

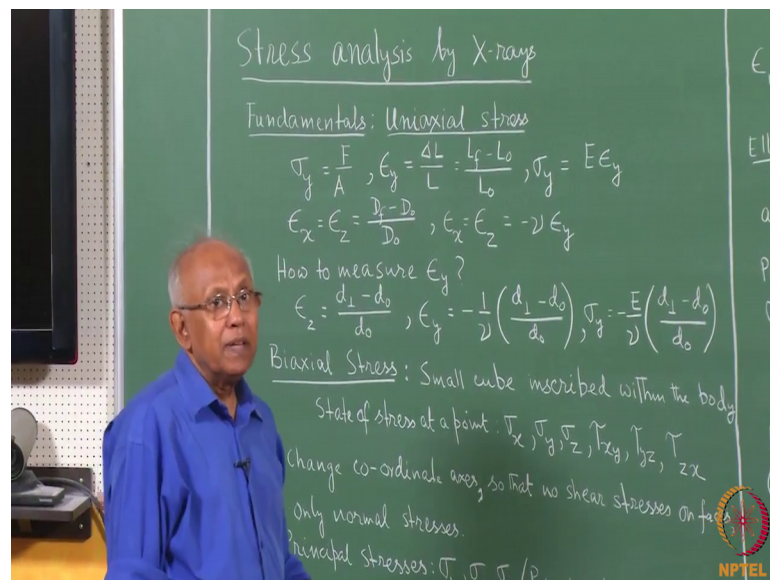
X-Ray Crystallography
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Lecture - 29
Stress Analysis by X-Rays

In this lecture, I will deal with stress analysis by X-Rays. X-Rays provide a very elegant method of stress measurement in a material. Whenever there is a homogeneous, elastic deformation in a material, we find that when a particular set of planes within a grain in that material are properly arranged for taking a diffraction pattern, there is a change in the inter planar distances in that set of planes from the value in the unstressed body.

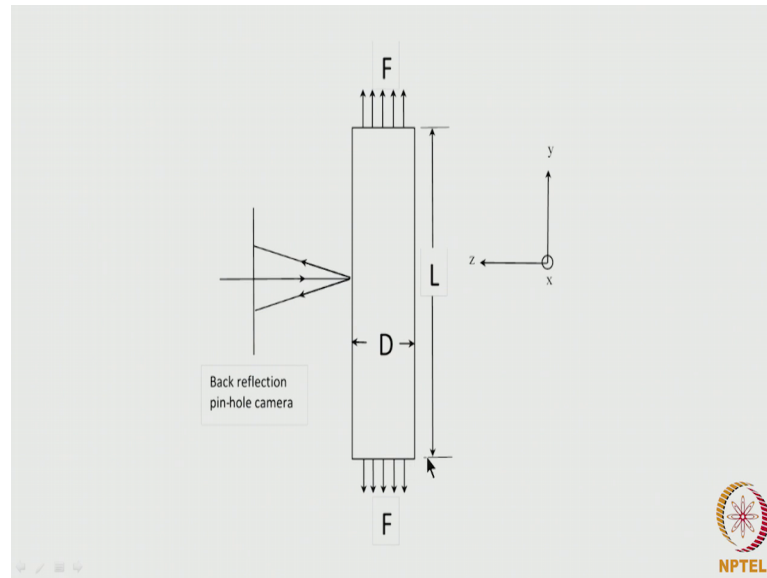
Now, this change in the value of the inter-planar distance will give us a measure of the stress that has been put on the material. Now normally in the X-Ray method we do not measure stress directly. We measure the strain and then from the strain, we calculate the stress. Now before I get into the techniques of stress measurement by X-Rays, it will be better to deal with the fundamentals a little bit.

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Say for example, if we have uni-axial stress as shown over here; say this is a cylindrical rod of diameter; D and length; L ; say for example, we have a force; F acting in these directions as shown.

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In that case, the stress will be equal to F by A . A is a area of cross section of the rod then strain in the y direction. This is the y direction; strain in the y direction will be equal to ΔL by L which nothing is, but the final length minus the initial length divided by the initial length.

So, from this strain, we can calculate σ_y as $E \epsilon_y$ is the Young's modulus. Now you see if we want to measure the stress in the y direction, then the methods could be that we have some gains within this rod over here and there are planes lying a perpendicular to the direction of F . So, if we have a grain over here with atomic planes lying in this fashion you know perpendicular to the direction of F , then what will happen due to the tensile stress; the inter planar distance will increase there will be an expansion in this direction.

So, because of the expansion the inter-planar distance will increase and if we can measure the inter-planar distance after applying the force and also can measure the inter-planar distance when there is no force on the body then we can calculate the strain in the y direction. However, the problem is when the rod is under tension as shown over here;

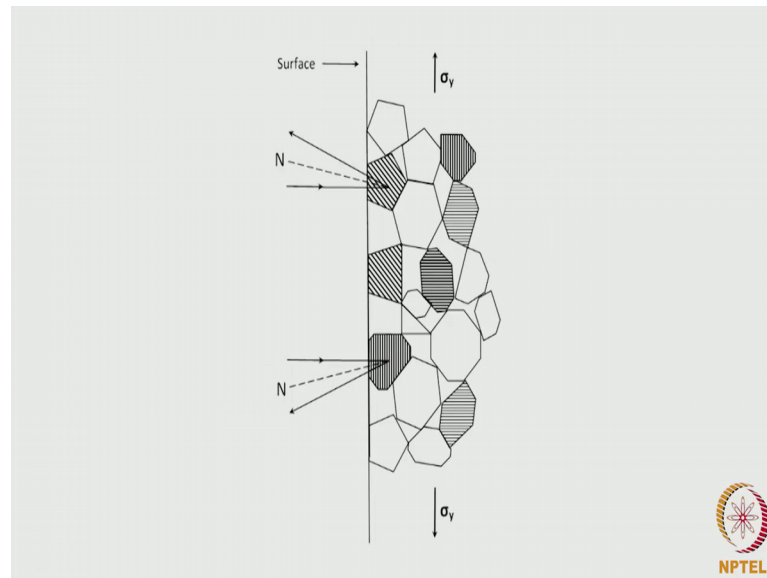
we cannot measure the inter-planar distance of the planes lying perpendicular to the direction of F because it will be difficult to put an X-Ray equipment over here.

