Acoustics & Noise Control
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Module - 03
Lecture - 06
Plane Wave 2

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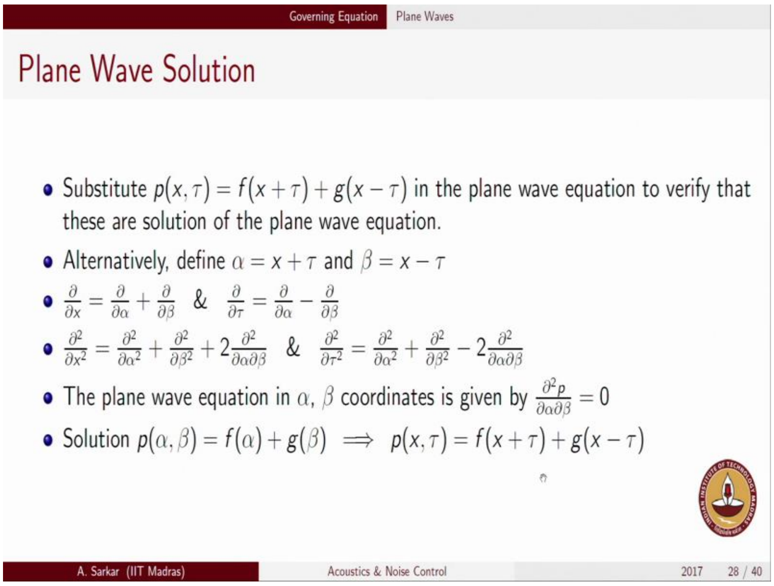

In the last class, if you remember we talked about the solution for to the plane wave equation and it was shown that the solution of the plane wave equation can be given in this form; p x comma tau is equals to f of x plus tau plus g of x minus tau. So, then we set up on ourselves the objective of determining the characteristics of these 2 solutions f of x plus tau and g of x minus tau.

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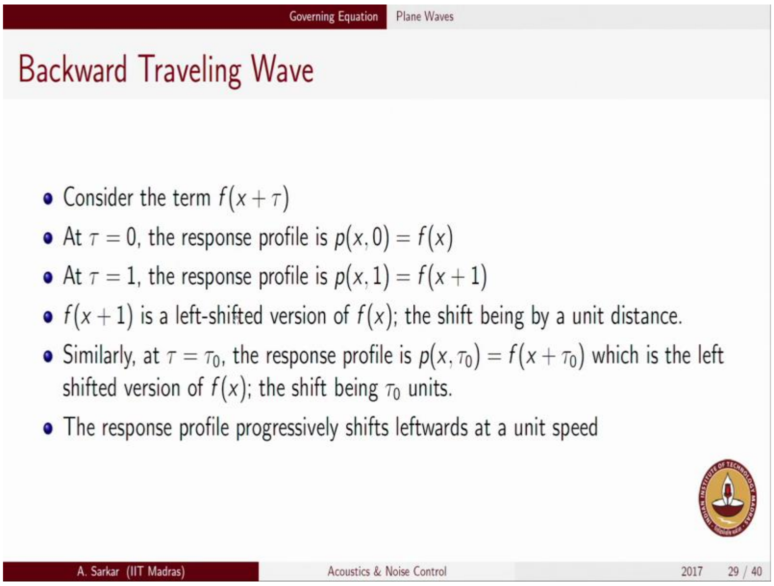

Towards that end, we had looked at the first term which is f of x plus tau right. So, we understood that if we set tau equals to 0 p of x comma 0 is let say f of x; we are not considering the j part of it right now with only considering in the 2 of the f part of it. So, at every progressive instant of time tau we get to see that this f of x which was corresponding to p of x at tau equals to 0; we will get shifted and the shift is exactly the same as inter numerically the value of the shift is same as a time value.

So, progressively the profile of the response is getting shifted backwards. So, that is what we understood as a backward traveling wave. So, f of x plus tau corresponded to a backward travelling wave which we analyzed in some details.

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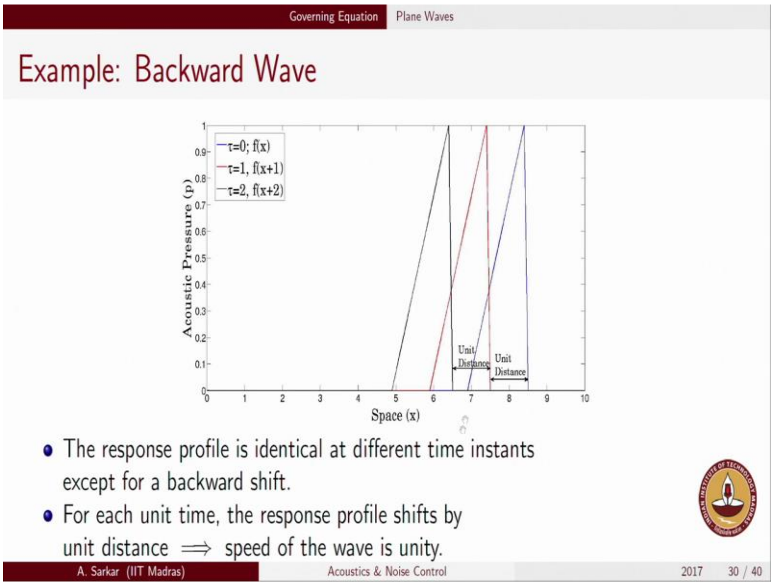

And this was the example that we looked at. So, what I have plotted here in blue is p of x comma 0 which is basically f of x that is happening at tau equals to 0. So, at each time instant time tau equals to 1 time tau equals to 2 each of these time instance if you again re do the graph for p of x which now will be a f of x f of x plus 1 f of x plus 2. So, progressively you will get to see that the profile is getting shifted left towards and it is shifted by a unit distance because there is a jump of time by a unit value.

So, from tau equals to 0 we have going to tau equals to 1 tau equals to 2 and that is responsible for this unit shift. So, the response profile is identical at different time in step instance except for a backward shift and for each unit of time the response profile is shifting by a unit distance which is implying that the speed of this what we now understand to be wave because it is the response profile which has a fixed shape, but it is just moving backward.

Or in the next example we will see it will move forward and it is moved the response profile is moving a unit distance ahead per unit time which means that the speed of the wave is unity that is no surprise, because the wave equation also was having was rescale such that time was rescale to tau such that you had any you had no presence of the c term which was essentially from that premise we also argued that in the x tau space as compared to x t space we will have a unit space.

