Transport Phenomena in Materials
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Lecture - 11
Flow Problem Statements

Welcome to the session on flow problems statements as part of the NPTEL MOOC on transport phenomena in materials.

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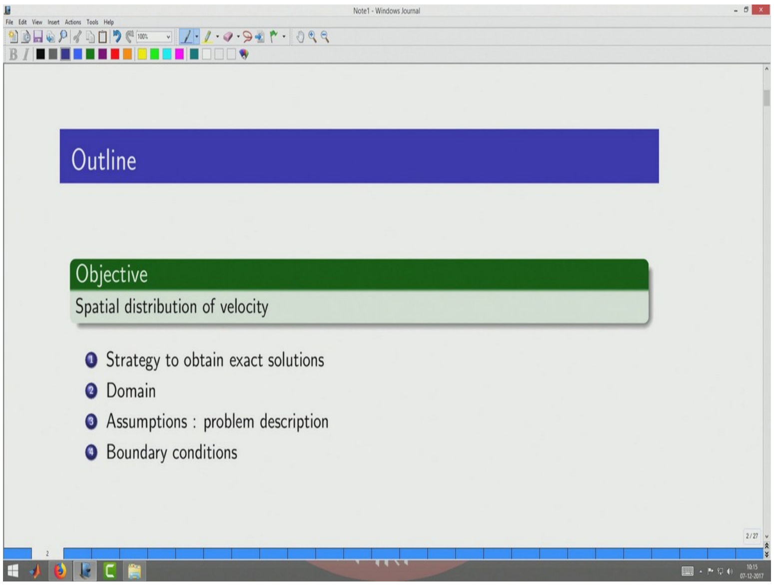

The objective of this session is to define the problem in a way that will help us arrive at the spatial distribution of velocity very often when the maximum velocity or average velocity are required usually we can use some empirical correlations or some energy balance equations to arrive at it; however, when we need spatial distribution of velocity we normally will need to solve Navier Stokes equation to arrive at the solution. So, what we will cover in this session is a strategy to arrive at the exact solutions a discussion about the domain how we choose and make it simpler.

So, that we can solve the problem easily and then what are the various assumptions that we can take so that, the problem will become a well defined problem and then some discussion on the boundary conditions. So, that we can actually use it integrate the PDE is partial differential equations that will be coming in a simple form in the end, so that we can come with the solution.

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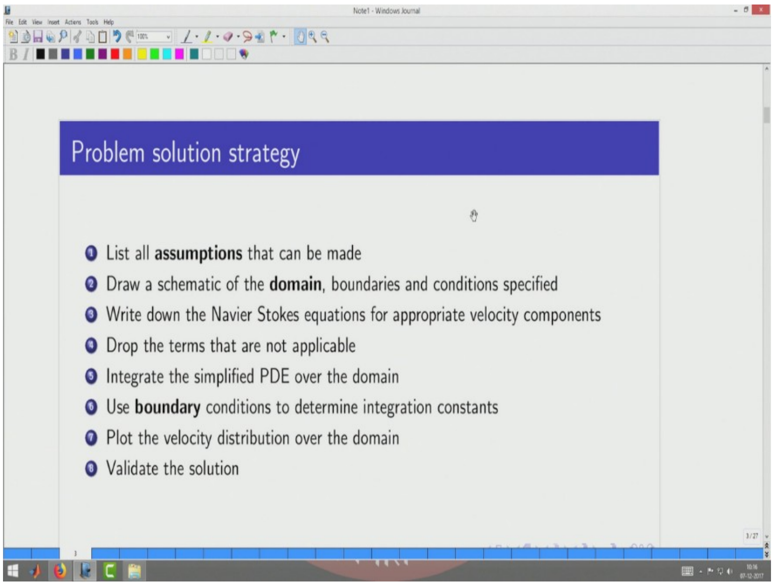

So, here we have the problem solutions strategy. So, these are steps that we would be following for most of the problems which will be at the undergraduate level in transport phenomena courses in metallurgical and materials engineering curricula. So, first thing we need to do is list all the assumptions that can be made in that particular problem. So, we have to make the assumptions to make the problem as simple as possible and then we need to draw the schematic of the domain and then identify the boundaries and what are the conditions that are specified at these boundaries half of the problem is actually solved by the time we come up to this stage and then we write the Navier Stokes equation for the appropriate velocity component and the appropriate coordinate system.

And then we look at all the terms and drop the terms that are not required because of the assumption that we made and then the equation will reduce to just a couple of terms which we can then integrate over the entire domain to arrive at the solution and while writing the solution we will always usually have integration constants, which we can determine using the boundary conditions.

And once we have this solution in the analytical form we can plot the velocity distribution over the entire domain and examine whether the distribution looks reasonable and appropriate for the problem that we started off and then in the end we validate the solution because the entire process is subject to assumptions that the velocity distribution is given by solution of Navier Stokes equation and this is usually when the laminar assumption is valid. So, we need to check the Reynolds number being within the range that is applicable for that particular problem.

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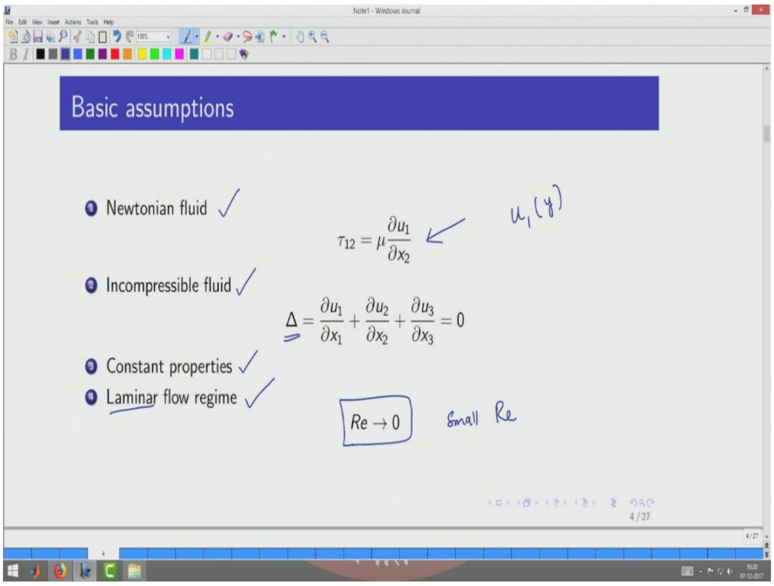

So, some assumptions that are usually made are listed. So, these are the assumptions that we no longer discuss for every problem because these are there for almost all the problems in the metallurgical material scenario. So, most of the liquid metals and slags are Newtonian fluids under normal circumstances in industrial applications. So, we normally assume that Newtonian fluid assumption is valid.

So, which actually means that this is the assumption that you are making which means that the Navier Stokes equation, we are writing is taking the Newtonian fluid assumption which we already done that and it also means that once we have the velocity distribution as a function of the spatial coordinates for example, in this case let us say x_2 is y. So, once you have the velocity distribution then the slope of it multiplied by the viscosity will give you the shear stress.

So, we can also use this assumption to arrive at the shear stress distributions within the domain the second assumption that will be made which is again very common in metallurgical scenario is that the liquid metals or slags or incompressible fluids. So, which again is very common and what this implies is that the rate of dilation given by this symbol δ is 0 which means that the divergence of the velocity is 0 this actually helps us in making some simplifications on the type of velocities forms that we can take and we will come to it in a moment.

We will also make the assumption that the properties are constant which actually means that the diffusion term on the right hand side of the Navier Stokes equation can be written in simpler manner and usually this is reasonable and where specifically we want the viscosity to be a function of location or any other parameter then we will have to watch out, but otherwise by default we are assuming that the properties are all constant they are also making one assumption implicitly whenever we try to solve the velocity distribution using Navier Stokes equation and then that is that the flow regime is laminar and normally the laminar regime is when the Reynolds number is very small.

So, small Reynolds number and it also acts as if one layer of a liquid is smoothly flowing over the other layer. So, that the wall effects the effects due to the boundaries are penetrating in the entire domain. So, this actually means that the velocity distribution we arrive at is applicable for the whole domain and that is exactly the purpose of coming through this particular exercise.