So, instead what we do we try to figure out the variation of the inter-planar distance from grains which have atomic planes lying parallel to the side of the rod in this manner and then if we have a beam of X-Rays incident on these planes, then by the back reflection by measuring the back reflection diffraction pattern, it is possible to figure out what are the values of the inter planar distances.

In fact, we can measure the strain in this direction and from that we can find out the strain in this direction. And finally, calculate the stress in this direction. So, this is exactly what we do. So, we know that ϵ_x ; the strain in the x direction will be equal to the strain in the z direction because it is a cylindrical body which has been under a force along the y direction and that should be equal to $\frac{D_F - D_0}{D_0}$. D_F is the diameter; the final diameter over and D_0 is a original diameter of the rod.

So, ϵ_x will be F equal to ϵ_z and this will turn; will be equal to $-\nu$ ϵ_y ν is the Poisson's ratio of the material which is being tested now. So, how we measure ϵ_y ? I have already said the problems of measuring ϵ_y directly. So, what we do? We measure the value of ϵ_z by finding out the value of the inter-planar distance of a series of atomic planes in a grain of the material; you know by having the X-Ray beam falling perpendicularly on those planes and then subtract D_0 ; the inter-planar distance in the unstressed material divided by D_0 again. So, the situation is like this.

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So, if we have some grains within the material, where the atomic planes are more or less parallel to the side of the sample and if we have a beam of X-Rays falling more or less perpendicularly over; you know it will make an angle small angle, but it will still be called a normal incidence.

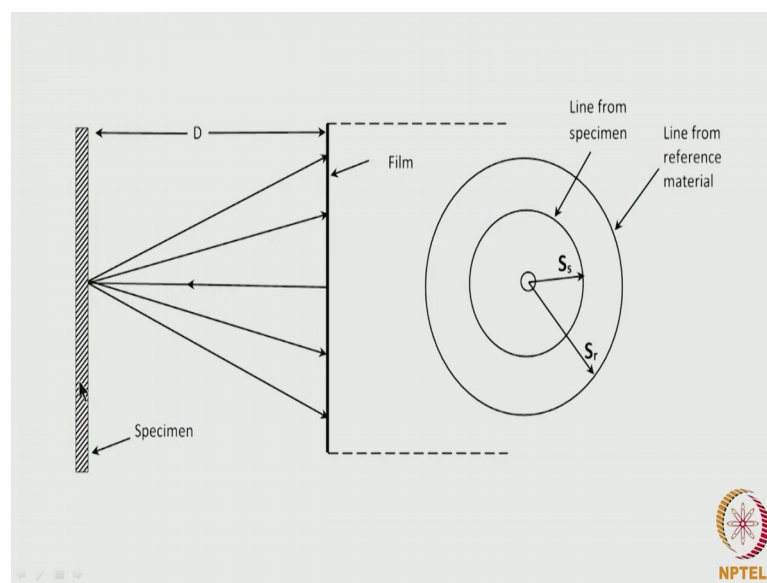
And if we can get the diffracted beams like this; this is the normal to the planes, this is the incident beam, this is the diffracted beams although; it will be making some angle. We can consider it to be a normal incidence of X-Rays. So, by having the diffracted beam from this grain we can figure out; how much is the value of the inter-planar distance when the material is under load. So, that is the value; we call the perpendicular here and then have the body in the unstressed condition and from the same place find out what is the value of D_0 .

So, ϵ_z will be equal to $D_{\perp} - D_0$ by D_0 . So, ϵ_y which is you know related to ϵ_x and ϵ_z by this equation can be found out as $-\frac{1}{\nu} \frac{D_{\perp} - D_0}{D_0}$. So, once we get ϵ_y the strain along the y direction, we can find out the value of σ_y ; the stress in the y direction as $-\frac{E}{\nu} \frac{D_{\perp} - D_0}{D_0}$. So, you say that x is provided an elegant method of stress analysis by finding out the inter-planar distance of a set of atomic planes in a particular grain which is properly arranged within the material. So, that we can find out that value of the inter-planar distance in this stress condition and

also in the unstressed condition and knowing these 2 values; it is possible to figure out the value of ϵ_y and from that you can calculate the value of σ_y .

In actual material; the stress condition may be quite complex; may be biaxial or tri axial stresses can exist in over there. Now you see the normally the technique that we use is what is known as a back reflection we know all method which I will come to later on. So, what essentially we do is in the; we all know about the arrangement of the specimen and the film and the X-Ray machine in case of a back reflection pinhole method.

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What happens in a back reflection pinhole method is we have got our specimen and X-Ray beam is allow to fall on the specimen in this direction then; we have got the recording film. Now whatever is recorded on the film is laid out in this portion and we can see what the; you know recording looks like the X-Ray beam is incident from this side through this pinhole and strikes the sample at this point.

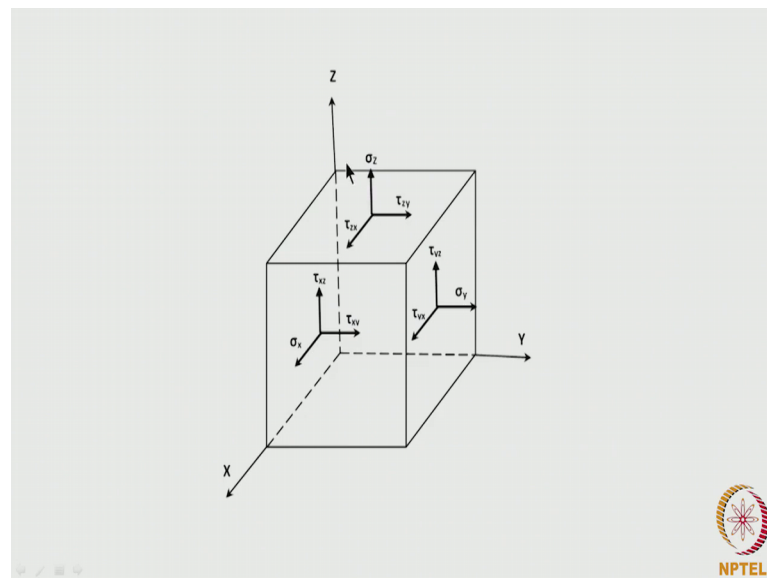
So, what normally we do is we smear powder of a known reference material on the surface of the sample and then take a back reflection pinhole pattern from the composite. So, what will happen? We will get the Debye ring from the specimen as well as we will get the Debye ring from the reference material for which the lattice constant and everything are known. Now provided we know the value of D the distance between the specimen and the film just by knowing the; you know displacing of the reference material from this recorded Debye ring for the reference material, we can easily find out

what should be the displacing for the specimen you know by measuring the radius of the Debye ring.

