Acoustics & Noise Control
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Module – 28
Lecture – 33
Analysis of Industrial Mufflers

We will see today some other muffler configurations which sort of exploit that idea of creating a bypass line and thereby increasing the velocity ratio and the insertion loss. So, towards that end, we look at extended inlet outlet resonators.

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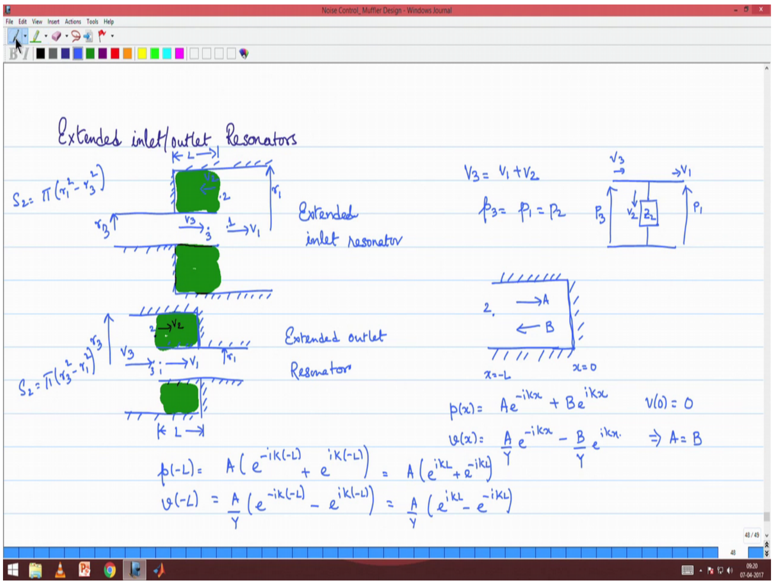

So, the idea is again pretty simple; you will have this is an extended inlet resonator wherein you have an incoming condition which is denoted by 3, the outgoing condition is denoted by 1 and here you have another bypass line which is denoted by 2. And similarly you could have a situation in the of extended outlet resonator which is as follows.

So, here you will have an incoming which is v 3 and outgoing which is v 1 and a bypass which is v 2. So, this is an extended inlet resonator and this is an extended outlet resonator. Note the similarity between these kinds of configuration with the simple expansion tube. In the simple expansion chamber muffler we had the situation where, the pipe was suddenly opening out. So, there was no extension that is given here, we are having an extension. In both these configurations please note that we have v 3 is equals to v 1 plus v 2.

So, if the incoming sound wave gets splits in to two parts; one which proceeds further downstream which is denoted by the subscript 1 and the other which is sort of by passed in the outer shell. So, we have v 3 equals to v 1 and also we will have p 3 is equals to p 1 is equals to p 2. So, p 3 is equals to p 1 is equals to p 2. So, the 0.3 is here, the 0.1 is here and the 0.2 is here. So, all these three points are just contagious points. So, therefore, the pressure at each of these three points marked as 3, 1 and 2; have got to be the same.

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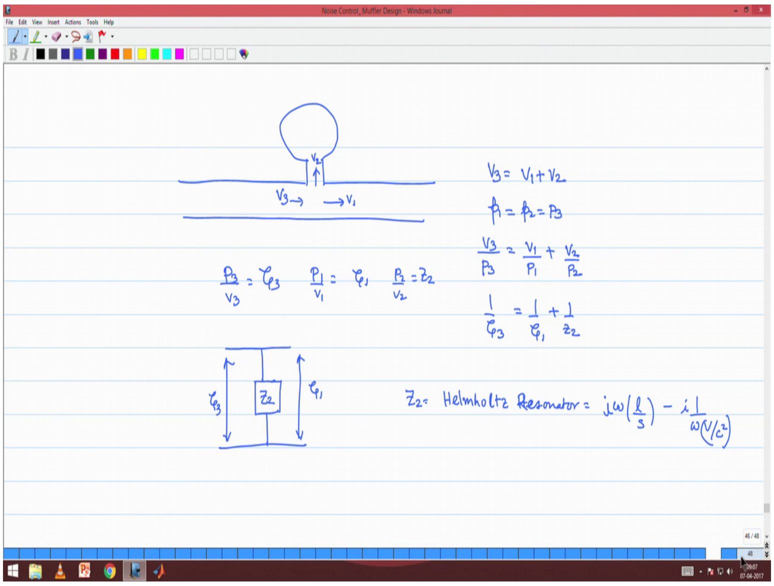