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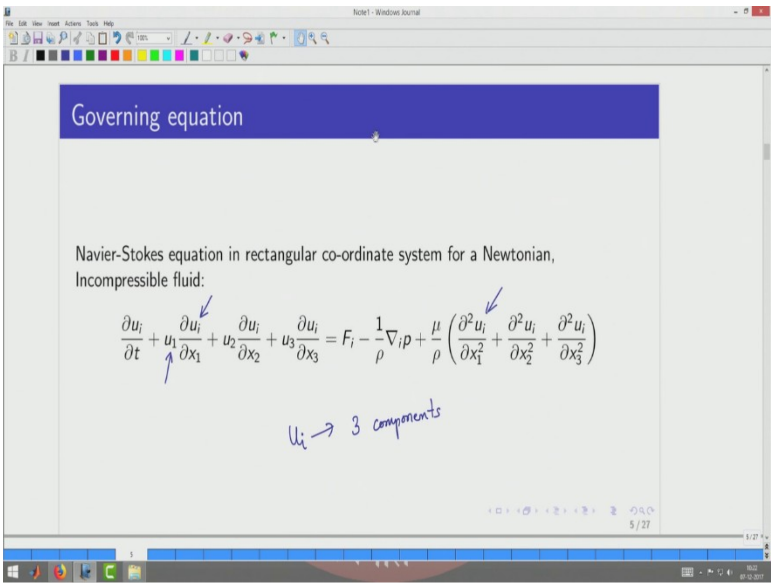

So, here we start of by looking at what is the equation that we are solving. So, most often when we simplify the Navier Stokes equation and look at the terms and then use them for the solution, we often lose track of what is an equation that we started off. So, as a practice in this particular course we will always start with the Navier Stokes equation. So, that we are a very conscious of what equation we are solving whenever we want flow distribution as a function of the distance in the domain. So, here is the equation that we are solving and if you notice the equation is written for u_i which means that it is for the 3 components of the velocities you could actually write this for other coordinate systems we have already seen those equations in the previous sessions and which means that we now have basically 3 equations.

And you can see that the velocity is appearing both in the absolute manner as well as in the differential manner on left hand side as well as on the right hand side. So, it is a very strongly coupled equation and there are many many terms. For example, you have the body force term you have the pressure gradient term and you have got the Laplacian term and the advection term and the transient term. So, you can actually imagine that the analytical solution for such an equation in 3 dimensions will be formidable. In fact, it is impossible for many of the situations. So, what we are going to do is reduce the situation much simpler by assuming a list of conditions that we will discuss as you go along ok.

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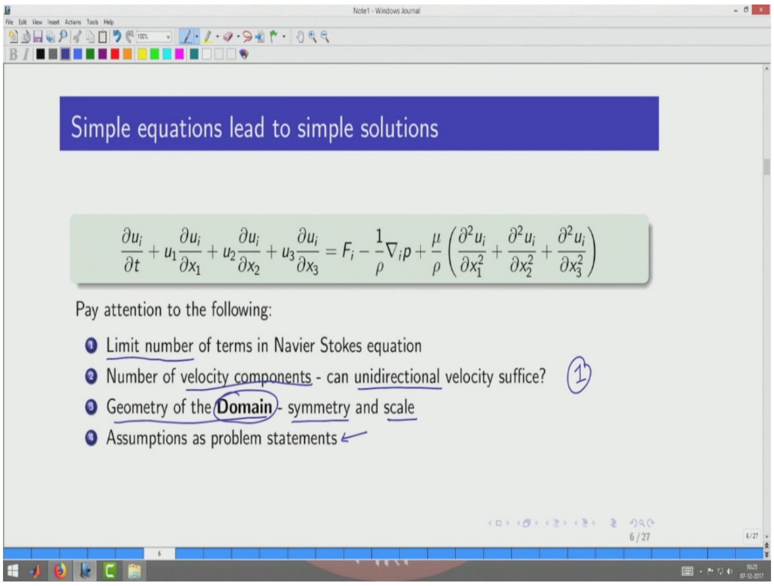

So, the equations are going to be made simpler as follows the first thing is that instead of solving 3 equations for 3 components we will see whether we can actually limit the number of components. So, by saying that we just see whether just one component is enough for the problem that we are looking at and if that is the case then we can arrive at unidirectional velocity profile. So, very often this is actually sufficient and in situations where the cylindrical or spherical coordinate system is taking up then we can actually also change the coordinate system and still retain the unidirectional nature of the velocity profile and instead of having. So, many terms that are playing a role we can actually inspect if some of these terms can be dropped.

So, we also going to consciously see how we can limit the number of terms in the Navier Stokes equation and then the domain. So, often problems that are actually correctly solved in let us say cylindrical coordinate system can be simplified to rectangular coordinate system by inspecting the domain the size and the curvature effects etcetera. So, we actually going to spend some time on looking at the geometry of the domain and seeing how we can actually make it as simple as possible and symmetry principles sometimes help us. For example, asymmetry will help us reduce the number of variables in cylindrical coordinate system the θ can be just dropped out of our problem because the solution appears to be axisymmetric in nature.

And by scale we mean whether some of the features of the domain can be ignored or not. So, this are the discussion that we will go through as we solve the problems. So, that the objective is to make the equation as simple as possible and we know that simple equations need to simple solutions which is the objective of arriving at the analytical solutions of Navier Stokes equation and of course, we are going to make a lot of assumptions which will actually form the problem statement. So, in defined problem which is the realistic scenario will be rendered as a well-defined problem by coming up with a list of assumptions and which means that this is going to be very important.

So, what we are going to now take up is this the domain. So, we will make little bit of discussion about the domain.

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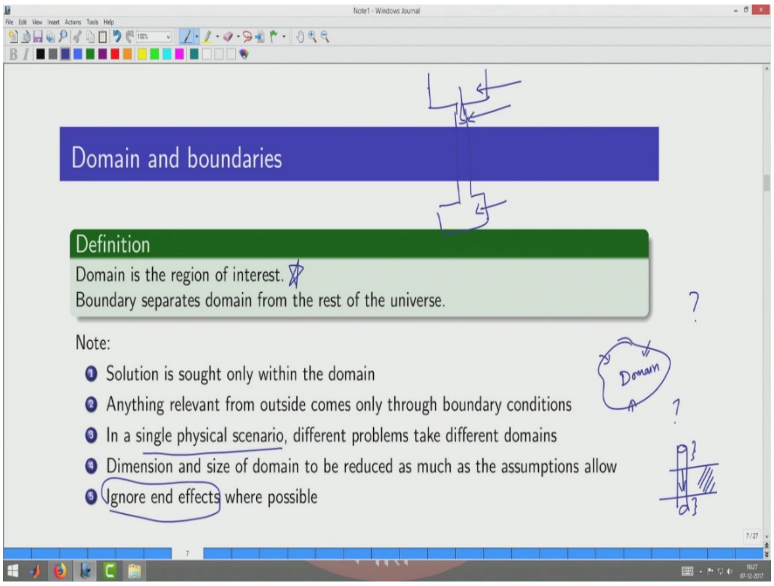

How do we choose the domain for a given problem? So, what we mean by domain is as follows domain is basically the region of interest. So, it just means that it is the region in space where the velocity profile is being sought we are not interested in the velocity profile outside of the domain and that is how we limit out problem to the domain and whatever is actually going to play a role on the modification of the velocity from elsewhere from the rest of the universe is only going to happen through the boundary of the domain. So, we draw the domain and once we say that this is my domain and which means that whatever happens elsewhere I do not bother it is effect through the boundary is only that is going to be important for me.

So, identifying the right domain in a given problem is also half the problem solved. So, we must actually take this very seriously and see how simple we can make the domain as possible. So, sometimes it may happen that in a single physical scenario you have different problems that will be addressing different domains for example, I will just scribble something here you could actually see that let us say a container you have the liquid that is coming out and then you are trying to fill it in to a casting system.

Now, what happens is that you could actually look at the flow in the melting furnace you could actually flow through the cylindrical tap hole you could also look at the flow into the caster. So, you could actually have the same problem, but look at different domains at different situations and that actually changes the problem also. So, often we have to ask which part of this problem is of interest for me and that would be the domain that we will take and then rest of the details will be all merged in the boundary conditions and often we also say that end effects should be ignored.

