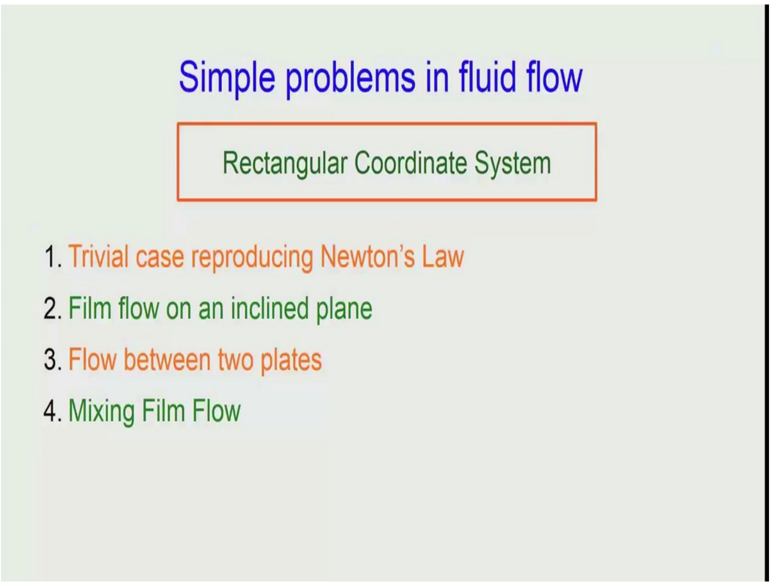
Transport Phenomena in Materials
Prof. Gandham Phanikumar
Department of Metallurgical and Materials Engineering
Indian Institute of Technology, Madras

Lecture – 12
Simple cases in fluid flow - Rectangular coordinate system

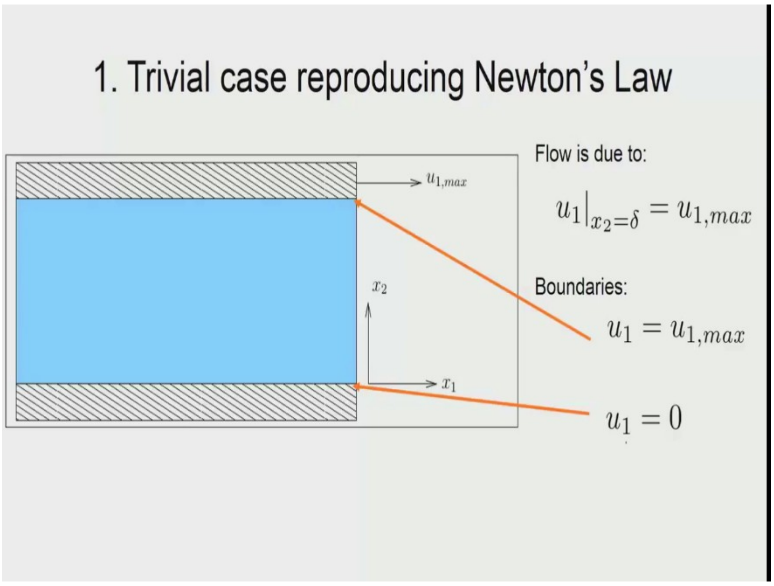
Welcome to the session on simple case in fluid flow, as part of the NPTEL, MOOC on Transport Phenomena in Materials. In this session we will be looking at simple cases where the problem can be reduced to one diffusion term on the right hand side of the Navier Stokes equation and so that the analytical solutions are very easy for us to see with just couple of steps of integration. We will be taking to the rectangular coordinate system in this session. So, the cases that we are looking at are as follows.

(Refer Slide Time: 00:44)



So, we will take a very trivial case where we will get the Newton's law back which we have put in at the stage where the constitutive equation is require and we will then take couple of problems which are very popular in the metallurgical materials curricula namely the film flow on inclined plane, flow between two plates and the mixing film flow.

(Refer Slide Time: 01:06)

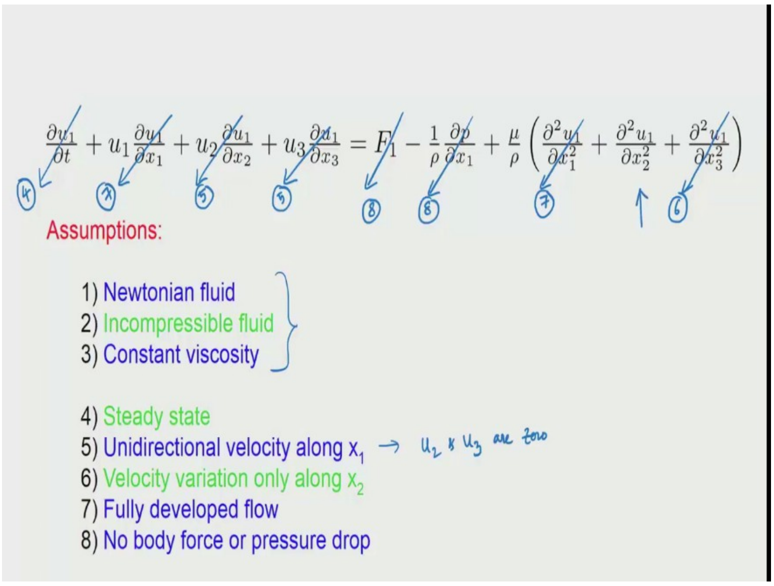


So, here is the first case. So, here is the trivial case reproducing the Newton's law. So, the situation is as follows. We have got liquid between two parallel rigid plates and the top plate is then moved at a velocity $u_{1,max}$ and because of the motion the liquid is taking to the top plate. So, we have got basically the fluid moving in the right hand side direction that is in the x_1 direction.

So, the fluid is only due to the motion of the top wall and there is no other term that is active. In other words there is no pressure drop we are talking about there is no body force we are talking about, and the boundary conditions are as follows. At the top wall we have got no slip condition with a moving wall. So, the velocity at the top surface will be the same as that of the wall and at the bottom we have got no slip condition for a stationary wall then we have got the velocity equal to 0 at the bottom.

So, in this case then what we are always do for many of these problems is as follows.

(Refer Slide Time: 02:03)

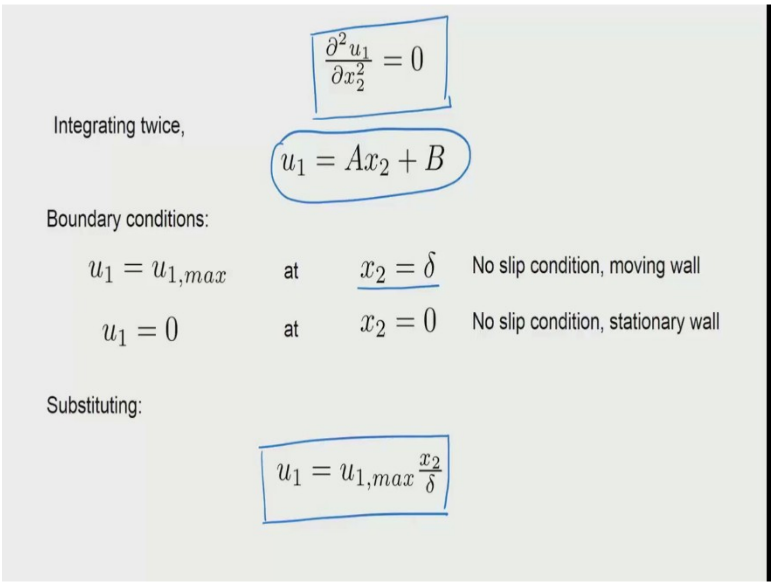


We start with the Navier Stokes equation for the velocity component that is relevant. So, we always write the full equation. So, we are conscious of all the terms and then we start striking all the terms as we do not need them later on. So, u_1 is the component that is relevant. So, that is what is written here.

