Acoustics & Noise Control
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Module - 05
Lecture - 09
Frequency Analysis 1

In this class we will be talking about harmonic analysis or frequency based analysis. So, just to recapitulate till now we have been able to derive the wave equation we went into some length in deriving the solution and also explaining the physical import of the solution.

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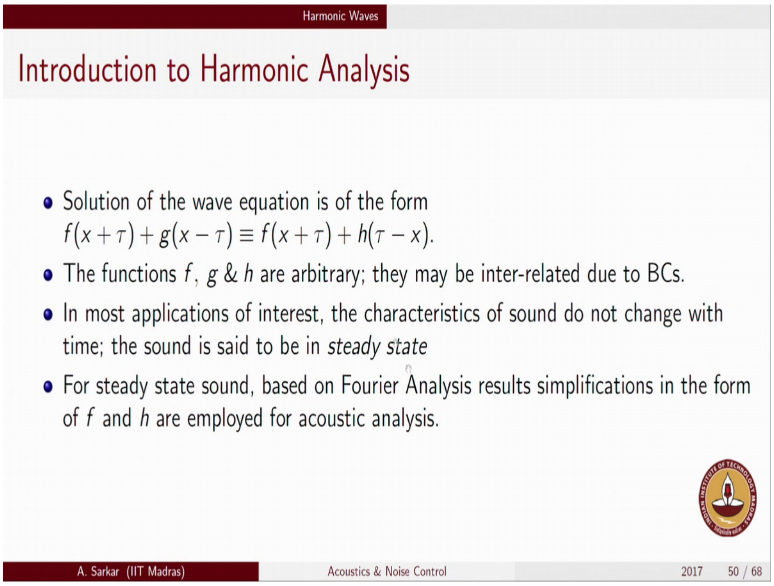

So, the solution of the one dimensional wave equation in particular was dealt with in great details and we came up with this inference that the wave equation solution is of this form if which is a function of x plus tau, and g which is the function of x minus tau. So, this f and g where basically left undetermined. The general solution is such that f and g could be anything at that point, the actual functions f and g where specified only through a proper specification of initial condition which was done in the case of let say (Refer Time: 01:12) solution and also when there is a reflection at the boundary that sort of interrelates the f and g which is what we saw when we study the reflection.

Alternatively we could have just transform the g function into an h function, the 2 being just the flipped versions of each other. So, g of x minus tau is basically h of tau minus x. So, any of these methods adjust perfectly feasible, it is just that we use one of these formalisms probably to understand one aspect of the wave solution in better details and we may use the other form to understand some other aspect, but both of these are completely equivalent.

The important import as I said is that the functions f g and h are arbitrary and they may be interrelated due to certain boundary condition due to Dirichlet and the Neumann boundary conditions is what we talked about and also we found out how to actually evaluate this function from the initial conditions which was the d Alembert solution. But from here on we will specify a specific form of f and g, we will not go for arbitrary forms of a f and g we have in the examples demonstrated the triangular waveform as one specific form, but usually in most acoustic applications that we are going to deal with from here on we are not going to deal with such arbitrary functional form the other we are going to deal with a very specific form of this function f and that is dictated by the application.

So, I make an opening comment and I will elaborate in due course that in most of applications of interest the point is that the characteristic of the sound do not change with time and when that happens we say that the sound is said to be in steady state. Please note that I am not saying that the sound is invariant, I am just sort of loosely qualifying that the characteristic of the sound does not change. What do I mean by characteristics of sound characteristics as we understand colloquially is something about loudness, is something about speech, is something about quality of the sound, but anyway without getting into technicalities at this point it suffices to say that when we mean the characteristics of sound should not change we simply means that these aspects of sound should not change.

For example, when you whistle at a constant tone such as this, in all through this you are hearing a constant characteristics of sound right it is not changing it is not changing in loudness, it is not changing in terms of its frequency or peach tonal characteristic right. Whereas, my talking or if I actually start whistling in here for a song then; obviously, the tonal characteristics and the loudness will change.

So, that would be an example of a non-steady state behaviour or what can be qualified as a transient behavior. Whereas the steady state characteristics implies that perceptibly the perception of sound does not change over the duration that it is being played with a more realistic example from let us say the machine acoustic point of view, would be if you are looking at engine which is running at a constant speed and nothing is changing constant speed constant load everything is same. Obviously, the engine makes noise, but then the noise of the engine does not change provided that the conditions at which the engine is running is not changing right, but then if you are leaving up the engine let us say you put your vehicle in neutral and just keep threatening up here accelerator, then you can hear that the sound characteristics of the engine is changing.