So, what we mean by that is whenever let us say for example, you are looking at a situation where a flow due through a tube is actually being looked at. So, in the initial and final portions of the tube what will actually happen in making the flow stabilized is going to be ignored and most of our analysis is valid for a length where the start and the end effects are actually negligible. So, this kind of a assumption will actually help us make the steady state condition also possible and we are going to make this in almost all the problems in this particular course.

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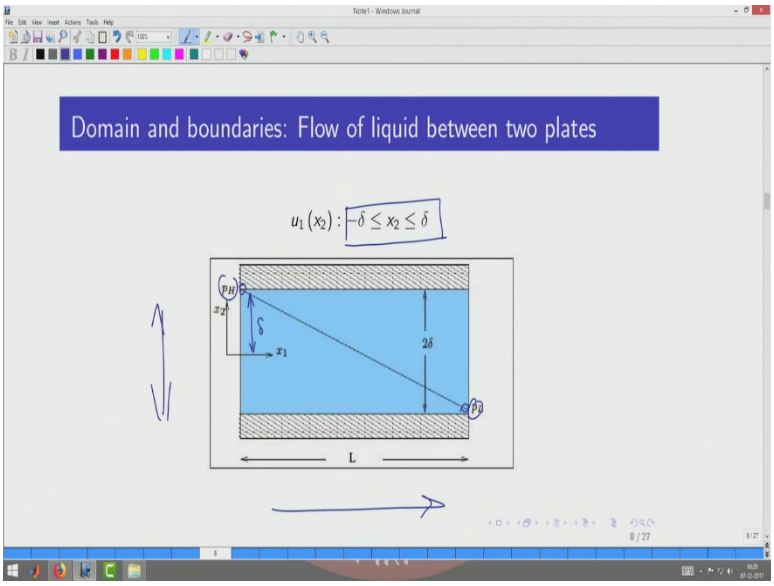

So, just for practice; I am now showing you some domains and looking at how we actually define. So, here is a situation of a flow of liquid between 2 plates. So, this could be a situation where the slag is being tapped and it is being flown into another container from a furnace and it is then covered with another plate on top. So, realistically how closely it represents a given problem is not our discussion at this moment, but let us just look at what the domain is to be looked at as here. For example, you have got the 2 plates and the domain is basically the liquid that is in between. So, if you notice the coordinates systems are placed in a way that is extracting the symmetry out.

So, you have the spacing between the 2 plates is 2 δ. So, I place the coordinate system midway. So, that the distance here is δ which means that the domain is going from - δ to +δ. So, this way the symmetry is captured in a way that the solution also will look symmetric and because there is nothing that is happening in this direction you have only the flow in this direction because pressure here is high and here is low. So, which means that the flow is going to be from left to right, which means that there is nothing happening in this vertical direction, there must be a symmetry that is happening.

So, we normally define the domain in terms of the distance variable across the domain in this manner. So, the variable is x_2 which is y and is going from - δ to + δ. So, that is our domain and everything else is of no relevance for us at this momentum.

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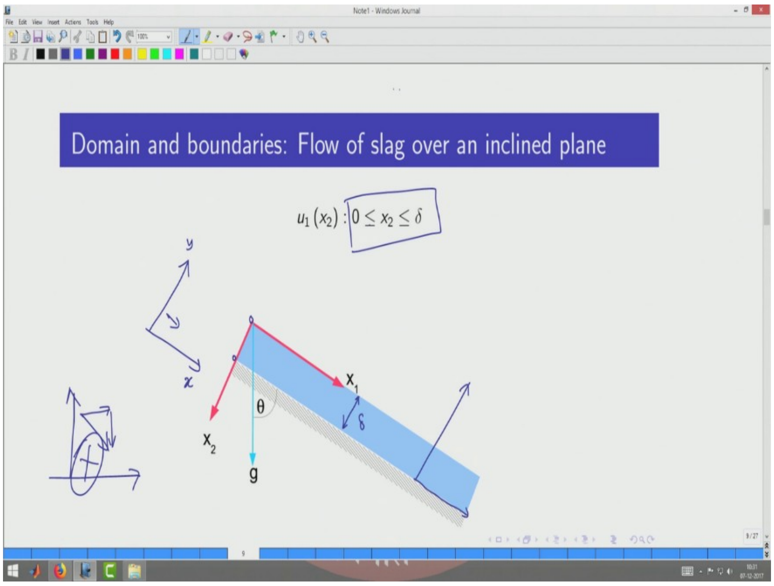

So, take another problem which for example, is flow of slag over inclined plane. So, let us look at another situation where we are seeing slag flowing over inclined plane. So, here if you see the plane it certain angle to the vertical axes. So, if I were to for example, choose the coordinate system in this manner then we have a problem because the velocity is in this direction which means that there are horizontal components as well as vertical components which is unnecessary because the flow is only down the inclined plane.

So, if you then rotate the coordinate system in such a way that one of the axes is parallel to the inclined plane then we can see that the flow is actually only unidirectional. So, in this case for example, x and y, if I write, then the flow is only along the x direction and then we can also choose the origin of the axes to be at the surface of the plane or at the free surface of the liquid slag; so that is a choice that we can exercise it we are free to choose either way.

So, you could actually make the axes either this way or the way that is shown in the red color. So, between these 2; the one in the red color actually gives the solution in a more symmetric form. So, that is what we write, but we are free to choose this way this way is wrong. So, we should not do in this manner because it unnecessarily make the velocity come up with more components then necessary now the domain is defined here again you can see that the domain is defined in a very simple way from 0 to δ. So, you can then see that this distance is δ. So, the domain is only from 0 to δ.

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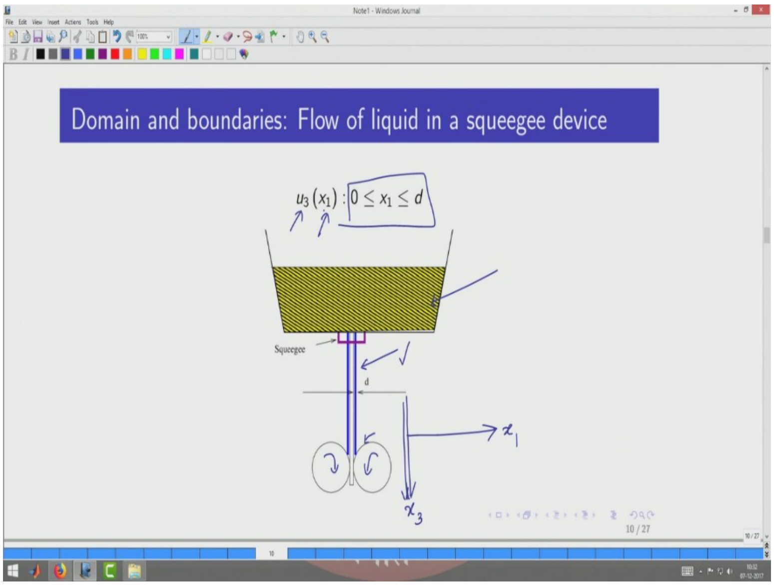

So, now another problem that we look at here is a squeegee device tapping a liquid on to 2 drums over a plate. So, you can see that again here you have a choice of the domain you could actually look at the problem of how the liquid is pouring from the bottom tap hole or how it is actually flowing on the plate below that and how it is actually flowing on this cylindrical rods that are rotating in this manner. So, you have a choice and in our case this is what we are actually choosing and if you chose like that then the domain will then be from the surface of this rod to the free surface which is given by this. So, which means that this is a distance x and the flow is actually happening in this direction ok.

So, it happening in the y direction and x is a distance over which the variation is happening. So, you have let us see if I want to call this as x_3 and this is x_1 the velocity will be u_3 and the variation is along the x_1 and that defines the component and the direction over the which it in varies.