So, these two conditions are identical to the Helmholtz Resonator Condition that we had described in the earlier class which was as we said sort of an academic exercise because you do not really find these kind of mufflers. But the point to note is that this condition that the velocity bifurcates into two parts v 1 and v 2 whereas, the pressure remains the same is exactly obtained in this situation also which is something, which you would see in practice.

So, therefore, this situation this the resemblance of this resonator or this configuration of the muffler with the Helmholtz Resonator suggest that, you will have a situation where in you have a bypass line given by Z 2 and the upstream conditions are p 3 and v 3 and the downstream conditions are p 1 and v 1 and this is v 1 and this is v 2. So, here in the electrical circuit you can see v 1 plus v 2 is equals to v 3 as is required by our configuration. The only difference between this system and the Helmholtz Resonator system arises in the fact that, while using the Helmholtz Resonator we found that the impedance of the Helmholtz Resonator was well known from our previous calculations. So, here this Z 2 has to be calculated a fresh, Z 2 remember is the impedance at point 2. While doing the Helmholtz Resonator cum calculation it was easy to evaluate Z 2 and that was evaluated earlier in one of our previous analysis where, Z 2 in the case of the Helmholtz Resonator had two parts which is; i omega l by s that is the past part associated with the mass and minus i 1 by omega V by C square.

That is the impedance associated with the Helmholtz Resonator which we had previously derived whereas, here we need to evaluate the impedance at this station 2. So, this is what the point of difference lies. So, let us quickly do that. So, I will do it just for one configuration and the other should be pretty easy. Please understand that the walls of this duct are rigid right. So, we all along we are assuming that all these structural walls are thick and rigid, if not it would lead to vibration of these structural surfaces which would emanate sound again and that form of sound radiation will be called as breakout noise. But here we are dealing with the situation that the structure is fairly rigid and does not vibrate in response to these acoustic pressures.

So, what we really have to figure out is that in this chamber, let me highlight this chamber in this chamber and also in this chamber we wish to find out that given that the downstream end; the end which is which is having a rigid termination that is this end. Given that there is a rigid termination in this chamber, what is the impedance at the point two? So, we here we are simply drawing this portion afresh, the highlighted chamber is drawn again here with the understanding that this end is completely rigid and the objective is to determine the impedance at this point which is 2.

So, towards that end we can put a coordinate x equals to 0 here and x equals to minus L here. Again as usual we only assume that there are plane waves which are going to reside within these highlighted chambers right. And therefore, the pressure within this chamber is going to be A e to the power minus i k x plus B e to the power plus i k x and the velocities, the mass velocities to be more appropriate is going to be A by Y to the power minus k x minus B by Y e to the power i k x. But now we need velocity at x equals to 0 to be 0 right; because here you must have a rigid termination.

So, therefore, if velocity at the point 0 has to be 0 then you must have A equals to B, from the second condition. So now, the deal is to find what is the ratio of p at minus L and v at minus L? Given that A is equals to B. So, p at minus L therefore, reads as A which we can pull out, e to the power minus i k minus L plus e to the power i k minus L. So, that would read as A e to the power i k L plus e to the power minus i k L. We have already go the result that A is equals to B by putting the condition of rigid termination. Similarly, velocity at the very same point which is the point 2 is denoted as A by Y e to the power minus i k, instead of x I have to put minus L minus B need not be considered because B is same as A, e to the power i k minus L. So, that would read as A by Y e to the power i k L minus e to the power minus i k L and using these two relation which I will copy in the next page.

So, using these two relations we could write the impedance at the point of interest. So, Z at minus L which is basically Z 2 has got to be p at minus L divided v at minus L.