So, by comparison with the reference material for which the lattice constants are known we can find out from the Debye ring of the actual specimen what should be the lattice parameter lies.

So, this will be an accurate measurement of the lattice parameter. So, this is what we do over here. Now I will go to bi-axial and tri-axial stresses which normally we find in case of an actual material. In fact, in an actual material, if we inscribe a small q within the material; the state of stress at a point will be given by these components.

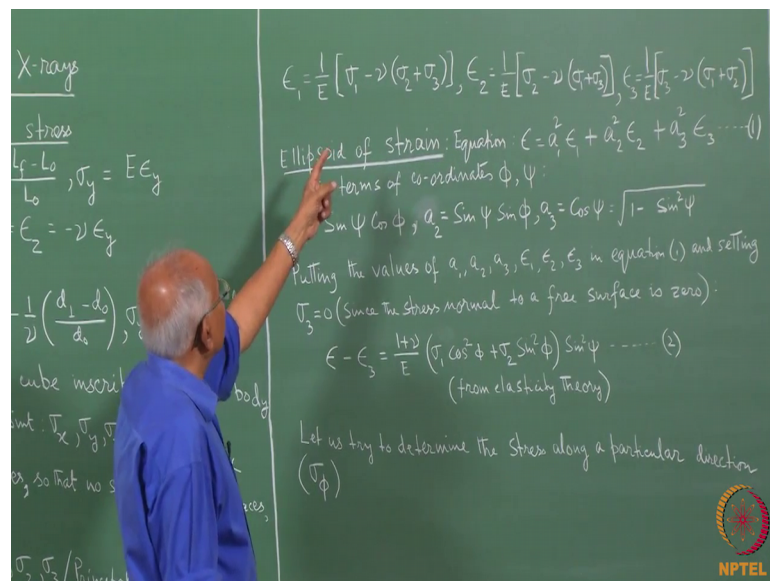
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Say for example, on each phase of the cube inside within the material we will find; there will be a normal stress acting perpendicularly to the surface and there will be 2 shear stresses given by that and this will be true for all the phases. Here for example, this is σ_z . Here it is σ_y . This is σ_x and you have got all the shear stresses $\tau_z x$, $\tau_z y$, $\tau_x z$, $\tau_x y$, etcetera, etcetera. Now if we consider all this 6 phases, we will see that there will be some of these stresses are equal to some other stresses within the on the surfaces. As a result, the stress at a point within the material can be given by only 6 stresses. So, if we know the values of σ_x , σ_y , σ_z , $\tau_x y$, $\tau_y z$, $\tau_z x$, then the stress at a point will be completely described within the material.

Now, if we change the coordinate axis for example, here there will be a situation when there is no shear stress present on the surfaces and there will be only normal stresses. So, it is possible to change the co-ordinate axes of this cubic body from x, y, z to some other. So, such that they are only shear stresses present on the surfaces; only normal stresses. Now in that case, we say that the normal stresses will be sigma 1, sigma 2 and sigma 3 along the axes 1, 2 and 3. So, under that condition, the state of stress will be given by only 3 principle stresses; sigma 1 sigma 2, sigma 3 and accordingly the principle stress will be say epsilon 1, epsilon 2 and epsilon 3.

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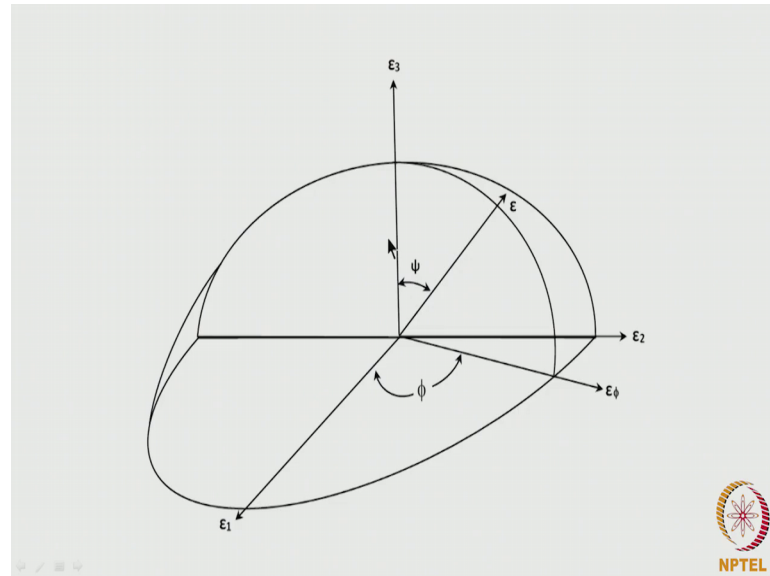


We can write down the principle strains in terms of the principle stress values as shown here. So, epsilon 1 will be 1 by E, The Young's modulus sigma 1 minus nu Poisson's ratio sigma 2 plus sigma 3 sigma; epsilon 2 will be 1 upon E sigma 2 minus nu into sigma 1 plus sigma 3 and epsilon 3 will be 1 upon E sigma 3 minus mu into sigma 1 plus sigma 2.

Now, we can talk about what is known as an ellipsoid of strain. Say for example, if there is a homogeneous elastic deformation of a material; then a sphere within a material; sphere element within the material will be turned into an ellipsoid and the equation for that ellipsoid of strain is epsilon equal to A 1 square epsilon 1 plus A 2 square epsilon 2 plus A 3 square epsilon 3. So, this epsilon is any strain in any specified direction and A 1,

A_2 and A_3 are the direction cosines of that particular direction with the directions of ϵ_1 , ϵ_2 and ϵ_3 .