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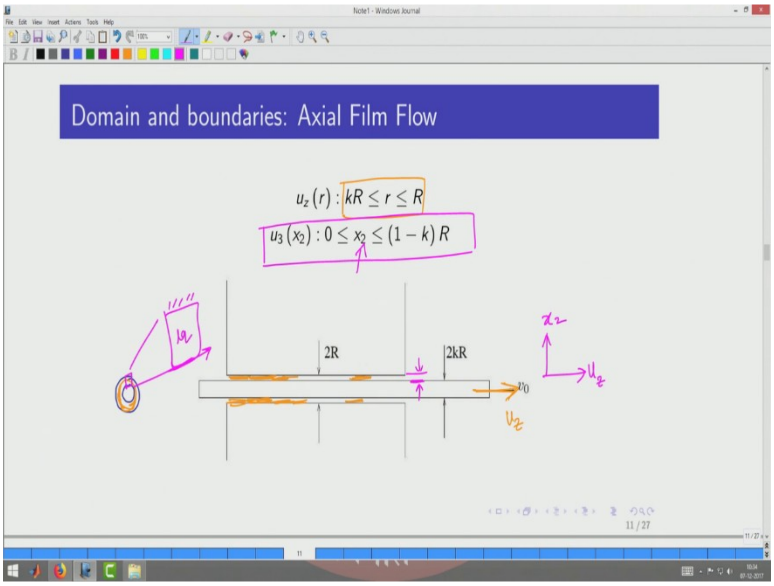

So, here one more problem to look at you can see that here we are looking at the flow in a annular tube. So, the where is the liquid the liquid is here? So, it is in this annular tube and if you now want to look at from the side, so the liquid actually is here in the annular region and it is actually flowing in this annular region because the rod is actually being pulled out. So, because of this the liquid in between the shaft and the annular hole is actually moving in the in this direction the direction I have indicated. So, if I want to call this direction as z, then the velocity will then become u_z.

And the domain is the annular region which actually can be defined in cylindrical coordinate system in this manner this is going from kR to r. So, it is only this gap that is our domain. So, rest of it is not of our concern at this moment. So, sometimes it may happen that the value of k is very close to one which means that the gap is very small compare to the diameter of the rod which means that the curvature is actually not very significant in such a situation, you can actually also make an assumption that a rectangular coordinate system can also make this particular domain possible which means that if you look at this region alone then you can think of it as here you have a motion here and this is stationery and you have got the liquid.

So, you can actually pretend that the domain is rectangular in nature and say that the domain is going from 0 to one minus k R where the 0 actually starts on this surface of this shaft and one minus k R actually ends here. So, this gap is now our domain and the variation in the aerial direction which actually is in this case x_2 variable. So, we choose that this is the x_2 direction and this is the u_z flow. So, that is now our problem in the case of an axial film flow where the gap annular gap is very small compare to the diameter of the rod ok.

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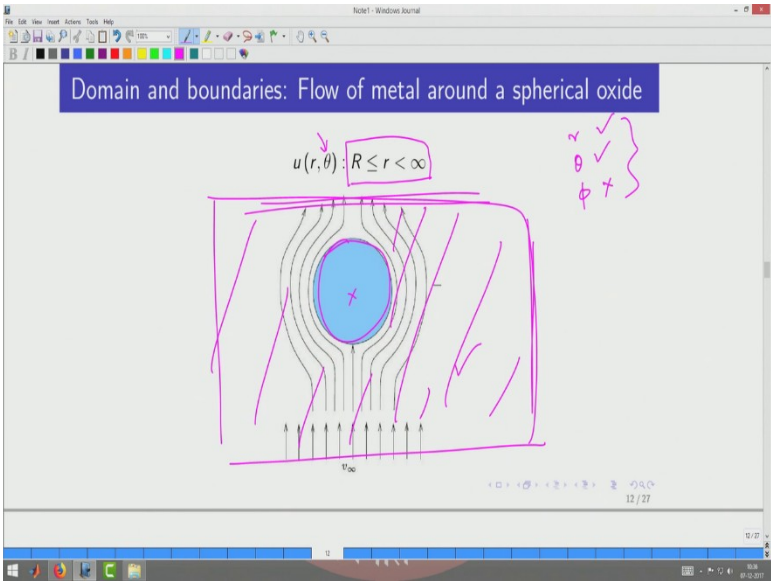

Here is one last scenario for us to practice the domain. So, we are now looking at the flow of liquid metal around a spherical oxide. Now what it implies is that the flow is actually of our interest in the region that is going from r to infinity which means that the everywhere else except the blue region is our domain; obviously, we cannot have an infinite domain you know to depict, but mathematically it is actually very simple to do. So, you could actually think of the domain as very large kind this is all our domain and this is not part of our domain.

So, what our boundaries the boundaries are basically the infinite distance and also the surface of the spherical oxide. So, that is our domain. So, this is all our domain and this is not our domain. So, like this you could actually choose the domain and solve the problem. So, in this case, for example, if the flow is around the sphere then anything outside the sphere is our domain. Now we are also choosing the R to be the variable over which the domain is refined and we can also choose θ to be one of the variables for which the velocity is varying and we do not have to take both the angles because we can assume that the velocity is symmetric over the vertical axes.

So, that the ϕ can be dropped off and θ will be there and R will be there. So, like that we can reduce the problem from a 3 dimensional one to an axisymmetric one in this case and then look at the problem solution, like this in a given situation we make the domain as simple as possible and without losing the essence of the problem.

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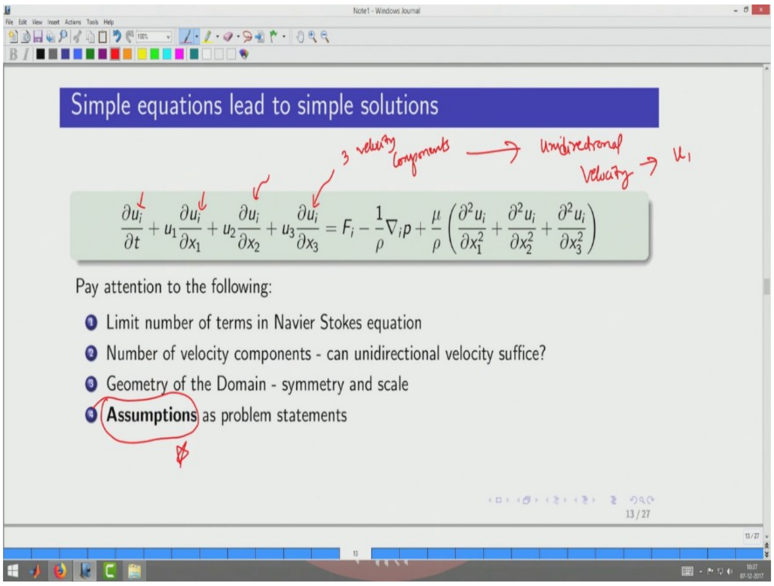

So, here what we are now looking at is to check how the assumptions can now lead to some help in reducing the complexity. So, we have actually seen that the equation will have large number of a components and you can see that there are many many terms. So, how do we reduce? So, we need to make many assumptions to make this kind of a simplification possible. So, we are going to make some more discussion on the assumptions.

So, we it is also clear that by now we have already seen that this actually means that we have got 3 velocity components. So, we are already assuming that we are interested in unidirectional velocity problems which means that the only u_1 for example, or u_2 only one velocity component we are interested. So, we write only one equation not 3 equations as we have written here and in that one equation we have already got. So, many terms, we want to reduce a number of terms. So, how do we go about that is the discussion?

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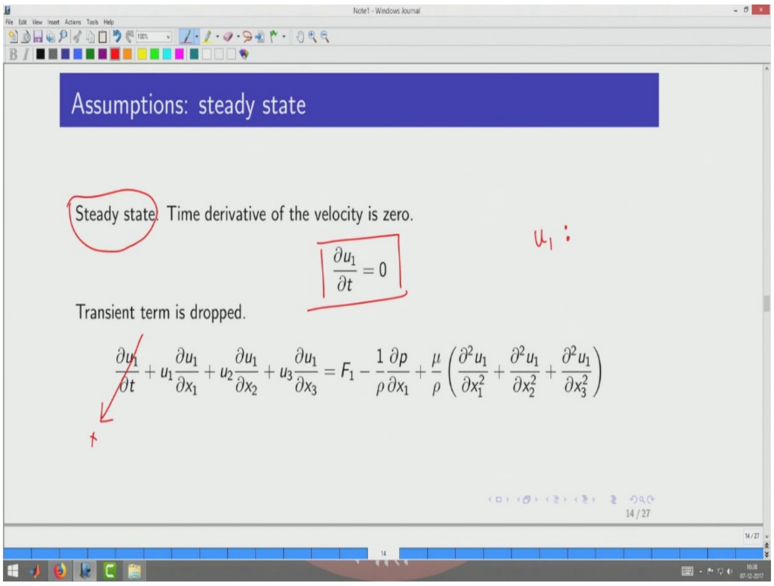

So, the very first assumption that we will do in most of the metallurgical problems means because of the long duration over which the flow takes place in many of the industrial problems. So, the problem is actually over such a long duration that we can assume that the steady state is prevalent. So, steady state prevalence means that the time derivative of the velocity is 0 so; that means, this is actually applicable and what this actually means that immediately in the Navier Stokes equation we will drop the first term off.

