you have see that the same term is it will being added. So, you can actually when substitute and they get σ_{rr} and σ_{rr} is then available in this particular form. So, you now have pressure and σ_{rr} which means that we have the normal pressures that are acting on the sphere pressure and stress that are acting and therefore, its vertical component can be taken.

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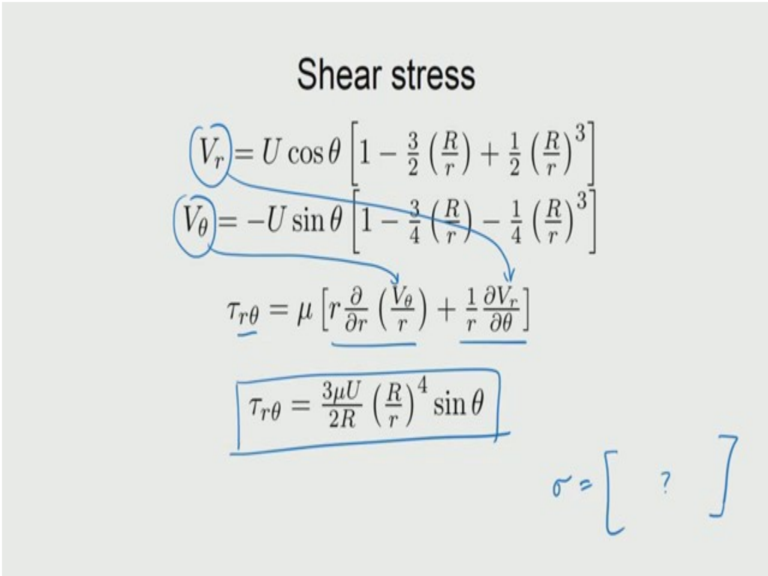

Similarly the shear stress can also be looked at and you can see that it is written in a symmetric manner using r θ and you can see that these two terms are available. And you have the r component and the θ component of the velocities that are present. So, you can then substitute them and get what is the $\tau_{r\theta}$. So, the functional form for $\tau_{r\theta}$ can also be obtained readily by plugging in.

So, once we have the velocity components so then you can see that the pressures; the normal stresses as well as the shear stresses can all be obtained. Technically, now you have all the terms to write for example, σ is equal to what. So, you can write it in a matrix form because all the terms are now available, but we only are requiring the r θ and r r components for this particular problem.

So, once we have; so, which is there, then we have to see how to calculate the drag. Drag is nothing, but basically the integration of stresses that are acting on a solid surface over the entire surface. So, we will get basically when you integrate stress with an area; you get basically force. So, that sum of all those forces is basically the drag; so that is what we are going to look at now.

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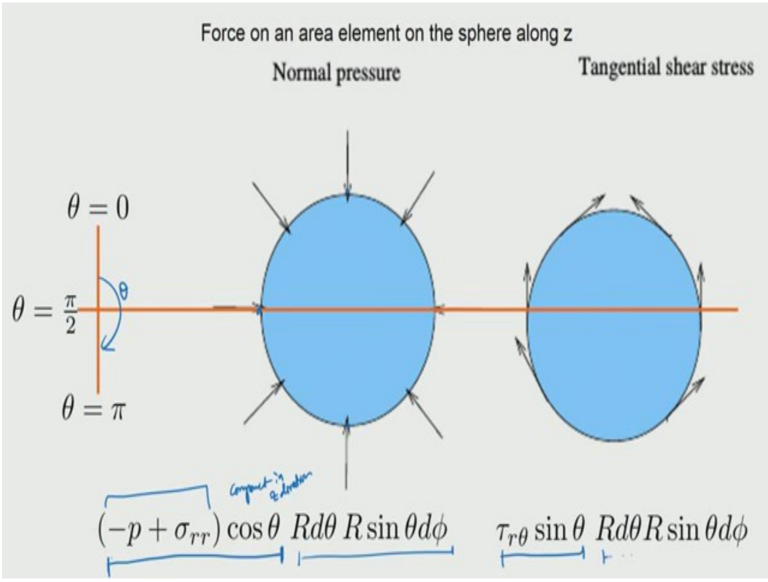

So, now we have already seen that the θ variation in this manner. So, if you look at how the normal stress has to be computed in the vertical direction you could see that here and the pressure would come straight away as -p and cos θ is 0. So, -p it will come.

