Transport Phenomena in Materials
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Lecture - 14
Simple cases in fluid flow – applications of pipe flow to porous bodies

Welcome to the session on simple cases in fluid flow as part of the NPTEL MOOC on Transport Phenomena and Materials. In this session we will be looking at applications of the pipe flow equation that we derived earlier to porous bodies.

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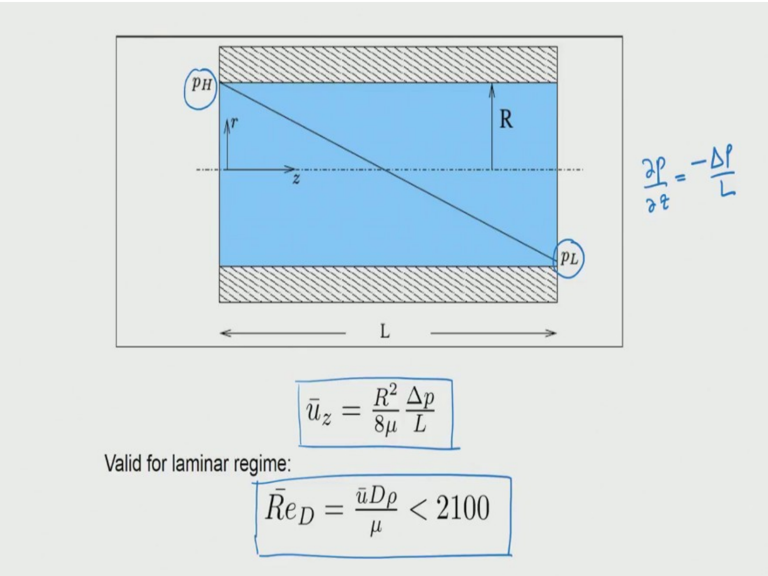

So, here we have the solution that is written already we have derived this in the previous session where due to the pressure drop per unit length that is shown here the pressure at left hand side of the domain is shown as p_H, right hand side is p_L. So, the pressure drop is given as $-\Delta p/L$. So, therefore, that is a driving force for the fluid flow in the axial direction and the average velocity that is which is half of the maximum velocity is given $\frac{R^2}{8\mu}\frac{\Delta p}{L}$. So, this is also referred to as Poiseuille flow and when we look at the mass flow rate by the same problem then that would be called as the Hagen Poiseuille equation.

So, this equation should not be applied for any range of pressure drops that we like, but when the Reynold's number that comes out is less than 2100. So, after we solve a problem to determine the average velocity through a pipe due to pressure drop or body force ρg. For example, we need to multiply the velocity with the diameter and the density divide with mu and check the Reynold's number that comes out is small. So, that is when the laminar flow assumption is valid and therefore, the results will be correct. So, beyond this number then we cannot use this expression.

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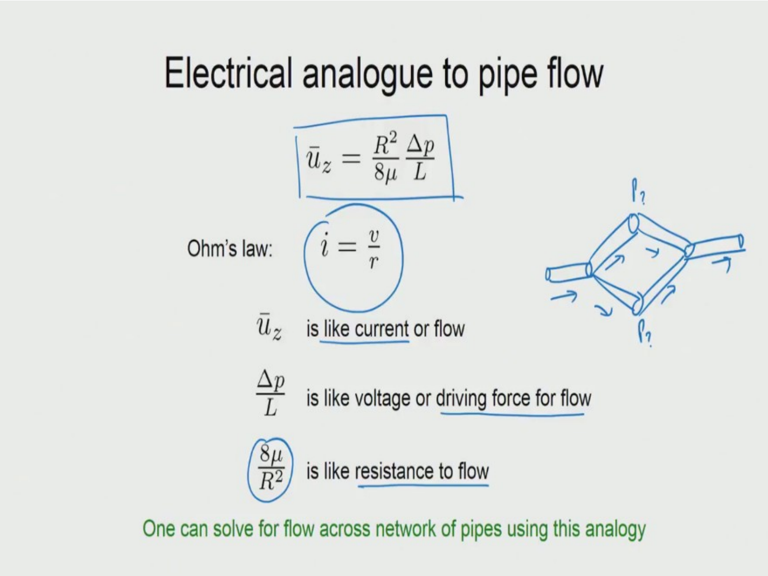

So, here we want to just look at this equation in a slightly different manner the same equation which is written earlier you can then compare that it looks somewhat like the Ohm's law. So, the velocity the average velocity of the fluid through the pipe is like current and the Δ p/L is a reason why the flow of the fluid is actually happening. So, it is like the voltage or the driving force for floor and what goes as a denominator that is $8\,\mu/R^2$that is proportional to $1/R^2$ that is like the resistance to flow. So, it means that pipes which should have a narrow diameter would be requiring more pressure drop to have the flow take place at the same magnitude etcetera. So, once you compare you could already see that we have a analogue electrical analogue. So, the current is like the flow the pressure drop per unit length is like the voltage and we do have a quantity that comes like a resistance. So, just like one would solve the circuits problems in electrical engineering problems then you could also do the same thing with the pipe flows.

So, whenever you have a network of pipes for example, in situations like this for example. So, in situations like this what happens to the flow and what would be the pressure set different junctions etcetera these can be solved using the same analogy. You could also solve them piece by piece, but that will be little more tedious.