So, that would be an example of a non steady or a transient acoustic behavior; as oppose to the first example where the perception of sound does not change right. So, that is what at this stage we will call it as a steady state behaviour of sound, but steady state does not mean that if you where to take the acoustic pressure plot overtime, it does not mean that the pressure will be date constant that is not imply it simply means that whatever is the variation in time that variation in time is identically repeated right. So, will come to all those aspects as we go along, but at this point I hope you agree that at least in machine noise application there will be a lot of applications where the characteristics of the sound is not changing with time and therefore, it is said to be in steady state. As I am talking in the background your hearing this a air conditioner noise which is sort of just perceptible right, but the characteristics of that ac noise is not changing with time right it is sort of held constant. So, that is again an example of a steady state noise

So, from here on we will be interested in steady state noise as opposed to transient noise right, transient noise is music or music is not noise, but examples which are ruled out from steady state acoustic behaviour is music, voice these are all examples of non steady state signals and you need a different framework to analyze them which is not possible in the framework that we are going to go for. In the present framework we are only be will only be able to analyze the steady state acoustic behavior, and it is pertinent for you to know under what contexts the steady state acoustic behaviour is visible; to that extend you must be very careful in using this assumption in real application problems which you encounter. The reason why steady state makes it very simple is that, what we are going to show right now in today’s lecture is that for steady state sound we will be able to use

Fourier analysis results to simplify the form of these f g and h functions, in particular will here on look for look at the solution in the form of f and h.

So, we will see that based on the Fourier analysis results we will be able to simplify the form of the f and h function, it will not be left as arbitrary or a general from we will now tie it up to a very specific form and that will be based on our results of Fourier analysis.

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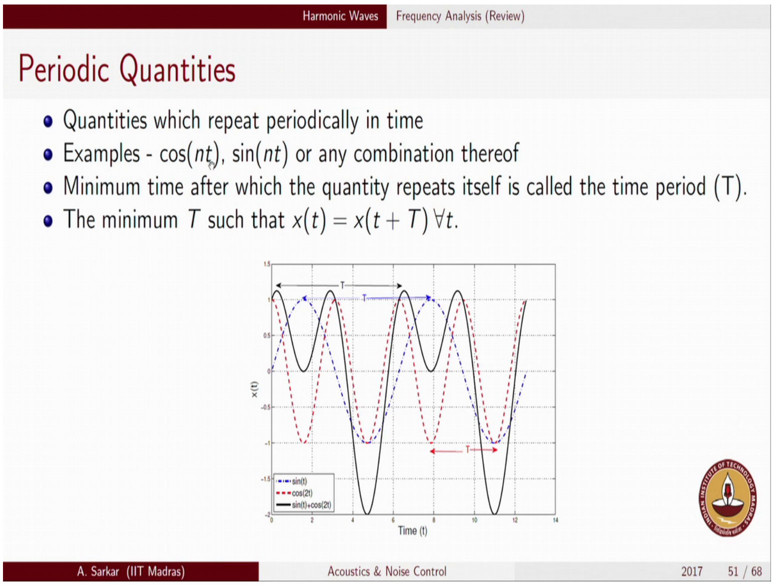

Before we get at the Fourier analysis in depth, let us have a quick recap of the build up to the Fourier analysis and will start with periodic quantities. As the name suggest quantities which repeat periodically in time are called periodic quantities, the obvious example are cos of n t n sin of n t and if you can even have combinations all of this cos and sin functions all of these are periodic quantities, but a more general example would be a arbitrary function f such that f of small t plus capital T is equals to f of T.

So, rather this is the definition of the periodic quantities that any quantity x of t which satisfies this relation that is x at any time t is equals to x at that time plus a shift at time of capital T, this function which is represented in this form is called as a periodic function right and this periodic functions are very important from the concept of analysis of steady state acoustic behaviour or any other statistic steady state quantities. So, this capital T which is the minimum time after which this signal repeats is called the time period, and we will see that frequency analysis is essentially sort of built on this aspect. You should be able to convince yourself that cos of n t plus sin of n t does fit this bill because let us say if you take cos of n t then the minimum time after which it repeats is 2 pi by n right.

So, that should be cleared to you, I will do that calculation on the same.

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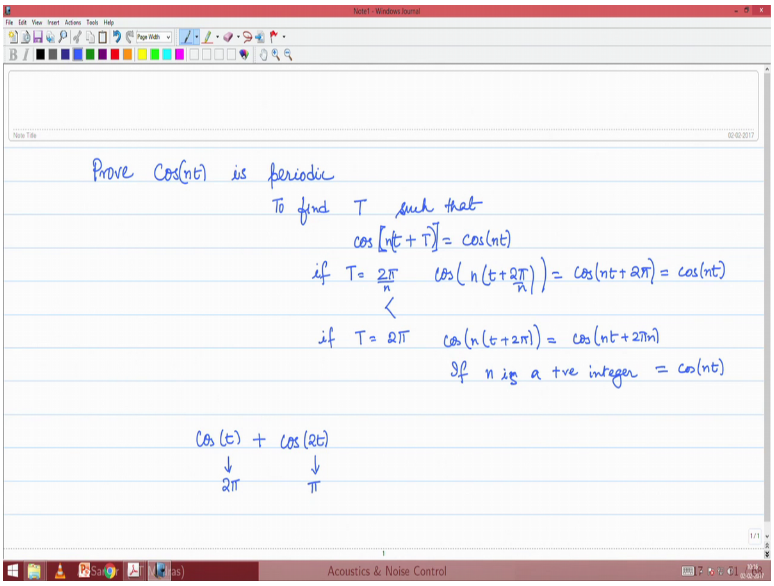

So, we will see that cos the question is to prove cos of n t is periodic right. So, the proof is quite simple all that we have to find is, we have to find some capital T such that cos of n t plus capital T is equals to cos of n t right. So obviously, we know from properties of a trigonometric function that if capital T is chosen to be 2 pi by n, I should say small t plus capital T. Cos of n times if I change this small t to small t plus capital T that should give me the result which is same as the argument being small t. So, if I choose this capital T to be 2 pi by n we get to see cos of n times small t plus 2 pi by n is going to be cos of n t plus 2 pi and that is; obviously, cos of n t from properties of trigonometric function.