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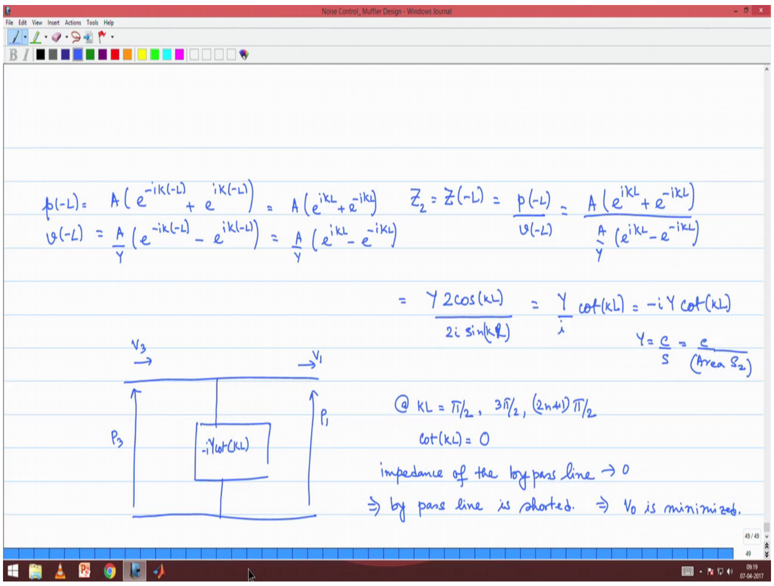

So, that should read as A e to the power i k L plus e to the power minus i k L divided by A by Y e to the power i k L minus e to the power minus i k L and then we could simplify this further. The numerator could be written as 2 Cos k L and the denominator could be written as 2 i sin k L and then you could put the Y on the numerator because you have A by Y in the denominator of this fraction and this would read as Y by i cot k L, which could also be written as minus i Y cot k L.

So, important point to note is that the impedance of the bypass line is going to be minus i by Y cot k L. What is Y in this case? Y is C by S where, S is the area of cross section of the concentric chamber right. So, please understand that if the radius of this part is r 1 and this or I will call this as r 3 and if the radius of this part is r 2 right r 1 sorry. If this part is r 1 then, the area appropriate to the section 2 would be. So, S 2 would be pi r 1 square minus r 3 square.

Similarly here, S 2 would be, this is r 3 and this is r 1. Here S 2 would be r pi r 3 square minus r 1 square. And the Y that I am dealing with is basically the area associated with that concentric chamber.

So, S in this case would be the area of area S 2, what I have marked as S 2 in the previous diagram. So, with that correction I could find the impedance of my bypass line. So, the impedance of the bypass line will be given as minus i Y cot k L. Again different situations can arise where in this bypass line will actually get shorted. For example, when you have, what is L by the way here? L is this distance right, because this is the length of the chamber. So, L is this distance right and S I have already said how you should calculate the associated S, C is the sound velocity. So, that does not change yeah. So, interesting cases happen when let us say k L goes to pi by 2. If k L goes to pi by 2 then what happens? 2 cot k L, cot k L will go to Cos k L by sin k L, k L Cos of pi by 2 will be 0. So, the impedance of this line will be going towards 0, which means this line will be shorted and that is precisely what you want because if this line is shorted then the entire current will move through this line and no current will escape in the downstream end and the v 0 quantity, that is the current which is in the load side will go to 0.

So, this is P 3 and this is P 1 and this is V 3 and this is V 1. So, at k L equals to pi by 2 cot k L turns 0 which means impedance of the bypass line goes to 0. This implies the bypass line is shorted. So, which implies V 0 is minimized. This will again lead to a good insertion loss because you have a bypass line where in you are having the situation that the entire current flow is occurring through the bypass line, it is not going downstream. So, in contrast with the Helmholtz Resonator, this resonator also has this same feature that it offers a bypass line which at least for some frequencies you are able to tune. Let us see if you are able to tune the length of this parameter capital L and bring it in to a form such that k L equals to pi by 2. You will achieve the exactly the same condition as the

Helmholtz Resonator condition, but the advantage of this resonator is that now you could have this not only at pi by 2, but also at 3 pi by 2 and.

Student: 5 pi by 2.

5 pi by 2 and in general any odd multiples of pi by 2 will give the same condition, whereas the Helmholtz Resonator was tunable only at one frequency because Helmholtz Resonator was, after all a lumped system approximation for our acoustic system. Whereas this system is going to lead to zero impedance path, but not just at one frequencies, but many many different frequencies. So, if you if its. So, happens that all of your problematic frequencies are these then, this sort of a configuration will sort of improve the insertion loss characteristics at all these frequencies.