So, again what we have seen here is that in x tau space the response profile is in the nature of a wave which travels at a unit speed in the x t space it is also going to remain a wave, but it will travel at a speed of c because there is a rescaling of time between t and tau and the relation between that was tau is equals to c t as was talked in the previous lecture. Now we take a different perspective very really at least if we are doing experiments very rarely will you be able to measure the response at any fixed instant for all the spatial points what we are showing here is this is the response at all the spreads point spread over 0 to 10.

So, all these points you are picking up the response, but usually you will not have. So, many sensors such that you are able to pick up a number of response at a fixed instant of time right let us now look at converts picture where you have a limited number of senses let us say 2, 3, 4, but for these 2, 3, 4, points you can pick up the time response at all time instance right here the picture shows response at all the points for 3 instants we want to now construct the converts picture where we pick up the time characteristics of let say 3 point. So, this is the objective that we have in mind. So, we will now have a temporal perspective that is a time perspective this was the space perspective because we were showing the space plot for 3 independent time instants now we will look at a time plot of 3 different space points let us this is what will usually happen in our experimental facilities because usually we will have a limited number of sensors.

So, with these limited number of sensors we will be able to get the temporal characteristics of exactly the same number of points as the number of sensors which we have. So, if we have sensors mounted at let say this point 2, this point 3 and this point 4, we will be able to get the entire time history of all these points 2, 3 and 4. And we will not be able to we have to be able to work with just these 3 points we cannot work with all these other points which is as shown in this graphical plot this graphical plot shows the response at these are snapshots at particular instance of time, but the snapshots have been taken for all the space points we are just now flipping that picture.

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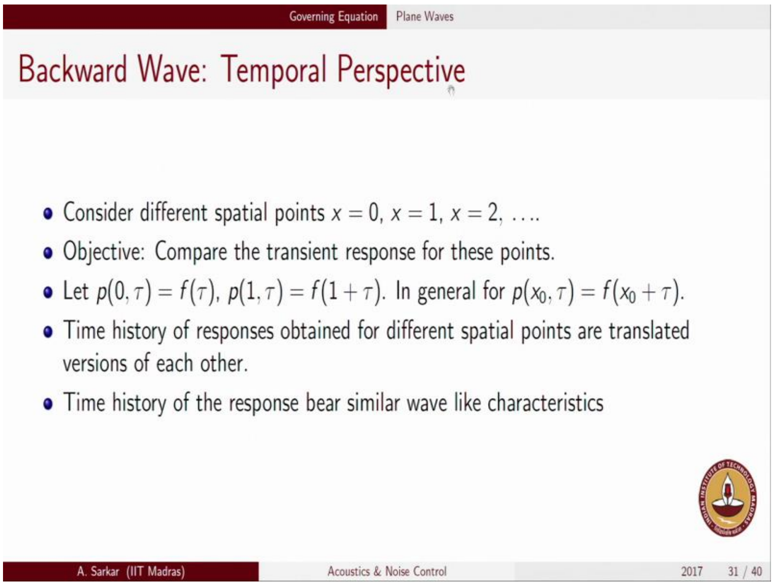

And now we are adopting a temporal perspective. So, we are going to consider different spatial points without loss of generality I have considered x equals to 0 x equals to 1 and x equals to 2 you could compare considered other points also.

So, the objective would be to compare the transient response for these points. So, we want to generate time plots corresponding to these points and we wish to understand how these time plots would look like at x equals to 0 x equals to 1 at x equals to 2 and whether there is any such relationship we understood that at different time instants the respective snapshots showed a very similar characteristics accepting for the fact that there was a shift. So, we wish to investigate whether such a characteristics will happen even for the transient response across 3 spatial parts.

So, let us take to start with the point x equals to 0. So, p 0 comma tau is we already know p x comma tau is f of x plus tau by our basic solution the basic solution that we are looking at is f of x plus tau. So, p 0 comma tau is f of tau right similarly p 1 comma tau is f of 1 plus tau and in general if you consider any point at x equals to x 0 p x 0 comma tau that is the temporal characteristics of the transient responds at the point x equals to x 0 is going to be f of x 0 plus tau. So, this is the manner in which different time history is taken for different spatial points can be analyzed.

If you now look at the difference between these 2 time histories you will again see that it is the same thing happening in time also the time histories at the point x equals to 0 and the point x equals to 1 an identical, but for the fact that it has been left shift. So, the left shifting perspective which we took in the snapshot manner of investigation when we investigated earlier we said that we are going to look at the response of all the points at progressively different time instance we understood that the response profile is progressively moving backward or leftward here 2 in the case of temporal perspective we are getting the same inference that when we are taking different spatial points and analyzing the temporal response they are 2 we get to see that the time history of the responses obtain for different spatial points are identical, but for a translation between each other.

So, this is what you will exactly see in your oscilloscope if you happen to take the sensor output input into the oscilloscope you will see a wave like feature. So, that is what one would again interpret like a wave, but typically this wave will be different from the wave that you see in a beach the way that you see in a beach is this where it is a form in space which goes translate which keeps translating in time whereas, if you are using the word wave even for your oscilloscope response obtained in the manner as mentioned then you will please understand that you are basically looking at time histories of different points and those time histories are having a translated version of each other and all this is happening because of the nature of the solutions that we have picked up.

The solution depends only on the argument x plus tau it does not depend individually on x or tau in any arbitrary fashion as long as x plus tau is constant you will get to see the same solution. So, this is exactly what happens; what is happening in the time perspective. So, again now if we where to plot these solutions this is how the plot would look like.

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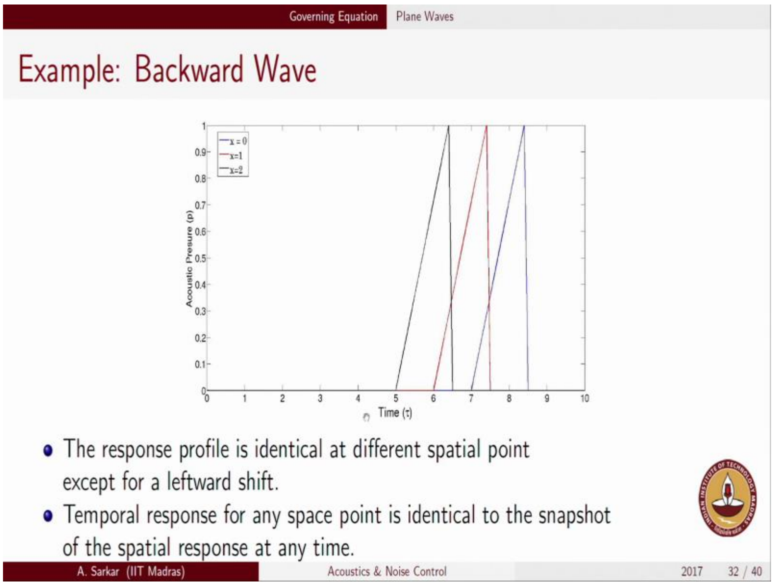

So, here what I have plotted in blue is the time plot of this solution at x equals to 0 right at x equals to 1 you are getting this time plot at x equals to 2 you are getting this time plot which is shown in black. So, again you get to see the same feature that as you pick up different spatial points which are at higher values of x is these corresponding time history plots are behaving like a wave in the sense that they have identical characteristics except for a leftward translation.