So, the equation is already assuming that it is a Newtonian fluid, it is a incompressible fluid and it has constant viscosity. So, the top 3 assumptions are already made. And so we are going to make some more assumptions which will reduce this equation to extremely simple form. So, first assumption here is a steady states, which means that we can strike off this term because of the assumption number 4 and unidirectional velocity along x_1 direction which we mean by saying that u_2 and u_3 are 0, which means that we cut out this term and the this term being dropped off because of the assumption number 5. And then we also say that in this situations the velocity variation of u_1 can be along the x_2 or x_3 direction. So, we choose that it is varying only along the x_2 direction, which means that the variation of u_1 along the x_3 direction can be dropped that is because of the assumption number 6.

And then we say that the fluid flow is well developed along the x_1 direction, which means that the $\frac{\partial u_1}{\partial x_1}=0$. So, we drop this term because of this assumption 7 and also this term goes away it is the second derivative of the same term. So, this one also goes away. And then we say that there is no body force or pressure drop in this particular problem it is a very simple problem. So, we basically knock off these two terms also because of the assumption 8, which leaves us with only one term here and that is equal to 0 and therefore, the equation appears in the simple form as written here.

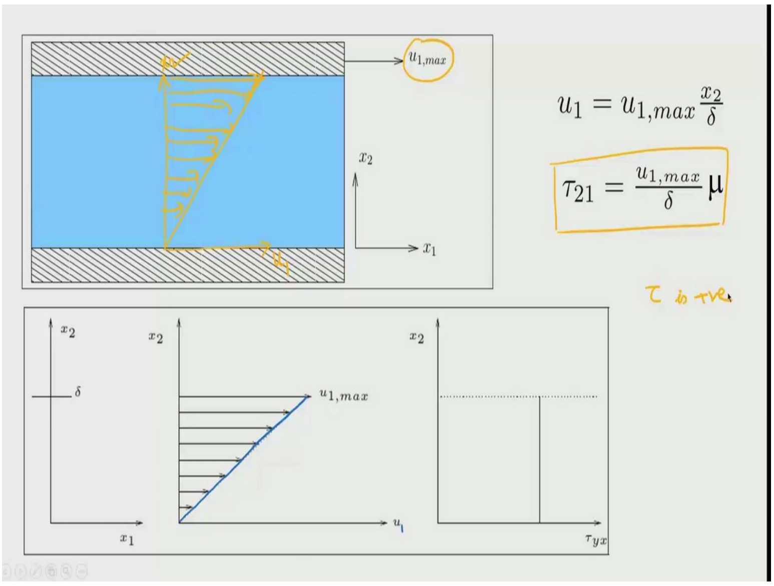
(Refer Slide Time: 04:06)



And you can immediately see that the solution should be a straight line and that is what is solution that is given here then we integrate this twice we get $Ax_2 + B$ and the integration constants are to be found from the boundary conditions which we have already been mentioned. So, at the top wall the $x_2 = \delta$ value you have got velocity equal to $u_{1,max}$, the bottom it is 0 so when you substitute you get the functional form for the velocity which is basically proportional to the distance in the y direction and scaling of the maximum velocity with the factor δ that is at the denominator.

So, this is the solution which gives you straight line profile which is a simplest profile that is possible for any solution of Navier Stokes equation.

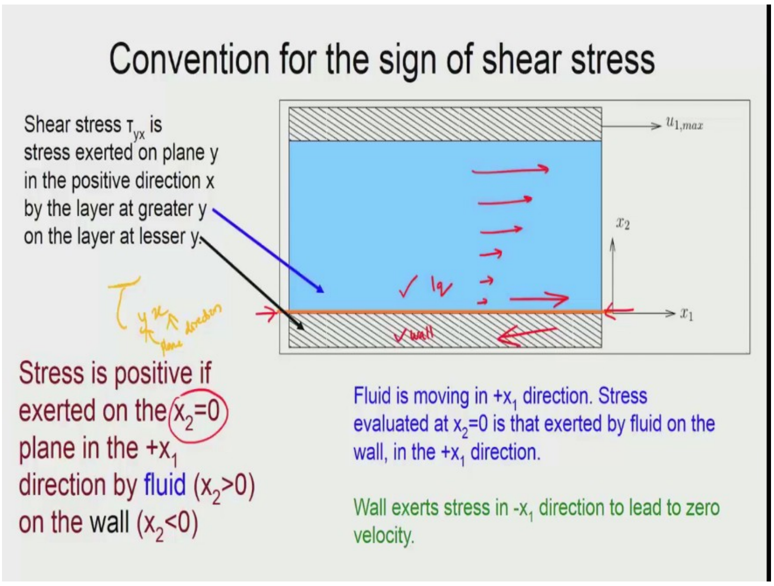
(Refer Slide Time: 04:58)



So, we see this plot, the plot actually shows that it is a straight line. So, here I am marking the profile of the velocity u_1 as a function of x_2 or y, which means you want to show this on the schematic itself then you could also do that. So, you could do that here in this manner the axis can be drawn and then this is x_2 and this is u_1 and then we show what is a maximum velocity there and so velocity would look like that which means that as you go up in the thickness of the layer, the velocity is picking up upto a maximum value given by this term.

And we can also see that the Newton's law we have retrieved because when we now see that the shear stress is calculated as per the same formula then it comes out to be a constant and we can see that here it comes as a constant because u_1 is constant and δ is in the denominator. And u_1 is positive because it is along the direction of x_1, which means that τ_{21} is positive and it is constant. So, that is why the plot τ with respect to x_2 is just a constant line vertical line that is going like this, which actually means that what is it when we say τ is positive. Of course, when we substitute any value of x or y if it is a functional form and then see that it goes to negative or positive there must be a meaning for that. So, for that we introduce the convention by which we define how the value of τ should be interpreted. The convention is different from the book bird (Refer Time: 06:44) Stewart these take through so called positive convention.

(Refer Slide Time: 06:41)



Here and it is as follows. So, these definition will be repeated every problem in this session, so that we can commit it to a memory. The shear stress τ is basically the stress exerted on a plane y, you can see the τ is written as this, so τ_{yx}. So, we just, so this and this. So, one of them is plane and another is a direction.

