**Transport Phenomena in Materials**
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**Lecture – 13**
**Simple cases in fluid flow – Cylindrical coordinate system**

Welcome to the session on solutions to the Navier-Stokes equation. We will be taking up simple cases and in this particular session we will be taking up the cylindrical coordinate system. In the previous session we have done the rectangular coordinate system and this is as part of the NPTEL MOOC on Transport Phenomena in Materials.

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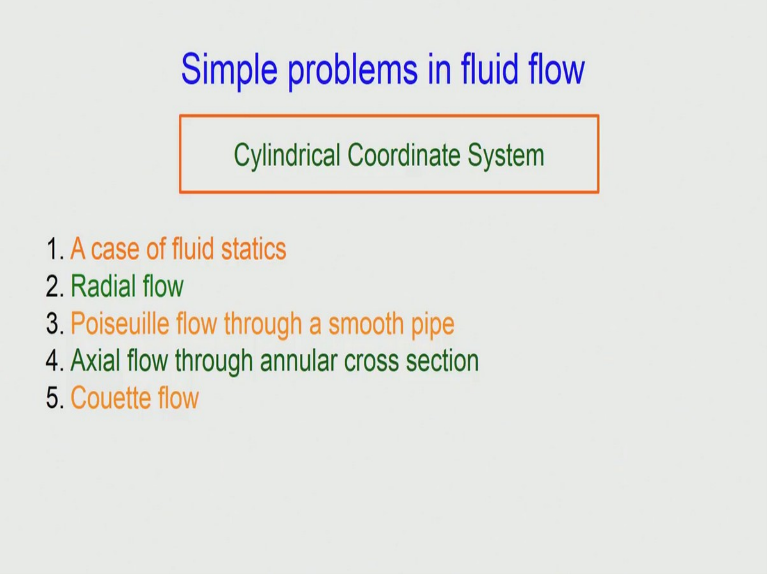


So, in this session we will be first looking at simple interesting case of a fluid statics whereby we will be contrast of the pressure term also being varied as a function of the special coordinates. Then we will look at how the 3 different components of velocity in the cylindrical coordinate system namely the radial component, the axial component and the θ component can be solved for in 3 separate problems. So, we will be taking up unidirectional flow in this session. So, we will take up only one component at a time in each problem.

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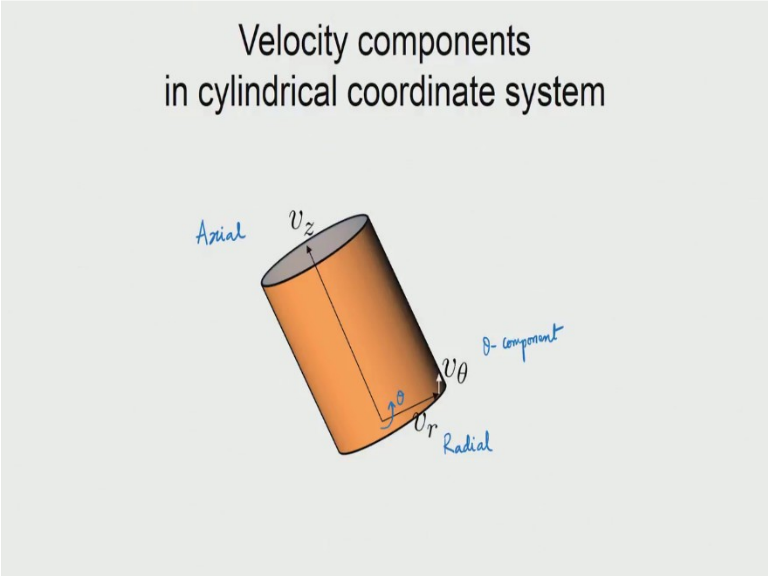


So, here is a depiction of how the components are looking like the, what we refer to by axial component is this which is along the z direction which is along the axis of the cylinder and the radial component is here, so it goes from the centre towards the periphery of the cylinder, and along the θ direction is the θ component. And we have problems in which one or more of this components will be present and we will take up the problems one after other to see how the equations can be solved for.

As you can see that the θ component is pegged at the end of the $\vec{r}$and as a function of θ which I am showing here the location of the vector $v_\theta$ would be changing. So, this actually means that in the cylindrical coordinate system the Navier-Stokes equation will have terms that are in addition to what normally are expected from the rectangle coordinate system and we must be conscious of those terms while solving the equations. So, here are the equations that we have shown in the earlier session on how the Navier-Stokes equation would look like for different components of the cylindrical coordinate system.

So, I was drawing your attention to essentially terms like this.

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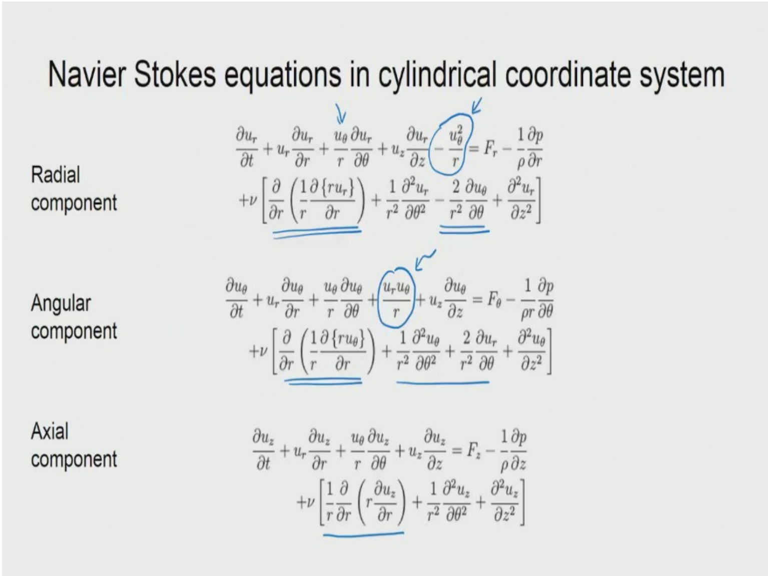


For the r component you have this term which is coming in and you do have for the θ component also some term that is coming in. And in addition the laplacian also would have a special appearance where you have got 1/r going in and here also there is an additional term. So, it is not directly substituting xyz with r θ z, but we need to watch out how the coordinate system transformations are being used. So, pay attention to this terms so that are being highlighted. So, that we can notice that the terms are looking a little different in the cylindrical coordinate system. So, we will take problems so that we will use these equations one at a time.

