Acoustics & Noise Control

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Module – 25
Lecture – 30
Helmholtz Resonator

So, we were looking at this problem of 2 connected springs and a mass.

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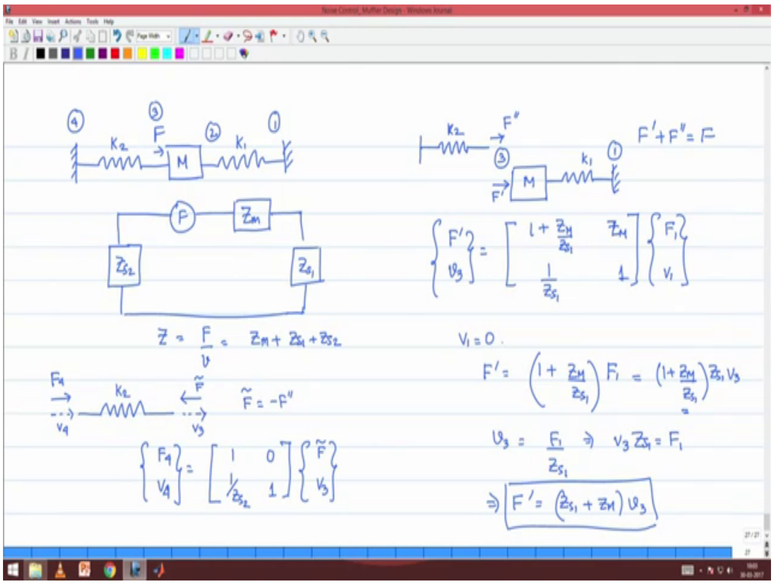

So, this is the configuration of the spring mass system that we were analyzing, we found the equivalent circuit for this situation to be that of the forces acting on the mass, the equivalent circuit for this situation was given in the following form. So, the impedance effective impedance or the drive point impedance was F by v which is Z m plus Z s 1 plus Z s 2, from the equivalent circuit that part we derived it cleanly; today what we will look at is the derivation through the transfer matrix method.

So, what we will do is again we will split this thing up into 2 parts, this sorry in the first part we will take only the spring and in the second part w will take the mass and the spring on the right this is the mass and this is the spring constant K 1. So, let us assume that effectively there is an F prime force on the right half of the sub system, and there is a force F double prime on the left half of the subsystem. So, F plus F prime plus F double prime is going to be the total force right. So, we need to find accurately what is the transfer matrix in this situation.

So, again we will define the station points in this fashion 1 2 3 and 4 right. So, this is 1 and this is 3 between station 1 and station 3 no big deal we can find F prime v 3 is given by 1 plus Z m by Z s 1, Z m 1 by Z s 1 and 1 right F 1 V 1. That is a transfer matrix for the right half of the system no big deal we also know that now we have to impose the condition that V 1 equals to 0. So, that implies F prime is 1 plus Z m by Z s 1, F 1 and V 3 is F 1 by Z s 1. So, if we make the substitution here. So, we get V 3 Z s 1 is equals to F 1 and putting this back over here we get F prime as 1 plus Z m by Z s 1 into Z s 1 times V 3 which effectively means I will write it here, F prime is Z s 1 plus Z m into V 3 this is what we have for the left half of the system.

Now, for the right half of the system. So, in the right half we go back to the first principle this is just a spring and this is station 4. So, there is as per our convention we have to take the forces in this fashion this is F 4 and this is F tilde, which is the negative of F double prime right and the velocities as per our convention is always directed in the positive sense. So, this is V 4 and this is V 3 right. So, this transfer matrix is known to be F 4 V 4 is equals to 1 0, 1 by Z s 2 where this is the spring constant K 2 and 1 times F tilde V 3 right.

So, what I have done here is that I have just looked at the element K 2 from the first principles I have just got the drawing back of a single spring element, at this stage I have not invoked the boundary condition that V 4. In fact, has to be 0, but I have written the transfer matrix none the same in a generalized situation involving V 4 V 3 and F 4 and F tilde. F tilde just resembles in a generic sense the force F 3, but in this situation I we understand that it does have we have got the situation exactly corresponding to where yeah exactly corresponding to this situation right this.