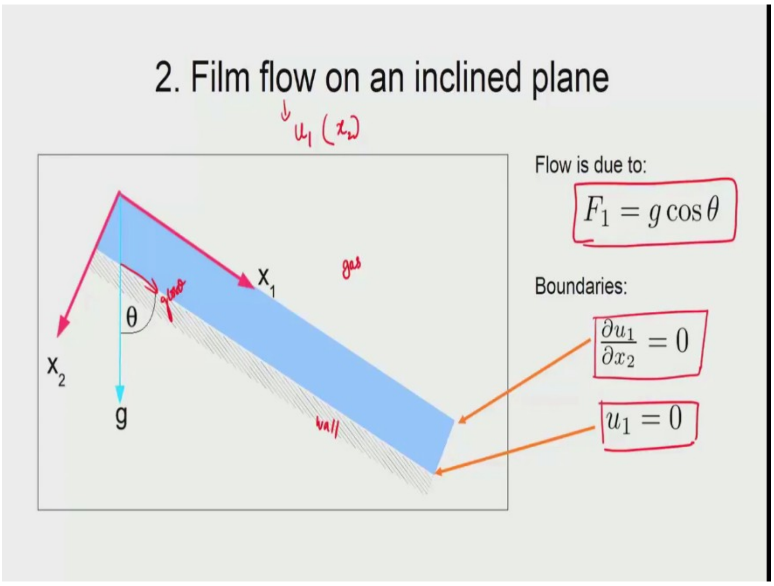
Of course, so, one of the plane and other is direction we can swap them because of the symmetry. So, the τ exerted on a plane y is in the direction positive direction x and it is exerted by the layer at a greater y on the layer at a lesser y. So, if that is the case then the shear stress is positive, in case it is the other way then it becomes negative. So, this is the convention by which the mathematical way it comes out when we substitute different values x or y as a case maybe it all come to be the same convention because we are using the same expression for the Newton's law. So, in this case when you take the plane x_2= 0, you would see that, you see here this plane this plane is here it is its shown here in this orange line.

So, if you take that plane for example, at greater y that is at a higher value of x_2. So, here this is the higher value that is liquid and lower value of y is basically the wall. So, the force or the stress that is exerted by the liquid on wall if it is in the positive x direction then this stress is positive which is actually is true the reason is that the top wall is moved along the x direction and therefore, the liquid is flowing in this manner and therefore, liquid is trying to move in the plus x direction the stationary wall is trying to stop it, the wall is exiting a stress in this manner the liquid is exiting in that manner and therefore, the tau that you get at the value x_2 =

0 would then be deemed as positive that is what we have also got from the a mathematical expression.

So, we check this kind of a convention even in other quadratic systems and very useful to remember this convention to apply in other situations also.

(Refer Slide Time: 09:13)



So, we now got to another problem which is quite popular and these also from the industrial application because when we want to transfer some liquid metal or slag from one place to another place then you may have inclined planes where the liquid would flow and usually the liquid is an exposed to the atmosphere. So, you have got here gaseous atmosphere and what would happen to the top surface would there be a pickup of oxygen or release of hydrogen etcetera these are all issues of concern for a metallurgist. So, you have got here basically a wall which is stationary and then liquid is exposed to the gas. So, if this is a problem then how do we arrive at the functional form for the velocity as a function of distance.

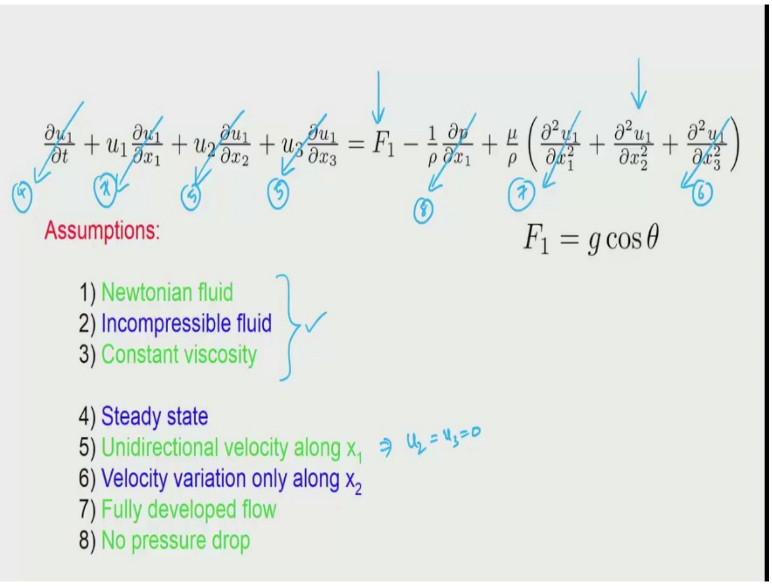
So, we choose that the coordinate system. So, that the velocity is single component. So, we see that velocity should be only along the x_1 direction. So, the flow is basically u_1 and it should be a function of x_2. So, this way we choose so that the problem is simplified. The driving force, reason why the flow is happening because of the body force term that is $g\cos\theta$ along this direction.

So, if you take the component in this direction. So, you have got gcosθ and there is a body for that is acting because of which the flow is taking place and the boundary conditions as we discuss in the previous session are as follows. On the bottom surface you have got a stationary wall. So, the liquid does not move is related to that. So, the no slip condition would come of use which means that this velocity is 0 at the bottom wall. And at the top you have got the liquid layer in contact with the gas the gas has very less density, so it cannot withstand any stress on the surface. So, the free surface you will have the stress is 0 and which means that according to the Newton's law

$$\mu \times \frac{\partial u}{\partial y} = 0$$

which means the slope of the velocity with respect to the y direction is 0 at the top surface. So, we have got the boundary conditions and we have got the reason why the fluid flow is taking place. So, we then can start the solution by writing the Navier Stokes equation with the component for the component u_1 ok.

(Refer Slide Time: 11:27)

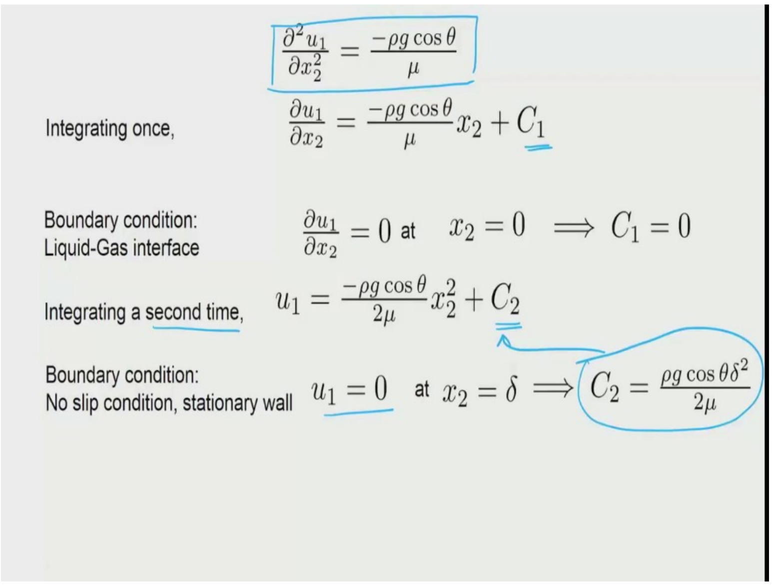


So, here we have the same situation we have got the equation written already for this 3 assumption. So, these already done because of which the equation appears in that form and we assume that this flow is taking place for a long time and at steady state. So, we knock of the first term because of the assumption 4. And we say that the velocity is only along x_1, so x_2

and x_3 directions there is no velocity which means that $u_2 = u_3 = 0$, which means that I can remove these two terms according to the assumption 5.