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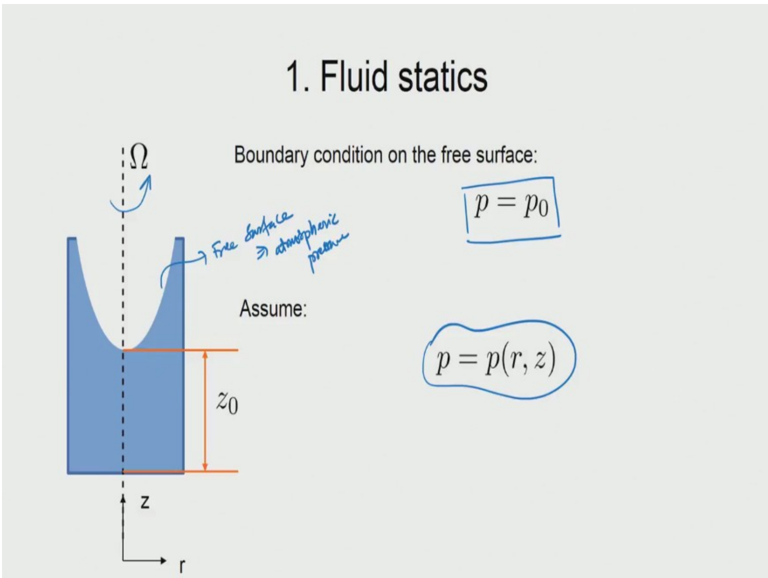

So, first problem is about statics problem namely the situation where we are only looking at how the pressure would change as a function of distance. So, here is a situation where you have a container of liquid which is made to rotate about its own axis. So, this is the rotation that is happening and because of this we expect normally that the liquid will be pushed away from the centre so it would climb up the wall little bit and it would take the free surface would take a shape.

Now, we are interested in determining this shape just from the statics. So, from the statics what we know that this shape is given by the condition that if because it is a free surfaces, so you say that this is a free surface. So, it means that atmospheric pressure is what is acting on the liquid at that location, which we determined here as a condition. So, $p=p_o$. $p_o$ is the atmospheric pressure and if we choose $\Omega$ then depending on the magnitude the $z_o$ also would change. So, we indicate that in the problem here $z_o$ . So, we want to assume that there is no $\theta$ variation it is an axisymmetric problem. So, we now have a situation where pressure is a function of r and z and we want to determine that. So, the equations are here.

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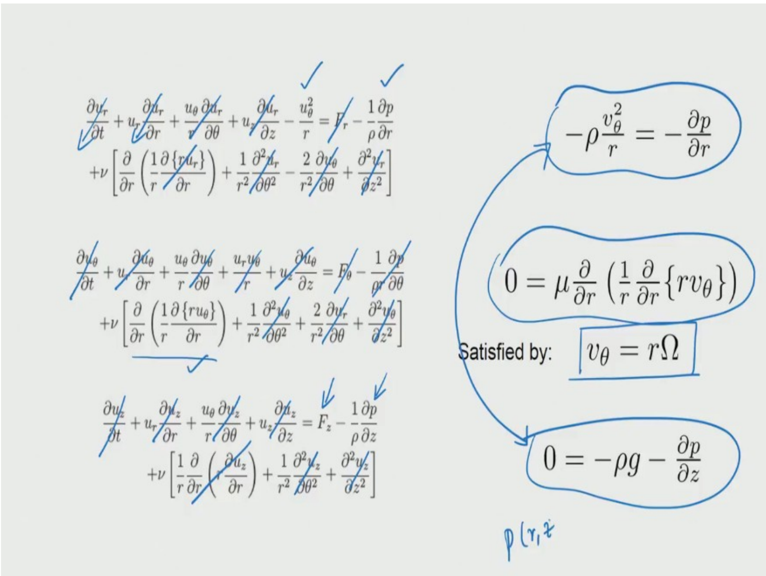


Now, this equations are going to be very trivial because we do not have velocity components except in the $\theta$ direction, because r velocity is not there, wherever $u_r$ is there those components will be all going off. So, I strike off all those and we also know that we have chosen the axisymmetric situations. So, we strike that off also and there is no body force term along the radial direction, which means that we have got only two terms which we have indicated here as the first equation which will give you the radial variation of the pressure.

So, similarly for θ we say that it is steady state and there is no $u_r$ and there is no $u_z$ and here also there is no $u_r$ and axisymmetry and we say that again from the axisymmetry and $u_r$ is not there and there is no z variation of velocity in the θ direction. So, it basically leaves us only one term and we also want to say that there is no axisymmetry also makes the p not varying along θ and there is no body force acting in the θ direction. So, we have a very simple equation that have only on term. So, that comes here.

Now, the boundary condition is such that the linear velocity at the end of the container is given by rΩ. So, that if you plug in then you see that the equation is satisfied, so $v_\theta$ can be taken as rΩ which we can substitute and see that it satisfies. So, in the case of z direction also the velocities are actually not there and so you have those terms going away and you see that this must be the body force which is basically the gravity term and this must be the pressure variation because of the z direction having a different heights of the liquid as you go along the r. So, you have this equation. So, you now have these two equations which are need to be solved. So, that we can get the p as a function of r and z and we substitute $v_\theta$ expression into this equation.

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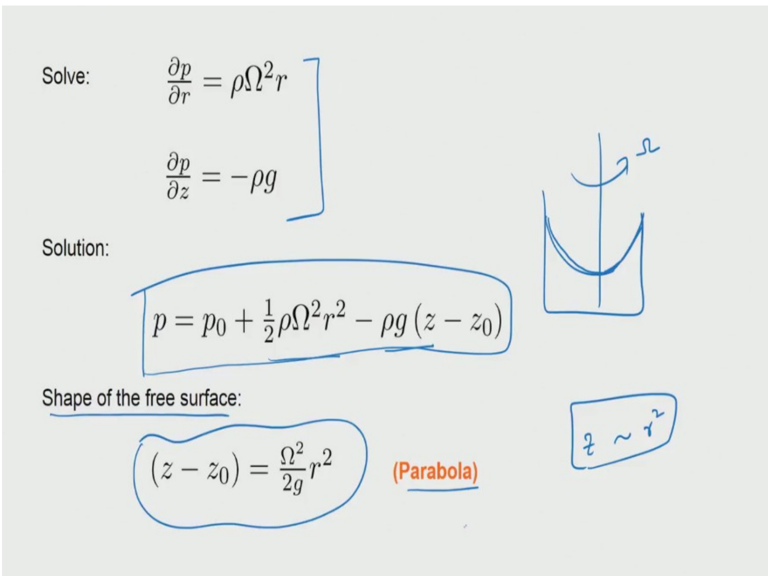


So, we do that, then we arrive at to these two equations which need to be solved simultaneously. So, there will be some integration constants that we are actually deriving from the fact that atmospheric pressure $p_o$ is there on the surface of the liquid, which when we plug in then we can clearly see from here itself that the first integration will give you $r^2/2$. So, there is this and second integration will give ρ g z that is this term the remaining ones are coming from the integration constants.

So, which means that the pressure variation in this particular situation where the liquid is being turned. So, this pressure variation is given by this particular functional form and therefore, wherever $p = p_o$ will give you the curve this particular curve. So, the free surface is then given by a functional form like this. So, z is going as $r^2$. So, z going as $r^2$ would imply that we have a parabola which means that when we rotate a container of liquid about its own axis at a velocity angular velocity Ω then depending on the magnitude of Ω the parabola will be a deeper high higher magnitudes and this kind of a situation will be used to make actually parabolic mirrors. So, we see that the idea of parabolic shape as come out only from the statics problem when there are no velocity components at all and by looking at the equations in the Navier-Stokes equation for the cylindrical coordinate systems. So, this is interesting and we are actually now seeing that the pressure variation can also be derived from these equations. So, now, let us look at the 3 components, we will take one component at a time.