So, which means that the steady state assumption has been used and we are writing the Navier Stokes equation for the u_1 component of the velocity which means that we have already chosen to have unidirectional velocity. So, steady state if it is possible immediately it means that one of the velocity equation terms the very first one the transient term can be just dropped.

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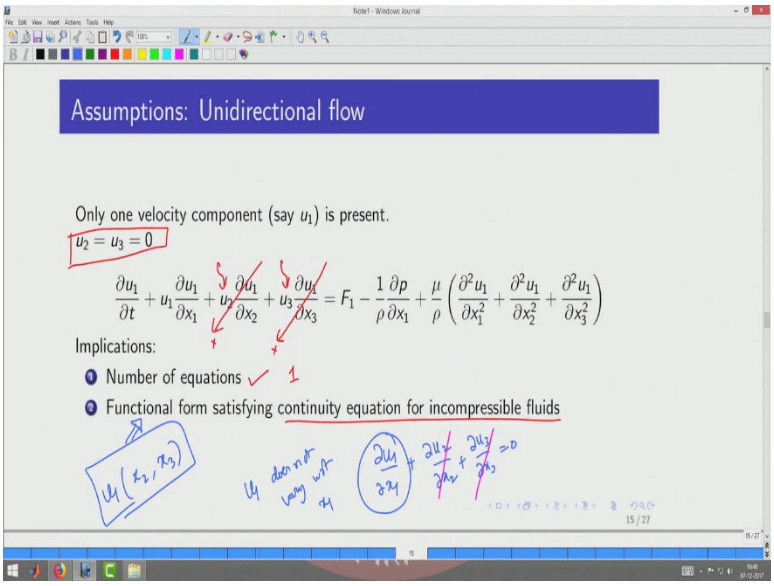

The other assumption that course is important as I just mentioned is unidirectional flow. So, unidirectional flow does not only mean that the number of equations has reduced from 3 to one it also means that the u_2 and u_3 are 0. So, what this implies is that here we have got u_2 and here you have got u_3. So, which means that the terms here are also going to be dropped off, which actually makes the number of terms in the Navier-Stokes equation much smaller. So, you can already see that the first term is dropped in case it is steady state. So, the third and fourth terms will be dropped in case u_1 is the only velocity u_2 and u_3 are to be dropped ok.

So, we have got simplification in terms of the number of equations it becomes one and also it also helps us in the functional form. So, what we mean by that is as follows the continuity equation for incompressible fluids we already know that it is written in the following manner. So,

$$\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}=0$$

and we are already now saying that it is unidirectional velocity which means this is this terms and this term does not exist which immediately means that; the u_1 does not vary with respect to x_1 which means that it can vary u_1 can vary as a function of x_2 and x_3.

So, you can already see what kind of a functional form is applicable whenever we choose the flow to be unidirectional assuming that it is an incompressible fluid. So, immediately we can see that simplifications are coming up.

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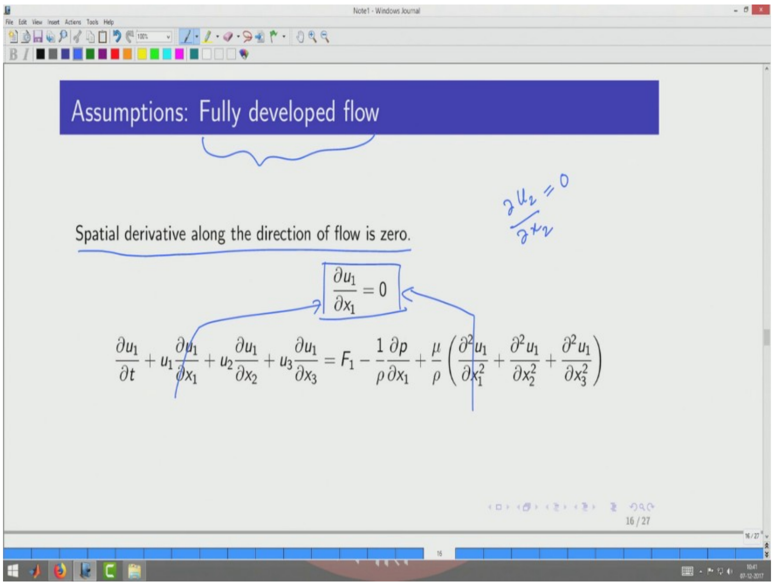

Now, let us look at the other assumption which is generally not discussed much in metallurgical text books, but it is an important assumption that has been made in most of the problems that is we assume that the flow is fully developed. So, what we mean by fully developed is that the spatial derivative along the direction of the flow is 0. So, it just means that if the velocity is along the x direction then the spatial direction is x whether is $\frac{\partial u}{\partial x}=0$. So, whichever component you take for example, if you take u_2 as a component that is along the x_2 directions.

So, if you say that $\frac{\partial u_2}{\partial x_2}=0$ it means that the velocities fully developed along the x_2 direction this does not mean that there is no variation with respect to other spatial variables like x_1 and x_3. So, it can still be there it is just that along the flow the flow does not accelerate or decelerate it has already achieved a particular value and stays put. So, this actually means that you can make this particular simplification which means that you can see that it is actually helping us in reducing this term and also this term. So, you can see that if a flow being fully developed implies that there are 2 terms from the Navier-Stokes equation that could be dropped off.

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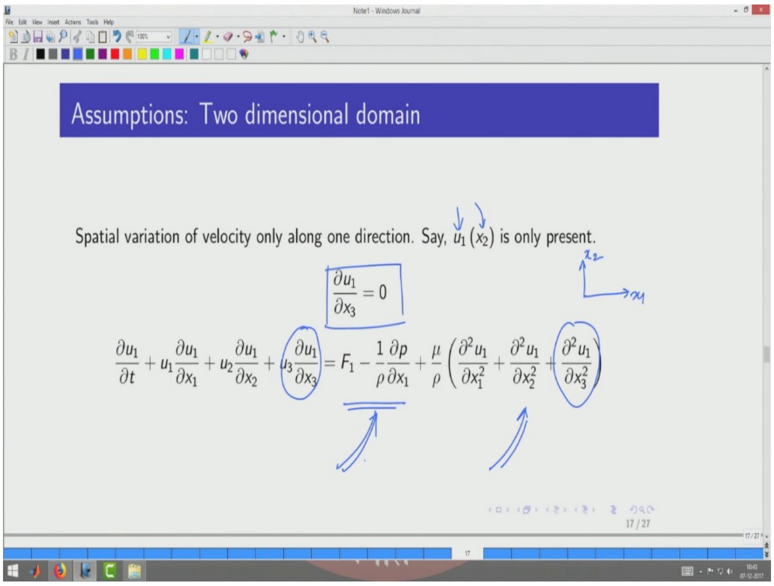

And we go further and see how it can be combined along with the other assumptions. So, we then make the flow a little more restricted we can say that not only it is unidirectional velocity; it is also varying only in one other direction; that means, that the problem is essentially 2 dimensional problem. So, let us say if u_1 is the velocity we are talking about and it can vary as we have just now seen as a function of x_2 and x_3 and we choose that it varies to only along x_2. It does not vary along x_3.

This is a choice, we are making with which means that our domain is in this particular coordinate system x_1 x_2 and x_3 direction nothing is happening ok. So, if we make that assumption then it means that you can drop the derivative of the u_1 velocity as a function of x_3 distance. So, it immediately means that you could actually drop this term off you can also drop this term off.