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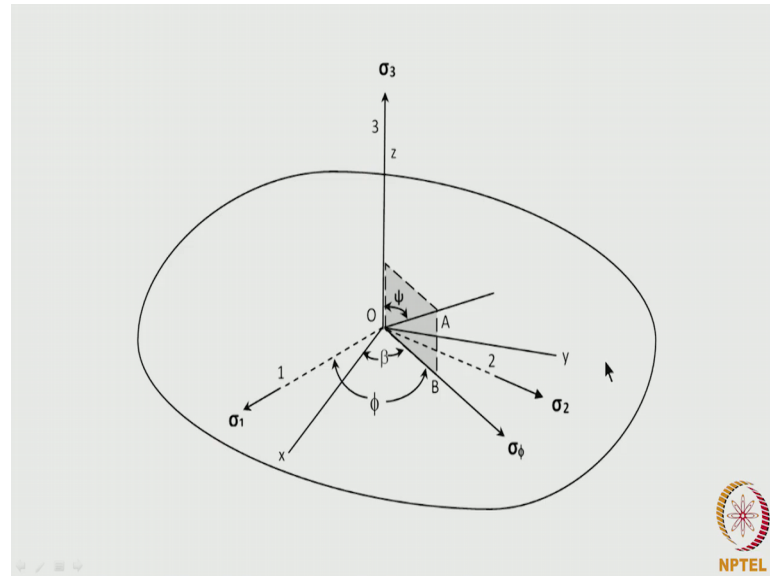
Now, if we look at the situation here. So, this is a ellipsoid of strain. So, this these are directions of ϵ_1 , ϵ_2 and ϵ_3 ; say we are considering a direction like this for ϵ and this is making an angle ψ with ϵ_3 and you know this particular direction. Here ϵ makes an angle ϕ with ϵ_1 . Now in that case, this equation will be valid. So, if we look at the direction of ϵ here and the value of ϵ will be given by this.

And now in terms of the coordinates as we have shown here; the ψ and the ϕ angles in terms of this coordinates A_1 , A_2 and A_3 can be written in this fashion. For example, A_1 can be written as $\sin \psi \cos \phi$. A_2 will be equal to $\sin \psi \sin \phi$ and A_3 will be equal to $\cos \psi$. So, will root over $1 - \sin^2 \psi$

Now, if we put the values of A_1 , A_2 , A_3 and ϵ_1 , ϵ_2 , ϵ_3 , in equation 1 and also put the value of σ_3 is equal to 0; as we know that the stress normal to a free surface is 0 then equation 1 becomes $\epsilon - \epsilon_3 = \frac{1 + \nu}{E} [\sigma_1 \cos^2 \phi + \sigma_2 \sin^2 \phi]$. The whole thing multiplied by $\sin^2 \psi$ or equation number 2. This is from; this can be derived from the elasticity theory. Now; our job now is to try to determine what should be the stress along

a particular direction as we have shown here say for example, you know along a particular direction what will be the value of σ_ϕ .

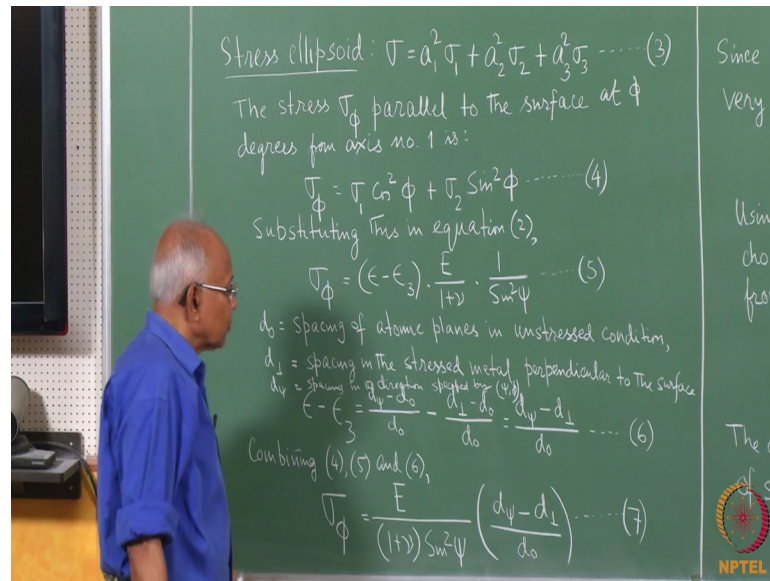
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So, this is what we are trying to find out. Now let us suppose that we have a free surface as shown over here and the angular relationships between x , y , z and the direction 1 for σ_1 , 2 for σ_2 and 3 for σ_3 are all shown here. Let us suppose that we want to find out the value of the stress σ_ϕ lying in this plane and also lying on the surface. So, we have got this direction which is the direction of σ_ϕ ; say we want to figure out the value of σ_ϕ stress. In this particular direction and the direction is specified by the angles ϕ as well as ψ .

Now, how to find out the stress in this particular direction that is what we are going to figure out now just as we saw the equation for a strain ellipsoid we can write down the equation for a stress ellipsoid in this fashion where $\sigma = A_1^2 \sigma_1 + A_2^2 \sigma_2 + A_3^2 \sigma_3$, A_1 , A_2 and A_3 just as before are the direction cosines of the direction of σ with respect to the directions of σ_1 , σ_2 and σ_3 .

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So, this is our equation 3 now the stress σ_ϕ parallel to the surface at ϕ degrees from the axis number one; that means, this one over here is σ_ϕ equal to $\sigma_1 \cos^2 \phi$ plus $\sigma_2 \sin^2 \phi$. This is our equation 4. Now if we substitute this in equation 2 then you find σ_ϕ is equal to $\epsilon - \epsilon_3$ into E divided by $1 + \nu$; the whole thing multiplied by 1 upon $\sin^2 \psi$. This is our equation 5.

Now, say for example, if we take d_0 as the spacing of atomic planes in a suitable grain in the unstressed condition and d_\perp is the spacing in the stressed metal perpendicular to the surface and d_ψ is the spacing in the direction specified by ψ then we can write down this equation you know as $\epsilon - \epsilon_3$ will be equal to $d_\psi - d_0$ by d_0 . This is the value of $\epsilon - \epsilon_3$ can be written as $d_\perp - d_0$ by d_0 that it equal to $d_\psi - d_\perp$ by d_0 .