So, the key idea again here is to have a measurement where in you know what is the problematic frequencies of your acoustic radiation and if those frequencies have to be knocked out you have to choose the parameter L such that k times L happens to be one of those frequencies and at each of these frequencies you can expect that the insertion loss will be maximum. So, this is how the Helmholtz Resonator system can actually be employed to understand the situation and then you can easily construct this type of a muffler where instead of an expansion chamber you just have an inlet tube and an out let tube. which is going into the expansion chamber. So, this is called extended inlet outlet resonators and these are resonators because you will understand that within this green cavity what is happening is that, you are having a resonance situation that is main created. Actually the cavity is not just this part it is also this part, I should mark this as the cavity right.

So, this is where because the area associated is this concentric region. So, with that let us study one more muffler configuration to sort of close this topic, but before we do that we will just revisit our Helmholtz Resonator problem once more.

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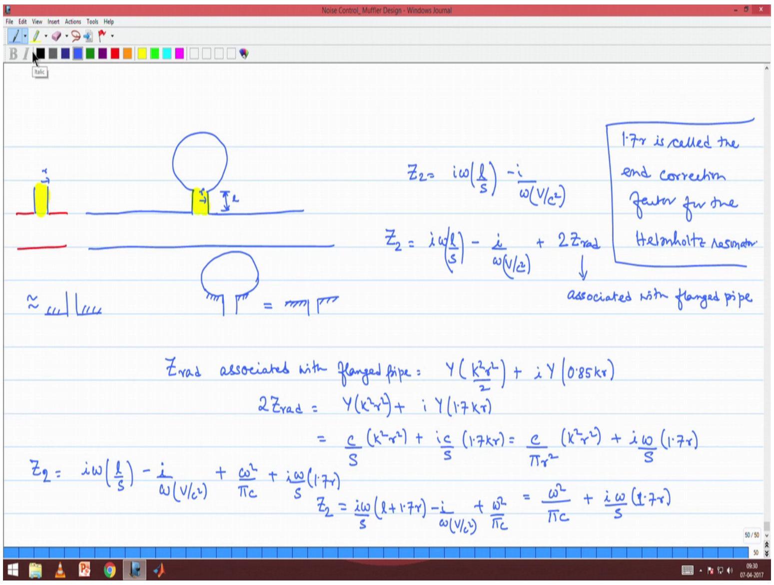

So, we did this Helmholtz Resonator within the acoustic line in the last class. So, there we said that Z 2 is the impedance of the Helmholtz Resonator and that impedance as was calculated earlier is i omega L by S minus i by omega V by C square.

So, the first term denotes a mass like effect and the second term denotes a compliance effect or a spring like effect. Now here one aspect that we did not incorporate in our first cut analysis, we will make amends for that. The difference is this the from this small neck there is going to be a radiation from both the sides one towards this large volume cavity and the other towards this tube. If I look at just the neck in isolation, this neck in isolation is going to see two large volumes; one on the outlet side and the other on the inlet side which is the tube. The tube is a large volume in comparison to the neck of the Helmholtz Resonator. Similarly this large spherical volume is a large volume in comparison to the neck of the annular construction of this Helmholtz Resonator.

So, the fact that the neck now sees two large volumes; the situation is just like that of a radiation impedance condition. Just like the tail pipe was opening into a large volume which is the atmosphere and thereby creating another impedance which was called the radiation impedance. The same situation is happening here if you consider just this tube, let me change the color, how do you change the color? Yeah, if I consider this tube; this tube is actually opening out into two large volumes; one volume which is associated with this cavity and the other volume which is associated with this tube right, the neck is far narrower then the tube itself.

So, it has to see two radiation impedances associated with these two large volumes. So, we will make that correction. So, the Z 2 corrected would be these two radiation, this impedance term will be as it is, but then we have to add two radiation impedances associated with these two effects right because this narrow annular cavity is seeing two large volumes on either ends of itself. However, you will recall that we have given two expression for radiation impedances; one for a flanged pipe and the other for an unflanged pipe. As opposed to the tail pipe which directly opens in to the atmosphere here you can see. For example, it is very clear if you think about this diagram, from here from this portion the cavity which is I mean the volume which is seen by this highlighted portion into this tube resembles that of a flanged pipe right because there is something which is sort of guarding this annular tube from the exhaust pipe right.