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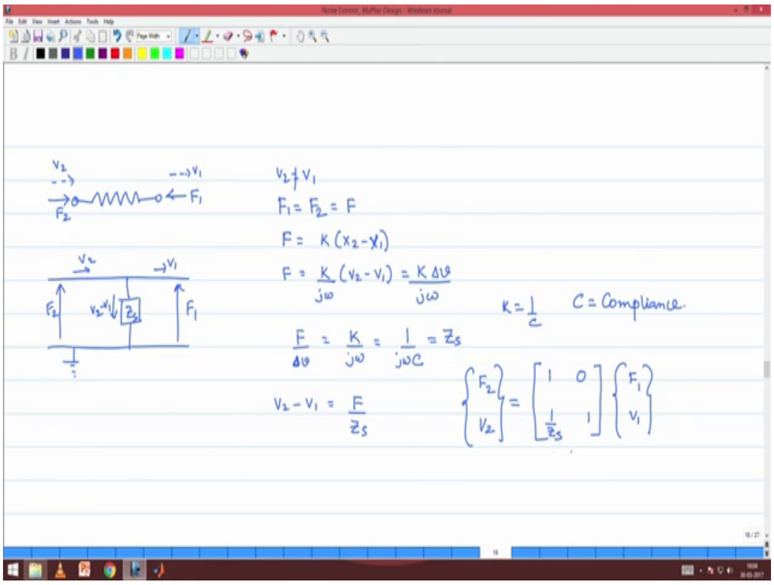

Was how we derived the elementary electromechanical analogies and also the transfer matrix method. So, here you see the transfer matrix is of this form diagonal elements 1 and 1, of diagonal element 0 and reciprocal of 1 by reciprocal of Z s.

So, in an identical fashion we have derived this situation now things we will be neat and clean if we just do a little bit of simplification associated with this matrix. So, that is what I will do in the next page.

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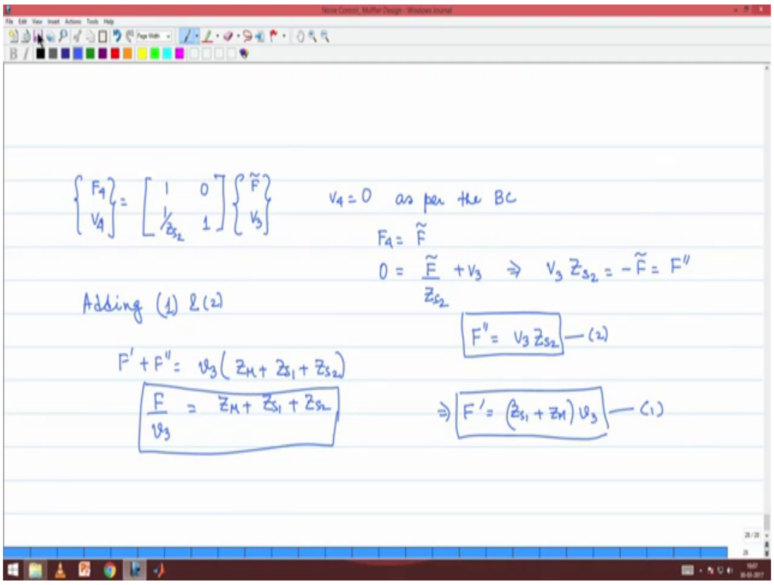

So, this is what we have now we invoke the condition that V 4 equals to 0 as per the boundary condition because it is fixed at the other end. So, in that case we will have F 4 is equals to F tilde, and V 4 which is 0 is equals to F tilde by Z s 2 plus V 3. That implies V 3 into Z s 2 is equals to minus F tilde which again is F double prime. So, what we get from here is F double prime is V 3 Z s 2; this is my second equation the first equation was F prime is. So, this is the first equation and this is the second equation if we put both of these side by side and then add it up it will be easy to see that the drive point impedance is again matching.

So, this is equation one. So, adding 1 and 2 we will get F prime plus F double prime is equals to V 3 into Z m plus Z s 1 plus Z s 2, which would also mean F by V 3 is equals to Z m plus Z s 1 plus Z s 2 this is the same result as was obtained using the electric circuit analogy. So, the electric circuit analogy and the transfer matrix method here in gives the same result as it should. So, now, we will move on to the acoustic case till now we are doing the electromechanical analogy, somehow to get comfortable with this analogy techniques and the transfer matrix technique itself, but now we will apply this method to the acoustic case, and towards that end our first problem will be to look at a duct.

