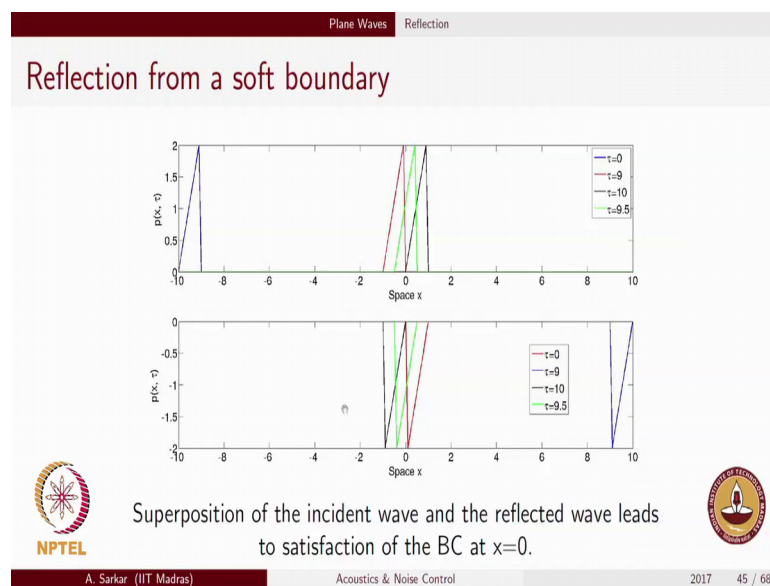


Acoustics & Noise Control
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Module - 04
Lecture - 08
Reflection Of Plane Waves 2

In the last class we talked about the Reflection of a Plane Wave.

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And in particular we talked about reflection from a soft boundary condition. So, the problem if I recall is going to be in this fashion. So, at τ equals to 0 there is an incident wave which is shown in the blue graph out there, and progressively as time progresses it comes in this at τ equals to 9 it takes this red form, but in order to maintain the boundary condition at τ equals to 0, which is the pressure equal to 0 condition and that is why we call it as a soft boundary, you must have another wave which just negates this effect of the incident wave.


So, as these 2 waves crossover the in totality they are in superposition they enforce the boundary condition at x equals to 0.

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Plane Waves Reflection

Reflection: Dirichlet condition

- Solution of the wave equation is given by
$$p(x, \tau) = f(x + \tau) + g(x - \tau) = f(x + \tau) + h(\tau - x).$$
- At $\tau = 0$, $p(x, 0) = f(x)$ is given for
 $x \in (-\infty, 0) \implies p(x, 0) = f(x) = 0 \forall x \in (0, \infty).$
- In order to satisfy the BC, $p(0, \tau) = 0$, we must have
 $f(\tau) + h(\tau) \equiv 0 \implies h(\tau) = -f(\tau) \implies g(\tau) = -f(-\tau).$
- Reflected wave is a flipped and sign reversed form of the incident wave.
- Reflected wave has the same amplitude.
- Power transmitted by the incident wave = Power transmitted by the reflected



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So, this is the origin of the reflection due to Dirichlet condition. So, we run through the slide in the last class also. So, we will now look at one very important fact that we will note is that the reflected wave has the same amplitude as the incident wave, that is though it is a flipped in its sign and it is also reversed, but the point is the amplitude of it is remaining the same; what has been plus 2 is now minus 2.


So, in terms of energy content in the incident wave and the energy contained in the reflected wave both are just the same. We will do a better analysis of energy calculation as we go along, but suppose I say to say at this point the amplitude of the waves is directly related to the energy that the wave is carrying from one direction to the other, in particular that term will be called as intensity, but at this point we just make a note that the amplitude of the incident wave and the amplitude of the reflected wave has got to be just the same.

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Plane Waves Reflection

Reflection: Neuman Condition

- Solution of the wave equation is given by
$$p(x, \tau) = f(x + \tau) + g(x - \tau) = f(x + \tau) + h(\tau - x).$$
- At $\tau = 0$, $p(x, 0) = f(x)$ is given for $x \in (-\infty, 0) \implies p(x, 0) = f(x) = 0 \forall x \in (0, \infty)$
- BC at $x=0$, $\frac{\partial p}{\partial x}(0, \tau) = 0$
- In order to satisfy the BC, $f'(\tau) - h'(\tau) = 0$
- As power transmitted in the incident wave = power transmitted in the reflected wave, integrating we have $f = h$
- The reflected wave g is a flipped form of the incident wave (no sign reversal)