So, this situation is similar to that of a flanged pipe which is opening to a large volume right. Similarly, if you look at the other picture which again I will draw separately. So, here you have a situation where a small narrow pipe opens into a large volume. So, this is again going to resemble as a small pipe which is opening in to a large volume, but then there is some blockage on this tangential side right. So, this situation is that of a flanged pipe. So, the Z radiation impedance is that associated with a flanged pipe not with an unflanged pipe. That is why I gave you both the formulas in the earlier class, so that you can use it appropriate. So, the radiation impedance associated with the flanged pipe is given as Y times k square r square by 2 plus i Y k r into 0.85. If you recall we had quoted this formula the derivation of this is beyond the scope of the current course, but we have just used this formula.

So, now what we have to use is that, we have to use twice this radiation impedance to 2 times Z rad should be Y times k square r square plus i Y times 1.7 k r and this if we recall should be C by S into k square r square plus i Y is C by S 1.7 k r and k C is or before doing k C, S is pi r square; remember this is now our r. So, this is the radius of the tube that we are talking about. So, S is pi r square and k square this is C by S and S is replaced as pi r square and k square r square leaves as usual plus i k times C can be written as omega, omega by S 1.7 r right. And then finally, we can have this condition to be omega square by pi C plus i omega by S 1.7 r.

So, therefore, this Z 2 factor needs a correction which is going to be i omega l by S minus i by omega V by C square plus 2 times Z rad is calculated to be omega square by pi C plus i omega by S 1.7 times r and then we could collate these two terms; the first and the last two terms if it is combined it would read as follows l plus 1.7 r minus i by omega V by C square plus omega square by pi C right. So, this is the corrected formula for the impedance of the Helmholtz Resonator which accounts for the fact that. it actually radiates sound within not only the cavity of the Helmholtz Resonator itself, but also the inlet pipe which is pretty large in comparison to the volume of the air content in the neck of the Helmholtz Resonator.

So, the import of this formula is this, that the length of this Helmholtz Resonator is actually changed to whatever its physical length is to a quantity which is 1.7 times the radius of the hole. So, one point seven times radius of the hole adds to the physical length of the Helmholtz Resonator through this calculation. So, 1.7 r is called the length correction or the end correction factor for the Helmholtz Resonator.

So, the physical length together with 1.7 r is the actual length which goes into the impedance formula. Now with this background, let us look at one of our final muffler configurations which is what you would have seen in a bike muffler.

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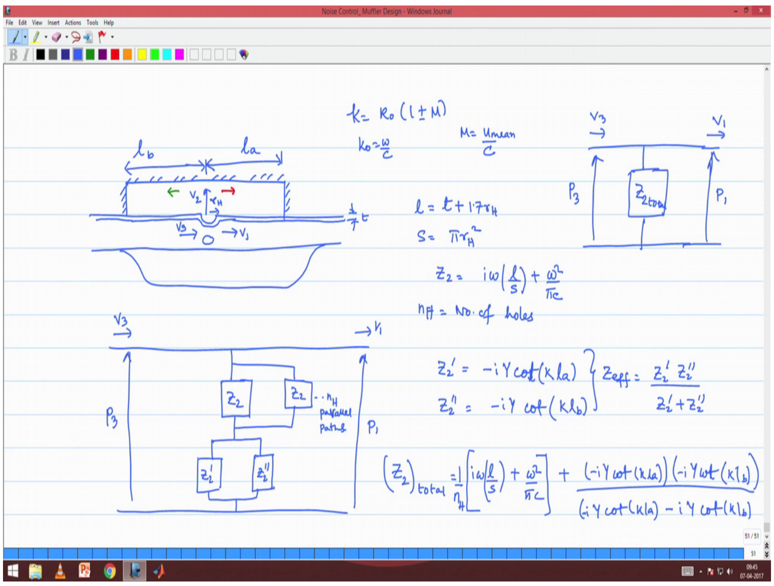

So, in a bike muffler you will have some an inlet tube and the inlet tube if you would have seen is perforated. There are number of holes in the inlet tube and then on top of it is a fairly rigid structure of this kind right. So, this is the schematic of a bike muffler where in there are perforations. To start with let us just handle one hole, if we understand what happens with one hole we can readily extrapolate the ideas to multiple holes. Now what happens because of this hole is that again you are providing, please understand the basic idea of providing these holes is to create a bypass line. So, not everything I will just put the hole on the top such that it is easier for me to draw this situation.