And the response profile that you see in this picture is just identical that you saw in this picture. So, it is just a translated version of each other it happens both in the time perspective as well as in the space perspective. So, this is what we will keep iterating for a while that the time and space part of the solution is actually inter related when you talk about waves if you know what happens in time for a particular spatial points you should be able to recover that what happens across different spatial points for a particular instant of time and for progressively different instants of time of your interest.

So, the moral of the story is that the response profile is identical at different spatial points except for a leftward shift the temporal response for any space point is identical to the snapshot of the spatial response at any time. So, that was what was presented in the slide number 30, this is the snapshot of the response at various time instance and what is presented in slide number 32 is a temporal response at different spatial points, but both these plots look identical because exactly the same analysis has gone in whether you take a time perspective or whether you take a space prospective and this is the reason why I have started with the backward wave though you may feel it offensive that my first example was a backward wave rather than the forward wave, but in forward wave this will not be true in forward wave there is going to be a slightly different perspective between time and space.

So, to make you are learning easier I have started with the backward wave where in you can actually see the time and space perspective is going to be identical that is because the sign of x and tau in this argument is same whereas, the sign will get flipped in the other solution which is in terms of g. So, let us now look at the other solution, but before we depart this topic of backward wave let us once more analyze what; say what is it that is happening? So, as we understood that it is the argument x plus tau is determines the solution it is not the value of x it is not the value of tau, but it is the argument x plus tau which determines the value of f at a point.

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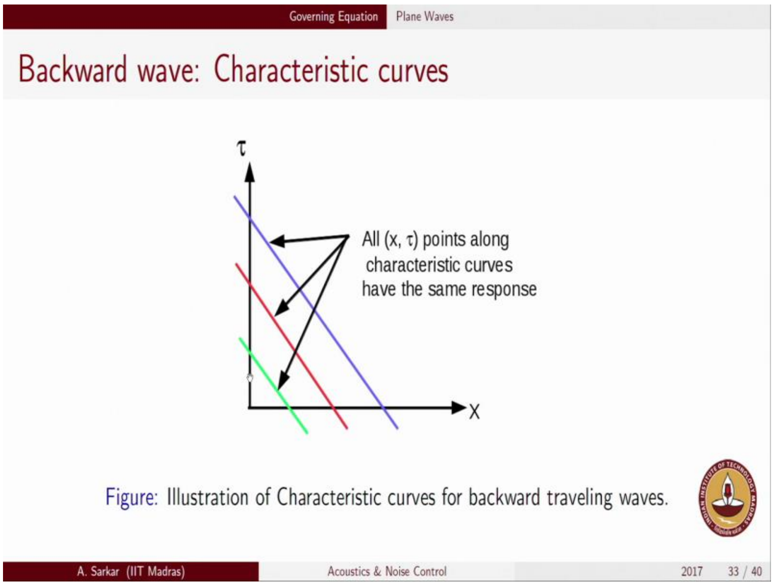

So, in other words for all these lines for which x plus tau is going to be constant these lines are lines of slope of minus 45 degree and this these lines on these lines all the points x; x are such that x plus tau is constant. In fact, in other words mathematically speaking these are the locus of all points having x plus tau equals to constant. So, on these lines you are going to see that there is no change in f value in other words whatever is happening at this point we will happen at this point we will happen at this point and so on and so forth. So, if I have a certain condition at this green line when it is intersecting with the x axis which basically means the tau equals to 0. So, if I have a certain condition here then this condition we will travel along these lines right in other words at tau equals to 0 if I have a certain condition along x.

Let us say I have given a certain profile here then this profile we will travel between the green and red lines. So, these lines are called the characteristic curves for the backward travelling waves similarly there will be the characteristic curves for the forward traveling wave which will be the oriented along the other way around let me just illustrate this once more to you in my notes.

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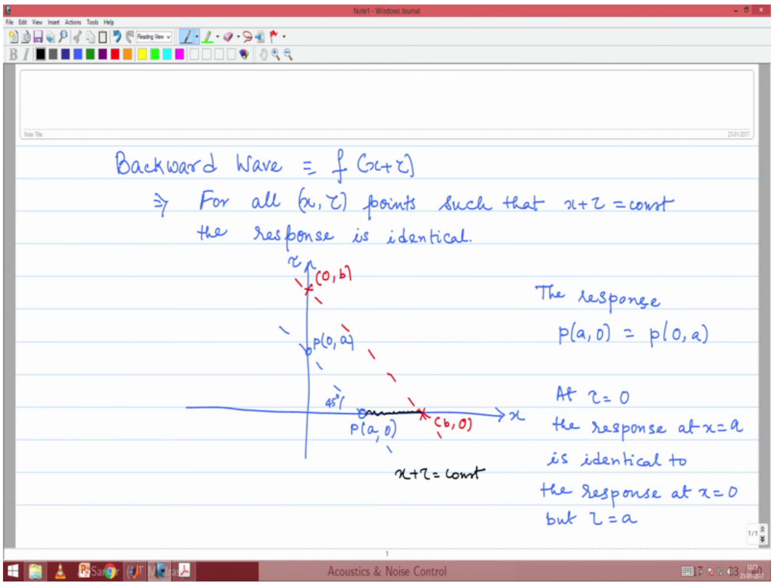

So, we understood that the backward wave is of the form f of x plus tau that implies for all x comma tau points such that x plus tau is constant the response is identical. So, let us take the x tau space this is x the space and this is tau the time in rescaled coordinates.

Now, these are the lines where in x plus tau remains constant these are the lines where x plus tau is constant these are the locus of all points x comma tau such that x plus tau remains constant. In other words if I am inter if I know for sure that this is a certain condition p x this is 0 comma 0 then whatever happens at p x 0 comma 0 will also happen at this point and this point because this is a 45 degree line is going to be p 0 comma x 0. So, if I know what happens at the initial time tau equals to 0 for the point x 0 the same thing will happen at the point x equals to 0 at the time t equals to x 0.