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So, we will see same condition now happening in the Neumann boundary conditions. So the Neumann boundary condition as was elaborated in the last talk is the boundary condition where in the gradient of the variable of interest that is pressure in this case, will be enforced to be 0. Again we pick up the thread from the solution of the wave equation the general solution as we know is an f function and g function, where both of them can be arbitrary, but the argument of the f function is x plus τ where as the argument of the g function is got to be x minus τ . But then, we sort of juggled around with this variable which is the argument of the g function and we said that if we redefine a new function h , and we say that h of minus x is basically g of x or equivalently g of minus x is h of x then g of x minus τ will be h of τ minus x .

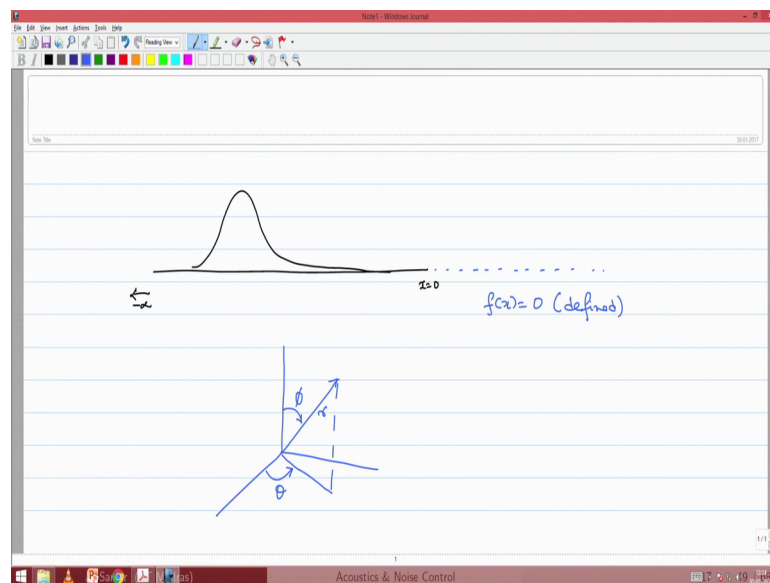
So, we might interpret the total solution in either of these 2 form f and g or f and h ; f has the same signs in both x and τ . So, there is no ambiguity here we have a negative sign accompanying the τ variable. So, that is flipped around when we go with the h function where in we say that the τ or the time variable has positive sign and the space variable has negative sign. But please remember both are forward travelling waves one implies the other, in a forward travelling wave the arguments are such that space and time has opposite signs and in the backward travelling waves space and time has the same sign.

So, anyway that was talked about in lot of elaborate details in the earlier classes. So, now what we wish to understand is the following that is what is the effect of a boundary

condition, but before we do that let us consider that at the initial time we have a just a incident wave the backward wave is not of interest to us. So, we take an incident wave which is travelling in the positive x direction. So, at time tau equals to 0 we are dealing with the condition which is given, but that is given over a semi infinite range that is in the region which is minus infinity to 0.

Remember we are dealing with at least one boundary which is x equals to 0. So, we are this time concentrated not on an infinite region, but rather on a semi infinite region. So, x between minus infinity to 0 is known to us which is given as a function of a we call it f of x. We simply extend this function to the entire domain by saying that p of x comma 0 will be f of x; and that f of x will be defined as 0 for all x in the positive. So, let me elaborate what I have done here.

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So, what we had to start with is a region of interest which is from minus infinity to 0. So, this is x is equal to 0 and we had the region of interest from minus infinity to 0. So, within this region there was some waveform which was the specified, and actual region of interest is minus infinity to 0, but just to do our mathematical calculations we will extend it even in the positive side and in the positive side f of x will be defined to be 0.

So, we are simply sort of 0 padding the function and on the positive half which is just a mathematical artifact we are considering a 0 padded f of x. So, that is what is meant by this statement and here comes the boundary condition. So, boundary condition which is

the Neumann condition at x equals to 0, is that the space derivative of the variable of interest which is the acoustic pressure for all times has to be 0 and this boundary condition has to be enforced at x equals to 0. So, that is why we write it in this form the space derivative of pressure at x equals to 0, but for all times for all normalized times τ will have to be 0, this is the boundary condition that we have to enforce. But then if you simply take a derivative of let us say p of x comma τ ; see always as I said the general solution is this or this in whatever way you may look at it, the point is this general solution has to be satisfied together with the general condition there has to be a satisfaction or enforcement of the boundary condition.