And then the velocity variation can also be chosen along either x_2 or x_3. So, in the depth direction of the plane we do not want to look at any variation. So, we would like to knock off this term. So, there is no variation along x_3 direction. And then we say that the velocity profile is fully developed that is as the liquid is falling down the plane it does not accelerate which actually is not true actually if the plate is quite long, but we may make the problem simpler by assuming this. So, we knock off this term from the assumption 7 and secondary rate of this term will also be vanishing. So, that should go away and we also say in addition to the assumption that we have listed till now there is no pressure drop. So, this will be going away because of this, which means that now our equation has reduced to having only two terms. So, we have got this term and this term the F_1 term is already known to us as $g\cos\theta$ because of which it is flowing. So, with this we can now write the equation that we need to solve appearing in this form.

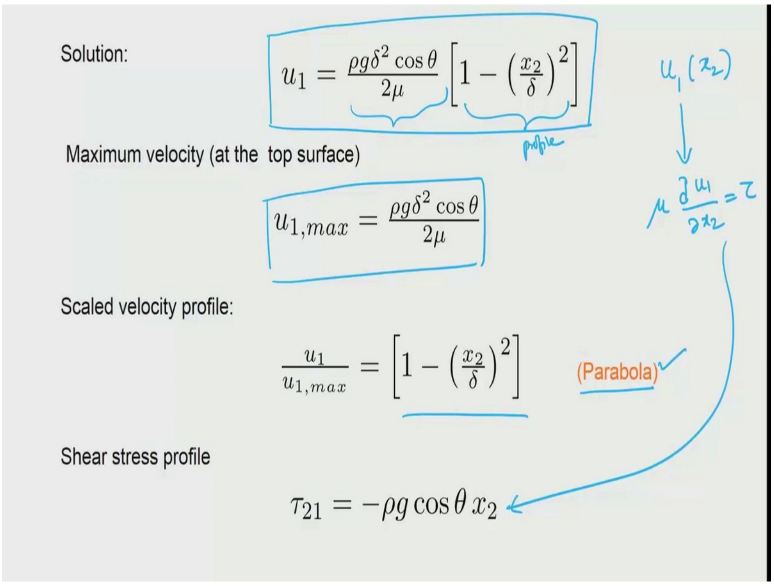
(Refer Slide Time: 13:17)



So, you can see that we have taken the μ and ρ to the other side multiply with the $g\cos\theta$ and so we have got this. So, we can see that the integration can be done once and the way we have chosen the coordinate is such that the wall is at $x = \delta$ and the gas is at $x = 0$. So, that is very convenient because when we integrate once and see the integration constant C_1 then the first boundary condition for the gas liquid interface would said that constant to be 0, which means that the form of the velocity is going to be very simple.

So, when we integrate a second time then we can see that it comes to $x_2^2/2$ and then from a integration constant C_2 we have and we need to substitute the boundary condition at the stationary wall, $u_1 = 0$ at $x_2 = \delta$ and that will give you the value of C_2. So, what we do is that this value we then substitute into this and therefore, we get the variation of the velocity of the function of x_2. So, u_1 as a function of x_2 is given here.

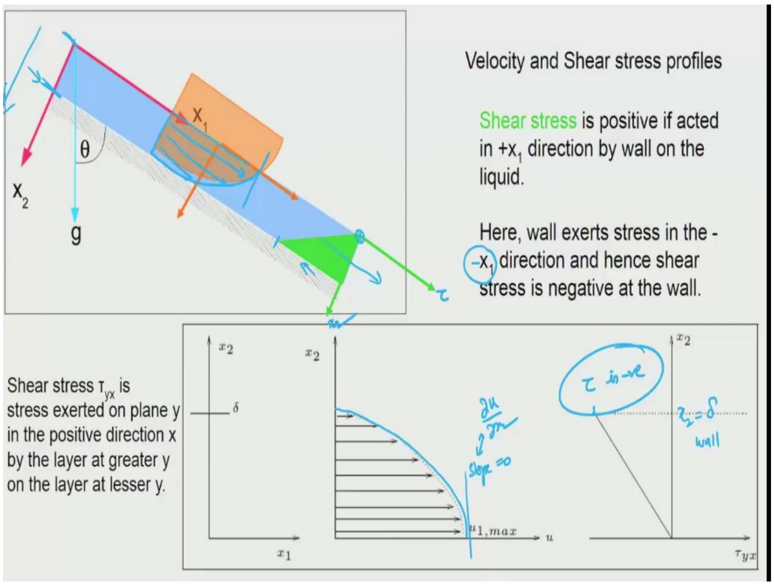
(Refer Slide Time: 14:22)



And we have intentionally take δ as a common term so that we can see the functional form with two parts of it. The first part is actually problem specific and the second part is basically the profile. And what would be the maximum velocity? Maximum velocity should be on the top surface and you could see that when you set $x_2 = 0$ you get the maximum velocity and that is given here. And once the maximum velocity is given we put the maximum velocity here and therefore, we can do the scaling which means that what is a velocity scale with the maximum velocity that is giving you the profile. So, the profile is then coming out neatly and that basically can recognize with the square term that it must be a parabola.

So, we have a parabola coming out as a solution for fluid flow when you have got this kind of a boundary condition. So, earlier problem that trivial case we looked it with a straight line, but here we have got a parabola. And once you have got the velocity distribution. So, you have basically got the velocity as a function of the distance. So, you could then from there, use the Newton's law to write for example, $\frac{\partial u_1}{\partial x_2}$ and that should give you the τ. So, that is what actually we used here and you can substitute and get the expression for the shear stress also. So, once you have these functional forms then you can plot, plot and see how they vary and also see whether they are meaningful.

(Refer Slide Time: 16:00)

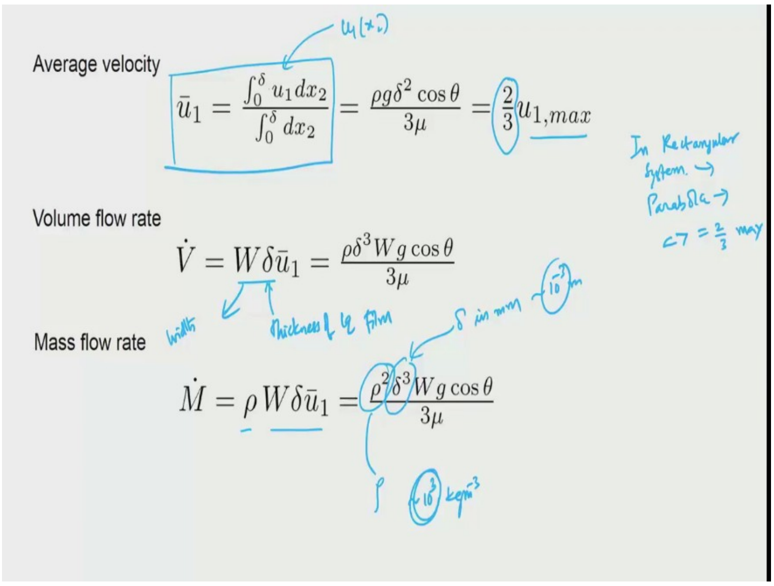


So, when we plot it looks like this the parabola has shifted is bit. So, this is how the velocity profile should look like. And here I have drawn the parabola in a flipped direction intentionally to show you the nature as a parabola and you can see only half of the parabola is actually applicable and the slope here is 0 when you look at here in this direction. So, $\frac{\partial u_1}{\partial x_2}=0$ and the slope here will have some value; that means, that a shear stress at the $x_2 = \delta$ which is basically here on this side will have some value. So, here the shear stress is 0 and here the shear stress has some value and the maximum velocity is here and the velocity at the interface between this wall and liquid you at this location will be 0.