Let me use the symbol a I think that will be less confusing. So, whatever happens at time t equals to 0 the response p a comma 0 is going to be identical as p 0 comma a right my first argument is space the second argument is time. So, initially whatever is happening at the point x equals to a, the same thing will happen at the point x equals to 0, but after a time gap of tau equals to a units. So, the same thing will happen.

So, I can as well right this statement at tau equals to 0 the response at x equals to a is identical to the response at x equals to 0, but tau equals to a. So, at a delayed time the same thing is going to happen. Now if you repeat this argument for other points same argument will go through between these 2 cross points I call this point let us say b comma 0 because it is at time tau equals to 0, but the same thing will happen for 0 comma b.

Which in other words means that whatever is the profile here right we will get the information we will get conveyed right that is how we are interpreting the wave the information is simply getting conveyed, but in the backward direction as time progresses the information is going in the backward direction because here what is happening at a forward spatial point x equals to a is now happening at a backward spatial point x equals to 0 at x equals to 0 the information is reaching only at a later time instant whatever is the information present at x equals to a at the initial time tau equals to 0 is reaching the point x equals to 0 at all later time.

So, the information is traveling backwards. So, this method of characteristics is very helpful in sort of visualizing the different features of wave. So, we will note this fact that for a backward wave the characteristic lines looks in this form they are inclined at minus 45 degree to the x axis now we turn to the forward wave.

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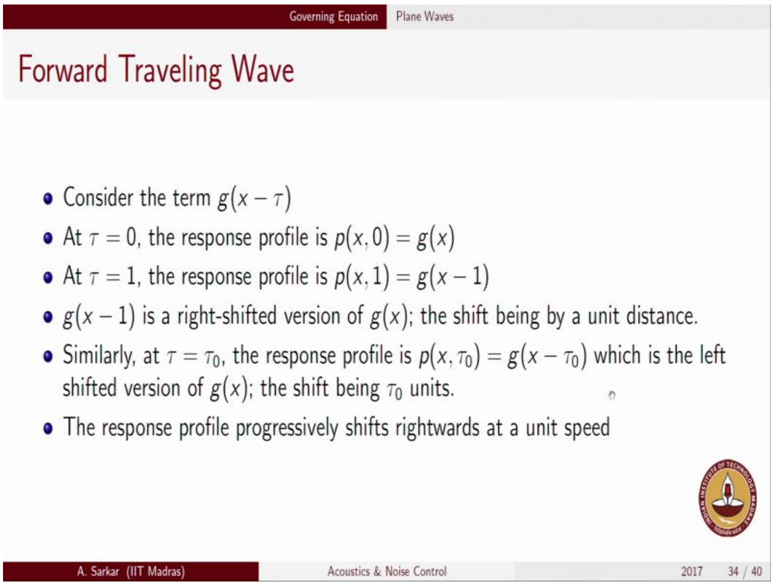

So, the forward wave is the other part of the solution you will recall that the solution of the plane wave equation was given in 2 parts f of x plus tau and g g of x minus tau by now we know what has what is the characteristics of this part of the solution f of x plus tau it is a backward wave it has the identical features whether you look at it from space or whether you look at it from a time perspective.

Now, we turn our attention to the second component of the solution which is g of x minus tau. So, let us see what we have in store for this turn. So, again we will re do the same analysis now considered the term g of x minus tau at tau equals to 0 the response profile is p of x comma 0 I simply substitute tau equals to 0 and I get g of x at tau equals to 1 the response profile will be p of x comma 1 is g of x minus 1 right. Now again let us relate how these 2 responses are looking like right this time we identify that g of x minus 1 is the right shifted version of g of x right it is not the left shifted version it is rather the right shifted version.

So, in other words 2 different snapshots taken off the response profile p of x taken at tau equals to 0 and p of x snapshot taken at equals to 1 will have a similarity in the sense that the response profile will look similar except for the fact that there is a right shift. The p of x comma 1 is right shifted version of p of x comma 0 and the shift being exactly by a unit distance right which means in tau equals to 1 unit time this response profile has shifted by a unit distance in other words it has a unit speak right this analogy can be continued for any arbitrary time what I have said for tau equals to for tau equals to 0 and tau equals to 1 can be extended for any arbitrary time I call that tau equals to tau 0 the response profile now will be g of x minus tau 0 which is the right shifted version a typo here this should be right shifted version of g of x.

And the shift being exactly equals to tau 0 units. So, there will be a right shift at time tau equals to tau 0 which is corresponding to the response profile p x come the p x comma 0 profile and the p x comma tau 0 profile will just be a differing in a term of in terms of a shift the p x comma tau 0 will be right shifted version of p x comma 0, I will show that in my next example plot that I have brought. So, the response profile shifts progressively rightwards with a unit speed or forward with a unit speed.

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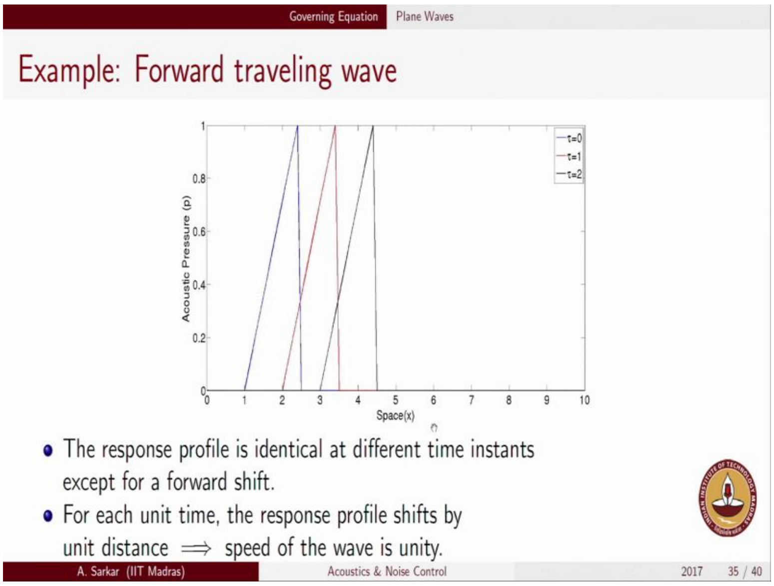

Let us see how all these things look when we plot it up. So, again in the blue graph what you see is the response profile across all points in space at the initial time tau equals to 0 at a later time tau equals to 1 this profile has been obtained which is shown in red the analogy between these 2 profiles is that they are just right shifted version the red plot has been obtained as a right shift of the blue plot.