So, whatever solutions you may think of it has to fit the bill that the solution has to be of this form right, and now when you take the space derivative of this form or you will get to see f' prime τ which is fine and h' prime τ come prime τ comes with the minus sign that is because the minus sign leads the variable x . Remember you are doing differentiation with respect to x , within here x comes with a positive sign there is no problem whereas, within here the x comes with a negative sign.

So, the derivative while using chain rule will lead to a negative sign here right; and the since we are evaluating this at x equals to 0. So, therefore, the variable x does not feature here. So, this is the derivative evaluated at x equals to 0 for all times. So, therefore, you have this condition f' prime τ minus h' prime τ equals to 0. Just like last time we needed to enforce f τ and h τ together should get you a 0 condition now you have to enforce f and h to be such that f' prime τ and h' prime τ should be equal such that they will negate each other in terms of their derivatives, and that will lead to be the satisfaction of the boundary condition.

So, exactly the same approach just that here you wish to have with have the negation in terms of derivatives rather than the original function itself. So, here before we can integrate this out what we could directly say possibly is that we could integrate this out in terms of the variable τ and that would have let us to f τ is equals to h τ , but possibly an integration constant which will still remain undefined because there is no way that you can calculate this, but here we will invoke a physical argument that we would recall that even for our Dirichlet boundary condition we found that the power transmitted the energy transmitted in the incident wave has got to be the same as the power transmitted by the reflected wave. So obviously, that makes sense in terms of our

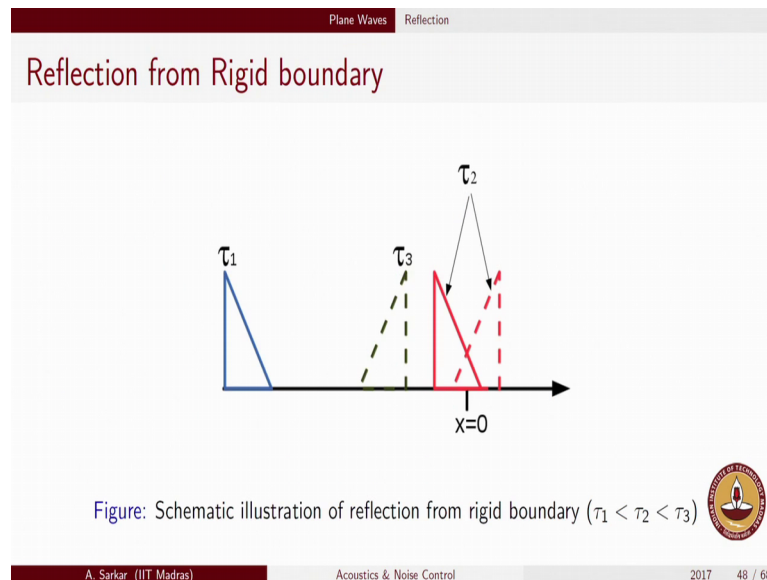
physical understanding also, we expect that the reflected wave should carry no more power than or no less power also then the incident wave was carried.

So, as a result the integration constant that will feature upon integrating this equation with respect to τ is going to be the integration constant will be basically 0, and that will lead us to the condition f equals to h which is just an integration of this equation, integrate this with respect to τ you will get f equals to h plus c , but that c has got to be 0 because if c is not equals to 0 then there is an imbalance in the amplitude between the incident and the reflected wave, and the imbalance in the amplitude of the incident and the reflected wave is going to lead to an imbalance of the power transmitted between the incident and the reflected wave that is not possible then there for that integration constant has got to be 0. So, the moral of the story is again we get f equals to h remember in the Dirichlet condition we got f equals to minus h right.

Therefore, we see that this time the reflected wave is just a flipped form of the incident wave and there is no sign reversal. The flipping does take place, but the sign reversal does not take place if you just look back once more to the Dirichlet condition here there was both flipping as well as a sign reversal, the reflected wave was coming in the negative value coming with negative values of pressure. But that was because the pressure had to be negated. Whatever positive pressures were getting built up by the incident values would have been negative by the negative values of the pressure, with the analysis that was just discussed in this slide we now see that the f and g .

Remember we are talking in terms of f and g and f and h interchangeably, g we understand is a flipped form of h right which is basically the way we are understanding a forward travelling wave, but whether it is h or whether it is g there is no change in sign, f is identically equals to h and g is a flipped form or a mirror image of h the mirror image being the vertical axis.