So, you could actually see that the parabola and the straight line are evident and we now again look at the definition of shear stress and see what we meant by the shear stress being negative or positive. So, you could see that at $x_2 = \delta$ which basically as at the wall we are saying the τ is negative now what is it mean. So, it means that here we say that this is tau and this is a negative value. So, you are actually having x_2 this way and τ this way. So, it has a negative value. And why what is it mean by saying that it is negative. So, you could see that when we apply the definition we see that the wall is exerting stress in the minus x_1 direction. So, the minus has come here because the wall is trying to prevent the liquid from flowing, the liquid is trying to flow this way and wall is trying to prevent and the way the x_2 direction is in this way, which means that what is the layer at a higher value of x_2, that is a wall. So, what is the layer at the lower value of x_2? That is the liquid. So, wall on the liquid is exerting the stress in the - x direction. So, therefore, the shear stress should be negative. So, that is how it is actually given.

So, in that sense the this consistency is there with us to the definition and we could also then imagine that the stress at x = 0 should be 0 here and that is also because you have got the slope of the parabolic profile also giving you 0 there because it is a maximum value of the velocity which is also at the top surface. So, from here the then can derive some more quantities which are very useful in metallurgical problems because we want to look at what would be the average velocity what is the volumetric flow rate of the liquid slag or liquid metal over that inclined plane and what is the mass flow rate etcetera. So, very often these flow rates are available and then we are then require to find out the average velocity there time durations spent on the plane and the oxygen pick up and so on.

(Refer Slide Time: 19:12)

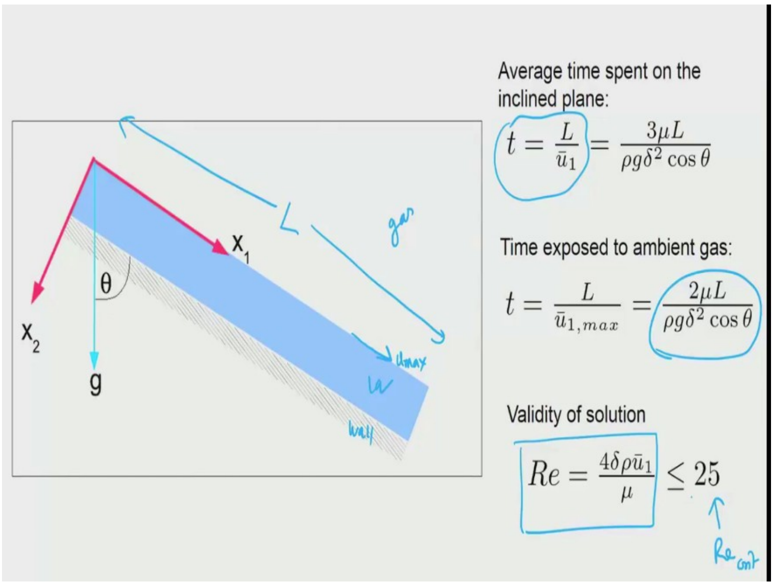


So, average velocity is always defined in this manner, you have any functional form if you have then you have the ratio of the integrations. So, the numerator will have the velocity the denominator will not have it rest of them should cover the entire domain. So, in this case the domain is 1D, so therefore, there is only one distance variable. So, in the case of cylindrical coordinate system which we will do in the next session we will see that we will have a differentiation that is done twice because of the way area is defined. So, you will have 2D access coming up here dr and dθ in this case of simple rectangular coordinate system we have only one dx that is coming here. So, you then substitute the u_1 as a function of u_1 as a function of x_2 substitute and then perform the integration and you would see that it would come out to be a ratio which is basically 2/3 of the maximum ok.

So, it is very interesting to note that for a parabola in rectangular coordinate system or rectangular coordinate system problems you will see that whenever other solution is parabola then you would see that the average is 2/3 of max. So, this actually will be evident when we do more and more problems and then it would also help us take a guess about the nature of the variations without even actually solving the problem. Now, volumetric flow rate is then available when we look at the cross sectional area through which the velocity is actually happening. So, the cross sectional area is this. So, you basically see that this is the height of the film thickness of that, film of the liquid film and this is the width that is the z directional width of the plate. So, that area you have got the velocity that is happening and therefore, volumetric flow rate is given by this expression which you can then substitute and see that the delta is coming as a cube here, here it is square it is cube here which means that when we substitute the values we must be very careful because often the thickness of the liquid layer is in millimeters which when we see here is cubed. So, if we do not make the substitution correctly then the volumetric flow rate can be quite off because of the cubic nature of the thickness of the film coming here.

The mass flow rate it is nothing, but the volume flow rate times the density. So, you can see that here density × volume flow rate and therefore, if you see the expression you will have density square coming and the thickness cube coming. So, here the thickness is generally in mm and the density in SI units with kg /m^3will generally will be in the order of 10^3 kg /m^3. So, you can see that there is a large number that is coming here in rho and there is only a small number that is come in the cube, which means that while we substitute numbers for this expressions we must pay attention to the units and do not make any mistakes because otherwise the values that we get for the $\dot{M}$ could be quite off.

(Refer Slide Time: 22:26)

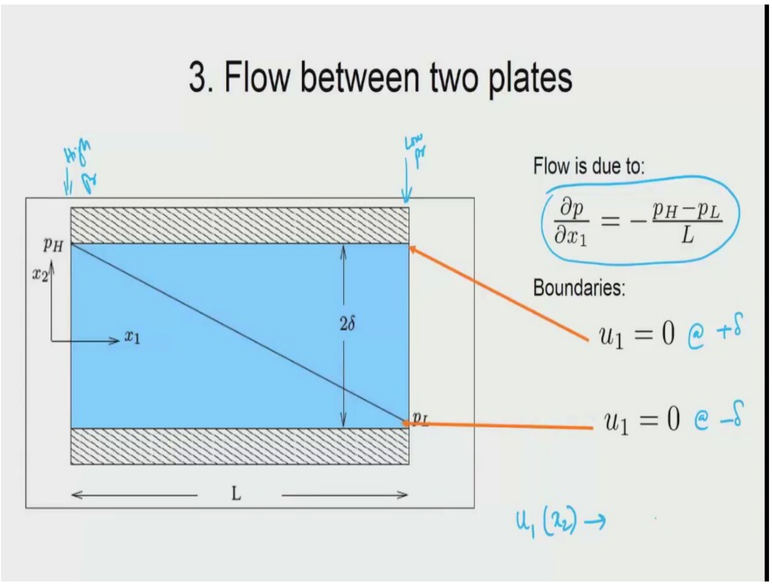


So we have also in metallurgy other expressions that are of importance for us. So, we are often required to find out what is the average time spent by the liquid on the inclined plane. So, average time spent is basically the time taken for the liquid to go through the entire plane and we take the average velocity and divide that below the length of the plane then you get the average time. So, length is available. So, the width is in the other direction the z direction. So, this expression will give you the average time spent by the liquid and if you substitute the velocity average velocity expression then you would get that.

