Acoustics & Noise Control
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Module – 02
Lecture - 04
Governing Equation 2

In the previous class we had looked at how to derive the acoustic wave equation. So, principally we had used the continuity equation, the Euler equations or the momentum equation, and the thermodynamic process.

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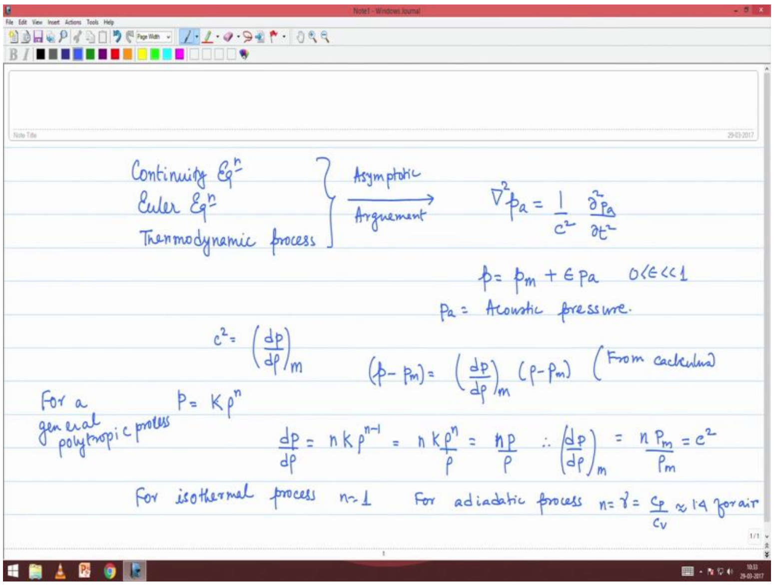

And these three equations together with an asymptotic argument give us the acoustic wave equation which was written in the following form. So, the acoustic wave equation turned out to be del square P a is equals to 1 by c square del 2 P a del t 2. Please recall P a is the acoustic component of the pressure. So, you will recall that the total pressure was broken up into two parts P m plus epsilon P a, where epsilon is a small quantity is a bookkeeping fictitious parameter introduced. So, as to enable the order tracking and P a is the acoustic component and acoustic pressure and P m was the mean pressure.

So, this is what we had done in the last class this is just a quick recap of the last class, we will also recall that this quantity c square by the thermodynamic process was given as the following, it is the derivative of pressure with respect to density evaluated at the mean conditions because the thermodynamic property give us the following that P minus P m is equals to d P d rho evaluated at the mean conditions into rho minus rho m right. So, this is from calculus. So, presently we will try to elaborate these quantities c square. Remember c square is precisely abbreviated as this derivative d P d rho evaluated at the mean conditions.

Now, the thermodynamic process will usually interrelate the pressure and the density. So, in general the pressure will be some constant call it capital K times rho to the power n. So, n for a general polytrophic process we have this condition which relates the pressure and the density remember this is the total pressure and the total density. If we take the derivative of this equation we get the following n rho to the power n minus 1, which could also be written as n k rho to the power n divided by rho, and since we know K rho to the power n is P we could as well right this as n P by rho. So, therefore, d P d rho evaluated at the mean conditions would read as n P m by. So, this is basically what we abbreviated as c square, later we shall see that these quantities c is just the wave speed.

But before we do that we must understand what value we should choose for n. Remember n is just an arbitrary number at this stage because we assume that thermodynamic process is a general polytrophic process, but now we need to really understand what is the number that we will choose for this quantity n. So, it turns out that for isothermal process you should have n equals to 1, and for adiabatic process through the value of n should be equal to gamma which is just the ratio of the two specificities and this turns out to be nearly 1.4 for air. And if it is neither isothermal nor adiabatic, but a general polytrophic process then it has to be determined through some experimental mix, but let us understand these two extremes and let us try to physically argue which of these two process that is isothermal or adiabatic which of these two processes is applicable for the case of acoustic wave propagation.