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So, at the end of the day this time I am just drawing a schematic plot. So, what you will see is that as usual the blue line represents an initial time instant I call that tau 1, this waveform would have propagated to a later time instant here right which is shown in solid line. Now at x equals to 0 if there were only an incident wave then you see there is a slope at x equal to 0 corresponding to the slope of this red solid line right. So, this cannot be true because the boundary condition x equals to 0 will be violated if there were only incident wave right. So, to save the day a reflected wave will have to be brought in to save the day at this stage and what is demanded is that whatever is the slope of this solid line the reflected wave should create the opposite slope, which means it has to be symmetric right it has to get the slope which is just the mirror image of the slope of the solid line.

So, that is what is shown in dotted lines here, accordingly as this pulse propagates you will get another reflected wave which is shown in the dotted line. Please again note that the amplitude at the instant that is shown here is such that the incident wave and the reflected wave has the same amplitude, it cannot have a different amplitude otherwise there is an imbalance of energy. So, for the instant tau to that is shown in this red colour you see that there is a red solid line which corresponds to the incident wave which has reached the boundary x equals to 0 simultaneously, a reflected wave has also started so as to ensure that at x equal to 0 the boundary condition is getting satisfied for all times tau right including the times that has been shown. So, therefore, you see this red dotted line

is taking birth right; and beyond the point it is this remember these dotted line is going to be a reflected wave which means that it has got to travel backwards. So, therefore, at further lapse of time this waveform which is shown in dotted lines will travel in the backward direction on the leftward direction and in tau three instant of time this is the waveform that has been shown. At tau three instant of time this way form the solid waveform which was corresponding to the incident wave would have surpassed the point x equals to 0, and now it is basically in the positive half of the x axis which is basically fictitious it is not corresponding to any media.

So, the incident wave has just passed from the physical domain to the fictitious domain we do not need to track it whereas, the once the incident wave dies its natural death that means, it has pass this cross x equals to 0, then the reflected wave takes birth and now the reflected wave will keep propagating in the leftward direction. So, this cycle keeps on going if you had another termination in this direction. So, now, in this would have become the incident wave again it would have led to a reflection, and that reflected wave will become an incident wave again it would lead to reflection. So, that is exactly what will happen in a finite domain problem if you really have domain which is terminated by 2 boundaries it becomes a proper finite domain problem, but now you may visualize that a finite domain problem is basically a problem of multiple reflection right.

So, whatever happens with one reflection has been elaborated to you in great details, you should be able to visualize that a finite domain problem in 1 d will basically involve 2 such boundary conditions and associated with each of this boundary condition there will be a reflection that will induced. So, that is about reflection from different kinds of plane waves. So, now, we will make quick transition rather from plane waves will go to spherical waves; you may think why I will be not dealing with cylindrical waves, it just happens to be cylindrical waves a little more complicated in terms of mathematical requisites, it will involve certain things like bessel functions which I do not wish to get into right now, may be later as a part of assignments one can think of such problems, but essentially you will get the idea that what is the difference between waves in 1 d and waves in higher dimension, and that will be quite adequately elaborated by this analysis of spherical waves.

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Spherical Wave



Spherical Wave

- 3 dimensional wave equation $\nabla^2 p = \frac{\partial^2 p}{\partial \tau^2}$
- In spherical coordinate system (r, θ, ϕ) ,

$$\nabla^2 p = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin^2 \phi} \frac{\partial^2 p}{\partial \theta^2} + \frac{1}{r^2 \sin \phi} \frac{\partial}{\partial \phi} \left(\sin \phi \frac{\partial p}{\partial \phi} \right)$$

- Consider a radially symmetric solution viz. $\frac{\partial}{\partial \phi} = \frac{\partial}{\partial \theta} = 0$
- Wave equation: $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) = \frac{\partial^2 p}{\partial \tau^2}$
- This can be simplified to $\frac{\partial^2}{\partial \tau^2} (pr) = \frac{\partial^2}{\partial \tau^2} (pr)$
- Spherical wave solution

$$p(r, \tau) = \frac{1}{r} f(r + \tau) + \frac{1}{r} g(r - \tau)$$

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So, till this point we were actually looking at a simplified version of this three dimensional wave equation, where we said nothing changes along y and z plane along y and z plane it depends only along the x direction; so that means, the wave is just travelling within either a duct or even in three d it travels in such a fashion that along y z plane, along the entire y z planes particles which are lying at different y z planes will have exactly the same propagation characteristics, exactly the same pressure, exactly the same velocity and so on. So, that is why we were able to simplify the mathematics from p d e or from p d e with the three variables in space and one variable in time, we were able to reduce the problem with one space variable and one time variable and from there on we saw that the solutions were pretty simple. So, we will do similar treatment now for a three dimensional wave equation, and in particular we will look at spherical waves.