So, you could see that the variation of u_1 with respect to x_3 appears twice and that you can drop off and you could already see that we have already strategized to reduce the Navier Stokes equation to very very simple form the first term is dropped for the steady state assumption and then you can see that the unidirectional flow will actually make the second and third terms drop off and then you can also drop one of them because it is a 2D domain and you can drop from the fully developed flow this term. So, we can actually see that we are actually seeing only these 2 term are going to survive. So, if you make all those assumptions ok.

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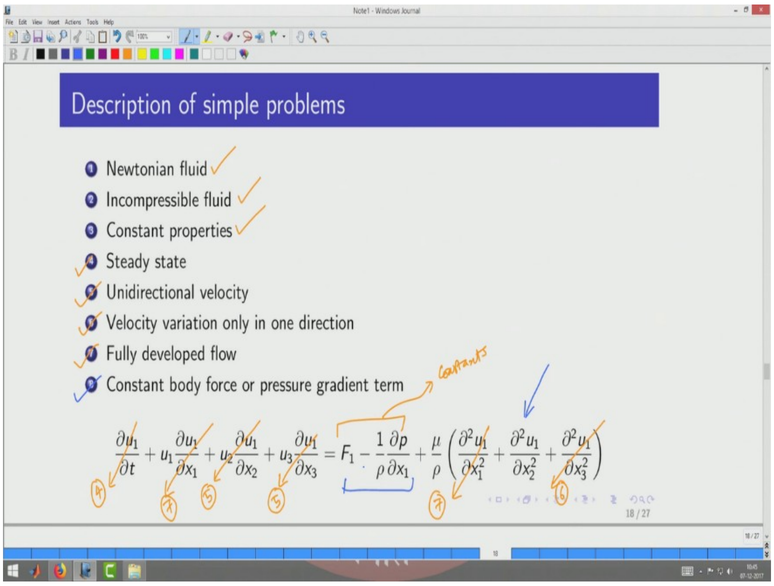

So, let us just see how a typical flow problem appears in most of the U.G. level text books in metallurgical curricula. So, these are all the assumptions that we will make so many many times actually, we are not making this assumptions very consciously we just simply make the assumptions and then write the equations without realizing that we have actually started from the Navier Stokes equation and then reduced it. So, here we are making it very conscious. So, first of all the equation that we wrote is assuming that it is a Newtonian fluid and that is an incompressible fluids. So, we are already made that assumption and then if you look at the diffusion term the viscosity is coming of this terms and then you have got a Laplacian here.

So, which actually means that you are assuming that the properties are constant with respect to the spatial coordinate systems, they are not changing. So, that is why μ has come out. So, we have made this free assumptions the moment we wrote this Navier Stokes equation and if we make the steady state assumption then we can strike it off and that is because of the assumption four and if we say that it is a unidirectional velocity for example,; that means, u_2 and u_3 are not there. So, I strike off here and here saying that it is because of the assumption 5 that I have taken and then we say velocity variation is only in one direction. So, let us say it varies only along the x_2 direction. So, it does not vary along the x_3 direction. So, I could then take the assumption 6 here. So, because of which that term is going off.

And then we can say it is a fully developed flow. So, the moment I want to say it is a fully developed flow then the velocity equation is written for u_1 component. So, which means that

$\frac{\partial u_1}{\partial x_1}$=0. So, we then make this term drop off and that is because of the assumption 7. So, it is done; now you can say that the constant body force or pressure terms. So, which means that these are all constants, this is how the equation will turn out to be many of the problems that is you basically end up with only the terms like this ok.

So, you have got this and this only 2 terms will be there in many of the times the problems are solved in the U.G. level metallurgy courses. So, you have seen that this is how it will turn out to be and which means that the equation would look like this just have a look at here we have got $\frac{\partial^2 u}{\partial x_2^2}$ is equal to something that is a constant or a function.

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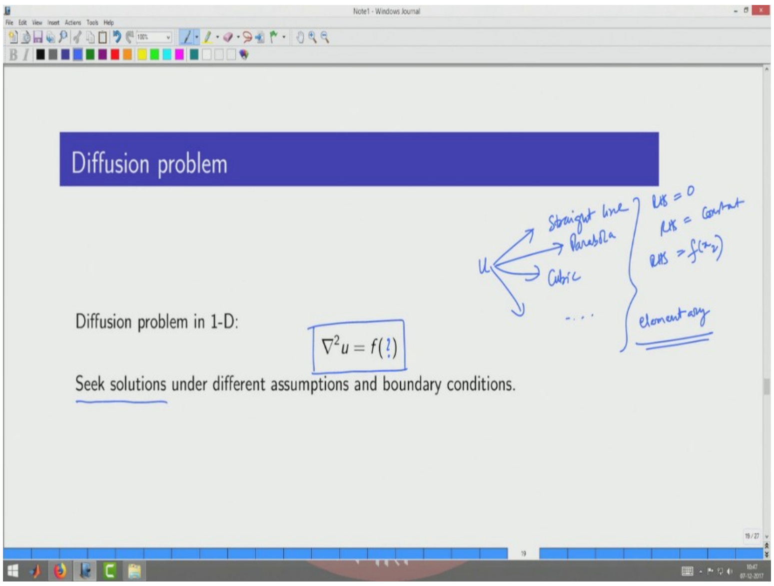

So, which means that it is going to look like this and this as you can recognize is like a diffusion problem. So, the function can be of anything. So, it has to be a function of x_2, but it can be in any polynomial order that we want to choose, but it has to satisfy this equation and then we can get the solution.

So, if you just pay attention to this let us say on the right hand side you have got 0 which means that the velocity variation would be like a straight line and here it is RHS is 0 RHS is constant and you would have a cubic polynomial if RHS is a function of only let us say x_2 and so on. So, you could have sinusoidal functions exponential functions and. So, on as a case may be for example, if your right hand side is a function of u_1, u itself then you can have sinusoidal or exponential functions. So, you can have a various elementary; elementary functions that could be looked as possible solutions of the velocity field.

So, what we do is that we actually go and seek solutions for this particular problem under various boundary conditions and see whether they can be used for the flow problem that we have at hand ok.

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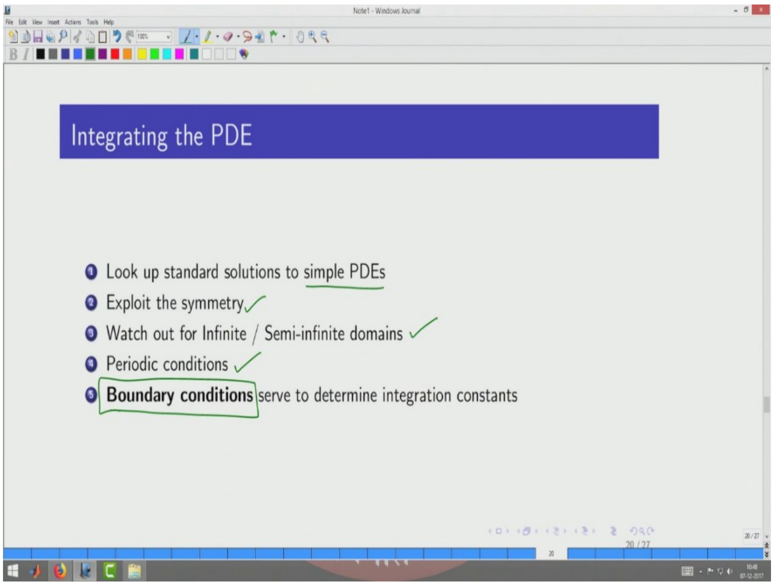

So, we what we do now is that once the differential equation is available in that particular form it is a very simple PDE. So, you can then go ahead and it is a simple PDE. So, you can actually go ahead and see what kinds of solutions are available for that. So, we basically then integrate and while integrating we always ensure that we exploit the symmetry what we mean by that is at the center of the domain is the problem symmetric, if it is symmetric then the slope of the variation of velocity will be at the center. So, some such observations we will make. So, that our problem solution is easier. So, we will do that.