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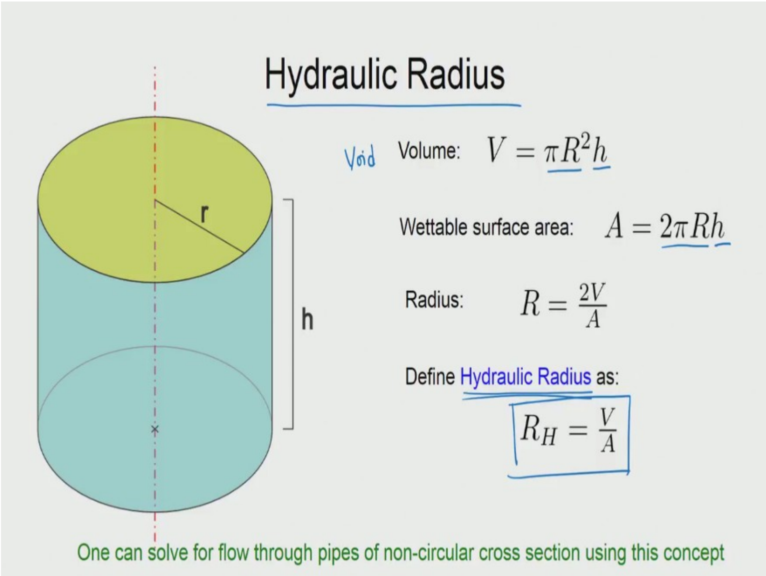

So, when we have the pipe not exactly a circular cross section then what do we do. So, here is a concept that could come of use for us, the concept is basically hydraulic radius. So, when we look at the volume through which the liquid is flowing which would be given by the cross sectional area which is $\pi R^2 \times$ h. So, that is the volume through which the liquid is flowing through. And the wettable surface area that is the area which is actually in contact with the fluid that is the surface area which is impeding the motion of the fluid so that would be basically perimeter $\times$ h. And if you take the ratio of these two would see that the radius will be arriving at from the volume divided by the surface area. So, volume is actually also the void volume. So, you could actually qualify this as void volume because it is through this void or cavity that the liquid is going through and once you have this you could actually imagine that you can already always call the ratio of volume to wettable area as a relevant quantity for the radius and which you would actually now define now.

So, we would like to define what is called the hydraulic radius. So, we are actually creating a new variable called hydraulic radius now we want to define it as a ratio of the void volume over the wettable surface area. So, once you have this then non circular cross section of the tubes can also be handled and you could also then try and see whether the expression that we derived earlier could be applicable when the deviation from the circular cross section is not very much.

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So, at this juncture we actually are going to apply the pipe flow equation to the very different set of problems and see how interestingly the relationships are coming out to be similar. So, here is where we are introducing the new concept called as the porous media or porous bodies.

In daily life these are very familiar to us we know from bread for example, these region you can see there is low of porosity and these actually are seen in engineering also and very common. So, you could actually see that you have a porous body an aluminum foam which is used to absorb a shock and it would collapse the porosity would help the material collapse and thereby absorb the energy before it the impact is passed on to the contact body which is behind. So, you also have situations where extended surfaces porous surfaces are available like here which can be used to remove the heat. So, the air that is in between these solid surfaces would take the heat and go away and thereby lead to a heat transfer taking place very efficiently. So, lot of heat exchanges would have their appearance looking like a porous body and you also have seen porous bodies in granular materials as you would see in bubbles or grains or sand particles or pluses when they flow.

So, in during all these phenomenon you would see that the domain basically is a porous body and the porous media and metallurgical scenario this is very much important because apart from heat exchanges the filters. For example, are very much porous bodies. Basically if you have a bed of alumina spheres then you could use it to filter the draws from the liquid aluminum melt and therefore, you could actually use porous bodies for filtering liquid metals liquid metals are very corrosive, so you could not use a metallic sieve to do that particular kind of a thing. And as you can also see from the appearance that porous bodies have a lot of surface area that is available, which also means that reaction that takes place on surfaces can also be enhanced. So, catalytic converters are also generally porous bodies.

And you also have multiphase reactors that would have porous bodies one of the reactants will be made as these porous medium the other ones will be like fluid going through and then because of the extended surface area that is available reactions can take place at more area and that can be useful in enhancing the productivity. And pressure reduces when you want to reduce pressure across a length then introducing a porous body in between generally helps. So, like this there are number of applications in metallurgical industry where the porous bodies are coming across and when we have a situation where a fluid is going through a porous body what kind of an expression can be used.

So, this is where for example, we can see whether what we have learned till now can be applied. So, a very generic way of representing porous body would be like this where you have got the white regions would be like for example, the solid and the black regions would be like the void.