So, therefore, 2 pi by n is the periodicity of this function right, remember it is the smallest time after which the repetition will definitely happen. You could also say if I had chosen capital T as just 2 pi, even then you would have seen that cos of n times t plus 2 pi would be cos of n t plus 2 pi n, and if n is an integer then if n is a positive integer then obviously, this will also be cos of n t. But then if you have an ambiguity that whether we should choose 2 pi by n or 2 pi, then you should fall back to the definition of periodicity which says that periodicity is the minimum time possible such that the condition is met the condition being a f of t is equals to a f of small t plus capital T.

So, the minimum time would be this. So, this is this is lesser 2 pi by n has to be lesser than 2 pi because n is a positive integer, and thereby you get this logic that 2 pi by n is the periodicity of this right. Similarly if you have a more involved function let us say like cos of t plus cos of 2 t right. So, the periodicity of cos of t is 2 pi, but the periodicity of cos of 2 t is pi right; however, when you look at the sum of it cos of t plus cos of 2 t the sum of it will have a periodicity of 2 pi right because if cos of 2 t is repeating after every pi intervals of time it is also repeating at 2 pi intervals of time, but cos of t repeats only at 2 pi it does not repeat at pi right. So, therefore, the sum of cos t and cos plus 2 t sum of cos 2 t and cos 2 t will repeat at 2 pi not at pi.

So, the periodicity associated with this 2 pi is called the fundamental periodicity right every other components will have harmonic multiples of these fundamentals. So, these are certain things which hopefully you have come across in a first floor in engineering mathematics in Fourier analysis or if you have taken vibrations these things would have been dealt with I am just briefly touching upon them. So, just to indicate what is the periodicity here I have taken three different functions sin t cos 2 t and sin t plus cos 2 t right. So, the one that is shown in blue is sin t as you can see the distance between peak to peak is this capital T, this is after this is the time interval after which this graph will repeat itself.

So, looking at the graph also sometimes when you are dealing with certain function which is very difficult to write it in terms of analytical form, but then you can always graph it. Looking at the graph also you should be able to decipher what is the associated time period by simply realizing at what time interval you should shift this graph such that the entire picture repeats itself. So, here in the blue graph we say that the time period is marked as capital T. For the rate graph which is cos of 2 t the associated time period will be this peak to peak distance it could be any 2 points which are at the identical locations on this graph.

So, here again we see that the peak to peak distance in the rate graph is halfed to the peak compared to the peak to peak distance in the blue graph right; that is because you are dealing now with a function which is cos of 2 t. Accordingly the periodicity of this is pi where as sin of t has a periodicity of 2 pi. So, the periodicity of the blue graph which is sin of t is more than the periodicity of the rate graph right. Now if you look at a superposition of both of them you see that the waveform repeats only from here to here right which is something like 2 pi right. So, this is 6 and it has started slightly of 0, so this would be roughly at 2 pi which is 6.28.

Similarly, here also this distance is 6.28. So, the black curve will have actually the periodicity corresponding to the lower frequency here you see sin t and cos 2 t, it is made up of 2 components one with a frequency 2 pi the sorry one with the periodicity 2 pi and the other with the periodicity phi with the periodicity pi the sum of the 2 functions will have a periodicity which is comprising which is exactly equal to the periodicity of the component we wearing the largest time period.

So, this is a adequately demonstrated through this illustration.

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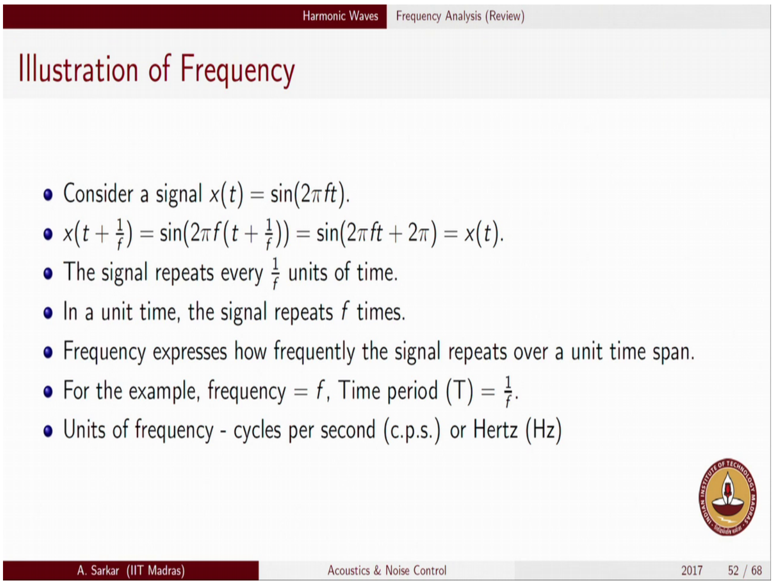