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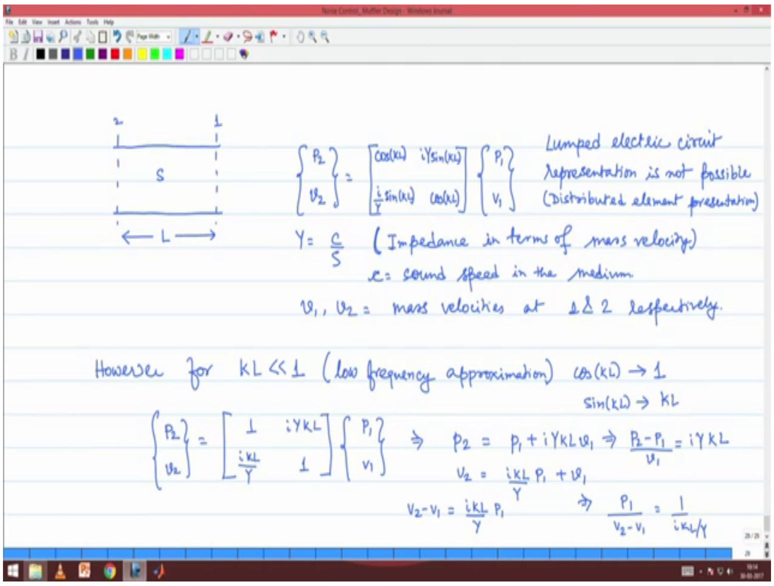

A simple uniform duct of cross section s right, we have already defined the transfer matrix for this situation when we did the analysis for the transmission loss of our expansion chamber muffler, we arrived at the transfer matrix between the 2 ends of this duct which is denoted by 1 and 2.

So, if you recall we had the situation P 1 P 2, V 2 is equals to P 1 V 1 is the inlet is the downstream condition and this transfer matrix was given by the following cos KL comes in the numerator in the diagonal terms, i Y sin KL and i by y sin KL comes in the of diagonal terms, this is of length L and what else Y is the impedance which is C by S the impedance is in terms of mass velocities, and yeah K is the wave number and c is the sound speed in the medium. And please note that v 1 and v 2 are the mass velocities at 1 and 2 respectively they are not the particle velocity we had changed over from particle velocity to mass velocity in dealing with duct problems.

So, this is the situation this is the transfer matrix which relates the upstream states and the downstream states in that fashion, it was done in one if the earlier derivation which is why I am not repeating it. So, if you look back at the nodes you will find the derivation here it go it is somewhere here yeah here. So, this is where it was derived that the transfer matrix for a duct. So, this is what I am repeating here. Now the point is this transfer matrix definitely is not of the form wherein you can present it as an equivalent electrical circuit in the lumped form, because as I said in the last lecture that if you have la electric circuit in a lamped form either in the in lined position or in the shunt position then you must have a very specific structure of the transfer matrix that being that 1 of the off diagonal term should be 0, but that this structure does not confirm to lumped electrical analogy.

So, lumped electric circuit representation is not possible; however, distributed element representation is possible, but let us instead try to think about some approximation under which this situation we will render itself to a lumped approximation. So however, for the special case that KL is much lesser than 1 what does that mean? K is the wave number L is the characteristic length of this of this of this problem. So, therefore, K time cell is the product of the wave number and length in other words 2 pi L by lambda must be less than 1 in which case it means that the length must be much smaller than the wavelength of the acoustic wave that is going in this duct.

So, in this case we will have KL lesser than 1 and if wavelength is large then we also or the other way to interpret it is that K is directly related to the frequency. So, if KL is small this we will also mean that this is the low frequency approximation. So, under low frequency approximation let us look at what happens to these entries of the transfer matrix cos KL will 10 to 1 and sin KL will tend to KL itself that is the first order approximation for these 2 trigonometric terms. So, therefore, if we now put this back we get the following P 2 v 2 is equals to 1 i Y KL, i KL by Y, 1 P 1 V 1 right. So, if you again open these 2 equations up we get the following P 2 is equals to P 1 plus i Y KL into V 1 and V 2 is equals to i KL by y into P 1 plus V 1.

The first equation could be written as P 2 minus P 1 divided by V 1 is equals to i Y KL right. The second equation could be written as P 1 divided by V 2 minus V 1 is equals to 1 by i KL divided by Y. I hope I am right V 2 let us do it in 1 more step V 2 minus V 1 is equals to i KL by Y P 1. So, P 1 by V 2 minus V 1 will have i L K in the denominator and y in the numerator, which is exactly what I have written let us take this 2 equations for further analysis.