And we will also see whether the domain is semi-infinite or infinite etcetera. So, that we will see what functional forms are valid for example, if it is infinite domain then as length variable goes to infinity you must not have the velocity blowing up which means that the length will becoming as 1/x or so on. So, some such insights will help us in arriving at possible solutions and if there are periodic conditions that are available which actually means that we will also use periodic functions to model the solution and whenever we do all this things we eventually end up with some integration constants and we have to actually use boundary conditions to arrive at what the integration constants are now just a couple of minutes on the boundary condition that we encounter in flow problems.

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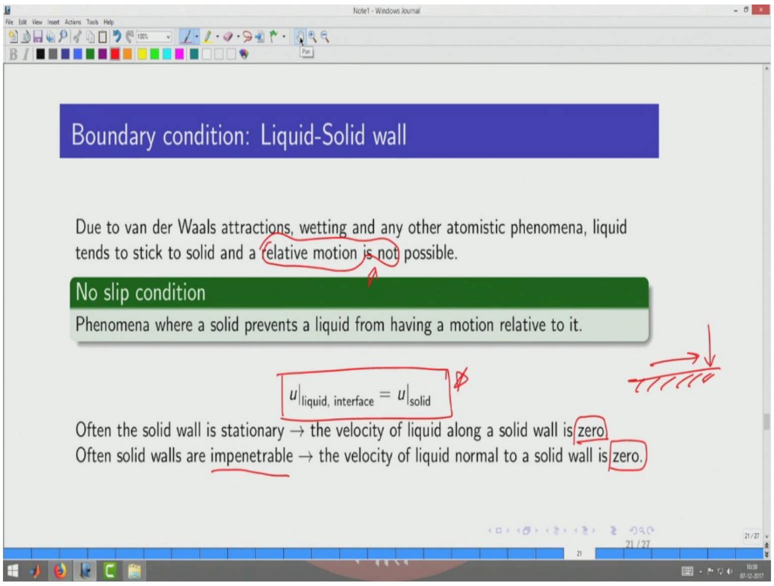

So, normally we have a container in which the liquid is flowing. So, most popular boundary condition that we encounter is that liquid is present near a solid wall. So, this boundary condition is very important and it is often to refer to as no slip condition the reason is as follows the liquid usually because of the Vander Waals interaction because of the wetting phenomena and any other problems that are associated at the atomic scale we can actually assume that the liquid actually stuck to the solid. So, if the liquid is stuck to the solid it means that the velocity of the liquid is not different from that of the solid there is no relative motion.

So, this actually is captured by stating here; here that is at the velocity of the liquid at the interface is same as the velocity at the solid and very often the solid wall is stationary which means that the velocity of the liquid at the wall is 0. So, sometimes the students are confused to assume that the no sleep condition means the velocity of the liquid is 0 at the wall it is true only when the wall is stationary if the wall is moving at a particular velocity then the velocity of the liquid is same as the velocity of the solid. So, this is very important there is no relative motion.

So, the idea is always that there is no relative motion. So, this is the idea of the no slip condition and most of the walls if you look at pipe flow for example, the pipe is actually solid material which does not allow the liquid to slip out. So, you can actually assume that the solid walls are also impenetrable which means that the velocity of the liquid normal to the solid wall is 0. So, you have a boundary condition both along the wall and along the wall and also normal to the wall. So, if both the components you actually have a way to actually limit the velocity at the wall. So, wall is basically of the boundary. So, it is a basically boundary condition.

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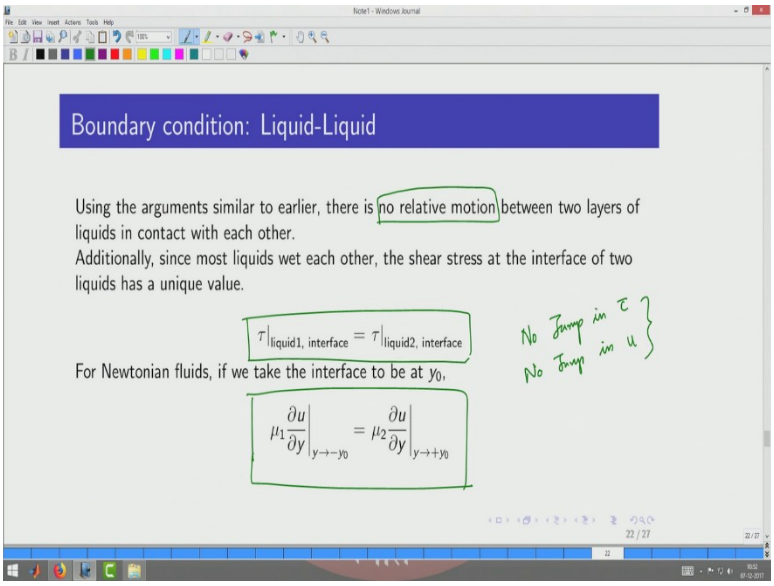

So, now there are situations where there are 2 different liquids that are present in the domain. So, in metallurgical scenario it will be for example, liquid metal with liquid slag on top of it and both of them are flowing down in inclined plane for example. So, we may have such situations again and again 2 different liquids which do not mix are actually flowing. So, whenever there is a liquid; liquid boundary, then the boundary condition that we use is that the shear stress is same for both the liquids at the interface that is there is no jump. So, there is no jump in the value of the shear stress.

Of course, the 2 liquids would also wet each other and there is no relative motion of the liquids with respect to each other which also is assumed. So, you also see that there is no jump in the velocity also. So, the velocity is continuous and then their slopes will also have no jump across and this is actually also useful now when you use the Newtonian fluid assumption then the shear stress can be expanded. So, then it turns out that the ratio of the gradients of the velocity this is the ratio of the viscosities. So, this is how the boundary condition is defined at a liquid interface.

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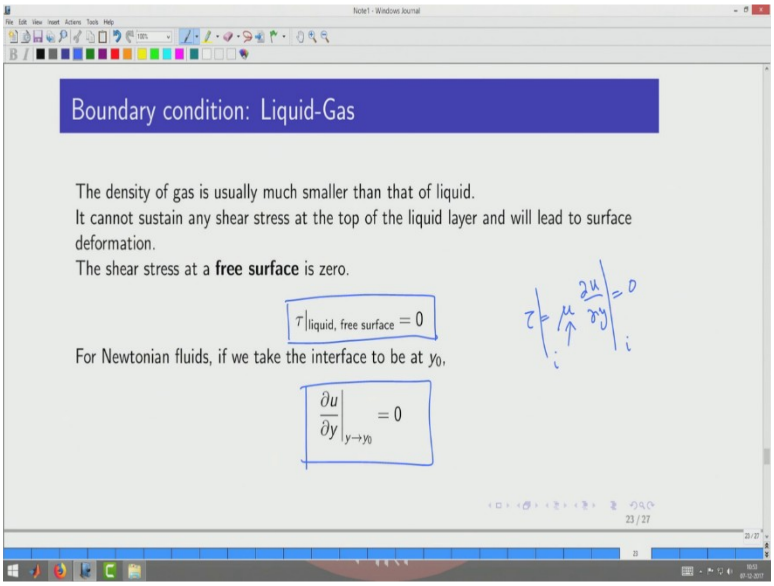

Now, sometimes we also have in metallurgical scenarios where the liquid is actually expose to gas. So, we are interested in how much of oxygen pick up is happening how much of hydrogen removal is happening, etcetera. So, in all those situations the liquid is actually having the boundary with the gas and that can be handled by using a boundary condition here that the shear stress is 0 at the boundary of a liquid and gas. So, the reason is as follows.

The gas actually has a density usually about a thousand times less than that of the liquid or solid. So, which means that there are enough number of atoms in the gas that can actually interact with those of the liquid or solid on the other side of the interface which means basically that there is no way to sustain the shear stress at the top of the liquid and therefore, it must be 0 at the top surface. So, at the free surface the shear stress is 0 and that is actually explained here in this form.

Now, if you use the Newtonian flow assumption then you can already see that you can write

$$\tau = \mu \frac{\partial u}{\partial x}$$

and that we say at the interface and you can already see that this being a constant you can drop off and you can say that the velocity gradient at the free surface is 0. So, you can now see that the boundary conditions of fluid flow are different for the 3 scenarios namely when it is actually with respect to the solid on the other end or another liquid on the other end or a gas on the other end. So, we need to keep this is mind and look at the boundary of a given problem and use appropriate boundary condition and that will help us in reducing the problem much simpler.