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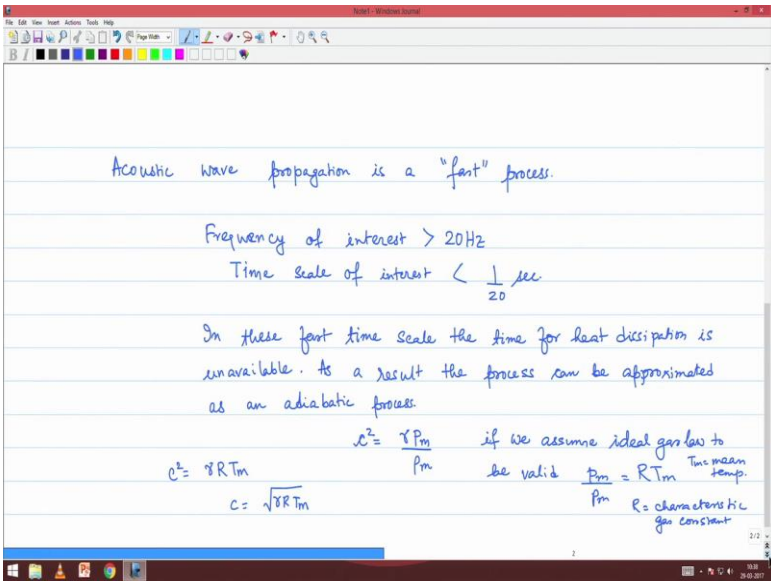

We will note that acoustic wave propagation is a fast process, but what do I mean by first I will just fast I will just elaborate. Typically the frequency ranges that are audible to our human ears are going to be 20 at least more than 20 hertz typically we cannot hear anything below 20 hertz. So, therefore, the frequency range of interest being at least 20 hertz or more.

So, the time scale of interest is going to be less than 1 by 20th second and therefore, it is going to be a very fast process. Because it is a very fast process the heat dissipation is not allowed, the time required for heat dissipation is just not available for the process as a result it is more mimicking the conditions of an adiabatic process. So, in this fast time scale the time for heat dissipation is unavailable as a result the process can be approximated as an adiabatic process typically here dissipation is a slow process and since the time scales of our interest are very fast there are rapid oscillations of these particles within these rapid oscillations within the time period afforded by this rapid oscillations of the particles there is no time allowed for the heat dissipation, and thus we can approximate this process as an adiabatic process rather than an isothermal process. Isothermal process is typically the other extreme which happens for a very slow process right.

So, therefore, from here on we can take c square as gamma P m by rho m, but now if we assume ideal gas law to be valid, then this quantity P m rho m could intern be related to the temperature right? R is the characteristic gas constant and T m is the mean temperature. So, the moral of the story is c square could be written as gamma R T m remember gamma is a property of the medium, so is R. So, in short this quantity c square which as I said rather the c quantity as I said earlier is going to turn out to be the wave speed. So, it turns out that the wave speed c would in turn depend on the square root of the temperature. This is a very important result and with this we can sort of be able to say that with fluctuating mean temperature or ambient temperature how does the sound speed change.

So, this is the elaboration of that quantity c which we initially in the last class took as just by rules of calculus just the derivative of pressure with respect to density, but then it could be simplified as is shown in this derivation to this quantity which is related to the mean temperature effects. Now we from here on we will be only interested in the acoustic components, we are not going to be interested unless otherwise mentioned in the mean components. So, therefore, instead of writing the equations in this fashion we will make a notational change just to is our life.

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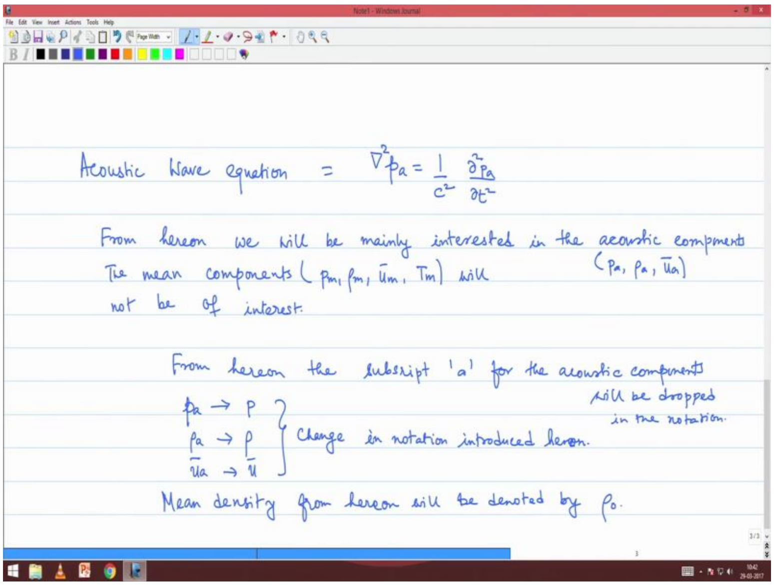

So, we understand that the acoustic wave equation is now derived as follows, but from here on we will be principally interested we will be mainly interested in the acoustic components which means P a, rho a, and u a the mean components which is P m, rho m, u, m u m anyway was not taken into account or even T m will not be of interest; thus to simplify the notation we will get read of this subscript a in our terminology.