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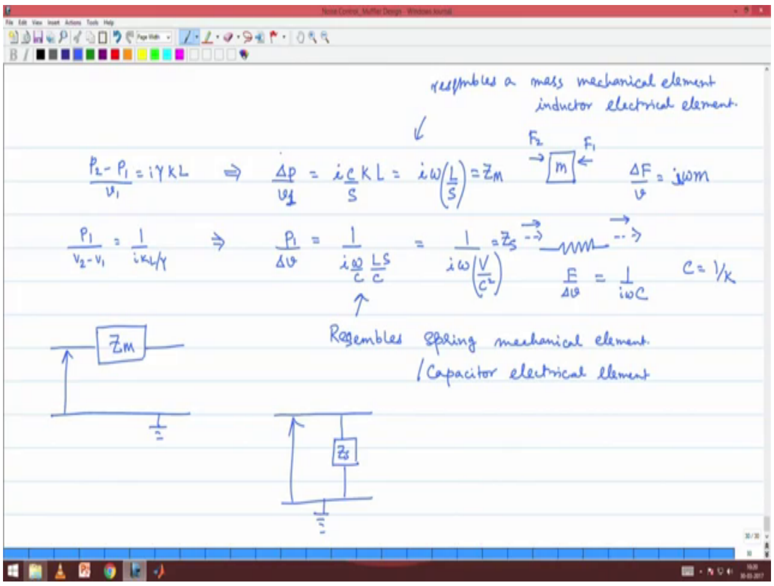

So, these are the 2 equations which I will grab. So, this could be written as delta P by v 1 is equals to i Y in fact, Y could also be turned out turns out as c by s into K into L and this could in turn be written as I omega L by S.

P 2 minus P 1 is delta p. So, delta P by V 1 is turning out to be this quantity right please note this is exactly of the same form as the impedance of the mass element right. You had the impedance when you did the impedance of the mass element you will said that the forces on the 2 sides need not be the same right and therefore, you said that delta F by v is j omega m this is x sorry I omega m right. So, this is exactly of the same form as this equation looks exactly of the same form as the equation of the mass element or that of the inductor in electrical circuit analogy right. What about the second equation? The second equation would read as P by delta v, in particular P 1 by delta v is equals to 1 by i again instead of K, I would like to write that as omega by C L c s instead of Y I am writing c by s.

So, this would read as 1 by i omega l times s is the volume s is the cross sectional area s is the sorry s is the cross sectional area l is the length. So, l times s is the volume. So, I will write l s as the volume by c square right. This resembles that of a impedance of a spring element while looking at the spring element we said that the velocity at the 2 ends of the spring element could jolly well be different, but the forces at the 2 n must be the same and it turned out that force divided by the change in velocity should be 1 by i omega times the compliance or we put capital c for compliance of the spring; capital C may was 1 by K right.

So, now you see V by c square therefore, resembles that of the compliance of the spring. Here we have exactly the same situation we are saying that there is we are finding the ratio of the pressure which is the force like term divided by delta V, we understand V is not they particle velocity, but it is the mass velocity. But the mass velocity and particle velocity are related by product of 2 constants that is density and the cross sectional area none of which are changing.

So, therefore, this numerator is a force like quantity, the denominator is a velocity like quantity. So, the ratio of this force like quantity to the differential of the velocity like quantity measured at the 2 ends of the duct gives us exactly the same form as that we got it for a spring element. So, this is resembling. So, this situation resembles spring or a capacitor electrical element and this represents a spring mechanical element.

In the first case this situation would resemble a mass mechanical element which is also an typical inductor in the electrical terminology, but then for the mass or the inductor we had the picture to be that of an inline. So, this was the ground and the inductor was to be put in an inline position. So, this is denoted as Z m because of this analogy and this is denoted as Z s because of the analogy, but the inductor picture is that corresponding to an inline position whereas, the other picture that of the spring corresponded to that of a shunt position right.

So, now the question is which of these 2 positions should we take. There is a very interesting analysis now that we will run into. So, what we have seeing is that the 2 equations of the 2 transfer matrices are presenting itself into 2 different forms, 1 is giving like an inductor the other is giving situation which resembles that of a capacitor or a spring element. Now we will consider 2 special cases case 1 S is large, S meaning the cross sectional area.