Similarly, if you go forward in time by another unit interval of time then what you get is another right shifted version from red to black. So, the conclusion therefore, is the response profile is identical at different instance of time except for a right shift or a forward shift therefore, this name comes as to a forward traveling its it is looking like this response profile is travelling in time if you are happy if you have some sort of a video camera and capture this response profile we will see that the profile is moving ahead in time right and that is exactly what you see even when you look at the waves as it flanges on the beach the response profile remains roughly identical it just moves forward from the sea side to the beach side.

But, that analogy has little problem that you know the amplitude of the way probably comes down as it breaks into the beach if you take that part a side this is what is a travelling wave that is the response profile is identical for progressively different time instance it has its own interpretation if you look at the temporal perspective also we have done that for the backward wave we will do that in a moment for the forward wave also. Now, let us look at the time perspective because as I said very rarely will you be able to x at least experimentally get this sort of a plot because here you will need lot of senses.

So, as to track the response for all of these points usually you will have limited number of sensors you can track the entire time history, but for a limited number of points you cannot do the other way round.

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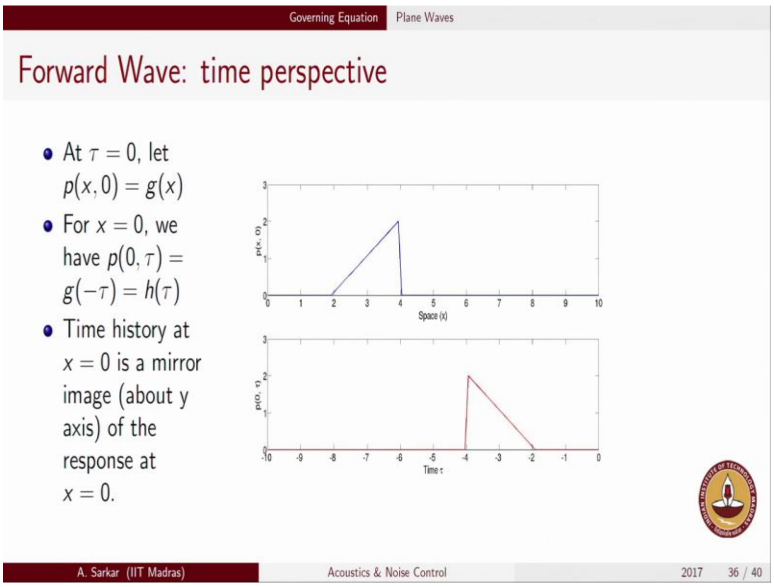

So, usually we will like to have a time perspective because or oscilloscopes will give that information rather than the information content in this graph. So, let us look at what is the time perspective for a forward wave at tau equals to 0 we know that p of x comma 0 that is basically the initial condition is g of x. Now, if we turn this around and thing that what is happening at x equals to 0. So, for x equals to 0 we have the solution as p of 0 comma tau and if p of x comma tau is g of x minus tau then p of 0 comma tau is g of minus t right and I just wish to call this g of minus t as g of minus tau I should say g of minus tau I am just re defining it has h of tau.

So, I have just again to get read of the argument minus within the sign of tau I have again redefined a new which I am calling it as h of tau right. So, if this is p of x comma 0 which is basically g of x right then what is this function p of 0 comma tau it has to be h right it is it is either g of minus tau or h this is the function g for you and the function h will have to be flipped version of the function g g of minus tau is h of tau which means whatever happens at value of 4 let say for g will not happen at a value of minus 4 for h whatever happens for the value of 2 for g will now happen for the value of minus 2 for h.

So, p of 0 comma tau is going to be h of tau and p of x comma 0 will be g of x, but the g function and the h functions are not identical as you would have seen in your backward travelling wave in the forward travelling wave they are flipped versions of each other that is they are just mirror images they are just mirror images. So, the space profile and the time profile does not look identical, but they look as a flipped version if you deal with a square pulse then probably you will not see this effect because the mirror image is going to be identical. So, on purpose I took a right triangular kind of a pulse to demonstrate this effect that once you have forward wave the space profile and the time profile does not look identical it is a flipped version.

So, time history at x equals to 0 is a mirror image about y axis of the response at x equals to 0. So, if you hold on to the response at the initial time snapshot of the response at initial time then you can recover the time history at the point x equals to 0 simply by flipping that image. And once you know what happens to the time history at a specific point let say x equals to 0 you can now recover the time history at any other point of your interest this is what I have shown how do you recover the time history at point x equals to 0 from the initial condition tau equals to 0 you have the space profile you can recover what is the time history of the point x equals to 0.

In the next slide I will show what happen at progressively different time instants. Now once you get to this plot of h of tau at x equals to 0 what happens at progressively different space points let me illustrate that to you in my nodes.

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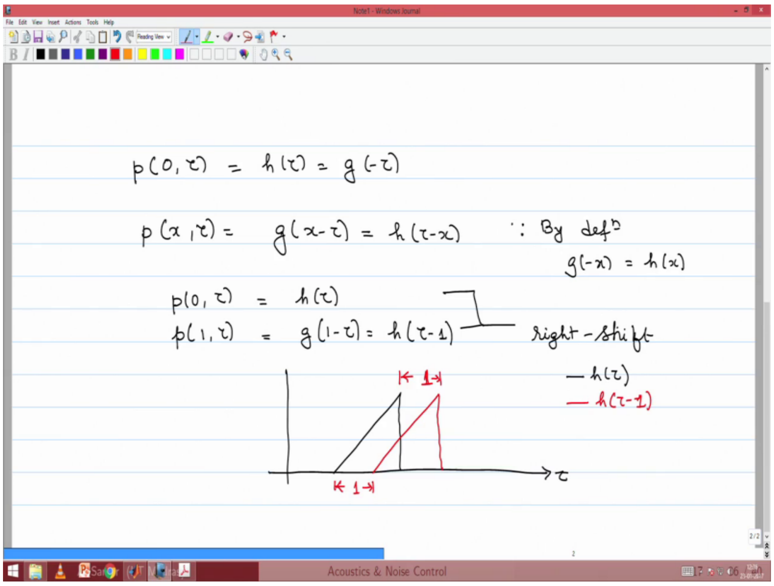

So, you know p of 0 comma tau is h of tau which is g of minus tau right you also know p of x comma tau the one that we are dealing with is g of x minus tau and this in other words can be written as h of tau minus x since by definition we have said g of minus x is h of x right. So, now, we know; what is p 0 comma tau that is h of tau?