So, from here on we will when we save P it will actually mean the acoustic pressure, we will not carry the subscript a because it is cumbersome to keep writing this. So, from here on the subscript ‘a’ for the acoustic components will be dropped in the notation. Because it will not create any confusion because we know that the only pressures that we will be talking of is the acoustic pressure there is no chance of any ambiguity. So, basically P a will now for notational purposes we denoted as P, rho a for notational purposes will be denoted as rho, and u a for notational purposes will be denoted as u without any subscript. So, this is just the change in notation that will be taken from here on. At times we will required to use the mean density, so the mean density from here on will be denoted by rho 0. So, with this change in notation we will do further derivations as we go along.

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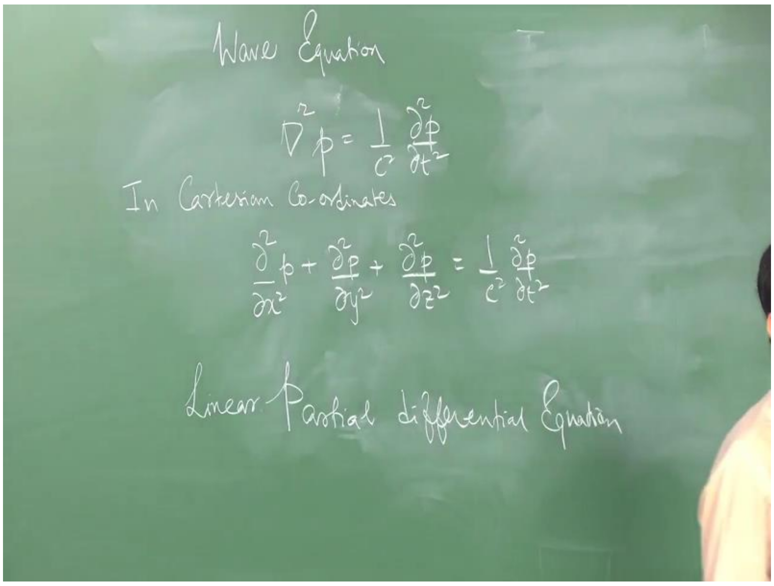

So, now that we have got our wave equation which states that del square P is equals to 1 by c square del 2 P, del t 2, this is the wave equation that we derive P here means the acoustic pressure not the total pressure right. So, here I am changing my notation because here on I will be interested only in the acoustic component not in the total pressure component right. So, if you open this up the laplacian operator if you open this up this would read as follows this is in Cartesian coordinates. In Cartesian coordinates this equation would open itself up in this form, this is firstly, realize it is a partial differential equation right; why is it a partial differential equation? Because the independent variables are more than 1 right even if you take one dimensional case you will have a time dependence and a space dependence.

Space dependence will be there will be one space dependence if it is one dimensional equation, there will be two space dependencies if it is a two dimensional equation and in its present form it is having three special dependencies. So, it is a linear partial differential equation. It is a partial differential equation that is the first observation, and it is also a linear partial differential equation. Why is it linear because every term what is the unknown here P right, each of these terms are some derivatives of P with respect to one or the other of the independent variables, but none of the terms have any non-linear factors coming in right.

So, where minimum look a glaze at this equation should convince you that it is a linear equation that is there are no other dependencies except for linear. So, in other words if I change from P to alpha times P right. So, then the alpha would come out both from the left hand side and from the right hand side and it is basically the same equation right? There are further test of linearity which I will come to in details as we go along, but at this point I would simply like you to identify that each of these terms are linear terms in the derivatives of P right; they are not quadratic terms, they are not cubic terms, they are not fractional terms, they are not trigonometric terms each of them being non-linear terms they are simply having a linear dependence on the derivatives of P.