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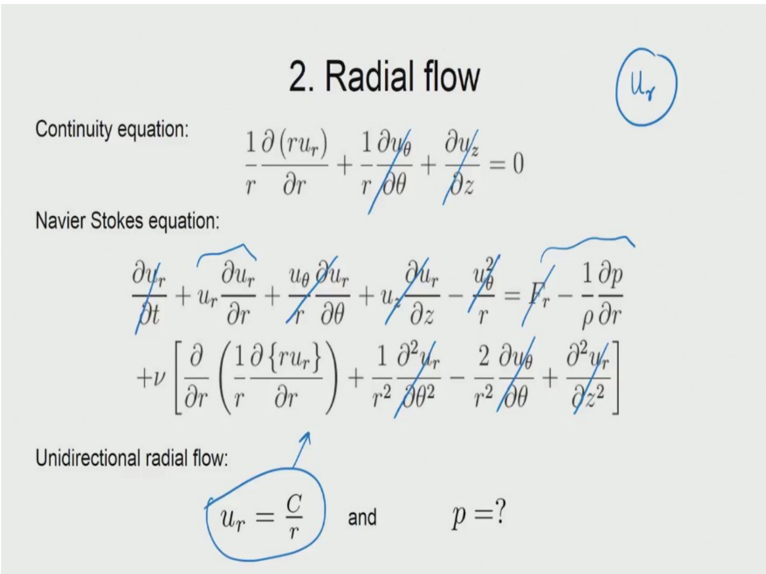


So, the radial component is being taken. So, radial velocity component is $u_r$ and the continuity equation needs to be satisfied and also the Navier-Stokes equation. So, we saw that if you look at only the continuity equation we already can guess what kind of forms are allowed. So, we see that if it is a unidirectional flow. So, then this functional form should give us that $u_r$ should go as a function of 1/r. So, that it can satisfy the continuity equation.

And when we plug in this into the Navier-Stokes equation and look at situations where I know it is a simple situation like a steady state and you do not have a $u_\theta$ and you do not have $u_z$ and there is no θ variation and there is no $u_\theta$ and there is no z variation. So, you could already see that here again you could see that this is already satisfied. So, this would go off, but what you see that these terms would still survive. So, which means that even if you want to drop this term saying that there is no body force along the r direction you can see that there must be a pressure variation as a function of r if you want to use a simple form of radial velocity like this. So, this can be immediately plugged in and you can derive it. So, I would leave that as homework for you.

But it just shows you that when you actually plug in the functional forms for the unidirectional velocity is that you can conclude from continuity equation then you may have actually pressure variations also to be taken into account as we call off.

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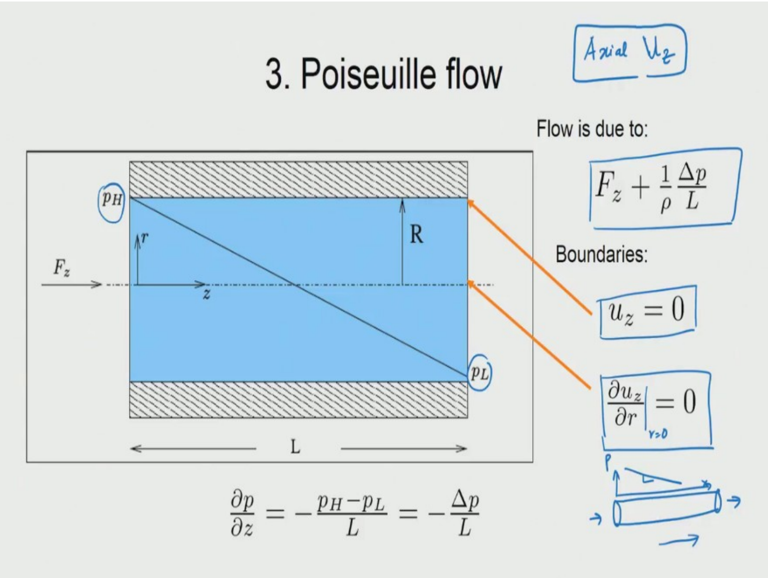


So, we now look at the second component of this velocity in cylindrical coordinate system that is the axial flow. So, that is $u_z$, along the z direction. Now, for that we choose a problem that is very standard that has to be learnt by all students who are studying this particular subject namely flow through a pipe.

So, you have a pipe and liquid is actually going through that and the reason why it is able to go through that is because either there is a body force that is acting along the length of the pipe, so within in situations where the pipe is vertically kept because of the gravity the liquid will flow through that that is one way another way is actually there is a pressure gradient along the pipe. So, when you have a high pressure here and when you have low pressure there, because of the negative slope of pressure then you have basically velocity is in the positive z direction. So, that is what is indicated here in terms of $p_H$ and $p_L$. So, you have got these two situations and so the flow is due to one of the two terms or a combination of the both.

So, $F_z$ is a body force term that is g and $\frac{1}{\rho}\frac{\Delta p}{L}$ is basically the term correspondent to the pressure drop. So, because of one of these two reasons the flow is happening along the axial direction that is along the z direction in this particular tube.

Now, the inner walls of the tube are assumed to be rigid smooth and. So, on and. So, the boundary conditions that are applicable are that the velocity at the inner wall is 0 this is from the no slip condition. And we also want to say that along the diameter there is a symmetry which would mean that the velocity profile should have a finite value at the r = 0 or you have a slope that is 0. So, both would actually mean the thing when we are solving the equation, but we need to use one of those two principles at the centre that is either the symmetry or the velocity being (Refer Time: 12:44).

So, what is the equation that we need to write for this particular problem? So, we have basically the equation Navier-Stokes equation for the z component of the velocity. So, that is what we are going to write.

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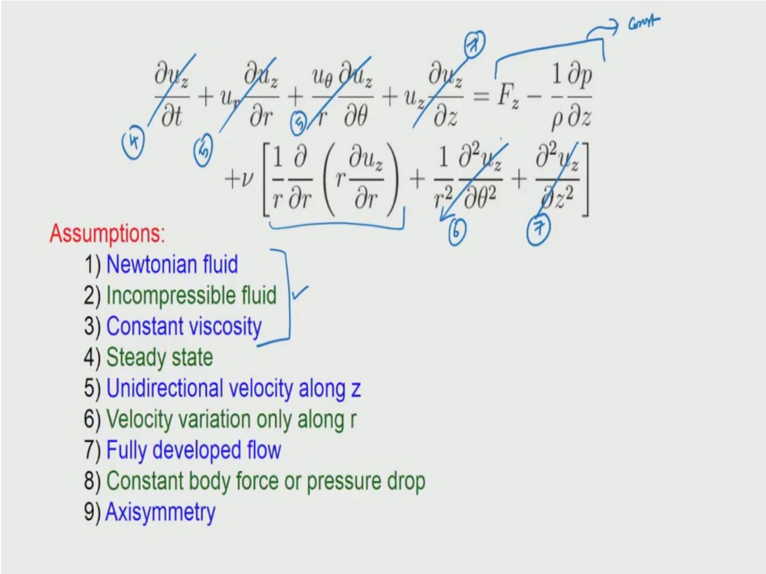


So, we have already written the equation which means that we have already made these 3 assumptions and more than that we are also going to make assumptions. So, we are going to strike out the first term because of the assumption 4 steady state and we are saying it is a unidirectional velocity along z. So, there is no $u_r$, and then there is no $u_\theta$ also and. So, that would be gone and we say that the velocity variation is only along r. So, variation along θ direction is being dropped. So, that would be assumption 6. Along the z direction we say that it is a fully developed flow. So, we want to strike that off saying that it is because of the condition 7, then this two will be gone.