So we will take this entire wave equation in its entirety, but since we are looking at spherical waves, spherical waves as the name suggests this time particles which lie on a spherical surface are supposed to behave in an identical fashion. Just like plane waves are waves wherein particles which lie in one plane have the same pressure same velocity same impedance and so on and so forth. For a spherical wave the simple definition of it would be particles lying on one spherical surface would have a completely the same characteristics. So, therefore, it is worth looking at this equation in spherical coordinates because we are finally, by the very definition of spherical wave we want to look at all particles which lie on a spherical surface. So, we will make a use of the spherical

coordinate systems which is r , θ and ϕ . So, in r , θ , ϕ or spherical coordinate systems will open up this laplacian and I must just give you a quick drawing of what are these r , θ , ϕ variables. So, the variables would be in this fashion. So, this is r , this is ϕ and this is θ . So, this is my r , θ , ϕ . So, with this spherical coordinates if you look at any book on vector calculus you can look at (Refer Time: 19:38) book for example, in any book on vector calculus you would be able to see how the laplacian in spherical coordinates is derived I have just picked up this formula, but if you wish you can just verify that it is true. So, this is the formula for laplacian as expressed in spherical coordinates, please note that the derivatives that appear are all second order.

So, this part of it is common between Cartesian systems and any other systems. So, del square operator is a second order operator and therefore, it features second order derivatives all of these derivatives are second order right. But now we will invoke a physical argument to make some deductions the physical argument is this, when we said that spherical wave by definition are waves where in, you do not expect any changes for all points which are lying on a spherical surface, which means for all points which have the same r , but maybe different θ and ϕ values.

For all those points you do not anyway expect any change so therefore, the physics of the process tells us that there is absolutely no changes associated with θ and ϕ variables; just like in plane waves we said since all particles lying on a plane yz will have identical characteristics which mathematically implied that all the derivatives with respect to y and z is got to be 0. In identical fashion we can now argue that since in spherical waves we are dealing with the condition that all the points on a spherical surface has got to be having an identical behavior. So therefore, nothing will change across the θ variables or across the ϕ variables.

So, accordingly the derivatives with respect to ϕ and θ will be set to 0. So, this is what we will call as a radially symmetric solution, the only variable which will have any role to play or only variable for which the things the pressure values will change based on which the pressure values will change is the radial variable. All points at the same radial value will have identical pressure value will have identical velocity values; whereas different points which have different radial value will have a different pressure value also. So, therefore, we will simply set these partial derivatives with respect to ϕ and θ to be 0 and when you do that these 2 last 2 terms are getting killed right, the

last to terms involves derivatives with phi and theta, but now because of our physical argument based on the reasoning's of spherical waves we can rule them out, because at this point we are interested to study spherical waves.

By the way one easy way to visualize spherical wave is that if you take a spherical balloon add think that spherical balloon is like you know it is pulsating it is like a football pump where in it will sort of breath in and breathe out right. So, that is called the pulsating sphere problem it is a very classical problem in acoustics, will solve that problem in great details, but at this point for your visualization just contemplate this idea that there is a sphere which is breathing in and breathing out.

And as it is breathing in and breathing out it will force the air outside this sphere and it will set up a spherical wave right and easier analogue probably in 2 dimension is throwing a stone in pond it that sets out cylindrical waves because those are surface waves those do not travel actually to the depth of the pond it just travels on the surface. Where as a spherical wave will actually travel within the entire volume of the air that is contained in the space, and it will it may be set up for example, when you have a sphere which is breathing in and breathing out. Now getting to the mathematics of it, so the wave equation we have reduced it in the following form the laplacian of p is just the left hand side of it is just this much and the right hand side remains as it is.

Now quickly we can look at how things can be simplified at this point. So, what we have is the following.