So, what I will do is, I will put the hole somewhere here. So, this is the hole right. So, what happens here is that at this junction there is an incoming velocity, there is an outgoing velocity, but then here there is a bypass line that is being created right. Because of this hole apart of the sound is expected to bypass and go in the direction which is not in the direction of the downstream end. So, again the same situation is being created you are providing a bypass line, but now apparently you do not see a Helmholtz Resonator here.

I mean it is not physical looking like a Helmholtz Resonator because there is no annular constriction or anything of that sort, but then if you remember that the length of, now that we know that the length of the annular tube of a Helmholtz Resonator is not just the physical length, but it is 1.7 times the radius of the hole itself that end correction factor is there. So, the length of the Helmholtz Resonator which virtually sits here; does not it is not there physical, but virtually sits there is going to be l is equals to the. Firstly, there is a thickness of this shell right, this shell has got a certain thickness. You have made a hole in that shell, so, there is a thickness to this shell I will call this t, which may be very small, but together with that there is a factor of n correction which is 1.7 times the radius of the hole itself; the hole is as shown here right.

So, that is the virtually the annular tube of the or the neck of the Helmholtz Resonator right and what is the S associated with it? S is again pi r H square. So, r H is the radius of the hole. I am really shot of space in this drawing here, but I will try to make amends. So, this is r H and from here there is a sound which is escaping at a mass velocity of V 2 right. So, therefore, definitely I can assume that there is an impedance at this point which is related to this annular cavity. So, one part of this impedance is going to be that due to this annular cavity which should read as i omega l by S where, l accounts for the thickness as well as for the end correction. The cavity in this case is not apparent. So, we will not use that cavity portion, the cavity of the Helmholtz Resonator is going to contribute to the second term; however, the cavity at this stage is not apparent. So, we will not use that term, but we will use this term because this term comes out from the radiation impedance factor. Just like the radiation impedance factor gave you the fact that there is an end correction. Similarly the radiation impedance factor gives a certain resistance. This is a resistance terms because it is purely real.

So, the you have to account for that pure resistance term and that term when accounted is going to give to rise to an term of this kind which is omega square by pi C right. But that is not all; there is also an impedance which is associated at this point because it is lying at certain. So, this is I will call l a and this I will call as l b. I will just make the drawing a little more simpler by assuming that this is of this form. So, this concentric cavity is again rigid right. So, because what we have seen in even the previous configuration of an extended inlet or an extended outlet resonator is that, if you have a rigid termination at the upstream then the impedance at the downstream is given by minus Y cot k L a.

Now, what happens immediately after the sound escapes from the hole? There are again two paths that are possible for this sound; one along this direction, one along this direction and the other along this direction right. So, basically if I have to now draw the electric circuit associated with this situation, it would look something like this. In the upstream you will have a situation of P 3 and V 3, in the downstream you will have a situation of P 1 and V 1. Exactly because of the hole there is a bypass line which is created right and that bypass line definitely has an impedance of Z 2 which is given by i omega l by S plus omega square pi by C right, but other than that there are two more bypass line, after the sound escapes from this hole there is again a bifurcation towards the left and the right of the chamber right.

So, again there has got to be two parallel paths for the sound to go to right and this is given by each of these impedances. I will call them let us say Z 2 prime and Z 2 double prime. So, Z 2 prime associated with let us say the path on the right is going to be minus i Y cot k l a and Z 2 double prime is got to be minus i Y cot k l b right. So, what is the effective impedance of these two combine? These two impedances are in parallel, so the effective impedance will be Z 2 prime Z 2 double prime divided by Z 2 prime plus Z 2 double prime right.