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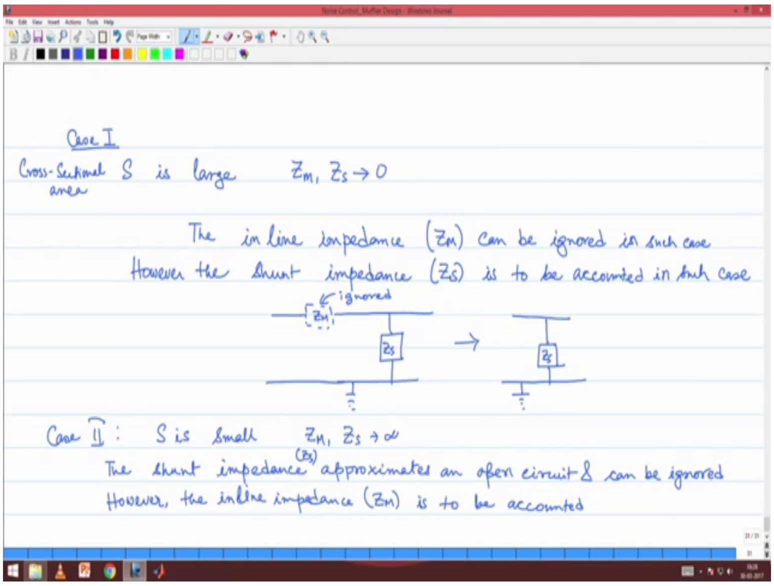

So, if cross sectional area is large what happens to Z m and Z s? If you look at this expressions both Z m and Z s have s in the denominator right look at this expression s as Z s has s in the denominator look at this expression Z m has in the denominator.

So, therefore, if S is large both Z m and Z s will go to 0 right if both Z m and Z s go to 0 then what happens to our electrical circuit, we are now confused that which 1 to choose we have for the first equation an inline position, for the second equation we have a shunt position, but then if we understand that both Z m and Z s are very small quantities then the effect of an impedance in the inline position is going to be like a short. So, you are basically shorting this right. So, you can as well delete this Z m and short it right.

So, the effect of Z m can be completely ignored whereas, the effect of Z s cannot be ignored right. If I short the circuit here then lot of current will move flow through this shunt path and therefore, the impedance of this the current that will flow through the other branch will be sort of minimal right. So, you should not minimum shorting the circuit, it is feasible to ignore the inline impedance provided it is small right, but it is not possible to ignore the shunt impedance provided it is small right that will be the other case.

So, in the inline impedance which is Z m can be ignored in such case. Please understand numerically both Z m and Z s are small right. So, if you think numerically both are small let me ignore both of them that is not a valued argument right. You can ignore the impedance the inline impedance provided it is small, but if the shunt impedance is small you should not ignore it. You should take it into account such that you are able to capture the effect that lot of the current we will actually flow through shunt path rather than the path which is parallel in the I mean path which is downstream the path which is after the shunt position. And this is where electrical analogy really helps that if you understand what is shorted and what is left open you can understand which way the current will be flowing, the extreme cases are quite easy to intuitively think of in the electrical analogy.

So, the inline impedance Z m will be ignored in case both of them are small; however, the shunt impedance Z s is to be accounted in such case right. So, that leaves us with the situation that in this case we will have only Z s in the shunt position, and virtually this Z m block is not there. So, this is ignored which means that we could resolve it. So, this Z m block is ignored leaving us with the equivalent circuit as this, which is only in the shunt position right, that is case 1 when the cross sectional area is large so, the other extreme case to when s is small. When s is small what will happen is that you will have Z m and Z s to be large right. Now if the impedance is are large and what happens in the shunt position if you have a large impedance, that effectively means it is an open circuit.

So, it is no current will go in this parallel path of this shunt line right. So, therefore, it is needless to consider this shunt line right whereas, if you have a large in line impedance then there is going to be a large potential drop across the line and you should consider it right. So, this idea that which impedance has to be taken under which circumstance is beautifully brought out hopefully through this explanation you will appreciate that this electrical analogy easily permits us to understand these 2 extreme cases, which is I thought a tag be difficult to understand from purely mechanistic point of view right. I am only appealing to the electrical analogy here to convince you that both this situations do happen and it is not so much about the numerical value of Z m and Z s, because strictly if you look at it the numerical values of both Z m and Z s go to either 0 or infinity.