Carrying on, let us now considered a signal of this form. So, instead of cos of n t and sin of n t we will choose to write it has 2 pi f into t, the argument within the sin or cosine function could be written as 2 pi f; then we can easily say that x of t plus 1 by f. So, instead of t I will replace t plus 1by f. So, this is what it stands out and when you do that algebra you get to see a 2 pi popping out here, 2 pi f multiplied by one by f is going to me a 2 pi. Therefore, a sin of 2 pi f t plus 2 pi is again sin of 2 pi f t which is back to x of t. So, what we have here is that if you have a sin function of this form then the periodicity of that sin function is exactly 1 by f. So, whatever is the time period that can be easily sort of memorized or if you can call derived by this simple formula that the time period is 1 by f. F is called the frequency right. So, f will be called as the frequency the units of which is cycles per second or hertz. So, the time period is related to the frequency in a reciprocal fashion as has been shown in this slide.

So, the crucial point to remember is that what has been shown actually here is that the signal will repeat itself, the signal or the function will repeat itself after every 1 by f units of time right; because x of t plus one by f is just the same as x of t, it is just that the entire signal gets repeated after such a shift. So, in a unit time therefore, the signal repeats f time. So, if let us say f is 10. So, in the signal will repeat itself 10 times in a second; if times unit is second right and conversely that the signal repeats every 1 by f units of time will now mean that the time period is 1 by 10, if f is assumed to be 10 just for illustration.

So, in this context if you now look at this interpretation of f that I am bringing out that f basically denotes how many times the signal repeats in a unit interval of time. So, in this context it represents how frequently the signal repeats over a unit time span, and thereby we derive this nomenclature that it is called frequency, because it denotes how many times the signal repeats itself in a unit time span right. So, that is the interpretation of frequency which I bring about at this stage and as I said the time period and frequency are just related trough a reciprocal relation which should be obvious, the units of frequency as cycles per second or hertz provided time is in seconds.

So, here on that will be assumed that the time that we are looking for is in seconds, we are not talking about normalized time as we did in the previous module where we were just trying to extract equation of we are trying to extract the solution for the plane wave equation where we rescale tim, here will stick to our usual definition of time. So, if the usual definition of time we have the units to be second and accordingly the usual frequency units will be cycles per second which is abbreviated as hertz.

There is another way in which we find it very useful to interpret this idea of frequency which is possibly that not obvious at least in the first quotes of engineering mathematics, and that is called the phasor diagrams and that is what I wish to illustrate to you.

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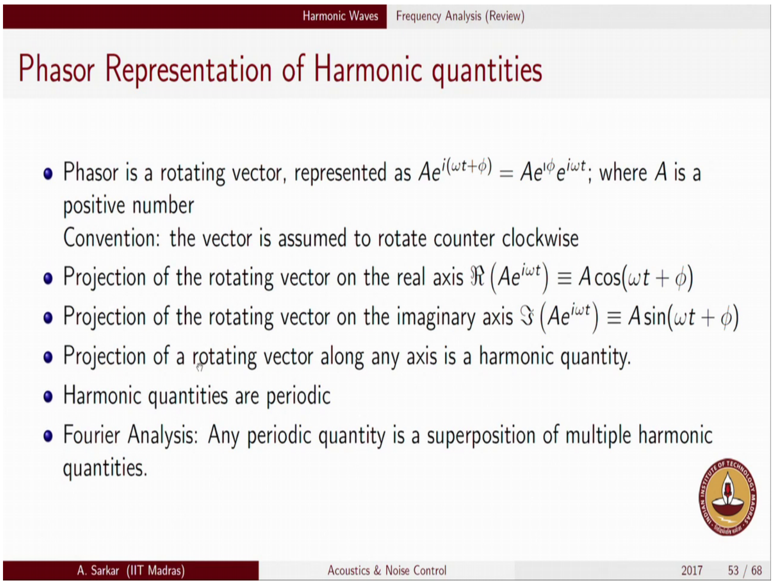

So, phasor is a rotating vector and it is represented in this form A e to the power i omega t, plus phi and as you know by using complex number theory this e to the power i omega t plus phi is also e to the power i phi in to e to the power I omega t right A remains as it is. A is has to be a positive number it cannot be a negative number, it cannot it is not a not even a complex number at this stage. So, what will do is that we assume that this there is a vector which is rotating and the vector length is A, and it is rotating as per our convention in a counter clockwise sense.