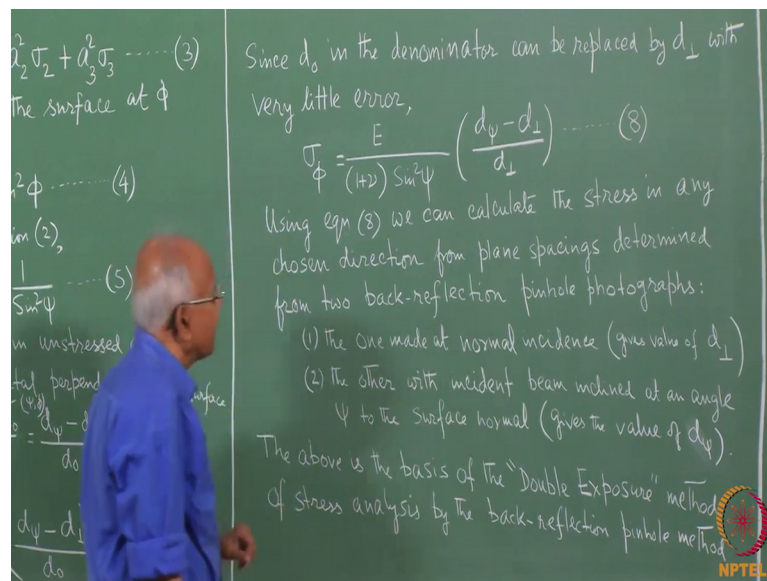
So, essentially what we are doing here? if this is the free surface; if this is the free surface and if we have the X-Rays incident on the surface in a perpendicular fashion and if from a suitable grain with the atomic planes; we can measure the inter-planar distance then that will be d_\perp . Now if we do the same thing in the material when the material is in the unstressed condition; then it will be d_0 and if again we allow the X-Ray beam to be incident on the surface in this direction as shown over here; then

that will give us the value of $D \sin \psi$ the spacing of that we planes in that particular direction specified by ψ and ϕ .

So, I will repeat in that case; what will happen? The epsilon will be the epsilon here is the strain corresponding to this condition defined by ψ and ϕ . So, this epsilon here will be equal to $D \sin \psi$ minus D_0 by D_0 and epsilon 3 will be equal to $D \cos \psi$ minus D_0 by D_0 . So, after subtraction it becomes $D \sin \psi$ minus $D \cos \psi$ by D_0 this is our equation 6.

Now, if we combined the equations 4, 5 and 6 then we get σ_ϕ equal to E divided by $1 + \nu \sin^2 \psi$ the whole thing multiplied by $D \sin \psi$ minus $D \cos \psi$ by D_0 . This is our equation 7.

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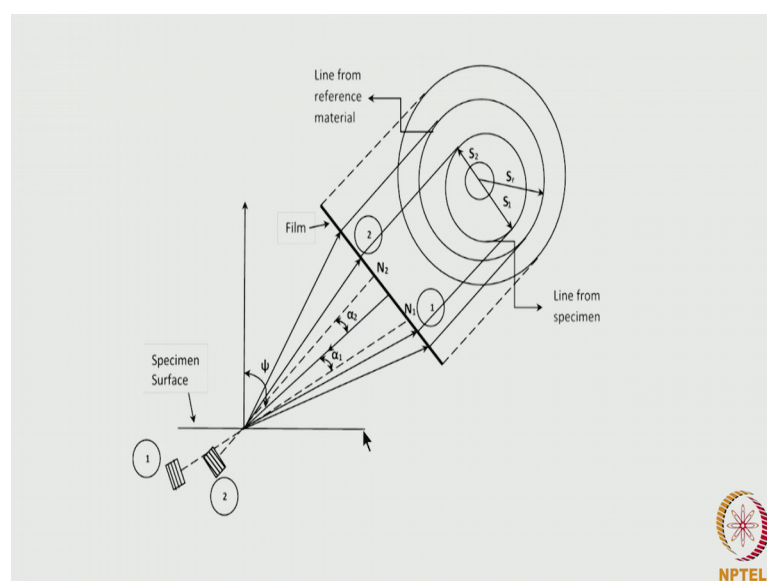
Now this D_0 in the denominator can be replaced by $D \cos \psi$ with very little error. So, this equation 7 can then be rewritten as σ_ϕ is equal to E upon $1 + \nu \sin^2 \psi$. The whole thing multiplied by $D \sin \psi$ minus $D \cos \psi$ divided by $D \cos \psi$. This is our equation A. Now this is a basic equation. This equation 8 is the one by using which we can calculate the stress in any chosen direction from the plane spacing determined from 2 back reflection pinhole photographs as I have already mentioned.

And what were those the one might at normal incidence the one might with normal incidence perpendicular to the surface; which will give us the value of D perpendicular the other with incident beam inclined at an angle ψ to the specimen surface. So, what we can do? We can make 2 back reflection pinhole photographs 1 by allowing the X-Ray beam to fall on the surface in a direction perpendicular to it and that will give us the value of the perpendicular and again a second back reflection pinhole X-Ray exposure in which the X-Rays are incident on the surface at this particular angle ψ and that will give us the value of $D \psi$.

Now, putting the value of $D \psi$ and D perpendicular over here; it is possible to figure out what will be the value of $\sigma \phi$ because you know ψ is a known value. In most of the cases, we keep it at 45 degrees. So, whatever the angle of ψ that is kept 45 degree; so, this value will be known ν is the Poisson's ratio that is also known. E is a elastic modulus that is also known.

So, we can find out the value of the stress in a particular direction ϕ . From this equation, simply by doing I by having 2 back reflection pinhole photographs in one case; the photograph X-Ray photographs made by having the X-Ray incident in a normal direction or to the surface and the second case by having the X-Ray incident at an angle ψ to the normal of the surface considered.

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So, from these 2 values, it is possible to figure out what should be the value of $\sigma \phi$. Now the above is the basis of what is known as the double exposure method of stress analysis by the back reflection pinhole method.

The figure here illustrates the set up for the double exposure method. Say for example, this is the specimen surface and this is normal to the specimen surface. So, what we do? we take the experimental stressed material and smear on that stressed material powder of a reference material. So, that when you do the back reflection pinhole photography we will have Debye rings from both the stressed material and the known reference material for which the lattice constants and other parameters are known. So, in that case, what happens is we do 2 exposures the first one by giving the X-Ray, then allowing the X-Rays to fall normally on this specimen surface in this particular direction. So, the first exposure is by allowing the X-Rays to fall on the specimen surface in this direction normally and prepare the pinhole photograph.