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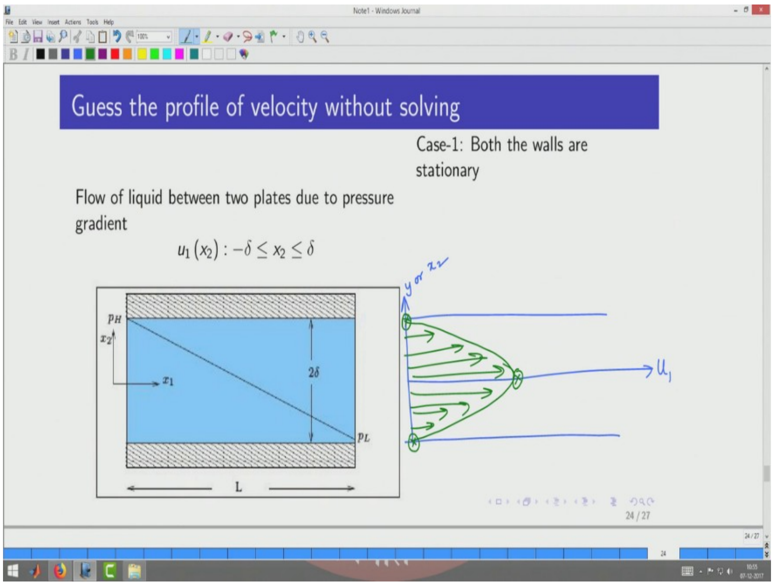

So, let us just practice a simple problem without having to solve. So, here what I want you to think of is look at the problem and see how simple the solution can be. So, here we have got a flow of liquid between 2 plates and it is happening only due to a constant pressure gradient and there is nothing else that is happening and under such situations how the solutions should be. So, from whatever we have discussed till now. So, the domain is drawn and we see that this axes is basically the x_2 which is y or x_2 axes and the velocity has to be plotted in this manner.

So, you have got the velocity in this manner u_1 and so, how the velocity should look like now from whatever we have discussed if the plates are stationary; that means, that the velocity must be 0 at the walls which means that we immediately mark that this is how it should be the velocity should be 0 on the 2 interfaces between the liquid and the solid wall and the liquid will have some velocity because of the gradient of the pressure that we have imposed and that velocity would be some value here. So, which means that we actually have a parabola that goes through these points, which already see that this is how the liquid flow is expected to be now we can see that this is already a nature of the flow we arriving at from just the boundary conditions and the knowledge of simple diffusion problems and we can actually get the exact functional forms when we solve this problems analytically in the next session.

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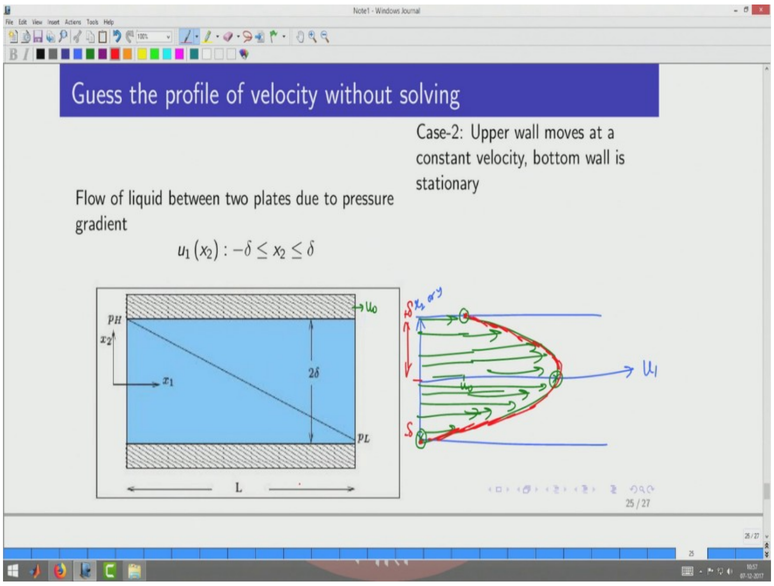

So, now we just change the problem a little bit and see whether we are able to handle that. So, what we are saying now is that in the same problem the upper wall moves at a constant velocity. So, which means that this is moving at u_o and then the pressure drop is there. So, which means that let us just see how the domain should look like. So, you have got this is x_2 or y and this is the velocity u_1, we are talking about and this is the domain. So, we can already see that from the boundary condition discussion the bottom wall is a stationary. So, we say that there is no velocity there and there must be some velocity in the middle because of the pressure drop that is present.

So, there is some velocity and the upper wall is moving at a constant velocity u_o. So, this must be u_o. So, which means that we must have a parabola that actually goes through this 3 points and that may come out like that. So, which means that we may have a maximum which is actually in the top half, you may have that the maximum is in the top half of the domain and this way we can actually see that we can we can see that the velocity distribution can be arrived at without actually even solving. So, I am just making the parabola and the parabola is shifted with a maximum slightly on the upper half to satisfy that the value on the bottom wall is 0 the value on the top value is some finite number and it has a direction that is given by the opposite to the pressure gradient which is actually going down. So, the velocity has to go towards the right.

So, this way actually we can already guess how the velocity field should have it is variation, but the actual analytical form of this can be arrived at from solution we will do that in the next session and then we can actually validate whether it turns out to be the way we imagine for this particular problem. So, the domain is - δ and + δ. So, you can actually mark that here and this will be + δ and this will be - δ for example. So, we expect the solution to be given in forms of δ and the PH and PL over the length L, etcetera.

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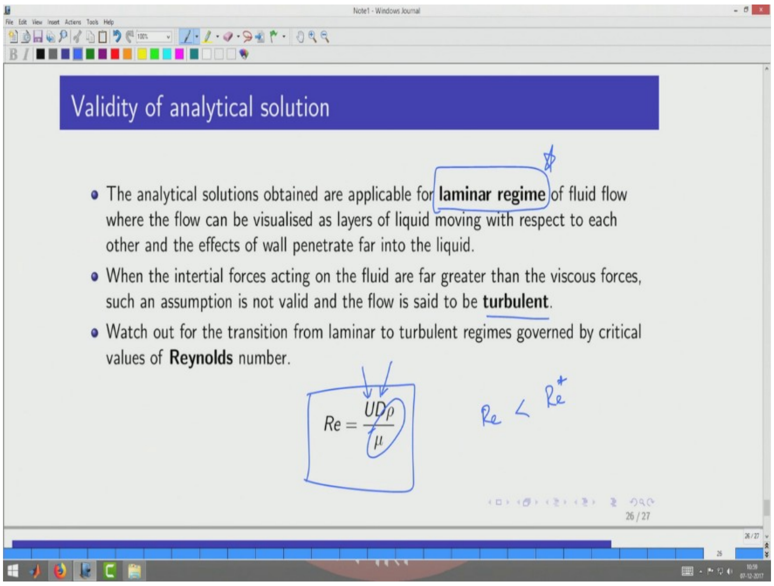

So, finally, whenever we get the solution in the form of a functional form, what we are expect to do is that, we should validate whether the solution is valid under the Reynolds number that is given for that situation or not. So, what it implies is that. So, we look at this Reynolds number and it has to be generally for a given problem it has to be some critical number. So, that number depends upon the particular problem and only when it is below at a particular critical number we can say that the regime is a laminar and only then we can actually expect the effect of a boundary all the way into the domain and that is exactly what we are going to when we integrate over the entire domain.

So, it is very important to ensure that the solutions that we get after we get the solution and plug in the numbers here. So, this is a velocity and this is the length scale distances and this is a properties we have taken. So, for this combination we evaluate what would be the Reynolds number and ensure that it is below a particular critical number for that problem. So, that the laminar regime is actually valid otherwise what happens is that the flow actually in the reality would turn out be turbulent in which case such a integration over the entire domain is invalid the velocity would actually be varying with time also and therefore, we need to then go to empirical correlation.

So, we will actually have an approach to handle that also as a part of this course later on, but this is one way by which we can validate the solutions. So, we validate not only by the plot, but also by the magnitude of the velocity that we get. So, with that the; we close this session and some of the practice assignments will be available for you to look up the notes will always be there on the course website.