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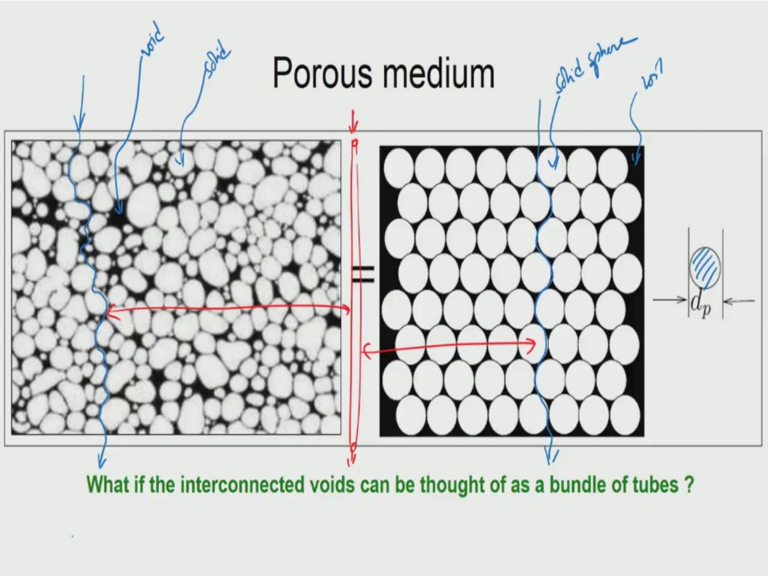

So, what this implies is that when you have a fluid that is entering from one end of the body. So, it would actually go through the void which is interconnected and eventually come out. So, could see that the path is actually tortuous nonetheless it is available so that the fluid can go through the porous body.

Now, it also means that you could also have a bed of spheres each sphere made of the diameter d_p and the black here is the void and this is solid sphere then you could also have the situation where a fluid can go through the a void that is available to go across the porous body. So, in both the situations for example, you could treat that there is certain amount of void volume that is available for the fluid to go through and one situation that we want to now use is the path that is taken can be then imagine that to be like a tube. So, we want actually check can this path be imagined as a tube through which the liquid is going through.

So, our idea of modelling the phenomenon is basically assuming that the void interconnected void is similar to a tube and then see if what we have derived till now can be applied for this kind of a problem. So, let us try if this works.

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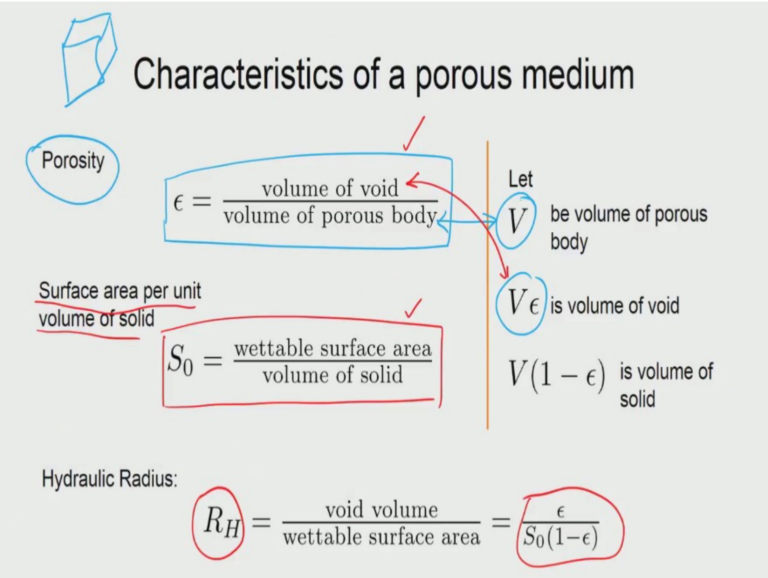

So, here we now we introduce some terminology to characterize a porous medium or a porous body so that these will be useful for our derivation. So, one characteristic of any porous body is to tell how much of porosity is there. So, you should tell the porosity as the first characteristic. If you take for example, a powder bed and then you are using a powder metallurgy principles to centre it to a solid block then depending upon the extent to which sintering is happening you would have porosity changing. So, as the density approaches the theoretical density the porosity is approaching 0. So, intermediate if you stop then you do can achieve the porosity to be something between 0 and 100.

So, like fifty percent porosity seventy percent porosity etcetera they are all possible and so you would like to define the porosity as the volume of the void to the volume of the porous body itself. So, by porous body what we mean is the entire body externally measured. So, if you use a calipers and measure the dimensions then that would be the volume of the porous body. And if you were to remove all the void then that would be this volume of the solid and together is basically what is coming as V here, what we refer to as V. So, $V\epsilon$ then becomes the volume of the void because from the definition V is this and therefore, $V\epsilon$ would then be the void because of the ratio becomes $V\epsilon / V$ that is ϵ which is basically the porosity because of the porosity is one characteristic.

Now, it is possible to have porous bodies which have the same porosity, but they may be made of particles of different diameter. It is possible to achieve that and in such situations what happens is that a body which is made of particles of a smaller diameter would have a more tortuous path. for the fluid to go through and which also have means that the surface area that is exposed will be more. So, it is possible to have a same porosity, but different particle diameter internally that would be actually give you more surface area. So, you need one more characteristic feature so that is basically the surface area per unit volume of the solid.

So, surface area per unit volume of solid is one another characteristic of the porous body that you can define and this can be actually measured experimentally and so we define one more quantity here. So, from whatever we have seen till now you could actually then use this two quantities to arrive at what would be the hydraulic radius of a pipe which we are imagining as basically equivalent to the inner connected void. So, hydraulic radius we have already seen it is ratio of the void volume to the wettable surface area. So, the void volume is coming from the first definition and the wettable surface area is coming from the second definition. So, that R_H is coming out to be an expression that involves two characteristic features of a porous medium namely ϵ and S_o .