But it is the location in the electrical terminology; it is the location of this impedance whether in line or shunt which decides which one has to be ignored right. So, in this situation the shunt impedance approximates an open circuit and can be ignored whereas, the inline impedance the shunt impedance was Z s, the in line impedance Z m is to be accounted. So, compiling this once again what I am doing is that the Z m is there, but Z s is virtually not there because it is large right.

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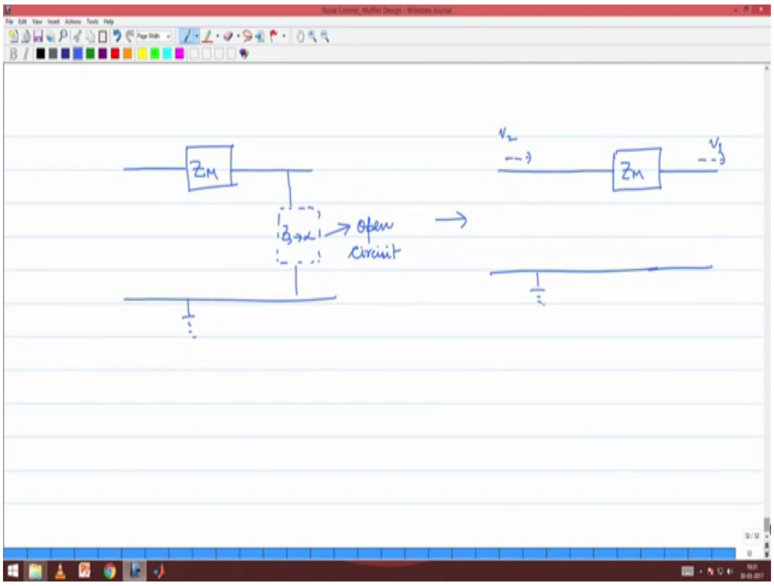

So, this is approximately an open circuit. So, if it is an open circuit you might as well choose to ignore it, and just have the Z m in line impedance that is it ok.

Now, using these 2 situations now we will build up what is known as a Helmholtz resonator.

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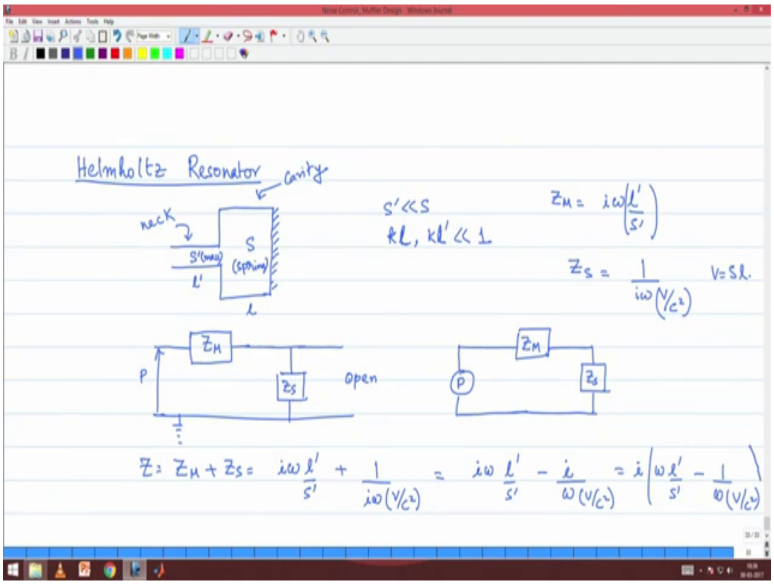

This is the analogue of a spring mass system in acoustics. Acoustics we actually way go the other way in the theory of vibration class first you to learn spring mass system, and then you learn about continuous systems, and then it is shown to you that using modal decomposition you can bring back a continuous system analysis to a spring mass system analysis. But here we have done it the other way we have always treated the acoustic system to be continuous system, but here for this continuous system we are bringing a special case wherein we are showing that special case of this continuous system boils down to a lump system as I will show you.

So, this Helmholtz resonator is simply 2 such ducts I mean in one of its in conditions which is easy to understand is just 2 ducts 1 with a large cross section, and the other with a small cross section. This is small cross section this is a large cross section. So, s prime is much much smaller than s and accordingly there are 2 links if you want l prime and l what is going to happen between these 2 ducts and we are going to look at frequencies which are small, KL and KL prime both are small ok.