Now, let say we wish to find out what is p one comma tau p one comma tau is g of one minus tau and that is also the same as h of tau minus 1 right. So, again we see between these 2 plots or between these 2 response time histories there is a right shift. So, if this is the tau scale this is h of tau then what will be h of tau minus 1 it will be right shifted version. So, I will do that in red color. So, this is h of tau minus 1 and this distance is one and so is this distance. So, it is the exactly the same profile the profile is going to get right shifted just like in a backward wave also we saw the time history was going to get left shifted here for the backward wave also the say sorry in the forward wave also we get to see the same idea once you get to know what is the time history at any point for any other point lying upstream that is for any other point which is having a value of x higher than this profile in time will keep moving.

So, you will in even in your oscilloscope screen your oscilloscope screen is ex exactly time history plot in your oscilloscope screen also you will keep seeing that there are if you have these 2 time plots taken you will also get to see that they are going to be shifted versions of each other. So, again the same thing has been demonstrated probably better by this mat lab plots.

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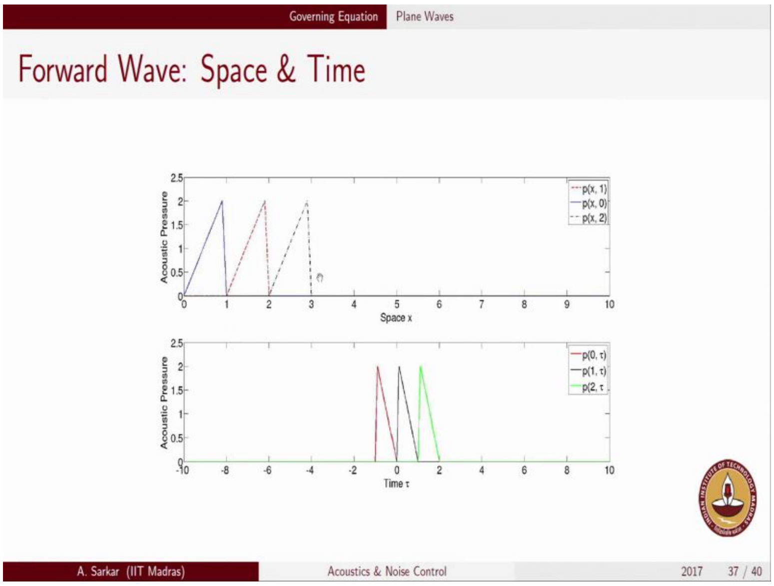

So, here in the first part of it what I have shown is that this is what happens for different space points across different time I have taken 3 different times 0 1 and 2 0 has been shown in blue one has been shown in red dotted and black shows what happens at the time instant 2.

So, for the forward wave you see that it keeps progressing in the forward direction that is the snapshot keeps progressing in the forward direction how do we attend this plot which is basically time history plots for different spatial points 0, 1 and 2 first we recover what is the time history plot for the point x equals to 0. So, towards that end we identify p x comma 0 is this point we just need to flip this blue curve and obtain p of 0 comma tau. So, if you see this has been mirror reflected and we obtain this rate curve here on purpose I have changed the color. So, that you will be careful to analyze the situation it is not that the red dot has been flipped to the red dot it is the blue curve here which has got flipped to the red solid curve here.

So, once I get p 0 comma tau to obtain the time profile for any other point 1 and 2 it is trivial task I just need to propagate this same profile forward right. So, different space points we will have identical response, but they will just be forward propagated or propagated in the rightward direction right, but please note while dealing with forward wave the time history picture and the space plot picture is not identical, but it is flipped version of each other right. So, that is the forward wave for you.

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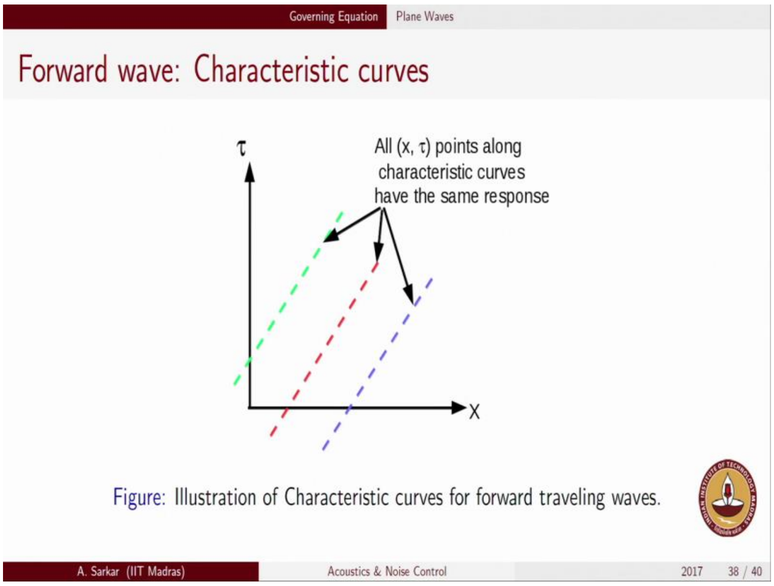

Figure: Illustration of Characteristic curves for forward traveling waves.

Now, if we take the argument from the perspective of characteristic curves we realize just like we had obtained the characteristic curves for the case of backward waves back in backward waves we realized for all x tau points as that x plus tau is fixed you will have a an invariance of the response.

See here the argument associated with g or h whatever you call it is going to be x minus tau right. So, x minus so for all x tau points as that x minus tau is constant the response is going to be identical, so you using exactly the arguments you arrive at these set of lines these set of lines are just the 45 degree lines in the x tau space and these will be now called as characteristic curves because whatever happens at one point the information will be travelling along these lines and it will happen at any other point. So, if you consider this point for example, whatever happens at tau equals to 0 and call it x equals to a right the same thing we will happen at this point whatever be its coordinates right. So, this is the graphical way of understanding what is happening and please understand in this case it is again very intuitive to understand that the information is being communicated in the forward x direction.

Time is always moving forward, but in the backward wave we saw that the information was being conveyed in the in this direction. That means, it is in the minus x direction whereas, here as you move ahead in time the information is being conveyed along the forward x direction and that is why we call it as forward wave. So, this is how we have analyzed the 2 distinctly different components of the solution f of x plus tau and g of x minus tau.

So, if I returned back to our starting point this is where it all started where we took the 2 parts of the solution the 2 sol components of the solutions where derived pretty much rigorously and we have by now analyzed the 2 individual components of the solution I will just like to show you an animation which I did in MATLAB and you will also do that as a part of your assignment to illustrate the concept of at least the forward wave you could do a similar thing for a backward wave also.