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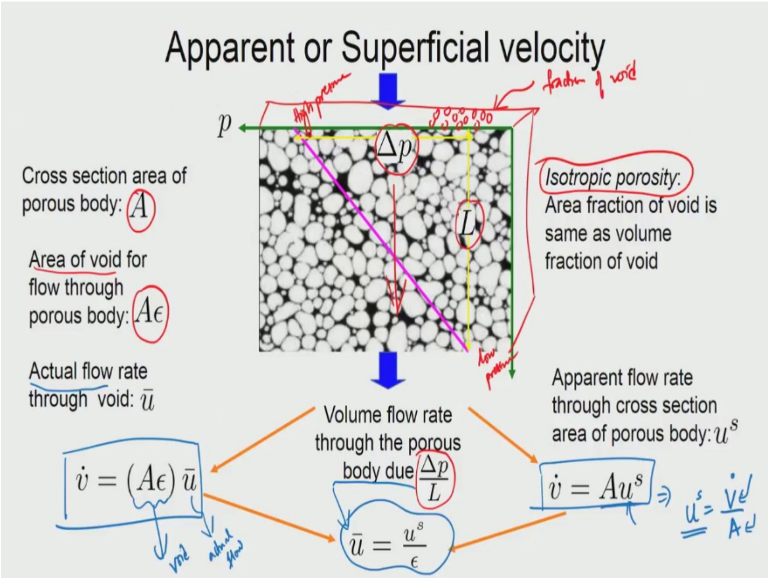

So, once we have the R_H then this is what we do. So, we then look at how the flow is taking place. So, here we have drawn on the top surface the pressure is high pressure and at the bottom surface we have got the low pressure. So, you could see that then it should lead to the fluid flow to happen in this direction.

Now, if the pressure difference from the top and bottom surfaces is given as Δp and the height of the porous body is L, then Δ p/L becomes a driving force which is causing the fluid flow or the fluid to go through this porous body. Now, the porous body would have a volume V and the ratio V and L would basically give me the surface area or cross sectional area of the body, so we call that as A. Now, this A is the area of the body which means that it can be measured by using calipers looking at the width and depth of the body, and this cross section when you look at the cross section you could then see that it is also made of voids and solids.

So, you could also see a fraction you could also look at the fraction of void in the surface there and if that is assumed to be same as the fraction of void in the entire body then we are actually calling this particular porosity as isotropic porosity. So, what we mean by isotropic porosity is that area fraction of void is same as the volume fraction of void. This is also the same as length fraction of void if you draw a line in random direction and then count how many segments of the line are falling in the solid and how many are falling in the void. So, isotropic would mean that in any direction the way of counting actually will give you the same void fraction. So, if that was true then A ϵ would then be the area of the void through which the flow can take place. So, A× 1 - ϵ would then be the area of the solid through which the flow cannot take place.

So, we now look at the entire volume of the fluid that can go through this body and see how it can represented. It can be represented in two different ways, on the left hand side we have one way of representing and here we write an expression in this form. So, there are two terms the first term is basically the void area and which means that whatever is going through the void is actually going at the actual velocity. So, this basically is a actual flow. So, u is actual flow, flow into the area of cross section will give you the volume flow rate. So, that is it is quite acceptable as we have seen till now.

However, this is not useful in the sense it is very difficult to measure the actual flow rate through the voids because voids are very small and they are of different sizes in different locations in the body. So, what actually is useful is to check what will be the superficial velocity. So, superficial velocity is defined in this manner that is it is a velocity as if the fluid is going through the entire cross section of the porous body. So, A × u^s should give us the volumetric rate. The reason why we call this as a useful quantity is because this actually implies I could actually calculate it as $\dot{V}$ / A, now $\dot{V}$ can be actually measured because you could actually collect fluid that is coming through this porous body for a duration of time and then divide the volume with the time and you get the volumetric flow rate. And you can actually measure the area of the porous body by using a callipers and therefore, this can be measured, which means that the superficial velocity of the fluid as if it is going to the entire cross section of the porous body can be experimentally determined. So, that is actually also determine the volumetric flow rate.

Now, the way you determine a volumetric flow rate in both ways is the same. So, it is the same fluid that is going through. So, if you want to then draw an equivalence between these two then there is possible when for example, the actual velocity is equal to superficial velocity divided by ϵ. So, this is how we are now able to relate the superficial velocity and the actual velocity.

Now, for the actual velocity we do have a relationship available because it is going through the void and for flow through the void which is roughly in the shape of a pipe then we do have an expression that is giving as a relationship between the velocity and the these two the

$\Delta p/L$ through the pipe flow equation. So, now, we want to relate these two and see how it comes about.