So, then we also want to say that there is an axisymmetry of course, that would also make this go away. So, one of the functions will actually you know take that is taken care. The constant body force term when means that this is actually constant that is only to make the integration easier for us otherwise there is no need for a that to be constant. So, you now have the Navier-Stokes equation having only two terms and that would look like that.

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$$0 = \left[F_z + \frac{1}{\rho}\frac{\Delta p}{L}\right] + \frac{\mu}{\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) = -\left[\frac{\rho F_z}{\mu} + \frac{1}{\mu}\frac{\Delta p}{L}\right] r$$

$$r\frac{\partial u_z}{\partial r} = -\left[\frac{\rho F_z}{2\mu} + \frac{1}{2\mu}\frac{\Delta p}{L}\right] r^2 + C_1$$

$$\frac{\partial u_z}{\partial r} = -\left[\frac{\rho F_z}{2\mu} + \frac{1}{2\mu}\frac{\Delta p}{L}\right] r + \frac{C_1}{r}$$

By symmetry:

$$\left.\frac{\partial u_z}{\partial r}\right|_{r\to 0} = 0$$

$$C_1 = 0$$

So, when we now integrate, so take this term to the other side. So, we get a minus sign and then you integrate once. So, then you can get the r coming in and then the integration constant is $C_1$ and when you integrate once more you can see that this would actually give you log term. So, that would be a problem because at r = 0, log r would actually blow up and you want the velocity at the axis centre to be finite, which means that we do not want $C_1$ to be present because $C_1$ /r will actually will give you a problem when we integrate the next step.

Another way to say it is because of the symmetry we want that the velocity profile will be having symmetry along the diameter. So, which means that at r = 0 the slope of the velocity profile in the r direction will be 0. So, which means that again this term should be 0 because if this is 0 at r = 0 then this must also be 0. So, either way we are arriving at we conclusion that $C_1$ is 0.

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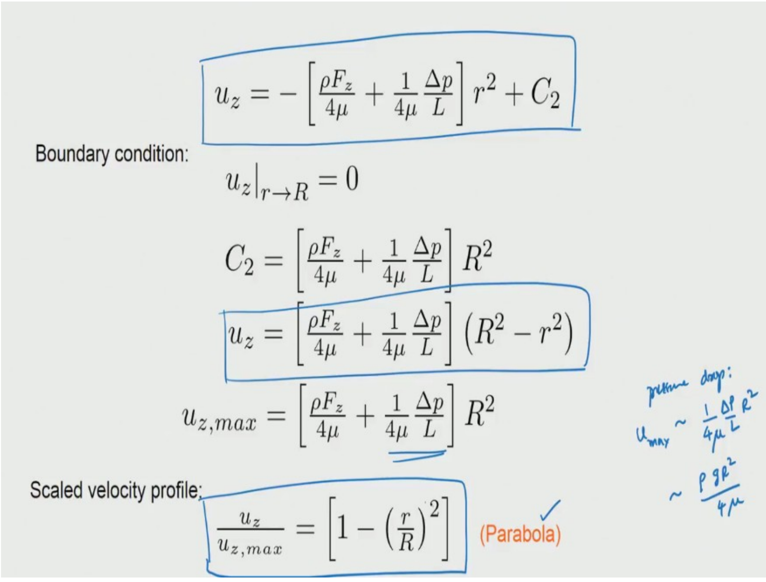


So, we substitute that and integrate once more. So, we get a functional form what looks like this. So, this is basically the solution of the z component or the axial velocity component in the cylindrical coordinate system further list of assumptions that we have mentioned. And when we apply the boundary condition that at the inner wall of this pipe you have 0 velocity because of no slip then we can determine what is the $C_2$, some of the $C_2$ is then determined here and when you substitute you get the optional form or the velocity which is appearing to be like a parabola because you already see that there is a r square variation that is coming here.

And what would be the maximum velocity? When you substitute r is equal to 0 you get the maximum velocity, which means that let us say you have got only one of the variations, because of only pressure drop. So, if only pressure drop is present then the maximum velocity is then given by $\frac{1}{4\mu}\frac{\Delta p}{L} \times r^2$ and you could also write for example, as $\frac{\rho g r^2}{4\mu}$. So, in the terms can be added if both of them are acting simultaneously.

Now, when we want to look at the velocity profile just by normalizing the velocity so that we only look at the shape of the velocity profile then it would come out on the right hand side with only profile form which means the scaled radius if you use that just courses $\frac{1}{1-r^2}$. So, which basically means it is a parabola and this actually is already encountered in a similar problem in the rectangle coordinate system as well. So, this functional form is very important because this will be then used in many many situations as we go along.

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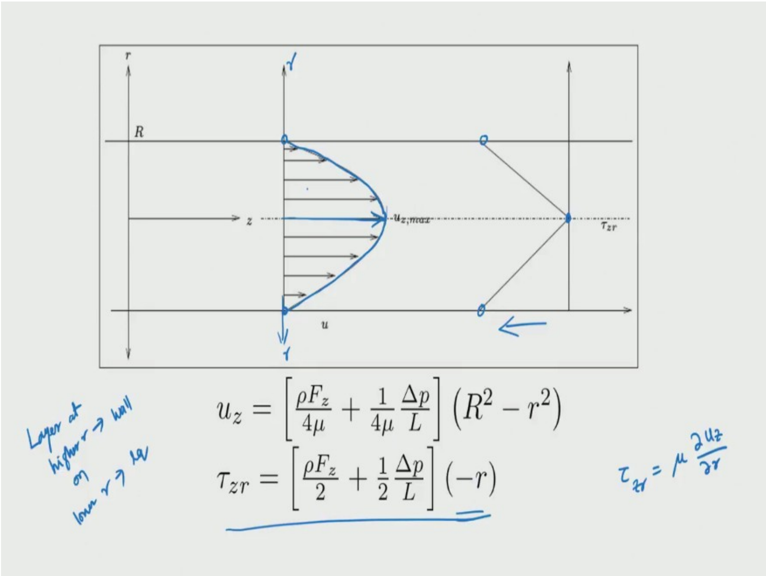


Now, when we plot this parabola it would look like here. So, this is the plot we are trying to show, which means that in the tube at this centre is the maximum velocity and at the walls it is 0 velocity and the slope at the centre will be 0 and the parabolic is a nature of the velocity as a function of r in the tube.

Now, you can already see that from the Newton's law slope of the velocity profile should actually indicate what kind of shear stress is present at that particular location. So, you can already see that this slope is 0 here which means that the shear stress is 0 and then the slope is actually here, so it appears as if the slope is changing the sign when we go to here to here, but please remember that this is actually a cylindrical case. So, this is r and this is also r. So, because of that the slope actually is having the same sign on both sides and that is negative. The reason why it is negative is evident when you actually mathematically show here and then the plug in. So, you can actually see that from the expression for $\tau_{zr}$ going as $\mu \frac{\partial u_z}{\partial r}$ for unidirectional component. So, you can already see that it comes out as minus r and from the convention that we have already seen we can actually apply that.