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The image shows a handwritten derivation in a software window titled "NOTES - Windows Screen". The derivation is as follows:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) = \frac{1}{r^2} \left[2r \frac{\partial p}{\partial r} + r^2 \frac{\partial^2 p}{\partial r^2} \right] = \frac{\partial^2 p}{\partial r^2}$$

$$\Rightarrow \frac{1}{r} \left[2r \frac{\partial p}{\partial r} + r^2 \frac{\partial^2 p}{\partial r^2} \right] = \frac{\partial^2 (pr)}{\partial r^2} \quad \frac{\partial r}{\partial r} = 0.$$

| | |
|--|--|
| $2 \frac{\partial p}{\partial r} + r \frac{\partial^2 p}{\partial r^2} = \frac{\partial^2 (pr)}{\partial r^2}$ $\Rightarrow \frac{\partial^2 (pr)}{\partial r^2} = \frac{\partial^2 (pr)}{\partial r^2}$ <p>This is identical to plane wave eqⁿ in the variable 'pr'.</p> | $\frac{\partial^2 (pr)}{\partial r^2} = \frac{\partial}{\partial r} \left(\frac{\partial (pr)}{\partial r} \right)$ $\frac{\partial}{\partial r} (pr) = p + r \frac{\partial p}{\partial r}$ $\frac{\partial}{\partial r} \left(\frac{\partial (pr)}{\partial r} \right) = \frac{\partial}{\partial r} \left(p + r \frac{\partial p}{\partial r} \right) = \frac{\partial p}{\partial r} + \frac{\partial p}{\partial r} + r \frac{\partial^2 p}{\partial r^2}$ $= 2 \frac{\partial p}{\partial r} + r \frac{\partial^2 p}{\partial r^2}$ |
|--|--|

One by r square let us look at the formula $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right)$, that if you do little bit of simplification we lead to the following $2r \frac{\partial p}{\partial r} + r^2 \frac{\partial^2 p}{\partial r^2}$ and this has to be equal to $\frac{\partial^2 p}{\partial r^2}$. So, what we will do is we will push one of this r that is sitting here along the other side and make it p times r, remember $\frac{\partial r}{\partial r}$ both r and tau are independent variables that is going to be 0. Therefore, one of these r squares will push it on the other side add that should be $\frac{1}{r}$, $\frac{2}{r}$, $\frac{\partial p}{\partial r}$ plus $r^2 \frac{\partial^2 p}{\partial r^2}$. So, we started with the equation this is equals to $\frac{\partial^2 p}{\partial r^2}$, and in the next step what we did is we pushed one of this r sitting inside r square on to the other side and we convert it this into a variable p times r, and this in the next step we could write it as 2 times $\frac{\partial p}{\partial r}$ plus r times $\frac{\partial^2 p}{\partial r^2}$, that has to be equal to the second time derivative of the quantity p r.

So, now, the left hand side also is equal to $\frac{\partial^2 (pr)}{\partial r^2}$ let us look at this quantity $\frac{\partial^2 (pr)}{\partial r^2}$ is $\frac{\partial}{\partial r} \left(\frac{\partial (pr)}{\partial r} \right)$. So, $\frac{\partial}{\partial r} (pr)$ comes out as p plus r times $\frac{\partial p}{\partial r}$, simple product rule that is I first take the derivative with respect to r that that is just identically 1. So, p remains as it is and then I take the derivative with respect to p. So, then I get a $\frac{\partial p}{\partial r}$ and then I have to take $\frac{\partial}{\partial r} \left(\frac{\partial (pr)}{\partial r} \right)$, and that has been already calculated. So, it is $\frac{\partial}{\partial r} \left(p + r \frac{\partial p}{\partial r} \right)$ right the first term is easy that is just $\frac{\partial p}{\partial r}$, the second term is again having a cross between or a multiplication between r and $\frac{\partial p}{\partial r}$.

So, we will apply the product rule will first take the derivative with respect to r that is just 1, so we will get another $\frac{\partial p}{\partial r}$ plus r times $\frac{\partial^2 p}{\partial r^2}$, that is going to be $2 \frac{\partial p}{\partial r}$ plus r $\frac{\partial^2 p}{\partial r^2}$ right which is exactly what you see on the left hand side of this equation. So, I can write this as $\frac{\partial^2}{\partial r^2} p r$ is $\frac{\partial^2}{\partial \tau^2} p r$. So, I have actually brought this spherical wave problem if you realize what I have done is that I have simplified this into a form where it now reads like a one dimensional plane wave equation, but the variable is not p the variable is p times r. So, I can simply say that this is identical to plane wave equation where in the variable p r plane wave equation was in the variable p, but this is the equation of this spherical waves happens to be identical to the equation of plane wave, but with the variable of interest being transform to p r right.