Now, the time of exposure to the ambient gas. So, the ambient atmosphere is gas here this is a liquid and we have got the wall here. So, the time of exposure means that the liquid which is actually exposed here and that is actually moving at the maximum velocity. So, the time of exposure calculation would require that we divide the length with the maximum velocity and therefore, you have got this slightly different expression from here and that would be basically two-thirds of the average time that is spent less than the average time spend time spend.

And when we substitute all the numbers we must always ensure that the solution we obtained is valid. So, that is from the Reynolds number. So, we see that Reynolds number as defined here can then be used to see that it is less than a critical number. So, this is a critical Reynolds number and here in this problem it turns out to be 25, so smaller better always. So, if you ensure that the numbers that we got are within this particular limit 25 then; that means, that the solution you got is reasonable. So, you could actually use those numbers as meaningful solutions or other problem that we have.

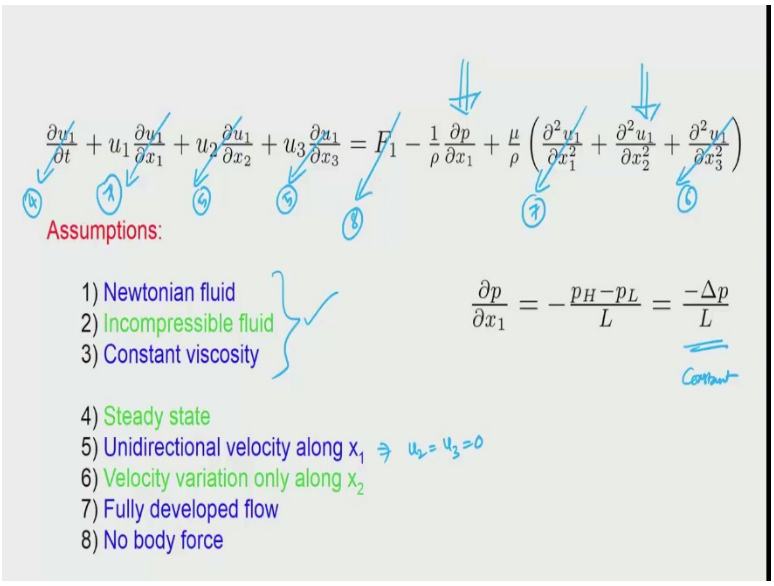
(Refer Slide Time: 24:21)



So, the other problem that we will look at is flow between two plates here again the solution is not going to be different from what we have seen till now, but we want to see that the similarity nature depending upon the term that we choose the solution can still look very much similar. So, that is what we want to illustrate from this. So, here we have got two plates and the two plates are stationary and the liquid is flowing mainly because of the pressure gradient. So, here is the pressure gradient that is given. So, PH is the high pressure that is applied at this end and PL is the low pressure that is present at the other end over the length L. So, (PH – PL)/L is a basically the gradient pressure you could see that the gradient is negative because it is decreasing with increasing x_1 and which actually also means that when you apply higher pressure on the left hand side the liquid should move in the right hand side which means that u_1 is actually in the plus 1 direction; that means, the velocities are positive. And y direction is x_2. So, you are asking this. So, this is what we want to seek.

So, boundary conditions are given as follows the velocity at the top wall and the bottom wall are both 0 because both are stationary walls and from the no slip condition you have that 0. Now, we need to choose how to place our access. So, the coordinate access are placed in the center. So, that the plus delta and minus delta will become the two walls. So, this you could say is at $+\delta$ and this will be at $-\delta$. So, there is certain symmetry in this problem we want to exploit that by simply also choosing the distances appropriately. It is not important when you plot it, but while deriving actually such simplifications make the algebra little easier.

(Refer Slide Time: 26:18)

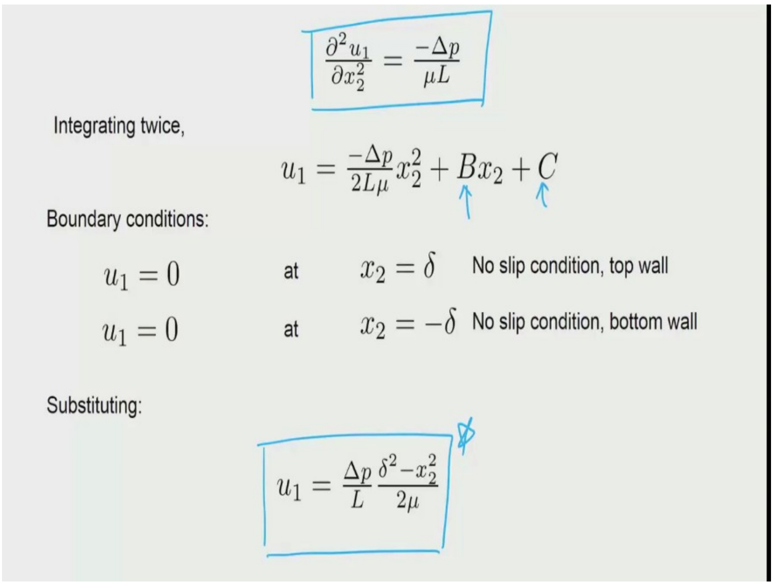


So again we go through this exercise of looking at the entire Navier Stokes equation and that is written for the 3 assumption that we have written here and making the simplification. So, we again assume that the pressure gradient is applied and the fluid flow is happening at steady state. So, we knock off this term because of the assumption 4 and we say the velocity is along x_1, which means that $u_2 = u_3 = 0$. So, then I knock off this term and this term from the assumption 5 and then the velocity is varying only along x_2, so we do not have variation along x_3 direction. So, we can knock off this term also and we say that the fluid flow is happening at fluid developed manner which means there is no gradient of the flow along the flow direction, which means that we knock off this term and this term is actually secondary rate of the same one. So, we knock that off also.

And now we have got only two terms here and we want to state that there is no body force term, which means that these are the only two terms that are coming of use. And of course, we also make an assumption that there is a pressure drop term is actually constant. So, we want to say that this is a constant because it could technically be a function of distance, but for our problem to we made simple we want to take that is a constant. So, the differential equation have only two terms and that is what we have written here.