So, if you have such a rotating vector which rotates in a counter clockwise sense the projection of that rotating vector on the real axis is just the real part of this complex number. So, the complex number is written as A e to the power i omega t, I have forgotten the plus phi. So, please bear with me on that. So, real part of A e to the power I omega t plus phi is a cos omega t plus phi right. Projection on the imaginary axis similarly will be imaginary part of A e to the power i omega t plus phi which gives me a sin omega t plus phi; we have already seen that sin and cosine are the fundamental examples of periodic quantities. So, what we are doing is that now instead of talking in terms of sin and cosine, we will talk in terms of real or imaginary parts of this sort of a complex exponential. So, this complex exponential is a representation is that which is associated with the phasor.

So, it is not just the projection on the real axis or imaginary axis which happens to be a having this sinusoidal kind of behavior, if you take projection along any fixed axis projection along any fixed axis will be having this sort of a harmonic nature as we will see this is therefore, a very fundamental example of periodic quantities and these are also called harmonic quantities, we will we will just now distinguish between what is difference between harmonic quantity and the periodic quantity. Periodic quantity is anything which repeats itself after a periodic period of time; for example, in here sin t plus cos cosine of 2 t is a periodic quantity it repeats itself, but it is not harmonic.

Harmonic means it has to have a phasor representation, alternatively it has to have a single frequency. Sin t and cosine t 2 t can be represented as a phasor you could talk of cosine 2 t as real part of e to the power i 2 t right you could have talked about sin of t as imaginary part of e to the power i t right you could have also talked about sin of t as real part of e to the power i t minus pi by 2 each of these things are possible.

So, sin t and cos 2 t can be expressed in terms of a phasor, which is in terms of a complex exponential and then take real or imaginary part as his appropriate, but sin t plus cosine 2 t or cosine t plus cosine 2 t you cannot express it in this form, because you required 2 frequency component which is not allowed as per the phasor representation. So, therefore, harmonic quantities are those quantities which can be represented in this complex exponential form. However, there are periodic quantities which are not harmonic; that means, they have multiple frequencies right they may be harmonically related the frequencies may be integer multiples of each other as you saw in the example of cos t plus cosine of 2 t it is periodic, but it is not a harmonic quantity. Harmonic quantity means there has to be a single frequency alternatively it should permit a phasor representation, I think I can write this part for you here.

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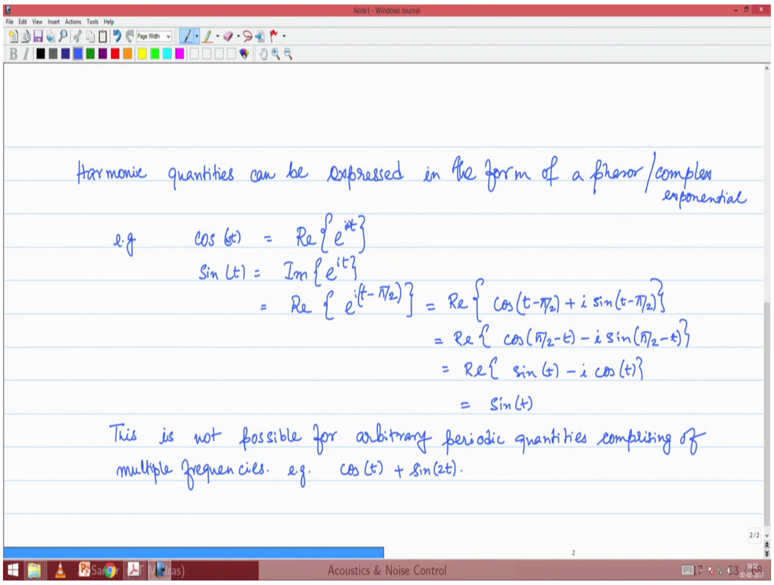

So, harmonic quantities can be expressed in the form of a phasor or complex exponential example cosine of t is just the real part of e to the power i t, sin of t could be expressed as imaginary part of e to the power i t it could also be expressed as real part of e to the power i t minus pi by 2 right just check that out.

Real part of e to the power i t minus pi by 2 is, cos of t minus pi by 2 plus i of sin of t minus pi by 2 right this can be written as real part of cos of pi by 2 minus t, i times sin of pi by 2 minus t. So, that would become real part of sin of t minus i times cos of t and that is sin of t. So, similarly if there are any other frequencies. So, if I put an omega here I could put an omega here also.

So, any quantities of the form cos omega t or sin omega t can have an complex exponential representation, this is not possible for arbitrary periodic quantities comprising of multiple frequencies if there are multiple frequencies; then example of that would be cos of t plus sin of 2 t, you cannot express that into a single complex exponential and thereby it will not be a harmonic quantity, but it will be a periodic quantity nevertheless. So, the important point is harmonic quantities are obviously, periodic, but it is more than periodic in the sense that it is periodic with the single frequency where as periodic quantities may have multiple frequencies also.

So, harmonic quantities are periodic as I said, but the beauty of Fourier analysis lies in this convert statement that it is trivial to understand that any harmonic quantity is a periodic quantity, but what Fourier analysis gives us is that any periodic quantity can be dealt with as a superposition of multiple harmonic quantities right. So, it is not a single harmonic, but it is a multiple harmonic. So, if you are saying on your graph that something is repeating in its pattern every capital T units of time, then what Fourier analysis assures you is that you can bring this up into multiple harmonic components which is what the result of Fourier analysis gives us.