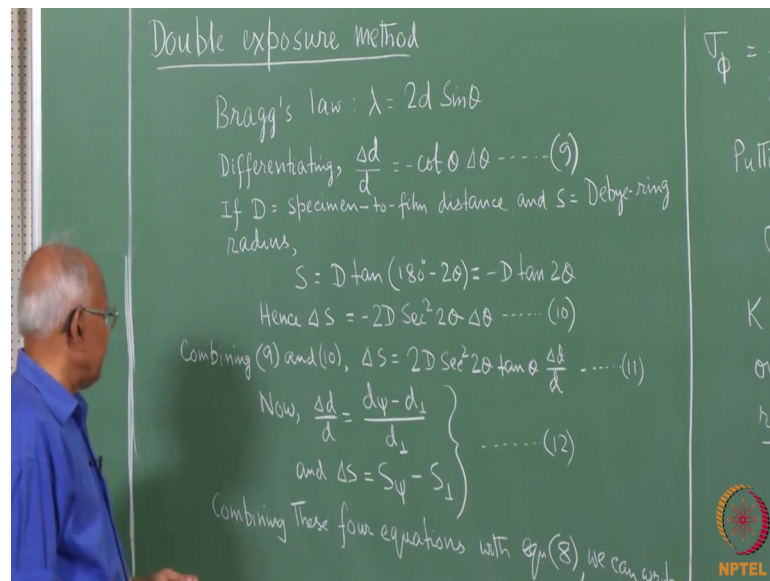
In the second case we allow the X-Rays to be incident at an angle ψ to the normal to the surface as over here. So, X-Rays are allowed to fall in a direction at a direction ψ to the normal to the specimen surface this is the film and on the film whatever you get is showed over here on a magnified scale. So, this is the pinhole through which the X-Rays come and go through and falls on the specimen now this particular ring dividing is due to the specimen sake and this particular Debye ring is from the reference material.

Now, if we look at the Debye ring from the specimen, we will find that it is not perfectly circular. The reason is this; you see when we do this incidence angle pinhole photography; then say for example, we have got 2 series of atomic planes. Now this is one series and this is another series. Now what happens is the strain in the direction normal to the atomic planes will vary with the angle from the normal direction. So, you say that in this particular case and in this particular case; the strains will not be the same and in fact, we will see that in this particular case, here you know when we talk about this particular case the plane normal is given by n_1 and in fact, this plane normal will not be at an angle ϕ , but at an angle $\phi + \alpha_1$.

Again on the other hand when we talk about this series of planes, in another grain giving raise to the back deflection pinhole pattern you know the normal is n_2 and this is making an angle $\phi - \alpha_2$ with the specimen normal now because of this we are not

going to have a perfectly circular Debye ring from the specimen. So, in this case we will find that the measurement of the radius of the Debye ring S_1 is not going to be the same as S_2 . So, normally when we want to measure the Debye ring radius; we measure it from S_1 because from this side we know the value of S is more susceptible to strain. So, this is much more sensitive to stress or strain and that is a reason why we always measure the Debye ring from the specimen from S_1 .

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So, this is what we do now if we go back to Bragg's law λ is equal to $2 D \sin \theta$ now differentiating we get ΔD by D this will minus cotangent θ $\Delta \theta$ our equation nine now if D is the specimen to film distance and S is the Debye ring radius then you can write for the back deflection pinhole method S is equal to $D \tan 180$ degree minus 2θ is equal to minus $S \tan 2 \theta$ hence ΔS will be equal to you know by differentiating minus $2 D \sec^2 2 \theta \Delta \theta$ this is our equation 10.

Now, if we combine the equations 9 and 10 then ΔS is equal to $2 D \sec^2 2 \theta \tan \theta \Delta D$ by D this is our equation eleven now we already know that ΔD by D will be equal to D_ψ minus D_\perp divided by D_\perp and ΔS is equal to S_ψ minus S_\perp ; now combining these 4 equations with equation 8.

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$$\sigma_{\phi} = \frac{E(S_{\psi} - S_{\perp})}{2D(1+\nu)\sec^2 2\theta \tan \theta \sin^2 \psi}$$
 Putting $K = \frac{E}{2D(1+\nu)\sec^2 2\theta \tan \theta \sin^2 \psi}$ ----- (13)

$$\sigma_{\phi} = K(S_{\psi} - S_{\perp})$$
 ----- (14)

K, known as the "stress factor", can be calculated once and for all for a given specimen, given incident radiation and given specimen-to-film distance.

(9) ... and $S =$ Debye ring
 (10) ... $\tan \theta \frac{\Delta \lambda}{\lambda}$

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We can write it down as σ_{ϕ} will be equal to E multiplied by $S_{\psi} - S_{\perp}$. S_{ψ} means what S_{\perp} means the Debye ring from the specimen during inclined incidence x radiation. So, this is the situation when we are having an inclined incidence photography that refers to the radius of the Debye ring minus S_{\perp} what is S_{\perp} stands for the Debye ring from the specimen when there is normal incidence photography from the material. So, now, when we have a normal incidence photography; that means X-Ray falls on the surface in a normal way and we produce the Debye ring. So, S_{\perp} refers to that and S_{ψ} refers to the value of this S_{\perp} over here during inclined incidence X-Ray photography.