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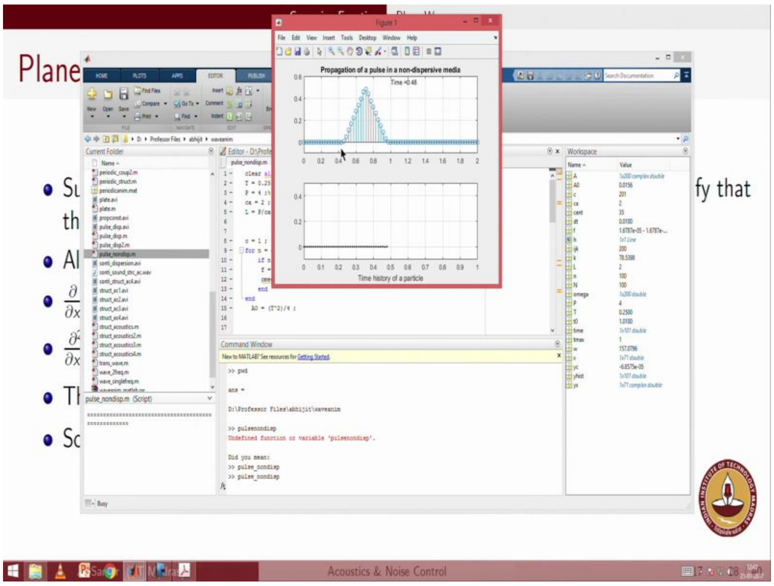

So, this is how it goes on the plot above what you see is basically snapshot at different tie this plot will show p against x for different time instants, whereas what I have shown in your plot below.

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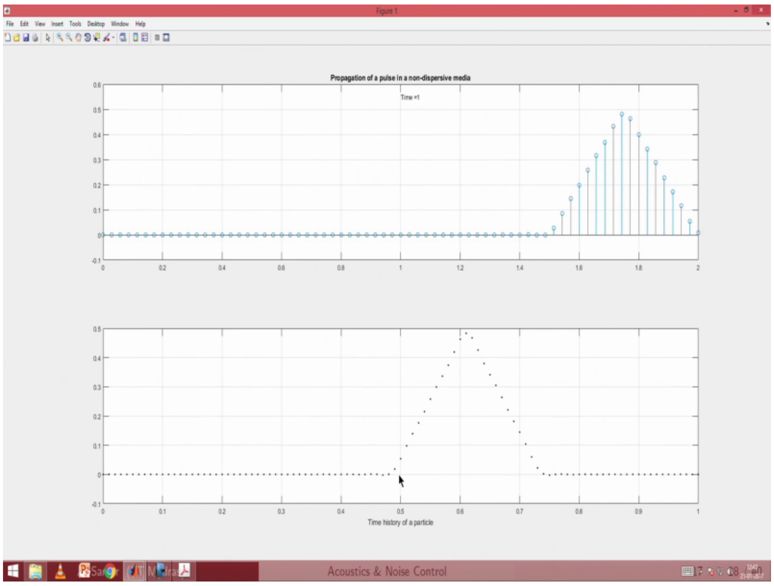

In the plot below what you get to see is I have taken a points somewhere here just one dot away from this point one x equals 1 I have taken a sensor here somewhere here and I am tracking what is the time history of recorded by this sense. So, I have taken triangular profile I mean a triangular looks like an isosceles triangular kind of profile is not the right angular triangle it is an isosceles triangle kind of a profile which is why you see roughly similar characteristics between this and this because there is no flipping effect that you will see in this image. So, that is why it will be a little bit different from what I showed in my slides, but none the same you should.

Firstly, understand that how you can animate different waveforms. So, what I have been actually doing in this animation is as simple as this that I have plotted at different time instance and I have generating this plots again and again in a loop and when you see this plots again and again it gives you an impression that it is a movie and you can very easily see that this profile is as if moving forward. So, I will play this once more for you then play it out.

So, this is the very profile as it goes down you can see this black particle is moving up and coming down and accordingly this is how you will see the time history of this black particle as it went up and came down maybe one more time to make things clearer. So, please notice there is a black particle sitting somewhere over here right. Now it will get active, it will go up and come down and then these black dots will be traced. So, if you where to placed a sensor right at this point the oscilloscope picture would look something like this right just one more re reminder that the plot on the space and the plot on the time looks exactly similar.

This is taken again taken on purpose to make things easy for you at this stage, but by now you should realize in general for a forward wave this is not going to happen it is just that the mirror image of this triangle happens to be a triangle of this sort had you taken a right angle triangle which is what problem we will do for re assignment you will get to see a flipping also right if had you taken a right angle triangle this way then you would have recovered this portion of the response right not the other portion. So, you will do that for your assignment this week we will go a little bit further now what we will go we are going to do is we are going till now we have seen 2 parts of the solution.

Now, we will see how both the parts can be obtained into in to in totality. So, towards that end we will recover what is known as D’Alembert’s solution. So, let us do the D'Alembert's solution right now.

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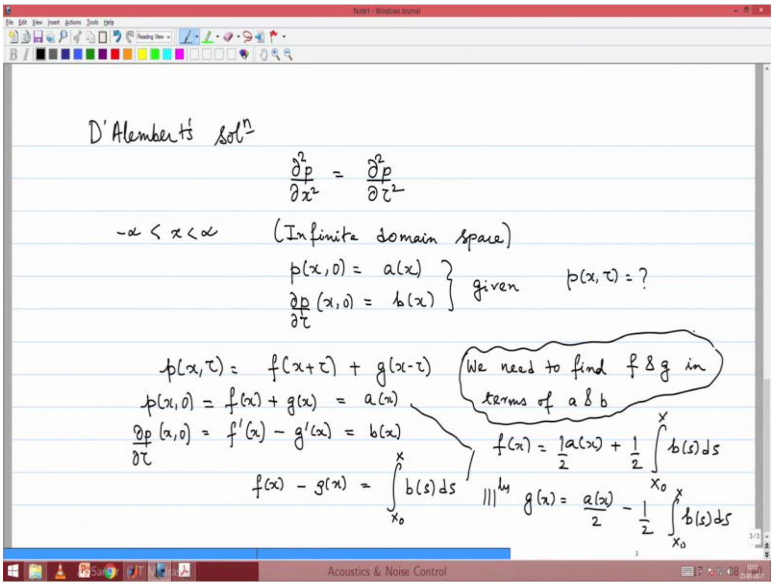

So, D'Alembert's solution is what we will study. So, the equations that we are looking to solve are this right, but remember this is a partial differential equation of second order. So, you can solve this only if there are some initial and boundary conditions given at present we are assuming that the region spatial region of interest is infinity right. So, this is an infinite domain in space right will soon enough come down to finite domain problems also, but let us starts with an infinite domain problem.