So, the layer at higher r and higher r is basically the wall or the layer at lower r and that is the liquid. So, the stress that is acted upon by the layer at higher r on the layer at lower r that is the stress acted upon by the wall on the liquid if it is actually along the positive z direction then the stress is positive. But actually as we can see here it is actually trying to stop the velocity from taking a magnitude at the wall and therefore, the stress is actually acted upon in this direction that is opposite to the positive z that is in the negative z, therefore, the stress has to be negative. So, that is how the convention actually is also you know coming out neatly even in the cylindrical coordinate system from what we have written earlier. So, mathematically what I shown if you plot it actually also makes sense with respect to the partial form that they have given.

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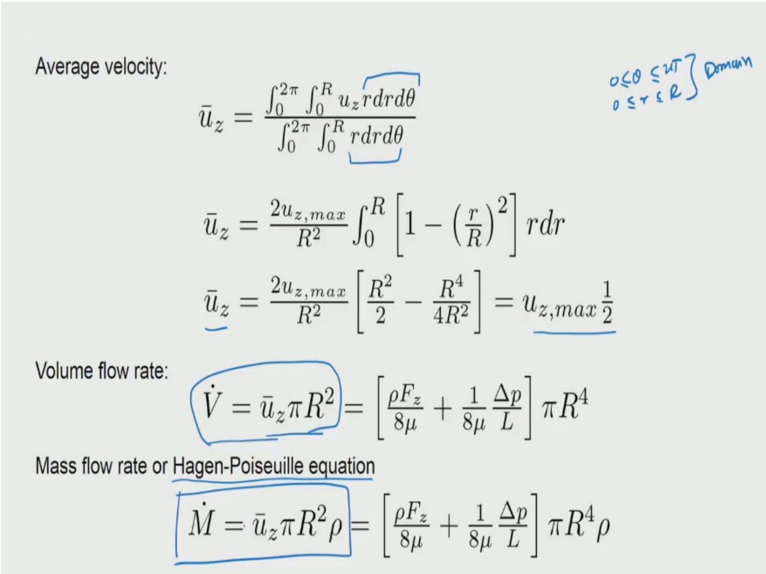


And what is the average velocity? This is very important because we want to determine the flow rates through pipes and junctions of pipes etcetera. So, the average velocity is always generally over the area through which the flow is taking place. So, you know that the area element is generally given in the cylindrical coordinate system as r dr dθ and, you just take that area element and integrate over the entire circular domain and, which means that the θ is 0 to 2π and the domain is defined by this and in the numerator you actually have a functional form from z. So, that also if you substitute then you do have expression coming a little more elaborate.

So, we go through that algebra it is quite straightforward you can just substitute and integrate its just integration of only r and $r^3$. So, if you substitute you basically then arrive at this conclusion that the average velocity is half of the maximum velocity. So, now, this is interesting because in the rectangle case where we had the parabolic flow variation the average was coming to be two-thirds of the maximum velocity, but in the cylindrical case the average is coming to be half of the maximum. So, the reason of course, is because the domain is in cylindrical coordinate system like this. So, the r that is going in is actually making this integration little different from rectangular phase. So, we then can use this to flip between the two quantities that we want to know. So, very often maximum velocity is not given average is given and then we can actually then determine the maximum velocity and then use that as a scaling factor for the functional form and therefore, get also the velocity variation as a function of r.

Now, often the volumetric flow rate is actually asked because we can actually you know collect the liquid that is coming through the pipe for a given duration and then from there determine what is the $\dot{V}$. So, how is the $\dot{V}$ related to the average velocity? It is basically average velocity into the area through which the flow is happening that is the $\dot{V}$and if you substitute it comes out to be this expression.

So, this is something that we need to use and if you multiply the volumetric flow rate with density then you actually have the mass flow rate and that is also straightforward you can use it here the average velocity into the area $\pi R^2 \times \rho$ . So, that would be giving the mass flow rate. So, mass flow rate has a name, Hagen-Poiseuille equation. So, this equation actually is quite popular and if you plug in the number and get the average velocity then that can be used to determine the flow variation.

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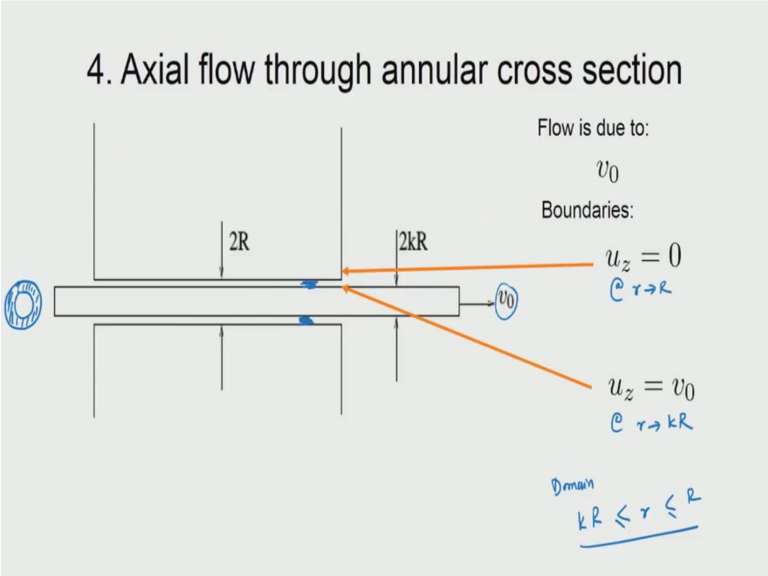


So, we now move to the axial flow, but in a small deviation the earlier had a type with the circular cross section, but in this situation we want to have annular cross section. So, we have our domain little different. So, the domain is looking like a (Refer Time: 22:53). So, the domain is defined in this manner. So, if you have kR as the inner radius, so this is a domain and the liquid is actually going in the annular region like this and in axial direction. So, it is still in the axial direction only. So, boundary conditions will be as follows.

So, the liquid is actually flowing because the inner shaft is being pulled out at a velocity $v_o$ and because the liquid would get stuck to the both the walls because of no slip condition. So, there will be a shearing of liquid. So, velocity gradients are being setup. So, the boundary condition on the upper wall this is at r is this. So, this is giving you axial velocity to be 0 because the outer container is stationary. And at r going to kR the velocity is given by $v_o$ because the situation is that of a moving wall. So, both of them are actually no slip conditions only. So, the flow is happening not because of any driving force, but only because of the moving walls. So, if this is a simple situation how would we go about arriving at the solution.

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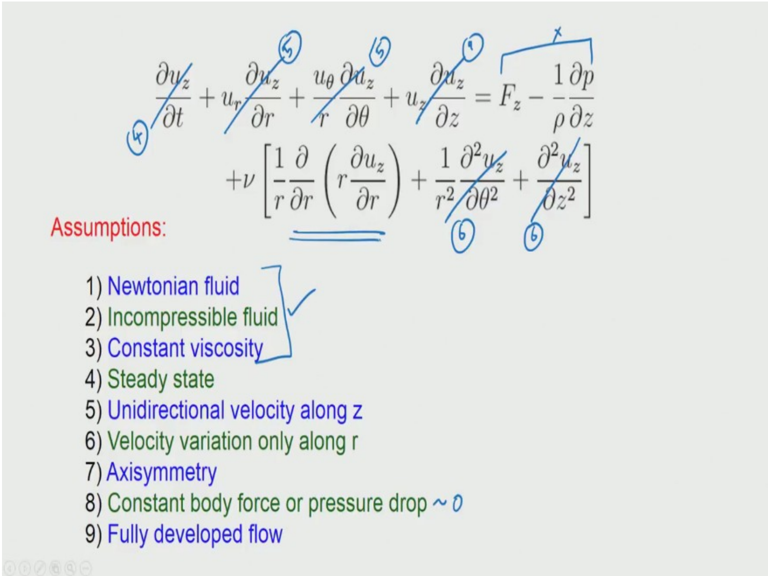


So, that is quite straightforward, we basically have the say equation that is written and we take the steady state assumption and unidirectional velocity along z. So, the θ is no there and θ $u_\theta$ is not there and then $u_r$ is not there and then the velocity variation is only along the r. So, variation along θ should be knocked off and along the z direction also to be knocked off. And axisymmetry also implies that this is gone and fully developed flow would actually I want you to drop this say term and then these are not there, so 0 because we say that the flow is because of only the boundary conditions and not because of anything else. So, we just drop this. So, which leaves us with only this term and so that term we basically write it and this is now the trivialized form of Navier-Stokes equation for a z component and when we integrate twice then we get the logarithmic term.