And the σ is actually going to be + σ_{rr} and here actually it will be 0 and here it will be again - direction. So, you could only see that this part is basically taking the stresses and giving a component which is in the component in the z direction. And now this is a stress component that is acting in the z direction; on an area element that area element is given in this form. So, Rdθ Rsin θ d ϕ is the area element for a sphere of radius r. So, this is already available when we integrate this alone over 0 to π for θ 0 to 2 π for ψ then you already get the surface area of a sphere.

So, we already know this from the high school geometry problem. So, so we can then substitute this part and we are going to integrate this entire term to get the force that is acting in this direction because of pressure and the normal stress. Similarly for the tangential, you can already see the components are actually sin θ. So, this tangential is normal; so, if this is cos θ it will be sin θ here and this is the area element.

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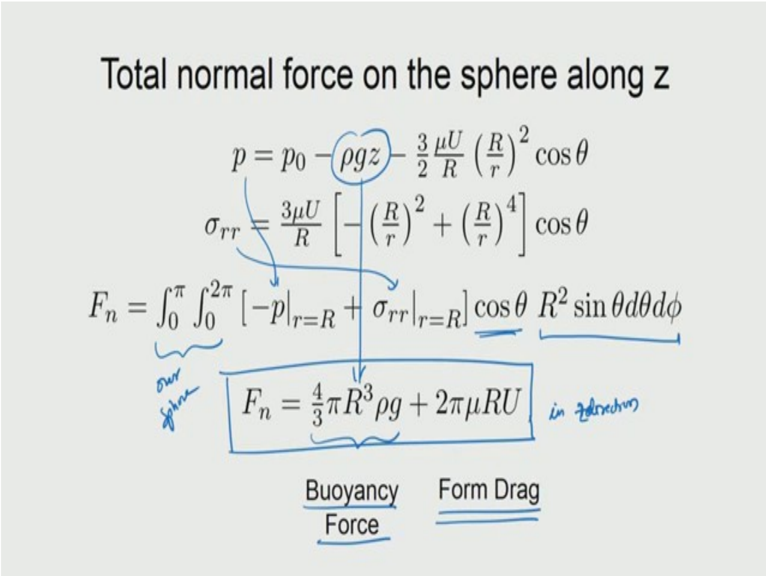

So, when we integrate this you get the normal in direction what are the forces that are acting. So, the pressure in σ_{rr} are available; so, you substitute those and then the cos θ is taken and then this is the area element and integration is over the sphere.

So, when you do the integration which again is not very complicated; when you go through that you will see that the normal force in the z direction is given by this. So, you can only recognize immediately that this term is coming out of with the volume× ρ at the mass into g. So, you can already see that this is coming because of this fellow. So, you can readily verify that when you take this term and then integrate over the entire volume you can already get this, but the rest of them also will be and giving you these quantities.

So, this actually is the buoyancy force because it is basically caused by the gravity and this is called the form drag.

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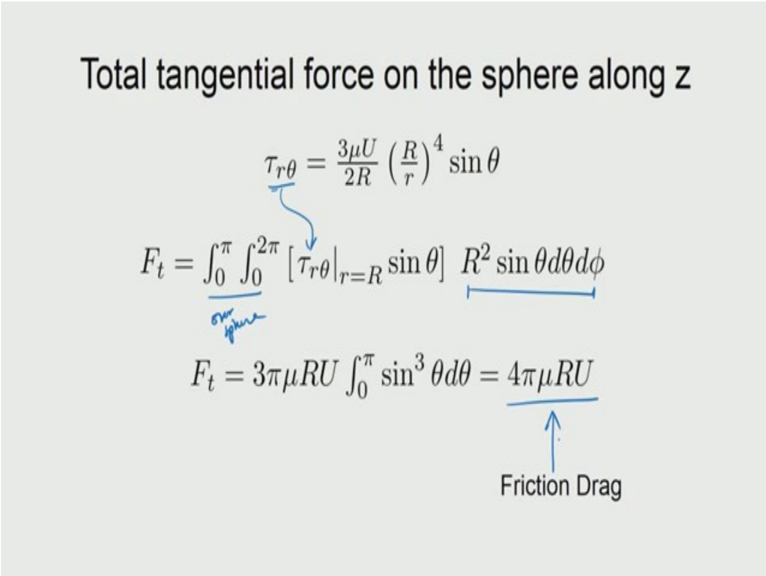