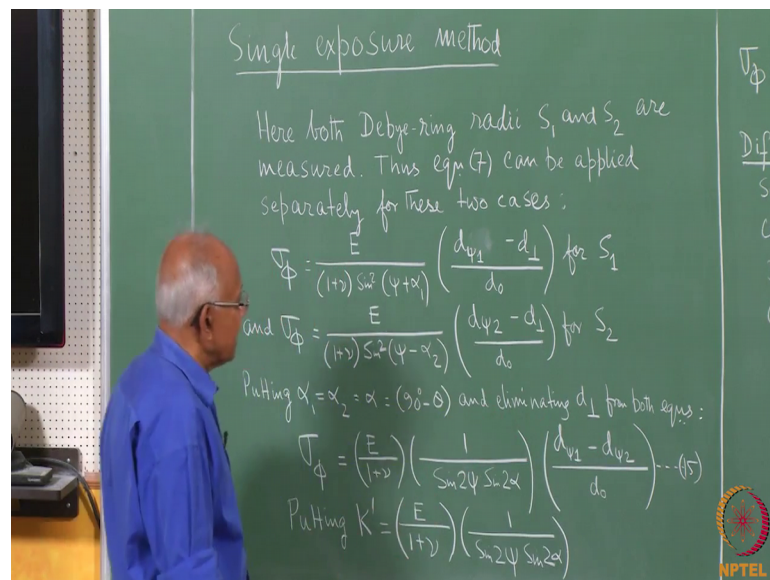
Now, if we put K is equal to E by $2D(1+\nu)\sec^2 2\theta \tan \theta \sin^2 \psi$ you see because all the quantities are fixed. So, if we do that we can write down σ_{ϕ} is equal to K multiplied by $S_{\psi} - S_{\perp}$ equation 14. So, this is basically the most important equation in order to find out the stress in a particular direction in a stress material by doing back deflection pinhole photography in 2 different ways one by you know the normal incidence another by inclined incidence of X-Rays. So, you say that here if we measure the Debye ring radius S_{ψ} .

So, in this case as I already said the Debye ring radius is normally shown by measuring the S_{\perp} because this side you know is more susceptible to strain and that is a reason why you know S_{ψ} is nothing, but this value S_{\perp} and exact value of S_{\perp} can be easily found

out with comparison with the line from the reference material for which the lattice parameters are already known. So, we can accurately measure S_1 by comparing with S_R and that value is put over here and S perpendicular is the value of the Debye ring which is obtained by taking a normal incidence x radiation and measuring the value of Debye ring radius they are from.

Now, this K is known as the stress factor and this can be calculated once and for all for a given specimen a given incident radiation and given specimen to film distance once we do it; we can use it over and over very often the double exposure method of stress analysis by back reflection pinhole method is not used instead we can have a single exposure method. So, to say which takes much less time than the double exposure method now in the single exposure method we get rid of the pinhole photograph using the normal incidence of X-Rays. So, if this is the specimen surface in the single exposure method; we do not use the normal exposure normal incidence of X-Rays, but instead we have only one pinhole photograph using the inclined incidence of X-Rays. So, in this particular method it is necessary to measure not only the value of S_1 , but also the value of S_2 ,

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Now if that is the case then equation 7 can be applied separately for these 2 cases of S_1 and S_2 .

Therefore we can write σ_ϕ equal to E divided by $1 + \nu \sin^2 \psi$ plus α_1 as we can see here for S_1 the angle; angle from the specimen normal is ψ plus α_1 then d_{ψ_1} ; that means, the inter-planar distance which has been measured from S_1 side minus d_{\perp} perpendicular divided by d_0 you know the equation 7 can be written in this manner for S_1 . We will see that the d_{\perp} perpendicular will be eliminated. So, that you know there is no need of having the normal incidence X-Ray method and σ_ϕ for the second case for S_2 will be E by $1 + \nu \sin^2 \psi$ minus α_2 because here the angle from the plane normal is this. So, it will be ψ minus α_2 multiplied by d_{ψ_2} ; that means, the value of inter-planar distance calculated from the S_2 side minus d_{\perp} perpendicular by d_0 , this is for S_2 . Now since α_1 α_2 ; these are very small; we can write α_1 equal to α_2 equal to α and this is equal to 90° minus θ the Bragg angle.

And if we eliminate d_{\perp} perpendicular from both these equations, then we find the σ_ϕ can be written as E by $1 + \nu$; the whole thing multiplied by $1/\sin 2\psi$ $\sin 2\alpha$ into $d_{\psi_1} - d_{\psi_2}$ by d_0 . This is our equation 15. Now if we put K' prime equal to E by $1 + \nu$ multiplied by $1/\sin 2\psi \sin 2\alpha$; you know all these things can be calculated and we can find out the value of K' prime; we can write σ_ϕ is equal to K' prime multiplied by $d_{\psi_1} - d_{\psi_2}$ divided by d_0

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method

radii S_1 and S_2 are
eqn (7) can be applied
two cases:
(1) $\left(\frac{d_{\psi_1} - d_{\perp}}{d_0}\right)$ for S_1
(2) $\left(\frac{d_{\psi_2} - d_{\perp}}{d_0}\right)$ for S_2
and eliminating d_{\perp}
 $\frac{1}{\sin 2\psi \sin 2\alpha} \left(\frac{d_{\psi_1} - d_{\psi_2}}{d_0}\right)$

$\sigma_\phi = K' \left(\frac{d_{\psi_1} - d_{\psi_2}}{d_0}\right) \dots (16)$ Here Knowledge of d_0 in Sample is needed.

Diffraction method
Since in a diffractometer, 2θ is measured directly, it is convenient to write the stress equation in terms of 2θ , no terms.
Differentiating Bragg Law, $\frac{\Delta d}{d} = -\frac{\cot \theta \Delta 2\theta}{2}$
Combining this relation with eqn (8),
 $\sigma_\phi = \frac{E \cot \theta (2\theta_{\perp} - 2\theta_{\psi})}{2(1+\nu) \sin^2 \psi} \dots (17)$
Putting $K'' = \frac{E \cot \theta}{2(1+\nu) \sin^2 \psi}$
 $\sigma_\phi = K'' (2\theta_{\perp} - 2\theta_{\psi}) \dots (18)$