So, therefore, the effective impedance of this parallel path Z 2 total is going to be Z 2 which is i omega l by S plus omega square by pi C plus Z effective which is minus i Y cot k l a into minus i Y cot k l b divided by these two quantities minus i Y cot k l a minus i Y cot k l b. So, this Z 2 impedance is what lies in parallel or in the shunt position in the circuit Z 2 total. And with that you are again creating a bypass line which is supposed to lead to a bifurcation of the current and thereby increase your velocity ratio and lead to a maximum insertion loss in this situation.

Now, what is the effect if I add multiple holes at the same location? At the same station instead of having one hole, if I have multiple holes what will happen? I will create more parallel paths right. So, what will happen is that basically this term is going to get divided by the number of holes n H will be equal to the number of holes because there are now the. So, the path will now have one more of this exactly equal situation that is the, sorry the first part of the path will get bifurcated.

So, this is the Z this each of these will be Z 2 right there by n H S of them n H parallel paths, but after the some sound emerges from the hole it will see the same impedance because in the concentric cavity nothing has changed right. So, if the, what we did is that we have first introduced one hole and then we said if there are multiple holes at the same station, we are not saying there are other holes at the different station, but at the same location if you have multiple holes it will provide more parallel paths. And as a result this quantity Z 2 which we said is i omega l by S plus omega square by pi C, this quantity will now be changed to accommodate that fact that there are multiple holes. And since each of these holes are hopefully identical you will have a division because this is a parallel path the impedance does not add up. You have 1 by n H sort of a factor which will come in.

So, now if you wish to understand what happens if there are multiple holes at different stations, again you will have to build the transform matrix associated with each station multiply those transform matrix or in the circuit theory you have multiple such shunt positions which are coming in at different stations. That is all the analysis that is left to do, but with this I hope you are able to really understand what happens in the computation of insertion loss and at least for these simple configurations you should be able to appreciate how insertion loss is to be done.

So, we will close our discussion of muffler design here, but please understand all the muffler design aspects that we have talked about was based on the premise of plane wave theory and it is true that its only plane waves will propagate, if the frequencies are lesser than the cut on frequencies. But if the frequencies are high then there can be some cut on modes which also propagate. This analysis precludes the effect of any cut on modes right.

So, the cut on modes will be an evanescent modes for frequencies which are less than the cut on frequencies. So, this analysis is going to be hopefully correct at the lower frequencies where in the effect of cut on modes should not be appreciable. And therefore, this analysis is strictly a plane wave analysis. There are many pitfalls or there are many assumptions I would say in this analysis for example, here we did not accommodate the aero acoustic effect. One arrow acoustic effect that we is very important, but we did not account for it is that here, the sound is actually getting generated in a moving medium. Because it is the exhaust gases which are moving in the pipe and these sound is travelling not in a quite medium, but in a moving medium. Remember our equations were valid for a quite medium right, but in the true situation the sound that is travelling within this duct therein the fluid is not stagnant it is not quite it is actually moving. Because of this movement of this bulk flow, you need to appropriately correct the equations of plain wave. The essential corrections that will happen is that, the wave number will get corrected to 1 plus or minus Mac number; Mac number is the particle velocity by sound speed.

So, sorry the not the particle velocity the mean velocity, the mean velocity by the sound speed; k 0 is omega by C which is what we have calculated. So, depending upon whether the sound is travelling in the direction of mean flow or in the direction opposite to the mean flow you will now get two wave numbers; one which is 1 plus m, the other which is 1 minus M times omega by C. So, this factor also would need to be incorporated this is also something that we have not taken into account. And then as I said in some earlier discussion also a muffler design only for the acoustics is not a good idea you have to make sure that the back pressures that are generated by the muffler is not high enough. Because if the backpressures are high enough the exhaust gases will not flow and that will lead to a very serious issue as far as the performance of the engine is concerned.

So, that is one aspect then there are space constraints you cannot have a large muffler with a large expansion chamber, with a large resonator. Those issues are not possible because there are lots of space constraints within the vehicle and as such you cannot, a large muffler is not of any practical usage. So, with these brief design guidelines, I think we will close the discussion on muffler design right here. See you again in next class.

Thank you.