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And you can do the similar kind of a integration for the tangential stresses to give you the normal force or vertical direction force, so r θ is available. So, the r θ is $\tau_{r\theta}$ is substituted and then this is the area element and then when you integrate it over the sphere, then you get the F_t as 4 π μRU. So, this is actually called as a friction drag.

So, there are two components of drag that are acting on the sphere because of the flow that is happening around it the form drag and the friction drag. And as you can see the form drag is coming from the normal stresses, the friction drag is coming from the tangent shear stresses. So, when you add these two; so, what happens?

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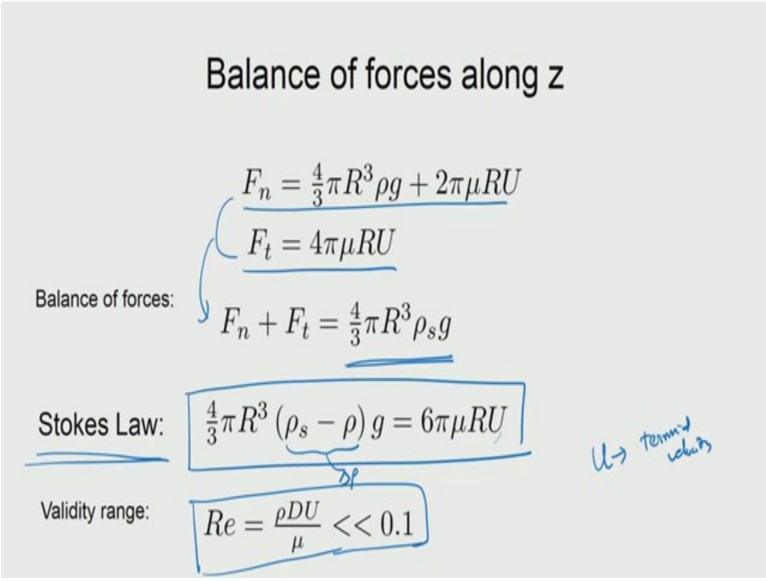

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So, you have got these two; when you add there is when you get this Stokes law. So, basically the normal and the tangential components of the forces in the z direction are available. So, when you then add them up; it should actually match the weight of the solid in case the total forces cancel and the solid is actually falling down at a velocity U which is a terminal velocity. Alternatively the fluid is actually moving upwards in the direction and the solid is a stationary.

So, it is only the relative velocity that we are looking at. So, when you put these two and the substitute into this then you can get this equation which tells you that of course, this can be taken as delta rho. So, which basically tells you that because of the density difference the buoyancy is actually compensated by a force and that force is actually going as 6 π μ×R× U

Now this U is then called as the terminal velocity. So, U is basically the terminal velocity; the terminal velocity when this balance is happening so if this balance it does not happen; then of course, you still are able to work with the fluid flow variations but it is just that this flow pattern has not stabilized and you still have the solid body accelerating etcetera and this is also given a name the Stokes law.

So, Stokes law is actually then valid only for creeping regime which means that the Reynolds number has to be very small. So, this is a validity range and whenever we want to arrive at with the velocity of the solid that is falling in the liquid column and if the solid is having a radius r and the liquid is having a viscosity μ; then we want to find out what is the U then use this formula we have to ensure that calculating the Reynolds number, it would give us a number that is very small. So, that is only when the creeping flow regime is valid.