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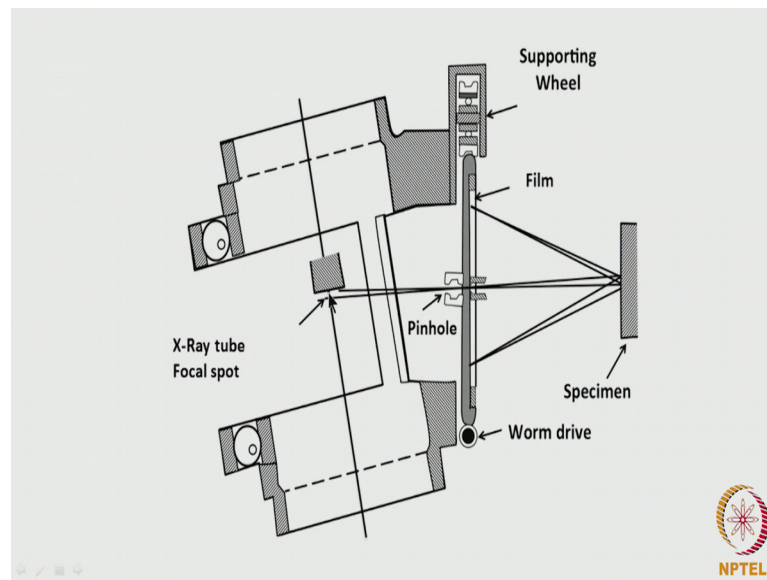
So, in this method, we need to know the value of D_0 in order to find out the value of σ_ϕ this is important; that means, D_0 is the plane spacing in the unstressed material. Normally if we know the lattice constants of the material it is possible to figure out the value of D_0 in any case. So, the advantage of this method is here; you know the total time taken for the measurement is roughly half of the time taken in the double exposure method because here we are not carrying out the normal incidence X-Ray back reflection pinhole photography we are using only the inclined X-Ray method for the back reflection pinhole photography.

Therefore this method's single exposure method about 50 percent of the time taken in the double exposure method, but the error in the single exposure method is 2 to 3 times the error that we encountered in the double exposure method. Normally if we use a steel sample by double exposure method; the amount of error is about plus minus 32; 35 MPA for steel samples, but as we can see that in the constant K values you know; there is always this factor E the Young's modulus. So, naturally the error will be much less for a material which has lower Young's modulus value; therefore, say if we take aluminum alloys the error in the measurement of stress by the double exposure method will be much less than in case of measurement on steel samples. So, to say that is another method by which we can measure stress and that is the diffractometer method.

Now, since in a diffractometer 2θ is measured directly it is convenient to write the stress equation in terms 2θ not in terms of D . So, if we differentiate the Bragg law $\frac{\Delta D}{D}$ is minus cotangent θ $\frac{\Delta 2\theta}{2}$ combining this relationship with equation 8, we can write σ_ϕ is equal to $E \cot \theta \frac{\Delta 2\theta}{2}$ perpendicular minus 2θ ψ divided by $2 \sin^2 \psi$. Now you see due to the presence of stress in the material; you know they are; there will be a shift in the position of the lines; you know there is a change in the D values and that will automatically lead to a shift in the 2θ values. So, this is $A_{2\theta}$ value when we do it under normal incidence or X-Rays and this is $A_{2\theta}$ value which we find out under the inclined incidence condition.

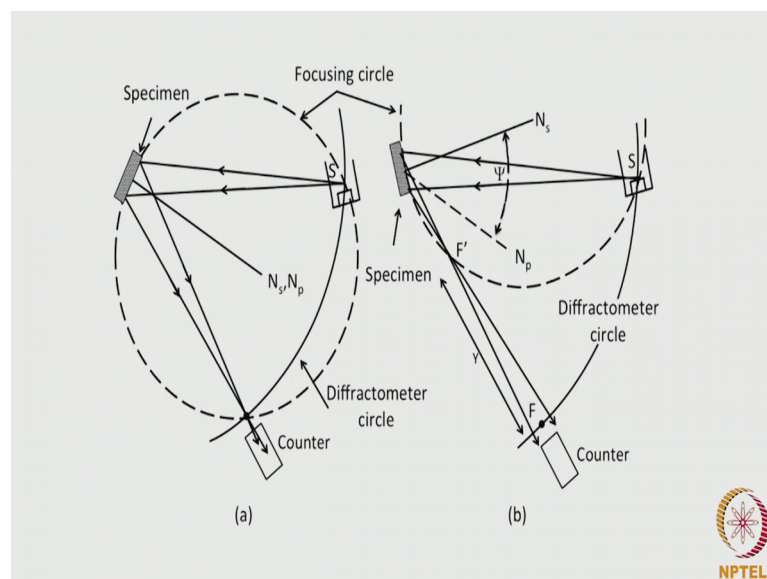
Now, if we put K'' is equal to this value which can be calculated; we can write the σ_ϕ is equal to $K'' \frac{\Delta 2\theta}{2}$ perpendicular minus 2θ ψ . So, in this case in the diffractometer method it is simply to figure out what is the; you know what is the shift in the value of the 2θ angle.

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Now, here you know it is a schematic of a back reflection pinhole camera this is the X-Ray tube focus spot X-Ray tube this is a pinhole this is the specimen and this is the film. So, the back reflected rays are obtained in this manner.

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So, this is you know the schematic of a back reflection pinhole camera and this shows the angular relationships between the specimen the counter you know the focusing circle etcetera in the 2 cases.

Here this is for in the diffractometer; here in the diffractometer, this is the condition you know for normal incidence diffraction and this is for the inclined incidence diffraction. So, we can see here under normal incidence; if this is the specimen and this is the diffractometer circle; you know, this is the focusing circle focusing circle is always tangent to the specimen and this is the location of the counter. So, in the normal incidence method you know the perpendicular to the specimen and perpendicular to the plane which is diffracting you know it is the same.

But what happens when we have the incidence you know inclined incidence X-Ray diffraction there the specimen is rotated by an angle ψ as we can see here. So, the N_P the normal you know and the N_S N_P ; the normal to the plane and N_S the normal to the specimen they are quite different here. So, N_S changes by an angle ϕ . Now in this case, the diffracted beam you know gets focused at F_ϕ you know much before it reaches the counter. So, what normally is done you know; if in this particular case, if it is focused at F' then in the counter; the intensity will be much less. Now in order to avoid that we can use a bigger slit here, but in that case the resolution will be very poor.

So, the ideal case will be; if we have a small slit over here and you can put the counter some over above there, but normally what is done we put the slit somewhere in between F' and f . So, that will you know in that method we make a compromise between the resolution and the intensity. So, this is this explains as I said how stress is measured by the diffractometer.

Now coming to a conclusion; stress measurement by X-Rays is quite useful when we deal with lot of high localized stresses in a material which varies over a distance from place to place. There X-Ray method is very useful compare to the; you know electrical or mechanical gauges. In fact, for residual stress measurement the X-Ray method is undoubtedly better than the electrical and the mechanical strain gauges.