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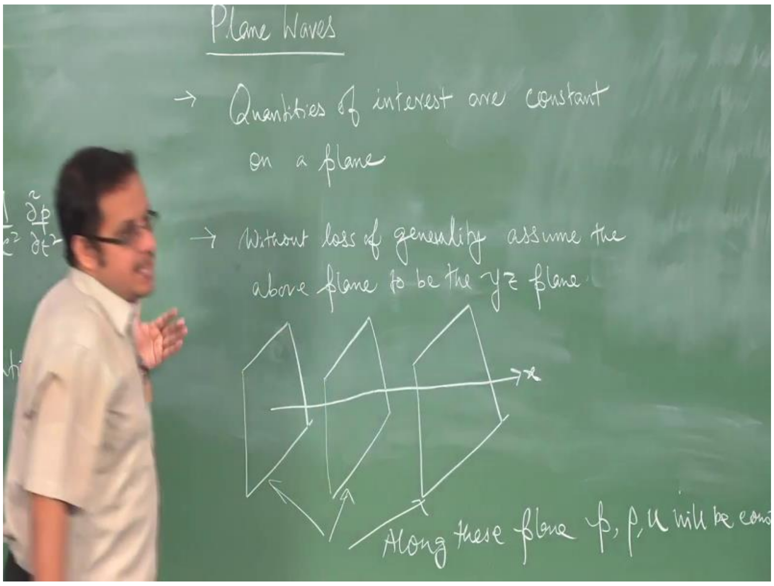

So, this is a linear partial differential equation what we will do now is that will spend some time in understanding a very special case of this equation which are called plane waves right. As the name implies plane waves stands for meaning that there are no variances in a plane, all quantities in a plane are constant there are no changes expected in a in any variable here the mean variable of interest is the acoustic pressure.

So, in other words acoustic pressure over a certain plane does not change. How does it change it changes only in the direction normal to the plane. On the plane all the quantities of interest primarily acoustic pressure, but also acoustic density acoustic velocity everything that we are interested in does not change on a plane right. So, the colloquial definition of plane wave would be as follows, quantities of interest are constant on a plane right let us say that this plane is the y z plane. So, any quantity that you are interested in when we are talking about plane waves will not change in the y z plane, if at all it changes it will change normal to that plane which means it will change only along the x direction. So, plane waves are this. So, without loss of generality assume the above plane to be the y z plane.

So, along the y z plane we are assuming that things are not changing just let me give you a idea as to what I am implying. So, these are different planes all parallel to the y z direction, and this is the x direction normal to this plane. Along each of these planes along these plane the acoustic pressure, the acoustic density, the acoustic velocity each of them will be constant will be constant. The only way in which acoustic pressure will change is between two points is if these two points have different x coordinates, if two points have the same y and z coordinate then the variables will change it will change only if the two points of interest will if it has different x coordinates only then it will change. And why do I make this statement without loss of generality? Because if you say what happens if; so here I am very sort of conveniently assume that things are changing only along the x direction.

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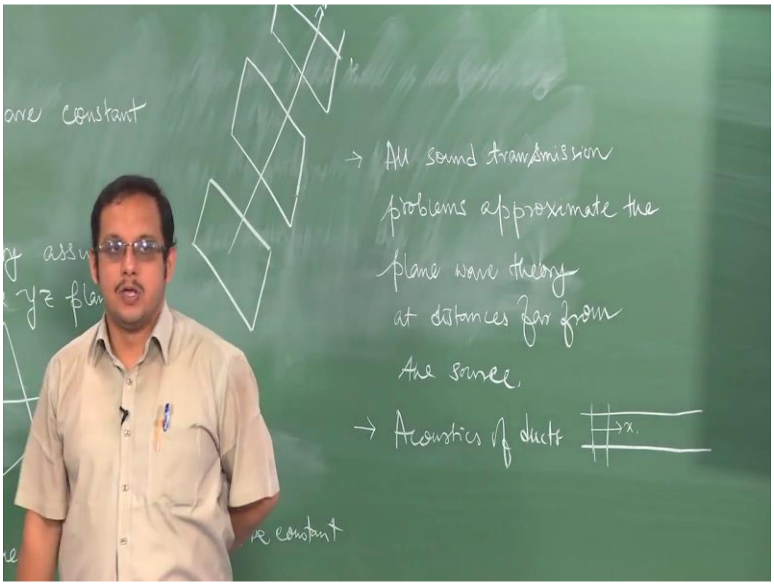

If you say now that the plane waves are going in this fashion. So, the planes of interest are somewhat in client right.