So, therefore, lumped analysis is valid and what we have shown is that when s is large we should have the effect of a spring, and when s is small we should have the effect of a mass. So, intuitively at this stage we are ready to understand that this volume will behave like a spring, and this volume will behave like a mass right. So, we are basically breaking down this collection of 2 ducts as 2 electrical system, the first one corresponding to s prime l prime, we will have and in line impedance. The second one so, there is a ground line which I should draw as usual, the second element corresponds to that of a shunt impedance Z s right and what happens downstream? If we assume that this is rigid; that means, no current has to go which means this is open, which means I need not consider this part of the circuit.

So, therefore, and I obviously, need to complete the circuit from this side also through a potential or a pressure right. So, therefore, this is this circuit is going to read as Z m and Z s. Now once you have the circuit you can talk about the resonances, because you are having a lumped electrical circuit nothing prevents you to talk about the resonances what is Z m by the way the Z m was calculated as i omega l by s right, i omega l by s and l prime by s prime to be more particular that is the impedance associated with the neck of the Helmholtz resonator this region is called the neck and this region is called the cavity and what is Z s? Z s is 1 by i omega V by c square right yeah 1 by i omega V by c square where V is S times l right.

So, therefore, what is the total impedance of the circuit? That is Z m plus Z s which is i omega minus or let me do it in 2 steps the total impedance of the circuit is Z is equals to Z m plus Z s which is going to be i omega l prime s prime plus 1 by I omega V by c square. So, this could be written as i omega l prime s prime and I could write this is minus i on the numerator divided by omega V by c square.

So, now I can pull out and i omega l prime s prime minus 1 by omega V by c square correct. So, this is the total impedance please note the impedance is purely imaginary. So, it is in the reactance form there is no resistance. Please understands resistance form of impedance will only come if there is some energy dissipation mechanism there is no energy dissipation mechanism in this problem. So, therefore, it is coming as pure reactance and as we know resonance is defined as a situation when the reactance part of it this is the un damped case. So, therefore, the reactance if it is going to 0 then we will have the un damped natural frequency.

So, therefore, the un damped natural frequency of this Helmholtz resonator can now be obtained by equating the reactance to 0 is given by omega n which is such that omega n l prime s prime, has got to be equal to 1 by omega n V by C square.

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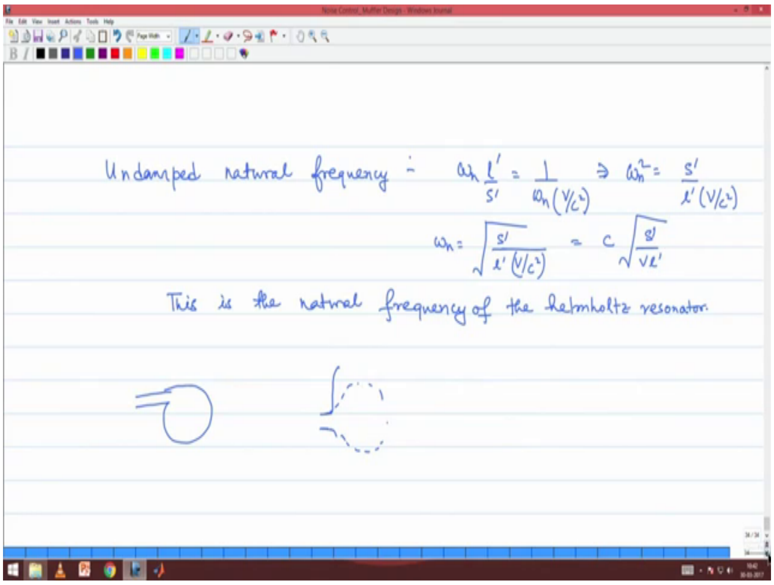

So, that would mean omega n square is s prime by l prime V by c square, which means omega n is s prime by l prime V by c square. I could make this little better also C s prime V l prime that is how the natural frequency of the Helmholtz resonator looks like. So, this is the natural frequency of the Helmholtz resonator. Referees whistle is the best example of Helmholtz resonator look at the construction of a referee whistle it is exactly this a large cavity a small construction right.