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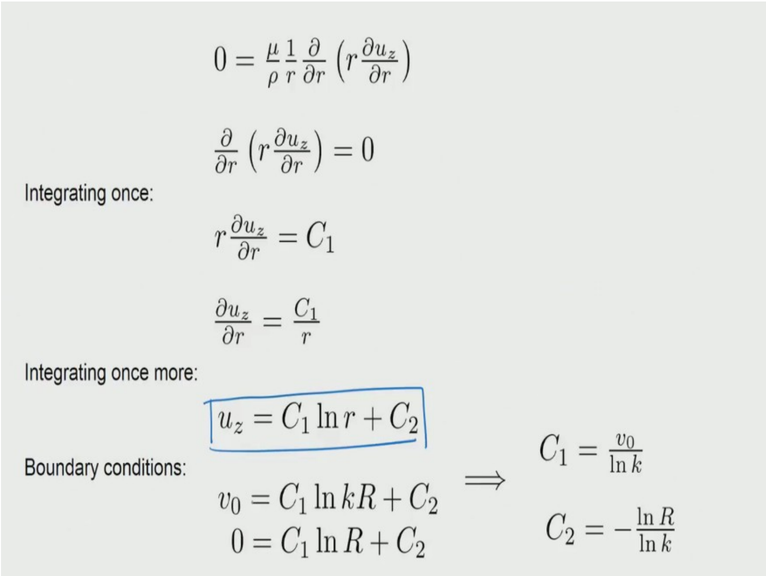


Now, in this situation we are not actually dropping that logarithmic term because there is no domain is actually not containing r goes to 0. So, we do not have a problem. So, we have this as a functional form. So, when we integrate twice. So, we get 1/r and then second term we get ln r and the boundary condition will be used to determine the values of $C_1$ and $C_2$. So, they are written here. So, you just have to substitute and then if you sub subtract the second equation in the first equation you will get the value of $C_1$ and you substitute the value of $C_1$ then you will get the value of $C_2$.

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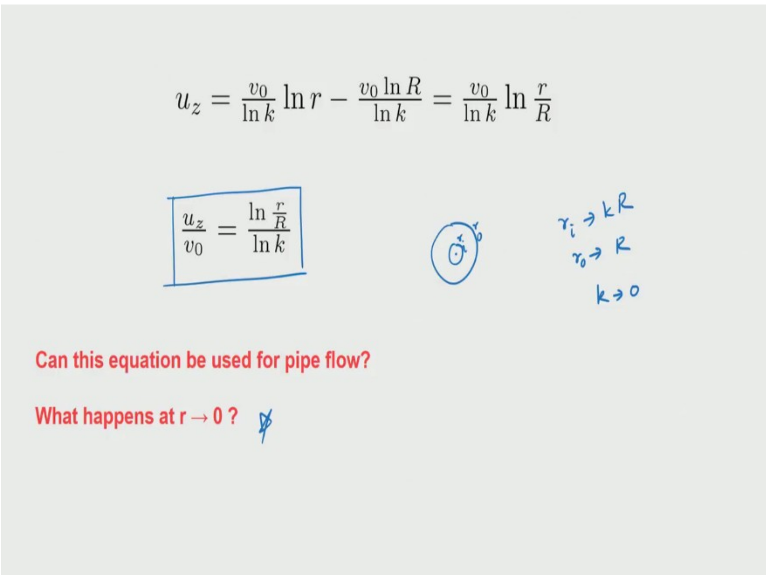

So, once you get then you can write the $u_z$ as a function of r and then simplify it and then it looks like this. Now, here can this equation be used for pipe flow that is situation where here we say that inner and outer, r inner and r outer. So, r inner actually is k × R and r outer is R. So, if k tends to 0 what does it imply? It means that you are actually making the angular region look like a cylinder pipe flow that we have come across earlier. So, can we use that equation for that say problem? So, evidently not, because we actually kept the logarithmic term with us when we did this integration and in the process actually now the solution has logarithm so when you actually want to reduce the domain then at the axis you have a problem. So, one should not blindly use this in such a situation.

So, that is what I want you to alert, saying that when we use a boundary condition while arriving at the equation then anything that actually violates that particular condition should be watched out for. So, a boundary condition in this case is that at r tends to 0 there is no problem and, but then when we extend the domain then there will be a problem if you actually apply it for a cylindrical pipe flow case. So, we should not use it. So, the solution for annular case should be used only for annular axial flow.

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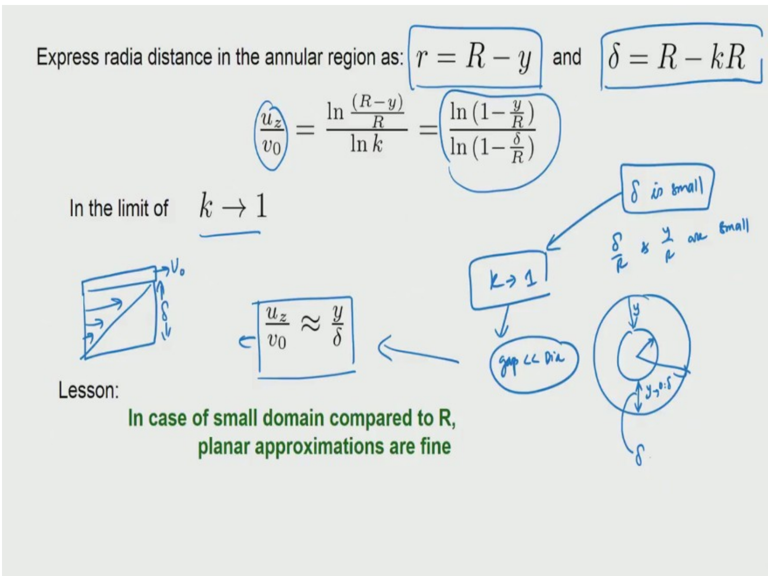


Now we can actually see under what circumstances this particular flow form can be approximated to a planar case. So, what we do is thus this annular region. So, what we try to do is that this distance alone we want to express and say that it goes, so going from here all the way from 0 to δ. So, we defined in such a way that δ is nothing, but the distance here and y is going from 0 to δ, so from either outer surface to inner surface or inner surface to outer surface. So, we define that way, which means that in the way we have defined $r = R - y$ would mean that it is going from outer to inner and that is a distance that we are taking up. So, what we do is we just substitute these two forms into the solution that we have obtained and then you see that the scaled form $u_z$ by $v_o$ is coming out to be this.