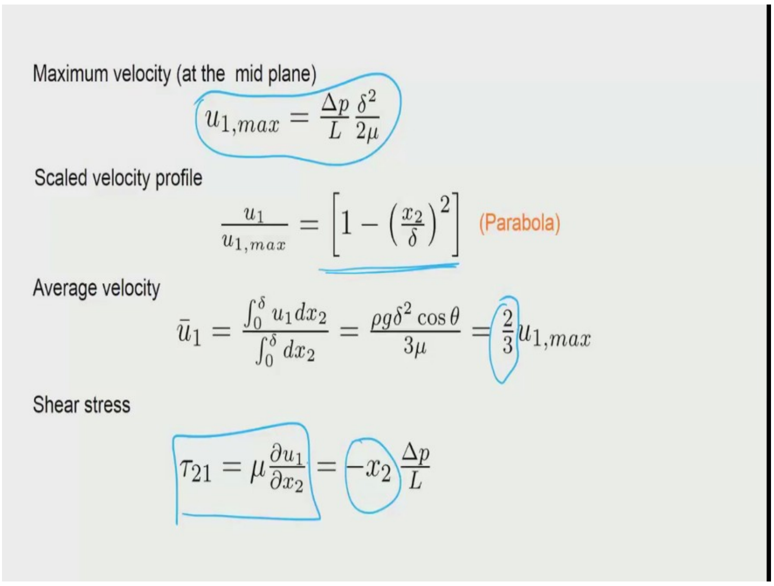
$$\frac{\partial^2 u_1}{\partial x_2^2} = \frac{-\Delta p}{\mu L}$$

(Refer Slide Time: 27:47)



So, now, for we can an integrate it twice to see that it would have $x_2^2/2$ coming here x_2 and then constant should be there. So, these are the integration constants. So, these constants are to be evaluated by using the boundary conditions. So, there are two boundary conditions that are available and we substitute them and we can immediately see that the solution will come quite symmetric such that the velocity immediately can be seen as 0 at $x_2 = + \delta$ as well as $- \delta$ and this is a functional form that we have sort. Now, we could see that again it is a parabola and when we scale it also turns out to be similar to what we have seen earlier.

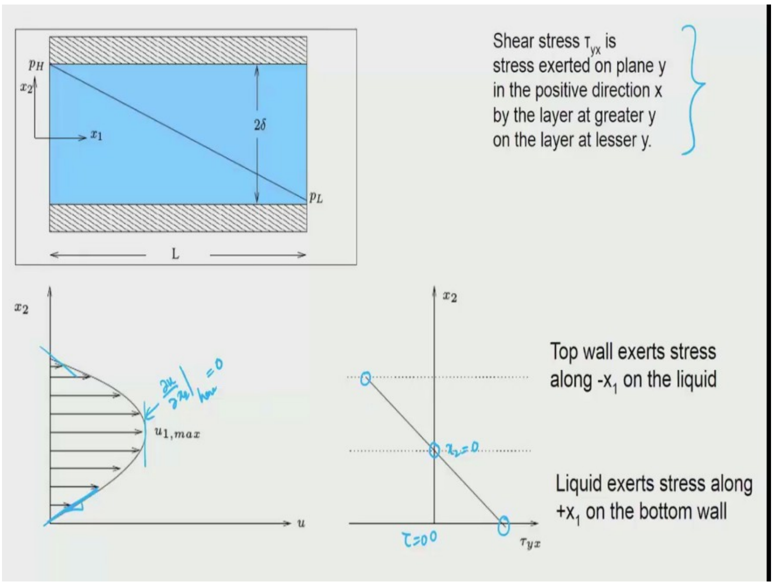
(Refer Slide Time: 28:33)



Now, the maximum velocity is at $x_2 = 0$ which is at the central plane, which will give you the expression here the pressure gradient × $\delta^2/2\mu$. Now, when we divide the solution with the maximum velocity we get the scaled velocity profile and we see the scaled velocity profile is actually giving you just the profile alone on the right hand side which is a parabola. So, interestingly the solution on the right hand side is identical to the previous solution, but the reason why the flow is happening is different if the earlier situation it was a body force term in this situation its actually the pressure gradient term. So, they both have the same way of affecting the solution and we also see the earlier problem had half a parabola, here you have a complete parabola its only matter of the domain definition from 0 to δ versus - δ to + δ otherwise the solution is identical, which means that sum of the solutions when we know we could actually transfer them to new problems by looking at the similarities.

The average velocity is calculated in the same way and you would also see that the same relationship is coming here also that is the average velocity to the maximum velocity has a ratio of 2 : 3. And we could also then apply the Newton's law to get the stress profile and you could see the stress profile comes as a linear function with a minus sign.

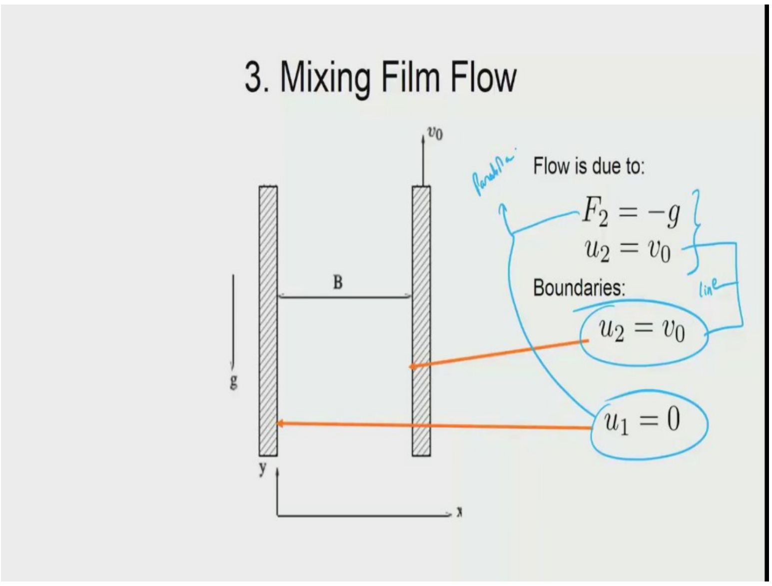
(Refer Slide Time: 29:54)



So, similarly we can also see that it comes in this manner and the stress profile will cross over the 0, τ =0 at x_2 = 0 and you could see that it is at the center plane that the velocity has a maximum. So, here you would have the slope 0 if you want to look $\frac{\partial u}{\partial x_2}=0$ at here is 0. So, you would then see this stress is too 0.

Now, you could also see that at the bottom layer then you have got a positive slope and you have got positive stress here, here I have a negative slope you have got a negative stress value there. So, you could actually see that you could predict the profile of stress profile from the velocity profile, you could also plug in the mathematical expression and arrive at the same thing. And you could also apply the definition here and convince yourself that the reason why the stress is given in this particular manner is actually valid.

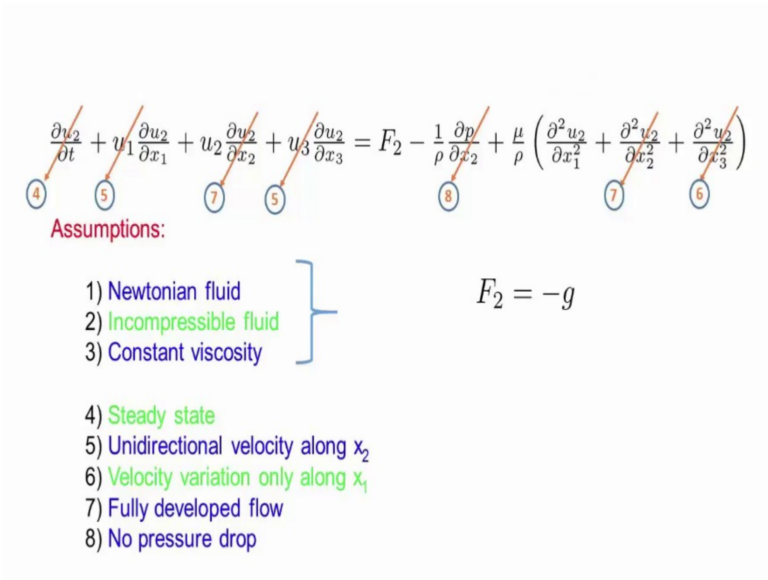
(Refer Slide Time: 31:07)



Now, the last problem we actually not going to solve we are only going to just combine the solutions from what we have done till now. Here is a situation where there are two walls these walls are fairly long in the bidirectional. So, the liquid that is in between the walls is getting mixed, the reason why it is getting mixed is because one wall is stationary the other wall is actually moving vertically in the direction of + y at a velocity v_0. So, this would mean that the flow is due to two reasons. So, earlier problems had the flow because of only one reason, but here it is because of two reasons, one reason is that is the body force that is acting which is in the - y direction is - g and the other reason is because of the wall being moved at a velocity v_0. So, these are the reasons why the fluid flow is happening the boundary conditions are as follows on the left wall you have got u_1=0 because of the no slip condition on the right wall you have got $u_2 = v_0$ again due to no slip condition.