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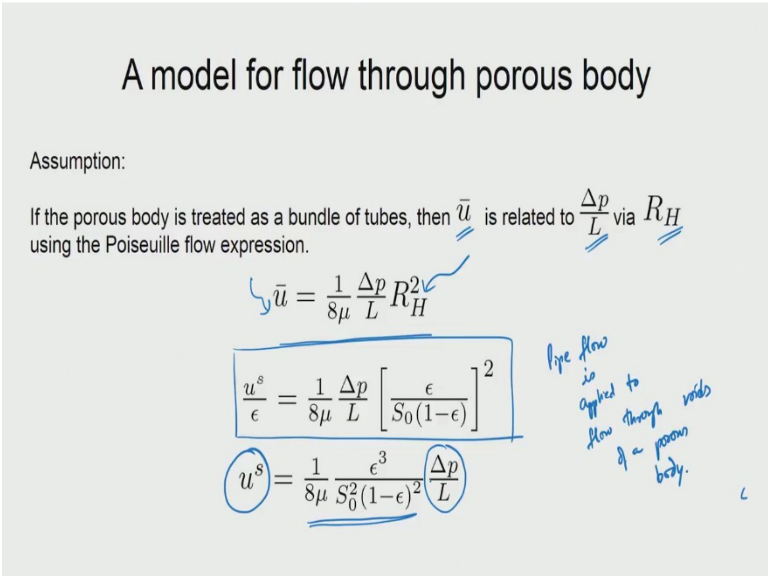

So, we are basically now making an assumption that is if the porous body is treated as a bundle of tubes, each connected void is like a tube then the actual velocity if it is related to the driving force for the flow through the hydraulic radius then maybe we can use the pipe flow expression or Poiseuille flow expression for connecting these quantities. So, we are just blindly going to apply that equation which we have already seen earlier. Only difference is that this is related to the superficial velocity and this is related to the porosity and this wetted area for unit volume of the solid.

So, we substitute those expressions and you see on the left hand side it is u^S/ϵ and on the right hand side $R_H = \frac{\epsilon}{S_o \times 1-\epsilon}$ which we have already seen just as while back. So, we basically use this expression here and then we basically are able to see how a pipe flow is applied to flow through voids of a porous body. So, we are basically modelling the porous body as a bundle of tubes and for each tube we are basically able to write this expression, which actually means that we can then take the quantities back and forth and see that we are able to relate the superficial velocity with the driving force and it is going through a bunch of quantities that are actually as a coefficient.

Now, this coefficient within inspect the trends when ϵ is going towards 1 or towards 0. So, when ϵ goes towards 0 means there is no void which actually also should mean that it is a fully solid and it would require infinite pressure drop to lead to any velocity and that would actually be also evident because they are coming and multiplication. So, for a given velocity u^s if ϵ is oing towards 0 then Δ p/L will blow up that is what we also expect.

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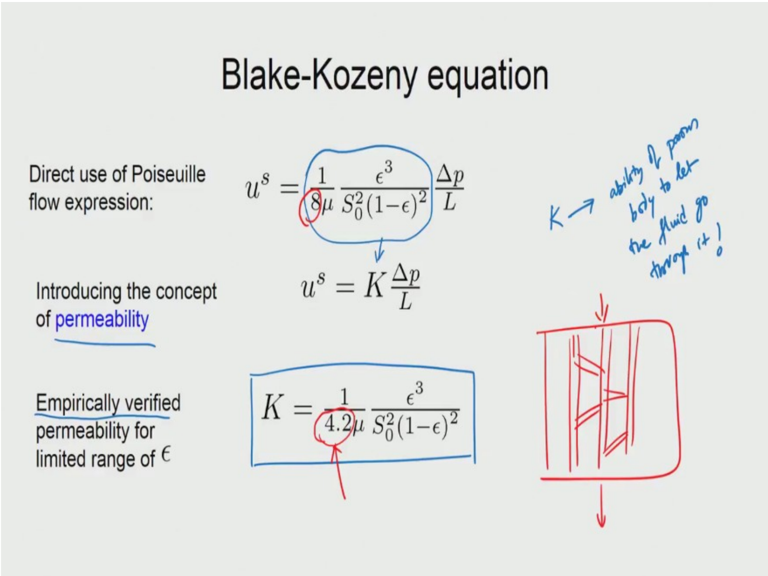

So, what we are now doing is to just see if we can make a generalization of this particular model to arrive at an equation that has a name. So, the equation that is available in exact metallurgical literature is the Blake Kozeny equation and we will see whether we can arrive at that. So, what we actually want to do is this entire coefficients that are there in front of the Δp /L you want to just simply call that as K, now what happens is that what does K do. So, K basically tells you that it is the ability of the porous body to let the fluid go through it. So, it is basically telling you how efficient will the pressure drop convert into the velocity. So, it converts to give you high velocity which means that it allows the fluid to go through very easily and if it does not do that it means that it causes lot of obstruction to the flow going through it, which means that it is nothing, but permeability.

So, permeability coefficient is what we want to call K, which means that we are now relating velocity to pressure drop per unit length through permeability. And the permeability coefficient which is written with empirically verified functional form is here. So, this is expression that has been derived separately. So, you could see that the only way place where we actually got it a little off is here. So, we see that this is the only thing we did not get right other than that the functional form is exactly how we have come about and this expression is empirically obtained and verified. So, which actually means that this quantity is a correction factor to take into account the fact that the void interconnected voids are actually not bundles of tubes, but they do have other effects. For example, whenever you have got bundles of tubes which are actually also interconnected then there must be some effect because of that, so some pressure drop will be modified because of that and therefore, it must lead to some correction factor. So, let us see that basically this is a correctional factor another wise our model seems to work quite well.

So, Blake Kozeny equation is basically expect for this correctional factor a model to treat the interconnected void as a bundle of tubes and then applying the analytically derived flow equation to it.