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The solⁿ of the above eqⁿ is identical to solⁿ of plane wave eqⁿ with 'pr' instead of 'p'. $p r = f(r+\tau) + g(r-\tau)$

$$p = \frac{1}{r} f(r+\tau) + \frac{1}{r} g(r-\tau)$$

radially inward wave radially outward travelling wave

$p \propto \frac{1}{r}$ Attenuation of waves in higher dimension

Therefore, the solution of this equation should be just like the plane wave solution but on the left hand side it is p r. So, the solution of the above equation is identical to the solution of plane wave equation with p r instead of p. So, accordingly p r this time would be f of x plus tau plus g of x minus tau. I do not need to work out this part of the solution because this has already been derived.

Accordingly p which is our final variable of interest sorry I should not give here x here, the variable of interest here is r right this is in terms of r and tau not x and tau. Therefore, this should be r. Therefore, the solution of the above equation is identical to the solution of the plane wave equation with variable p r instead of p, and p r this time will have the

solution f of r plus τ plus g of r minus τ , which in other words means p is f of r plus τ plus 1 by r g of r minus τ . As usual the interpretation of f and g remains identical, it is this wave is what we will call as a backward wave, but this time backward means it is radially inward it is not leftward. Leftward meant x is going in the negative direction or x is becoming smaller if you might. So, think it. So, this is associated with the same idea, but as that is that as time progresses the waveform goes into smaller values of r .

So, that that basically means that it is a radially inward wave, where as this g of r minus τ has the same characteristics as g of x minus τ , g of x minus τ is a forward wave by forward wave what we meant was as τ goes higher the waveform goes to higher values of x right. So, accordingly g of r minus τ would be interpreted in the following manner that as τ goes higher the values of r also will be high if you are looking at one if you are looking at the surface which has got to be at the identical values of the response.

So, this is going to be an outward travelling wave radially outward travelling wave and this is probably easier for you to visualize, if you have a pulsating sphere in open atmosphere and for a moment you think that you know even the ground is not there it is it is something like this that you have taken this football mid air and then this football is pulsating right. So, in that case it is going to lead to radially outward waves which will keep on expanding because the space is there to expand. If you keep the football just one meter above the ground and the football is asked to undergo a pulsating motion then what will happen is that, initially these spherical waves will come out, but this spherical waves will hit the ground, which is located just one meter ahead right or one meter below, in that case once it hits the ground then again transmits I mean there will be reflections which will started and you will have to deal with them separately.

So, at first glance we are going to talk about the situation where there is only a travelling wave right. So, this radially outward travelling wave is what you can easily visualize as the waves that are created by the pulsating motion of any sphere. Radially out inward wave may appear a little bit counter intuitive, but then the interpretation should be idea should not be too difficult if you realize that there cannot be an outward wave alone, the mathematical solution cannot be completed with just the outward wave for all possible boundary conditions.

For example, let us consider a spherical room if you have a pulsating sphere and this pulsating sphere will cause spherical waves to be generated all that is fine right. But at sometime this outward spherical wave will reach the boundaries of the room which is this right and because you have to let say enforce a certain boundary conditions this room could be rigid or this room could have soft, the 2 extremes that is Dirichlet or Neumann condition because the outward. Finally, spherical the outward travelling wave will have to hit the boundaries of the spherical room and again to enforce the boundaries of that spherical room there has to be a reflected wave which has to get generated.

So, after sometime when these spherical waves reach this spherical boundaries of the room there has to be a reflected wave that will be created, and this reflected wave has to travel inwards because the g function is the incident wave and if the g function is the incident wave it will lead to an f function which is the inward travelling wave or the reflected wave. So, there is no escape that if you have a boundary condition you will have to deal with the reflected wave, and that reflected wave is basically the brought out to satisfy the boundary condition right.

So, this is sort of one theoretical way in which you should feel convenience that this inward travelling waves also have to be there as a part of the solution, but again I reiterate if you are dealing with the domain which is completely infinite as I said that you are dealing with let us say at most propagation of sound as this aeroplane is travelling in mid air right then it is virtually sound propagation in an infinite domain right. So, there you do not need to consider one of these waves the inward wave is not there its only the outward wave if wave is basically 0 right, but the general solution of the wave equation must contain 2 components f and g and it is so happens that one of these components is the outward wave and the other component is the inward wave.