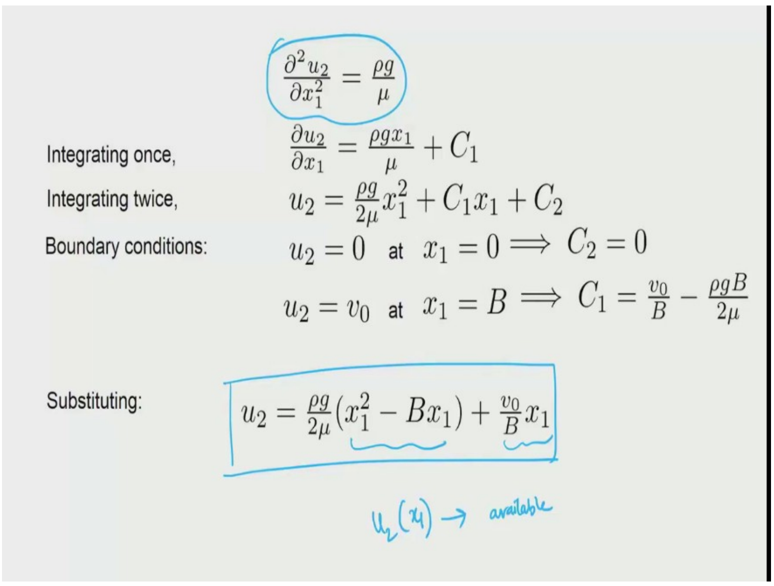
So, this problem we could actually see that if you look at the one combination that is if you look at this and this we saw this must be a straight line solution and when we saw this and this together we saw that it should be a parabola, which means that we can predict already what the solution should look like.

(Refer Slide Time: 32:29)



And we would actually do the same exercise and we see that the equation is written for them and we knock off the terms in the same way. So, that is because of the velocity being in only x direction whereas, variation along x_2 which means that I remove this fully developed flow then I remove this term, this term and then there is no pressure drops. So, I remove this term. So, you have got only this term and this term and this is already available there is - g there. So, that will it substituted. So, you can actually then integrate it twice.

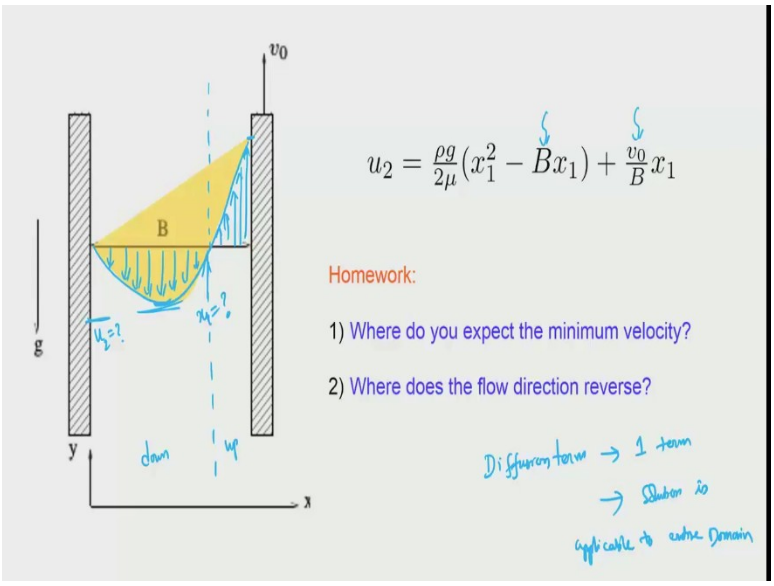
(Refer Slide Time: 33:11)



So, this is the solution and when you integrate a twice you get two constants of integration of C_1 and C_2 and these can be ablated using the boundary conditions $x_1 = 0$, you have got $u_2 = 0$ at x_1 is equal to B the right hand side wall which is moving at velocity v_0 in the + y direction. So, that would be here. So, then you could actually arrive at the integration constant. So, this will give you the velocity profile and, so u_2 as a function of x_1 is available. So, this as you can see it has the parabolic term that is coming here and there is linear term that is coming here.

So, you can already see that from our analysis just slide behind we can see that it must be a mixture of a parabola and a straight line and that is how it looks like. So, when you plot it would the profile would look like that.

(Refer Slide Time: 34:00)



So, that the velocity in the left part of the domain would look going down and the on the right hand part you see that it should go up and reaching the velocity exactly v_0 on the wall. So, you could see that fall down no. So, this is a plane over which the velocity profile is actually shifting going down on the left hand side and up on the right hand side which means that very far below if you had a closed wall. So, then the liquid has to mix it has to take a turn.

So, you could see that this kind of a arrangement would actually make the liquid mix up which is actually the principle that is used in apple juice mixtures or orange juice mixtures that are available in the vending shops in daily life that you notice. So, here we will not solve this on the slide now, but I want you to try this out in your leisure where is the maximum velocity you could evaluate it. So, you could actually check where is the slop going to 0 and find out the maximum velocity downwards and then where is the flow direction reverse. So, you could also check at what value of x_1 does that happen and. So, you could actually ask those things and then you could see that you have got variability that is across v_0 and B. So, by tuning v_0 and B you could then change these two conditions at your know wish.

So, we could actually go further by taking more and more terms, but at this moment we have already you know done enough to just check how the values variation of velocity profile would come like. So, basically we are looking at solutions that are coming from only the diffusion term and only one at a time and then getting the solutions in a simple form. And you can then see that the solution is applicable to and the entire domain and this is something that we are going to relook whether this is usable or not later on. But at this moment the diffusion profile is actually applicable to the entire domain and that requires the laminar assumption to be valid and that is the reason why we want to look at the critical Reynolds number so that the solution is valid.

And we are also going to then look at the reason why this is called the diffusion profile of velocity field the reason is actually what is being diffused is the momentum. So, the momentum that is actually being dispersed within the medium because of the wall conditions are the reasons why the viscosity is playing a role. So, these are all basically solutions for viscous liquids only which luckily ferrous in metallurgical domain is applicable because liquid metals and slags are generally very viscous.

So, with that we close this session at this moment and then we will have some numerical problems that will be made available to you as part of the course which you can plug in and then check whether you are getting those numbers right, some sample problems also will be available for you to practice.