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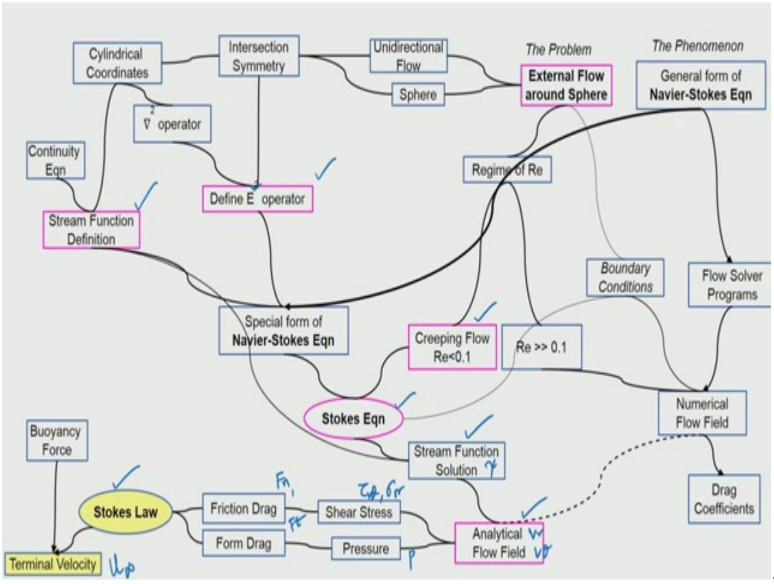

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So, we have then finally, arrived at the end of our concept map; we could see that we started off by looking at the stream function definition, the E^2 operator we have introduced and we have seen that the Stokes equation is being used for our solution because we are actually in the creeping flow regime.

And then from there we sought the solution of the stream function ψ and then that gave as the velocity compounds V_r and V_θ. And then from there we got the $\tau_{r\theta}$ and then the σ_{rr}, then you got the pressure from there we have got the forces and from there we got the balance with respect to the gate of the solid and then we got the Stokes law. And the velocity at which the forces are balancing; we actually are calling it as a terminal velocity; U_∞ very often or in our problem it is just U.

So, like this we are able to now cover the entire problem in the complete depth that we need and in the sample problems; in the course website you will have some numerical problems given where we are going to apply the Stokes law.

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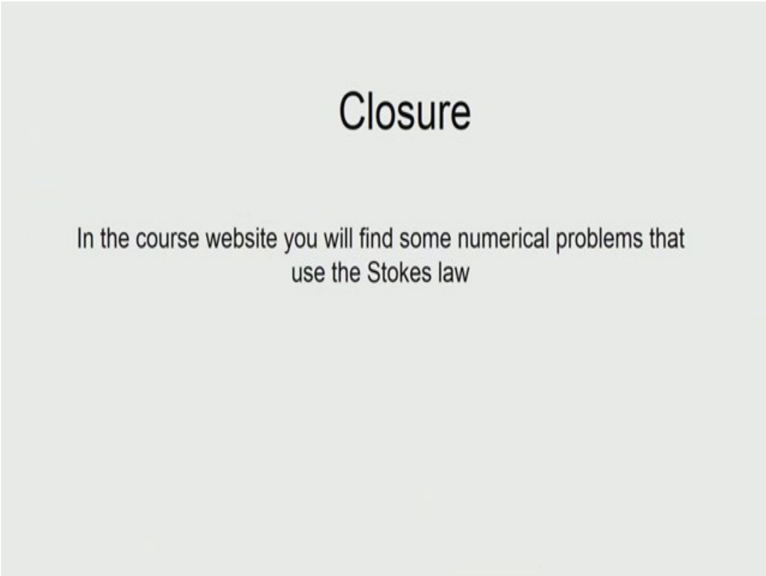

So, these are actually of very high importance in the extractive metallurgy problems. So, to know how these equations came about and which analytical solutions made this possible is important and I hope this session would have clarified the entire process of arriving at that.