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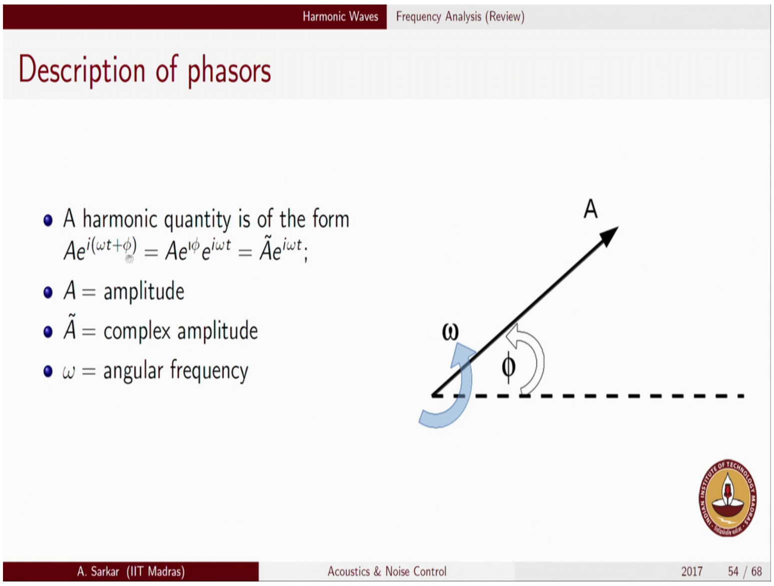

And that is what will quickly review as we go along, but before that let me just illustrate the concept of phasor once again with this diagram. So, what we are talking about is that we have this vector which is rotating and by our convention it is rotating in the counter clockwise sense as is indicated with this curved arrow, it is rotating at a speed of omega right. And phi denotes the orientation of this vector with respect to the real axis let us say at time t equals to 0, at time t equals to 0 the vector arrow is inclined in a counter clockwise sense with respect to the real axis by an angle phi right and what we could do is the following A e to the power i omega t plus phi could be written as A e to the power i phi into e to the i omega t, and e to the power I phi this time if we club these 2 terms we call this A tilde.

So, this A tilde is what is called a complex amplitude because if A is real because of the presence of e to the power i phi which is cos phi plus i sin phi, A multiplied by e to the power i phi is going to be complex it is not going to be real anymore. So, associatedly we call this a tilde as complex amplitude. So, as oppose to real amplitude which only bears the information of the link of this vector, this complex amplitude which is in its magnitude a, but if you take the phase associated with it the phase will be phi. So, the complex amplitude will not only indicate the length of the vector it will also indicate the orientation of the vector at time t equals to 0. So, this complex amplitude will be is what is going to be used predominately in Fourier analysis and also in our analysis of waves.

So, we will embed in other words the phase information right the phase information phi will be embedded in this complex amplitude as oppose to real amplitude. What we mean by complex amplitude is precisely this the complex amplitude magnitude of the complex amplitude is the amplitude which you would measure of in an (Refer Time: 33:01) scope being the amplitude of the signal, but the phase part of the complex amplitude will denote the phase of your harmonic quantity, which is in other words pictorially just the initial orientation of your phasor.

So, A is called the amplitude, but A tilde is called the complex amplitude, omega is called the angular frequency there is one needs to be a little careful in the usage of this word frequency there are 2 types of frequency that we have encountered by now, one is the angular frequency as I am talking here and the other was f which is circular frequency sorry omega is the angular frequency or circular frequency and f is the frequency in cycle per seconds or hertz, and it is not that omega and f are same. In fact, omega and f are related and I would like you to derive what is the relation between omega and f.

So, if there is 2 notions of frequency one is the angular frequency by angular frequency we simply denote what is the rate of rotation of this phasor right. So, in effect if you are taking the real part of this let us say phasor what you are going to get is a cos omega t plus phi right; and what we had shown earlier was that we had equation of this form representing simple sinusoidal or harmonic quantity. Therefore, omega turns out to be 2 pi f. So, the exercise is just solved in 2 minutes.

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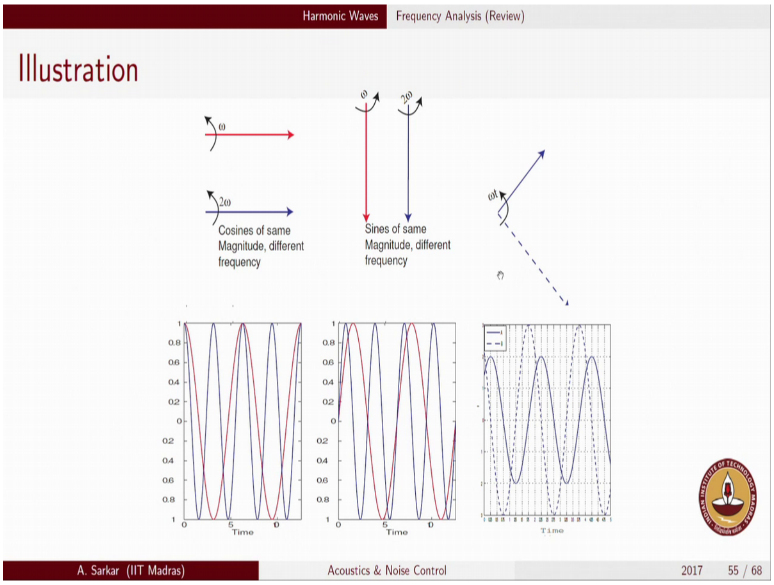

Phi is called the phase and let us take a few examples here. So, here I have a phasor which starts from the horizontal position as is indicated in this illustration, I also have a blue vector which starts from the horizontal position, but it rotates at 2 omega as oppose to omega for the red vector.