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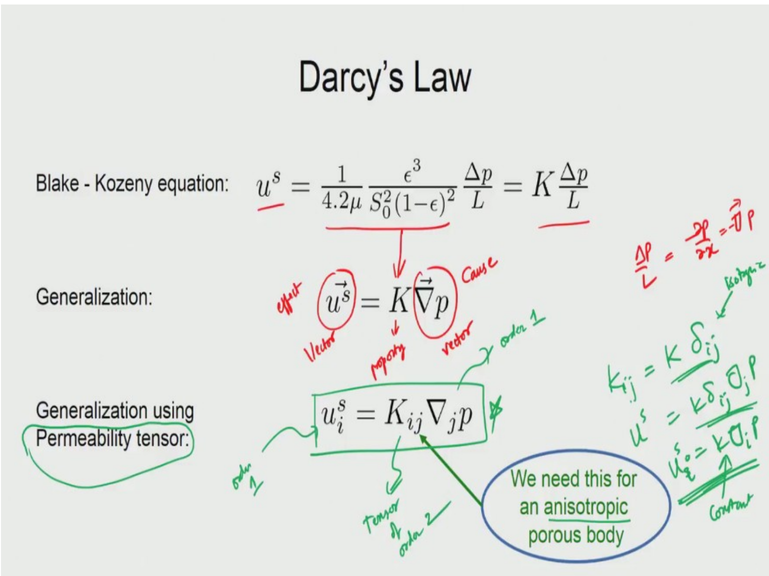

So, here this goes a little further. So, independently there is another equation that came about a little earlier which goes by the name Darcy's law and here basically the same equation we are writing and because we are now noticing that the superficial velocity is a velocity.

So, it is basically a vector. So, we have got a vector and this entire thing is nothing, but the coefficient and this is pressure drop per unit length. So, Δ p / L is basically in our case it is $\frac{\partial p}{\partial x}$ if the velocity happens to be in the x direction and so therefore, it is like gradient p. So, this is also a vector. So, from what we have studied earlier about the constitutive relationships relating cause and effect. So, if you want to call this guy as cause and this as effect that is due to a pressure drop and effect namely the fluid flow is taking place.

So, if you want to think of this as cause and effect then this must be a property and as per the tensorial relationships that we have seen that if cause and effect are vectors that is tensors of order 1 then the property can be in most general case a tensor of order 1 + 1 that is 2. So, we then basically go ahead and write that expression. So, this is a tensor of order 1 and this is a tensor of order 1. So, therefore, this must be a tensor of order 2. So, that is why we basically refer to the permeability as a permeability tensor. So, permeability tensor is a very generalized concept, which means that the equation that we came about by starting from the pipe flow equation and then modelling the porosity and then introducing the concept of hydraulic radius then once we got that equation if we want we can elevate it to the level of a constant into relationship between cause and the effect then we do arrive at what is called the Darcy's law.

So, Darcy's law is this equation and in which the permeability is coming as an anisotropic tensor of order 2 and you could actually call it as isotropic in situations where the medium is isotropic. So, if the porous body happens to be isotropic then you can go ahead and write the K_{ij}, as $K \times \Delta_{ij}$ and in which case then you could actually see that us is then given by $K \times \Delta_{ij} p$ that is basically $K \times p$. So, we could see that you actually see a vectorial relationship giving only a constant here for the case of isotropic medium. So, by Neumann principle the property should have the same symmetry as of the materials. So, if the material happens to be isotropic porous medium then the property also should be isotropic and in which case you get the relationship that we have already seen.

So, in that way we can actually see that starting from one equation we are landing up another equation which actually all came very early almost independently and the relationship is coming out quite beautifully thanks to the tensorial concepts that we have seen till now and the models that are interconnecting these. So, now, we see that we can actually check for the validity of these relationships. So, way to validate is the same as the pipe flow, so to look at the Reynold's number and checking whether the Reynold's number is within some range or not.

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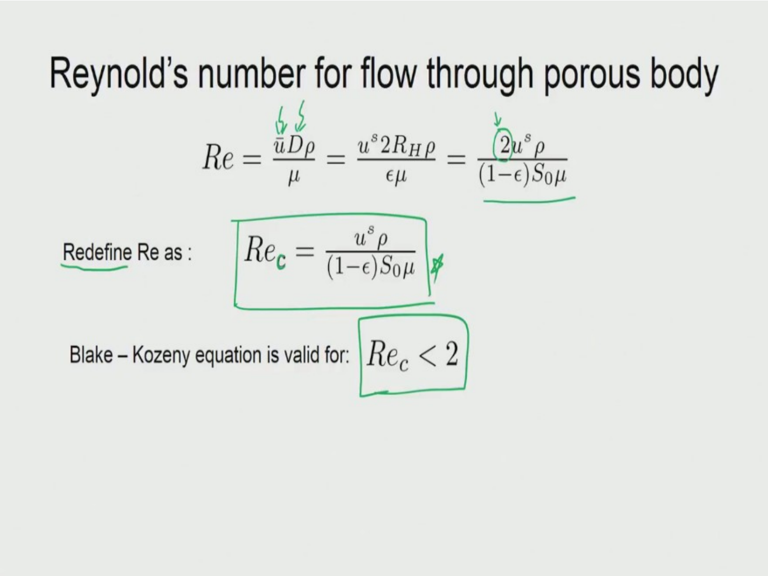

So, the porous medium has been modelled with these quantities being a bit different the u is basically the superficial velocity and here the Reynold's number should have actual velocity. So, we must then substitute properly. The diameter is basically twice the hydraulic radius. So, we must also bring the hydraulic radius expression. So, we bring that in and we see that the Reynold number will come out to the expression like this.