So, if you give me this sort of a framework wherein you say that these planes which are some strange oblique planes are the planes where in the particles have the same acoustic pressure, same acoustic density same acoustic velocity right. It is actually the same problem it is just that I will now define my coordinate x along this direction. So, the coordinate x can be appropriately oriented such that it happens to coincide in the direction of change right, and accordingly the coordinates y z will denote the direction where in the quantities do not change. So, this is the characteristics of plane wave please do not think that plane waves are only for academic interest and they do not happen. In fact, as we will show it is always that you can have a plane wave approximation to any complicated sound transmission problem right.

Eventually things will become a plane wave; it does happen in three dimension only the only associated assumption is that as the name suggest all the variables of interest are constant along a plane right. And since the choice of coordinate directions is arbitrary, you can choose this plane to be exactly the y z plane and choose the x direction along the direction of along the direction varying the quantities do change right.

So, this is a plane wave as I said the practical application of this plane wave theory is as follows, all sound transmission problems approximate the plane wave theory at distances far from the source right. At this stage I am I will not prove this statement, but you take it on faith eventually when we come to spherical waves for example, we will show that a spherical waves can be brought down to plane waves, it has very similar characteristics as plane waves the characteristics being that of impedance, the impedance of a spherical wave will approach to that of a plane wave when the distance is quite large in comparison to the acoustic wave length.

So, in other words what that implies is that when you are talking about transmission from distance is large far away distances from the source, everything becomes a plane wave right. So, that is a ready application, so therefore, you should not be mislead in thinking that a plane waves do not arise in reality. The other place where you may think plane waves are quite obvious is in the acoustic of ducts; what are ducts? Ducts are simply like pipes right as a name suggest it just a pipe where do you have acoustics of ducts in our automotive example whatever is the exhaust noise it actually comes out through the exhaust pipe which is a duct right. So, when we wish to design a silencer, silencer is basically something which has to be put on the exhaust pipe. So, the exhaust anyway by its very nature because if you ignore the cross sectional dimension in comparison to the dimension along the axis then everything here is in plane wave.

So, what we essentially as saying that along each cross section all the quantities of interest are going to be identical, if at all the quantities vary it will vary only along the axial direction of the pipe. So, ventilation ducts like you know centrally air conditioned room will also have ducts which carries not only the hot air and the cold air, but it also carries the sound associated with the air conditioning process right. So, acoustics of ducts and mufflers are very important topics. So, for both these problems this will turn out to be a very important topic. So, staying on plane waves therefore, what is the equation associated with plane waves? Since we said there is nothing with changes along y or z. So, these two terms will get killed and we will have the plane wave equation to be this.

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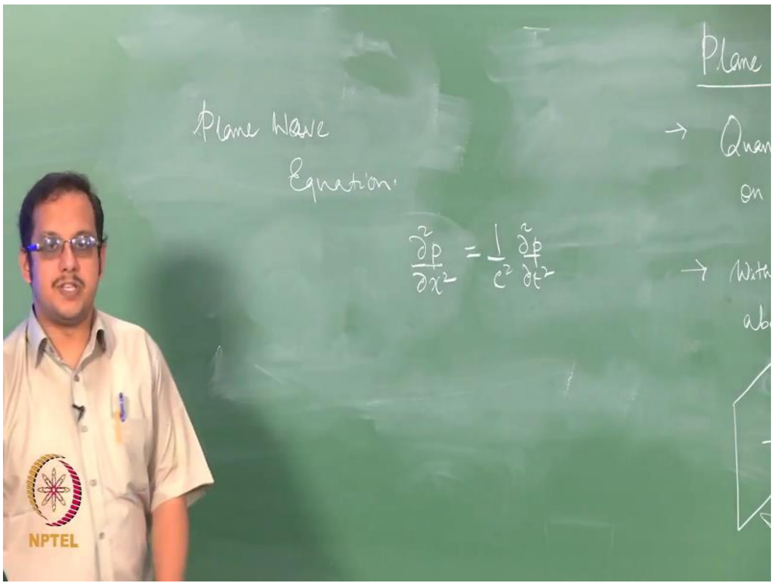

This is the plane wave equation which comes through, because there is no y dependence there is no z dependence therefore, you have the plane wave equation given in that form.