So, accordingly the real part of this rotating vector will be cos of omega t and the real part of this rotating vector will be cos of 2 omega t. So, this is the graph corresponding to this phasor. So, the real part of this rotating vector is shown in the red line, you say that the time period associated with the red graph looks very close to 6.28 right this is 5 this is 10. So, this is somewhere at 6.28. So, this is the time period associated with the projection on the real axis of this red rotating vector which is at rotating at omega, omega has been taken as one when I when I did draw this graph. Now if on top of it if I look at what happens when you have a blue vector which starts from a identical position at time t equals zero, but rotates a 2 omega; obviously, it will complete the cycle faster than the red vector right and it will complete the cycle at half the time because it is rotating at the double the speed.

Therefore, by the time the red vector comes through one cycle, the blue rotating vector would have come through 2 cycles, which is exactly what is seen here. If you look at this point by this point the blue vector has actually travel 2 cycles right and accordingly the time period for the blue phasor is half of the time period of the red one right. So, the time period is therefore, given by this quantity which is something like pi right. Now let us see what happens if instead of orienting these vectors at horizontally to start with we orient this vectors at minus pi by 2. So, the starting position this time is not aligned to the positive x axis rather it is aligned to the negative y axis or the negative imaginary axis.

So, in this case it starts from here just imagine as this rooted vector rotates the projection of it on the real axis will grow, at this instant what happens is that the projection of this red vector on the real axis is red 0, as it rotates the projection the shadow on this horizontal axis will keep increasing. So, it will behave like a sin right and after it has travelled pi by 2, it has actually taken the position of indicated by this diagram.

So, in other words this rate phasor is lagging with this red phasor by an angle of pi by 2. Whatever happened to this red phasor will also happened with this red phasor, but after a lapse of pi by 2 if that is omega is taken as one which is what has been drawn here right. So, what you must understand is that both cosines and sins are different; in condition of the same phasor the only difference being that you are you are starting time is different at the starting time rather at the starting time t equals to 0 the orientation of the phasor is different, but other than this the effect of cosine and sin is completely equivalent and therefore, it is we are actually going not going to make a big pass about a cosine and sin both are complex exponentials except for the fact that they have a phase difference.

So, omega here is a for omega equals to one I have again plotted out the graph, here you can see the periodicity is somewhere here. So, it started with 0 it comes back with 0 roundabout here which is again 6.28, and by that time it has come here the blue phasor which is now started with its alignment being in the minus y axis and rotating at twice the speed; by exactly identical arguments by this much time it would have crossed 2 cycles and which means that the time period associated with this blue signal would be half of it which is pi.

So, this is what are the some extremes that I have shown and here is one other example where I have taken 2 of these Phasors which are both at different both at different orientation as well as different amplitudes. So, both of them as you see are looking like a sinusoid except for the shift. Cosine starts with its maximum sins starts with its with the 0 value, but if you have an arbitrary orientation of this phasor at time t equal to 0, which is what is shown in this diagram it will neither start from its maximum point not start from its minimum point, but other than that the nature is completely same it is going to repeat again at the same rates.

So, same is taken for this plot which is shown in this dotted line right. By the way the omega that has been taken here is different looks like from here to here it is about omega is not taken as 1 here. So, the periodicity here is looks like 2. So, this is 0.25 and this is 2.25; this is 2.25. So, the periodicity is 2 here. So, accordingly the omega has been adjusted; periodicity remember is 2 pi by omega right. So, the 2 pi by omega quantity has been taken as 2 in this plot where as in these plots which has been generated using a different software the omega has been taken as 1.

Similarly for the dotted phasor: the dotted phasor you see now it has a greater amplitude also the fact that the length of the rotating vector is different has manifested with a higher values of the peaks and of the tracks of this signal right. This function has now shown a higher values because it is having a greater length, the shadow of this as it will fall on the real axis will be higher at some point of time and the phase also is different it has not started with the with the sin position or the cosine position it is started with some arbitrary position.

So, in some of the assignments you will be asked to plot the functions associated or the graphs associated with different phase and you should be comfortable with the phasor representation of harmonic quantities.