So, normally when we have non dimensional quantities then we do not want numerical things in there because there is no point in that. So, we can actually look at the introduction of non dimensional members has only a way to remove the dimensions. So, numerical values need not come. So, we redefine, we redefine the Reynold's number in this manner by knocking of that two and to remind us about the redefinition we actually also pay attention to the sub script which actually tell us that there has been some redefinition that has happened. So, the Reynold's number for a porous body is given by this expression. And the results that you have seen till now that is the Blake Kozeny equation is valid when the Reynold's number with the new definition is less than 2, which you can see that it is 1000 times lower than the Reynold's number for a pipe flow. So, definitely our model that we have applied is applicable only when the floor is very slow. So, the moment it increases then our model does not seem to work.

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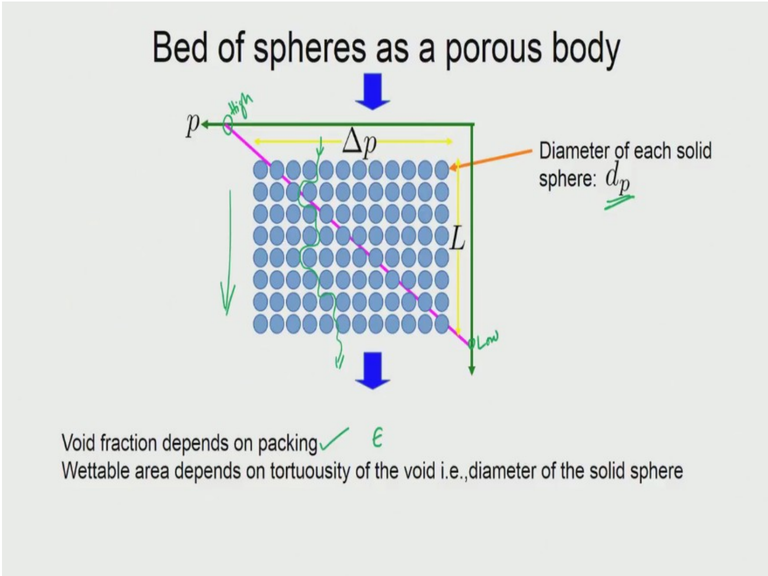

Now, we can then go ahead and apply the same concept to a bed of spheres that would act as a porous body. The only reason why we want to take it up is because we do have a simpler expression that can come out. So, the bed of spheres is depicted here. So, the pressure is high, on the top surface its low, in the bottom surface and therefore, the fluid flow will take place in this direction and that is happening in the void spaces that is interconnected and coming out. And we want to model that particular path as a tube and we are just basically going to use a same expression. So, the diameter of each solid sphere is d_p which actually makes us assume that the bed is made of uni-dispersed solid basically. So, if we have a distribution of sizes then we are not yet able to handle that.

And it is then also clear that the ϵ can still be varying because once you have got the particles you can arrange them differently to get different void volume. So, you can make them closed packed we get less volume we get loosely packed then we get more volume. So, it is showing that ϵ can be independently controlled without the particle diameter is already fixed. But once the particle diameter is fixed then the wettable area is also getting fixed and that is basically here.

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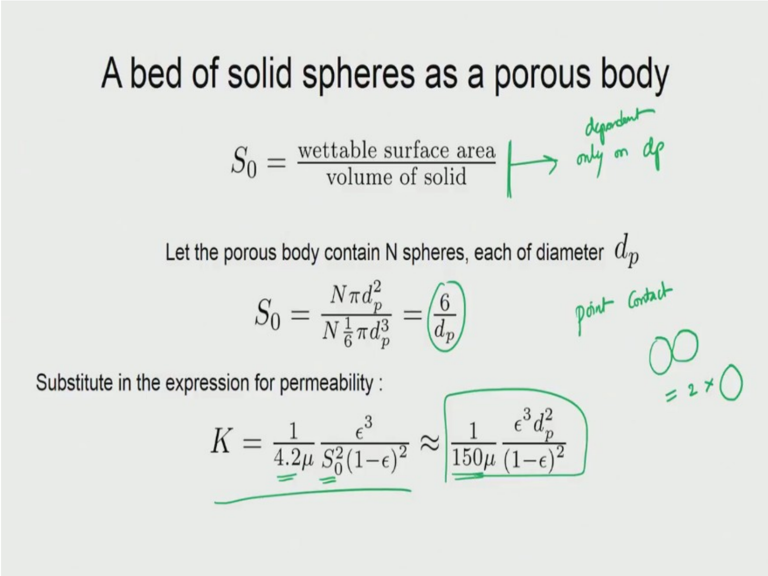

You can see the wettable area is given by ratio of is a point of S_o and volume of solid and we can see that if there are N spheres then N into the surface area of the sphere that is the numerator and N into the volume of the solid that is the denominator the ratio would give you the S_o and that comes out as a 6 / d_p, which means that this entire quantity is dependent only on d_p. So, powder diameter is directly controlling how much of wettable surface area is available per unit volume of the solid. So, here we are actually making a very important assumption that there is only a point contact. So, there is no loss of surface area because so many powder particles are coming together. So, we assume that the area of these two is exactly is equal to twice into this. So, there is no loss of surface area we are assuming.