Now, here in situations where the δ is small, if δ is small you know that then δ/R as well as y/R are very very small. So, if they are very small what happens is that logarithm of 1-x can be approximated to -x and so if you do that for both numerator and denominator. Then you have a situation that is stimulating what happens when k is tending towards one of course, you should not plug in k = 1 because you have a problem that will come in the de denominator, but in the limit you can actually see that it comes to be basically a ratio y and δ. So, what it implies is that when the δ is very small. So, this is the same as saying that k tends towards one this actually is a same as the gap is very small compared to the dia. So, under these situations we can see that the velocity profile can be given by linear relationship. But we already know this solution we already know that this solution is applicable for a problem where here is what we have originally looked at. So, this is for a problem that we have already come across, which means that the axial flow problem can also be reduced to planar problem in situations where the gap is very small compared to the diameter if the cylinder.

So, some problems you do not have to actually use a cylindrical coordinate system just because a domain is actually having a cylinder in it we can actually approximate it to be planar problems, but remember that is applicable only when this kind of limits are applicable. So, you know only in extreme situations such approximations can be made.

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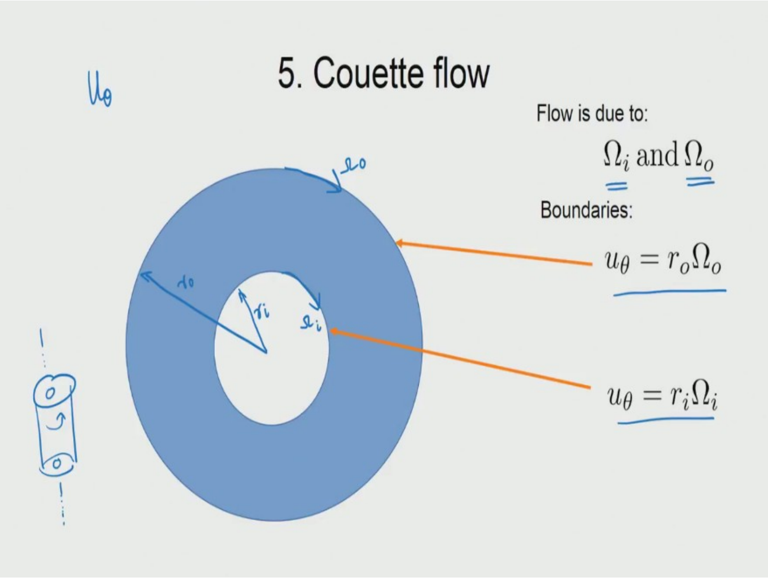


Now, we take the last component of the velocity the cylindrical coordinate system named the $u_\theta$ now $u_\theta$ of in this particular problem is actually called as the solution for the Couette flow.

So, the situation is as follows. So, you have got two cylinders that are rotating and the inner and outer velocities are different and the radii are also different $r_i$ and $r_o$. So, which means that $r_i$ - $r_o$ will be the annular region and the velocity is now not axial in the annular region, but actually along the θ direction. So, what we have drawn is basically something like this. So, the two cylinders are rotating above the centre axis and their velocities are different. So, therefore, the liquid between these two cylinders is getting sheared. So, this kind of a problem can now be looked at.

So, the flow is actually happening not because of any term we do not want to consider any body force any driving force there and we want to say this flow is happening only because of $\Omega_i$ and $\Omega_o$ that is inner and outer angular velocities. So, the boundary conditions are straightforward the no slip conditions are applicable. So, for the outer wall the linear velocity $u_\theta$ is given by $r_o$ $\Omega_o$ for the inner wall it is given by $r_i$, $\Omega_i$. So, these are the boundary conditions at inner and outer walls.

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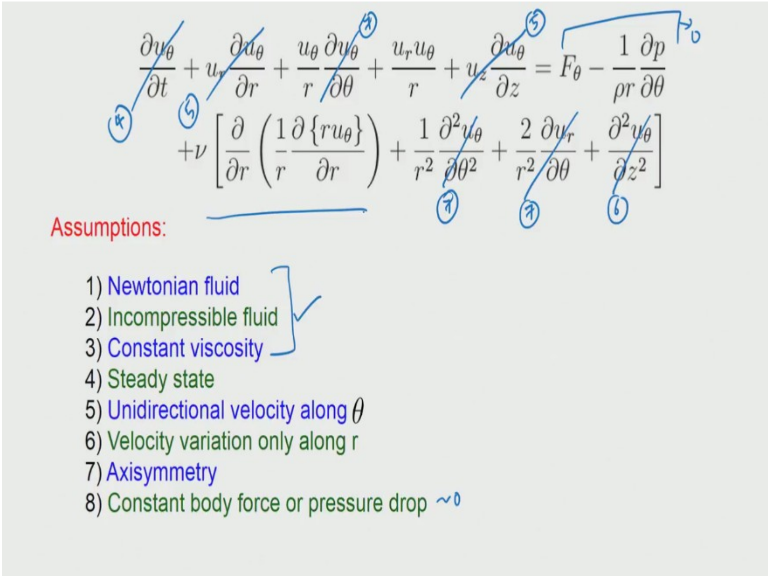


So, the equation that we need to solve is here. So, we already wrote the equation for a Newtonian fluid an incompressible fluid which also has a constant properties. So, that equation is already assuming the first 3 assumptions. So, we assume that the steady state is applicable and then unidirectional velocity along θ, which means that $u_r$ is not present and $u_z$ is not present and we want to also say that there is a velocity variation only along r. So, along the z direction it is not there and we also want to say that this is an axisymmetry. So, the θ variation is also been dropped. So, and this also because of axisymmetry and we also want to say that these terms constant and they are 0. So, because we say that the flow is not due to any of those terms it is only because of the boundary conditions, which means that we have got a very equation that is coming in here. So, that equation is written here and we then integrate. So, when we integrate twice then we get a functional form which goes as r + 1/r.

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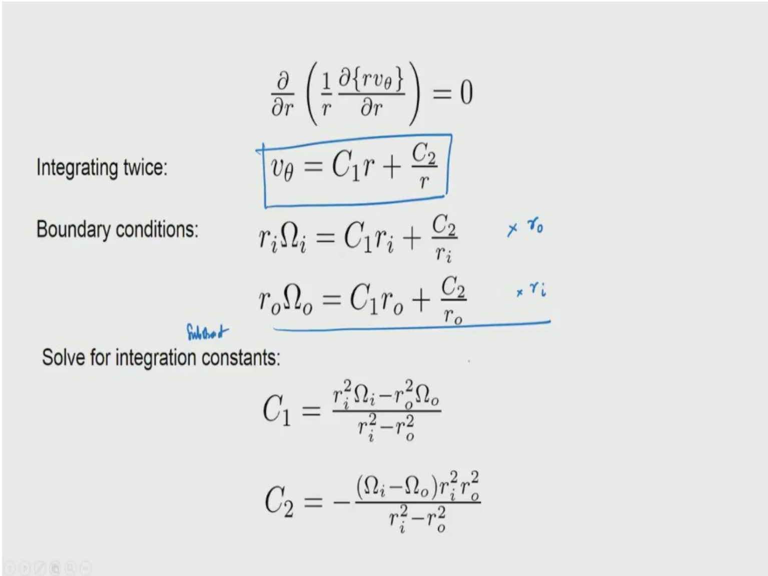


So, there are two integrate integration constant $C_1$ and $C_2$ and when we substitute the boundary conditions we can determine. So, the inner wall boundary and outer wall boundary conditions are listed here. So, what we do is at the first one if you multiply with $r_o$ the first second equation if you multiply with $r_i$ and then if you subtract then you could see that you would eliminate the terms and then start to see the solutions.