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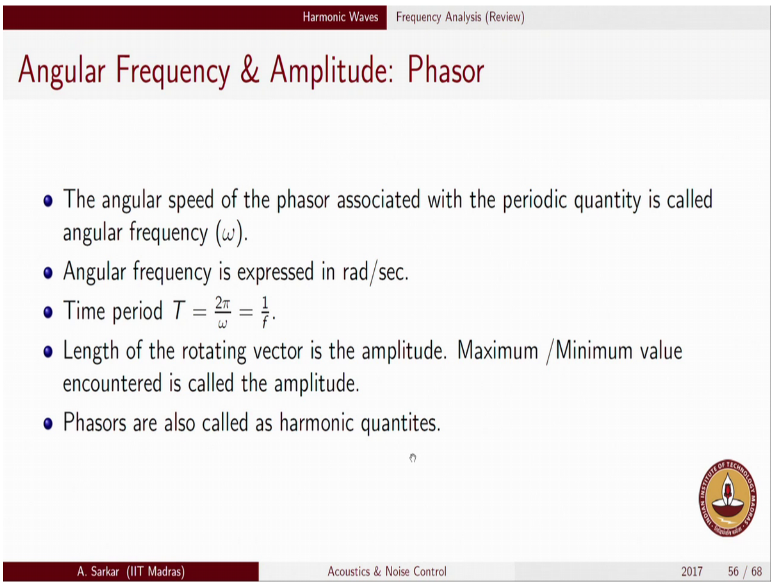

So, just to consolidate our ideas: so the angular speed of the phasor associated with the periodic quantity is called the angular frequency. Angular frequency it is just the speed of this rotating vector and therefore, the units of it will be radians per second not cycles per second not hertz. So, please do not confuse between frequency and angular frequency; angular frequency is in radians per second time being measured in seconds.

So, the time period is 2 pi by omega it is also given as 1 by f. So, omega and f are related therefore, has omega equals to 2 pi times f; and the length of the rotating vector is the amplitude maximum minimum value is in that is encountered in any graph of the signal is also called the amplitude the maximum and minimum value will remain the same because there is no dc component there is no constant component. So, the maximum and minimum value is also called the amplitude, which effectively is the same as the length of the rotating vector.

Phases are also called harmonic quantities right because as we discussed basically you can have phasor representation possible only if you have a single harmonic quantity if you have more than one harmonic quantity superposed then it is a periodic quantity, but it is not a phasor, it is not a simple harmonic quantity in other words.

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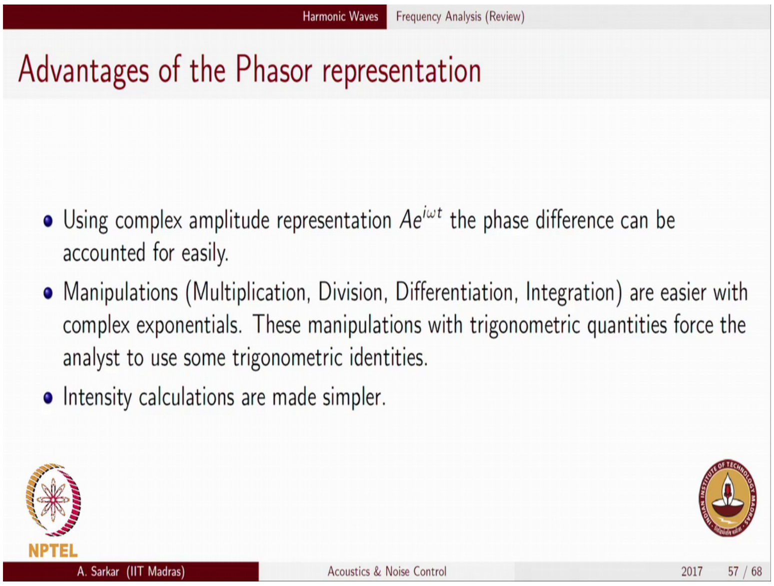

So, why do we go for this phasor representation as we will see that the use of this complex amplitude representation A e to the power i omega t, and I do not put this tilde again and again when I write A e to the power i omega t from here on it should be assumed that a is A complex number as opposed to a real number right because now we understand that there is an interpretation for complex amplitude. So, when we write it in this form the first advantage in at least the notational terms is that, we are making a very compact notation for our mathematical process and a compact notation clubs in this quantity A both the amplitude as well as the effect of phase.

So, the phase difference can be accounted for very easily, and phase difference is very important will dealing with dual channel measurements or in dealing with measurements which are actually product of 2 harmonic quantities and we shall soon see intensity is one place where this effect of Phasors will be leading us to a very important simplification. So, the algebra will be definitely much more simplified if we use this complex number algebra rather than the trigonometric form.

In trigonometric form we have to remember trigonometric formulas associated with sin square cosine square and all those things, but these formulas turn out to be much easier when we deal with complex number algebra. So, the complex number algebra hopefully you will find it easier to use them the trigonometric. So, you this manipulations with trigonometry forces quantities force the analyst to use trigonometry quantities if you do not use the complex algebra notation rather you use the trigonometric notation then you will have to sort of manipulate the trigonometry quantities which happens to be a little more difficult; and as we shall see probably a few classes down the line intensity calculations and impedance calculations will be turning out much easier with this analysis.

I think we will stop here for the class next class we will get deeper into the aspects of Fourier analysis.

Thank you.