So, for infinite domain problem, there is absolutely no boundary condition because there are no boundaries it is infinitely extended. So, there is no question of any boundary condition applicable to this problem once you make this point clear that x is going from minus infinity to plus infinity; however, there is a requirement of initial condition. So, let us say that the initial condition p x equals to 0 is a x and it is a second order in time. So, therefore, there will be 2 initial conditions which will be required one in terms of the variable itself and the other in terms of its derivative. So, this is b x and a x and b x are given the question is how do we recover p. So, p x comma tau is required.

So, we already know that the solution of this is of the form p x comma tau has to be f of x plus tau plus g of x minus tau this is the general solution there can be anything else we need to find f and g in terms of a and b because remember a and b are given the initial conditions are given they are prescribed a of x is a prescribed function it is already given we need to recover f and g in terms of a and b right these are functions these are not numbers. So, how do we do it? So, the idea is pretty simple you just substitute tau equals to 0. So, p of x comma 0 is f of x plus g of x just substituting tau equals to 0 what is the formula for del p del tau in terms of f and g we have to take derivative of f and g with respect to tau, but tau seeds within the argument of f with the positive sign.

So, it is basically the same as f prime and I am going to evaluate this at tau equals to 0. So, this will be f prime evaluated at x prime essentially means differentiation with respect to its only argument there is only one argument for f which is x plus tau, but then you are evaluating it at tau equals to 0. So, this is basically f prime x with g the situation is different with g there is a minus sitting inside the argument which means you will get minus of g prime x right I would have derived this in one of the previous classes also that how do you do del p del tau in these 2 cases when we did the verification for the wave solution exactly the same idea went through. So, if this is f prime x minus g prime x which is del p del tau this has to be b of x as per the initial condition and f of x plus g of x has got to be a of x.

Now, if we integrate this second equation we get f of x minus g of x has got to be integral of b by the fundamental theory of theorem of calculus, but I have to put limits of integration I will put x in the upper limit and I can put any arbitrary in limit in the for the lower limit right because I am this is what the fundamental theorem of calculus. In fact, says that f of if you wish to integrate this thing out then you have to have a definite integral and the definite integral permits you to put an arbitrary quantity in the lower limit, but on the upper limit you should have exactly the argument that you are looking for which in this case is x. So, f of x minus g x is b integral b s d s and f of x plus g x is a of x which means from these 2 solution what I can recover is f of x is a x by 2 plus half of x 0 2 x b of s d s.

Similarly, I can I have g of x to be a x by 2 minus half of x 0 2 x b s t s right. So, thereby I have found out what I wanted f and g has been found out in terms of a and b that is it.

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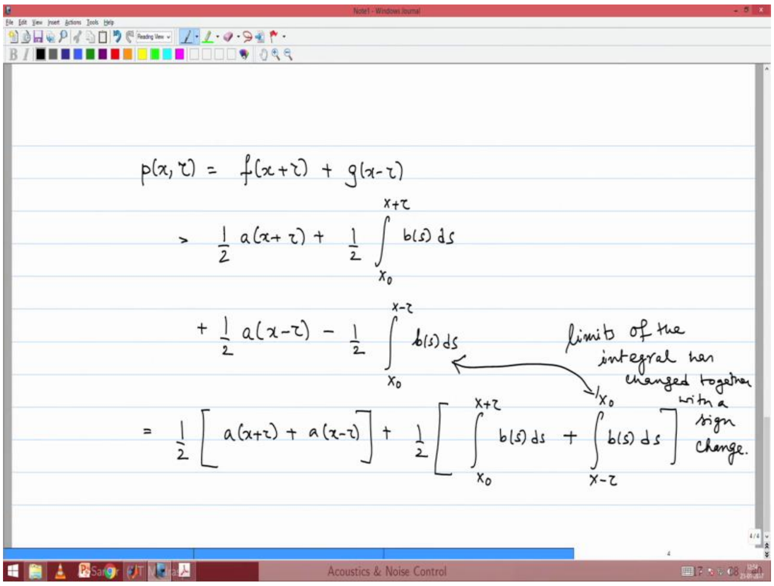

So, we can using this we can usually find p x comma tau as f of x plus tau plus g of x minus tau and what is f of x plus f of x f of x has been already evaluated. So, therefore, f of x plus tau can be evaluated by simply substituting instead of x x plus tau. So, that is exactly what I will do this is half of a x plus tau plus half of x 0 2 x plus tau b s d s and similarly g of x minus tau is half of a x plus x minus tau minus half of x 0 to x minus tau b of s d s right.

So, this can be turned around and written in the following fashion half of a x plus tau plus a x minus tau plus half of x 0 to x plus tau b s d s and I will flip the limits of this integral and make it the with the sign change. So, therefore, I will get up plus now b s d

s. So, here between here and here a limit of the integral has been changed with the sign change together with the sign change.

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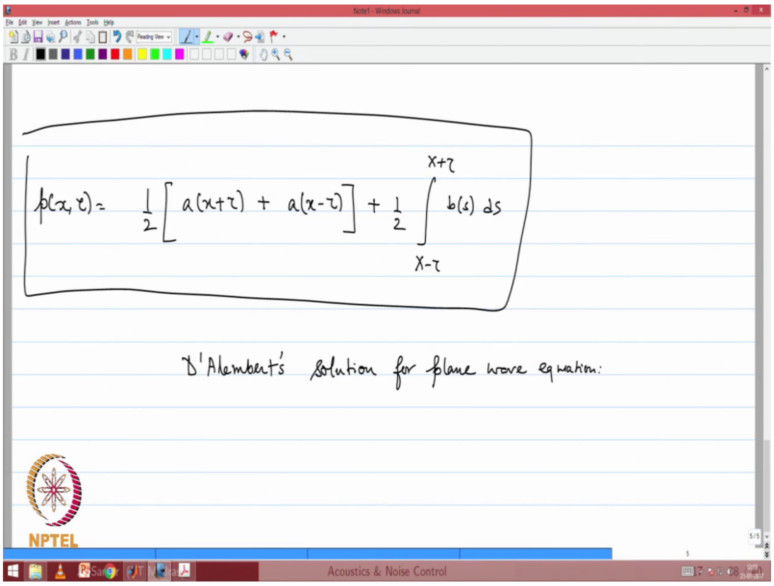

So, therefore, I can club this nicely as p x comma tau is equals to half of a x plus tau plus a x minus tau plus half of x minus tau to x plus tau b s d s I can club these 2 integrals using properties of integration definite integral x minus tau to x 0 x 0 to x plus tau. So, I can club it in a single integral of this form and that is my solution. So, this is the D'Alembert's solution for plane wave equation.

In the next lecture, we will interpret parts of the solution and we will take it up from here.

Thank you.