In totality both the components have to be present as a part of mathematical solution, there will be applications where one of these waves will not be there the inward wave for example, will not be there in case that I have just talked. So, that is all right, but the mathematical solution must be a both of them and physically both of them will be present if you are talking about a bounded domain right things will get really complicated if we start thinking about what happens as to these spherical waves if it right to be there in a rectangular or a cubical enclosure right then you have to deal with all of

this reflected waves will the reflections will happen at different times, you have to basically keep tracking those times.

This is exactly what is done in a ray tracing algorithm, but we will not get that far we will stick with the basics at this point of wave propagation. Another point of very stark contrast between the spherical waves and that of plane waves is in the appearance of this $1/r$ factor. The $1/r$ factor denotes that the pressure is not going to be constant as it propagates, right for an outward travelling wave as has been shown by these black lines these black dotted lines denote the outward propagating wave you because there is a factor $1/r$ sitting as of a multiplicative factor on the g function, it means as the wave propagate outwards the radius vector increases and as the result it will keep on decreasing in its amp amplitude.

So, that is what is called attenuation of waves in higher dimension, this is as I said in stark contrast to the plane wave propagation situation in the plane wave propagation you saw that the waves remain or waves propagate at the same amplitude there is no difference in propagation there is no difference in amplitude as the wave propagates, but here you will see that for the outward wave as it is traveling outwards the associated r value increases and $1/r$ value thereby decreases, but since this g function is getting multiplied with $1/r$ value, you are going to get a drop in the pressure value. It is the other way around for the radially inward wave; the radially inward wave as it is travelling inwards, the r value is decreasing and as the r value is decreasing $1/r$ value is increasing.

So, the associated with the radially inward wave you may think that it is actually the amplitude is building up, to bring out emphatically at this point of time is this that in contrast to plane waves the amplitude of the waves in spherical in the case of spherical waves is not constant rather it is changing. Even you may think that why is it that the acoustic pressures are decreasing for the case that you have an infinite domain propagation, as I said for an infinite domain propagation you will only expect a radially outward wave, but then again the question arises that even for such a radially outward wave is it physically justified that the wave amplitude continuously decreases, because you may believe that if the wave amplitude decreases the energy decreases and there by the law of energy conservation of energy is violated, but that is not so.

Because the wave amplitude does decrease, but you have to understand the wave spreads over a larger region. As the wave is spreading over a larger region therefore, the wave amplitude is decreasing in case of the outward wave propagation right. You can have a similar interpretation for the inward wave also, as the wave amplitude is increasing because now it is going to traverse within a region of space which is smaller because if you look at let say this region this magenta portion, it is a large area at this instant of time. At a later instant of time let us say the wave has reached here, now this spherical surface area is smaller.

So, the wave has now sort of got concentrated into a smaller area of surface area and as a result it is perfectly feasible that it will build up its amplitude within that smaller surface area right. Similarly in the case of outward wave if I look at any initial instant where the wave was located right here where I am solidifying the dotted lines, it is occupying a small area right? Now at a later instant of time when the wave has travelled a certain distance radially outward it is occupying a larger area which is this line which I have just solidified.

So, since the wave is now spread over a larger area in other words the amplitude has got to decay right. So, these calculations will again come back to when we do the intensity of the waves. So, we will return back quickly now to the slides and what we have done is that we have simplified the spherical wave equation into this form which is just the plane wave in the variables r and τ , but the unknown variable being $p(r)$ instead of p alone.

So, that we have shown is having a spherical wave solution of this kind, and we have talked about various physical aspects of this spherical wave solution. So, here we will like to stop for the today's class what we will do next time when we will meet is that from there on we will take another very crucial assumption into our (Refer Time: 43:40) which is the harmonic assumption. Till this point of time we have dealt with waves which are perfectly arbitrary in terms of its temporal nature, we have not posed any restrictions on the temporal behavior or the time domain behavior of these functions $p(r)$ right what we will do starting from next class on words will invoke a very crucial assumption and we will invoke that dictated by physical considerations.

Dictated by physical considerations we will find there are lots of problems which will arise in acoustics which can be very simply dealt with where wherein we can reduce the

generality of the solution make it a little more specific instead of having arbitrary f function and g function we look for functions of very specific form in particular we look at complex exponential forms of this f and g , because that will lead us to what is called the harmonic or steady state assumption in acoustics. So, that is what is lined up in the next few classes.

Thank you for the day.