So, subject to that then we go ahead and substitute wherever S_o is available we are actually putting $6/d_p$ and then we will see that the K expression would have a $4.2 \times S_o^2$ and that is approximately 150, which means that we now have an expression also where the porous body is made up of a uniform spheres of a same diameter d_p. So, the K that is permeability coefficient is now available for us as an analytical expression with ϵ and d_p that are there.

Now, ϵ if we already know that it is closed packed with let us say about 68 percent packing then you already see that 32 percent that is 0.32 will be the value of ϵ and then the diameter is available. So, d_p can be substituted and with the viscosity of the fluid available immediately we can get what would be the permeability coefficient.

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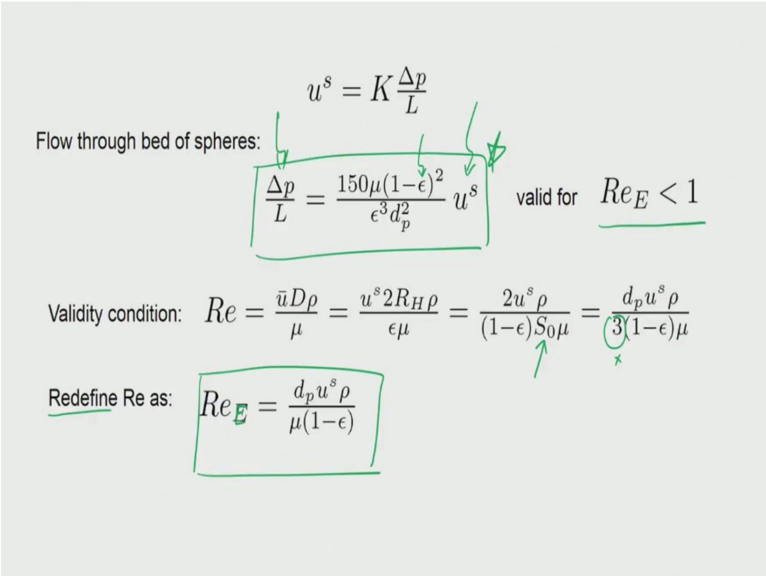

Now, once that is there then we can substitute in the Blake Kozeny equation and see how that would look like for the bed of spheres. So, for the bed of spheres I have just flipped the quantities around to show them in the way that most text books show. $\frac{\Delta p}{L} = \frac{(150\mu \times (1-\epsilon)^2) u_s}{\epsilon^3 d_p^2}$ So, this expression actually is there in many of the extractive metallurgical textbooks and we know from where this has come. So, this has come from our model and the fact that S_o happens to be $6/d_p$ for a bed of spheres.

Now, this is varies when the Reynold's number defined appropriately for this problem happens to be small. So, a very very lower Reynold's numbers and that actually is now modelled here. We already see that this can be modified to 6 / d_p, which we have done and we do not want in a Reynold's number definition. So, such things are actually not required, so we go ahead and redefine the Reynold's number and we pay attention to the subscript and we see that it has been defined in a different manner.

So, with this definition of Reynold's number then if it is less than 1 or very small Reynold's numbers then we can go ahead and use the Blake Kozeny equation for a bed of spheres here and this would actually relate how the pressure drop per unit length is changed because of the velocity that is going through which we desire. So, which also means that if you want a particular amount of pressure drop to happen then you can actually see that knowing what if be the velocity of the fluid going through then you can actually ask yourself what is ϵ that you would need to achieve that kind of a pressure drop. So, that way we can actually put these things to great use.

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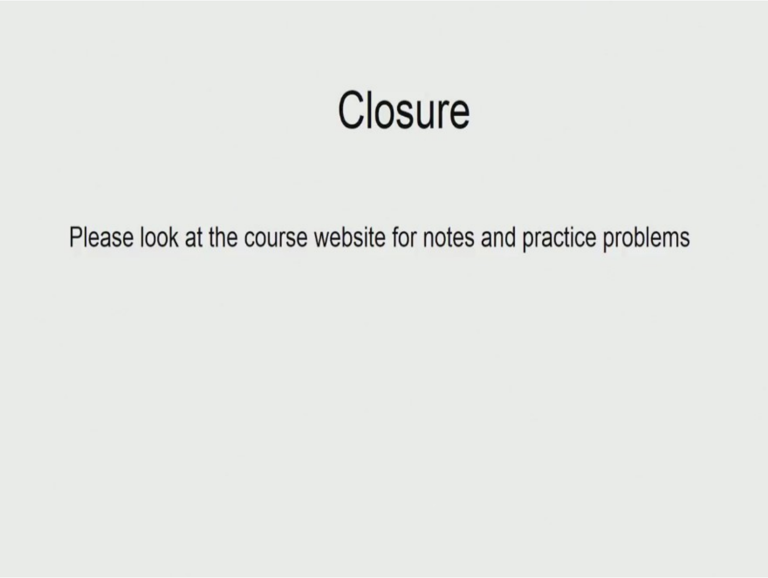

So, at this moment we will close. The numerical problems of using these expressions will be made available as practice assignments in the course website you could punch the numbers and see how we are getting through.