So, the solution for $C_1$ and $C_2$ are available and then when you plug in you have got the velocity with the θ direction.

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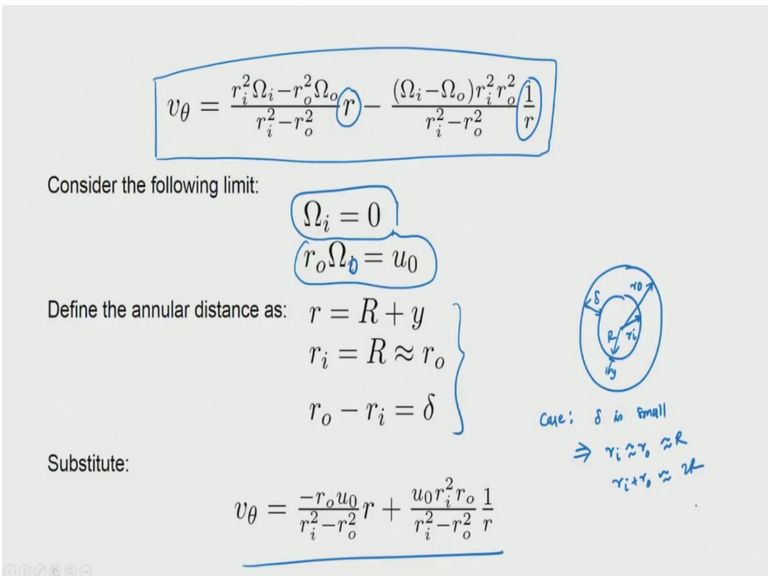

So, though the there are number of terms you can see that the functional form is essentially r and 1/r that is coming in.

Now, we want to look at under what circumstances such a solution can be simplified. So, again like we have done just now what happens if the gap between the inner and outer walls is very small compared to the diameter of the inner cylinder nor outer cylinder for that matter. So, we want to make the simplification like this. First of all we want to consider that only one of the walls is actually moving that is just to make it simpler. So, $\Omega_i$ is taken as 0, and the outer wall we want to determine the linear velocity and we want to give a symbol, so $u_o$ . So, $r_o \Omega_o$ is $u_o$ .

Now, the annular distance is being defined in the same way like we have done earlier. So, earlier it was like $r_i$ and $r_o$ . So, we want to define the distance as follows. So, we want to now say that this capital R and from here onwards it is basically y and the total distance here is δ. So, when we want to define that way then we say that for the case we want to do this for the case where δ is very small. And when δ is very small it also implies that $r_i$ $r_o$ are both very close to r and which also means that $r_i + r_o$ is very close to 2r and so on. So, we make those simplification and look at how the $v_\theta$ is varying. So, we just straight away plug in and then and get this equation and then simplify this. So, in the denominator $r_i^2 - r_o^2$ you write it as $(r_i + r_o) \times (r_i - r_o)$ . So, then they will be looking like $2r \times \delta$. So, that we write and then the equation would look like that ok.

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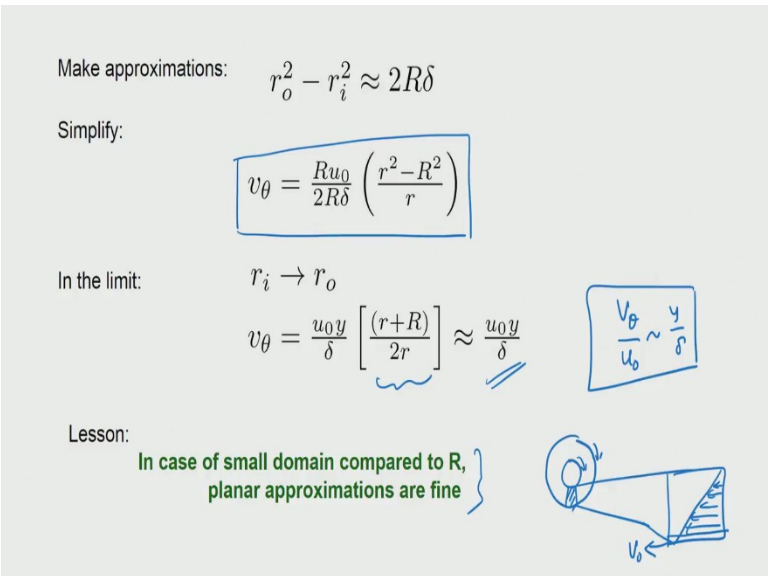

Now, in the limit that $r_i$ and $r_o$ are very similar to each other then we see that this can actually go as a unity. So, it tends to unity, which means that the $v_\theta$ /$u_o$ goes as y/δ. So, again we have come to the same equation as we have done earlier. So, which means that in a situation where actually this is what is happening then you can look at the domain and imagine that this domain is linear situation like this. So, flow would be like this and which means that a situation where you have to got a very small gap between the two cylinders compared to the diameter of either inner or the outer cylinder then the cylindrical problem can be reduced to planar problem. So, for small domains this is applicable.

So, you can already see from the solution that these simplifications are possible. So, it is not necessary that you have to solve problems in the cylindrical geometry by using the cylindrical equations under suitable situations you can actually reduce the complexity.

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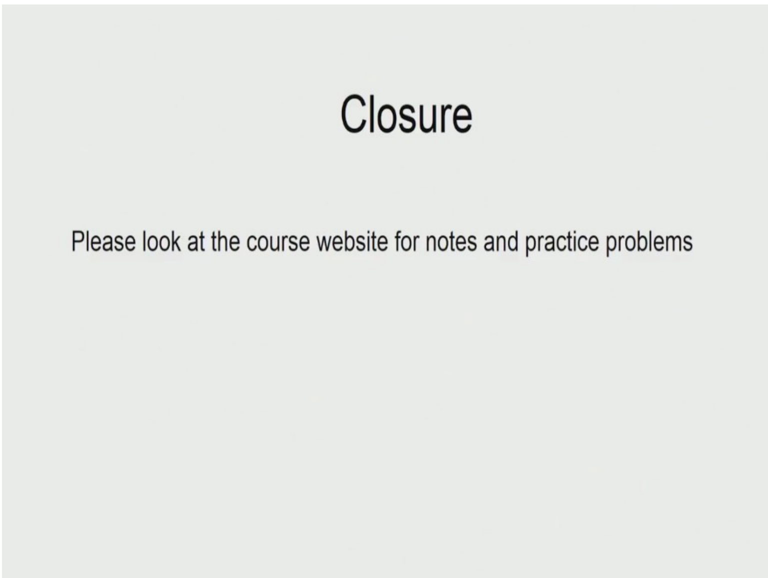


So, at this moment we just close this session. And in the course website you have got notes and practice problem where some numerical problems also will be available for you to plug in and see how these equations can be applied.