So, what happens is that for all the sound that is incident on to this construction of a whistle, it is only one frequency which gets amplified which is corresponding to the resonance frequency of the whistle right. Same thing actually happens when you whistle without this sort of a artifact, when you whistle you basically from the lip let me try to draw a face of at least myself which would look horrible, but this is the throat cavity and here is the lip region right. So, when we whistle we tend to construct a channel around our lip region, and create this small tubular zone right and our mouth cavity is that large volume right.

So, in between the mouth cavity and this narrow construction of our left lip region we have a Helmholtz resonator, and the effect of that is that no matter what air blows out through our vocal cords, it is only one frequency which will get amplified which exactly corresponds to the resonance of this Helmholtz resonator, and how do we change the tune when we do a whistle of a song with ten to change the diameter of our lips right very finely we will adjust the diameters of our lips or even the length of it which will affect one of these terms, s prime l prime as you see here is both there. As you change s prime l prime the frequencies change by some by exactly the fashion that you want such that the resonance is changed.

Remember that the incident voice which is coming out incident airflow which is coming out within the through our vocal cords remains just the same. It is just by tuning the resonance we are able to create different frequencies right. So, this is exactly what happens in time. So, these 2 are the classical examples of a Helmholtz resonator that of a referee. So, whistle and I think referees whistle these days have changed, but this is the good all times referee whistles, but hopefully the art of whistling will remain.

So, those of you who whistle please remember you are using a Helmholtz resonator, but by all means whistle by abiding all the social norms do not whistle unnecessarily and get into trouble.

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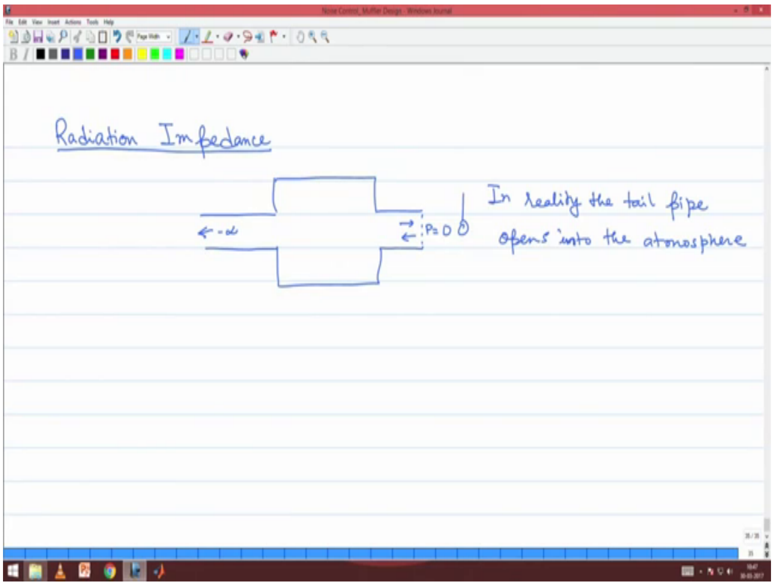

So, that is about Helmholtz resonator, we will quickly look at the next topic that of radiation impedance. We will see how Helmholtz resonator can be adopted in mufflers to create special situations wherein certain frequencies can get knocked out, because as I said that in the course of muffler design what we need is a special arrangements such that certain frequencies with wherein we know loud sound is existing should get filtered out or should get knocked out.

So, we will see how exactly. So, what we have in a whistle is just the opposite in the case of whistle, we are amplifying a certain frequencies in a muffler we need to do the reverse, we need to kill a certain frequencies. So, it can be done very nicely we will talk about it in due course, but let us quickly look at this topic of radiation impedance. Till now we have looked at a muffler which is infinite at both ends right, we had looked at the situation and we had looked at the transmission loss of the simple expansion chamber, but you will recall that it has been the analysis proceeds with the assumption that it is infinite at both ends.

What happens if instead of being infinite it is actually finite and it opens into the atmosphere right? This is a more practical problem and this is what we should look at. So, the actual tailpipe in reality the tailpipe opens into the atmosphere. So, if it opens into the atmosphere what should be the boundary condition that you should use? Till now we were saying that if it opens into the atmosphere you should have a boundary condition P equals to zero, but then there comes one more problem. If we really use P equals to 0 boundary condition what is the transmitted wave from this point onwards? Nothing should go out everything should get reflected because of P equals to 0; which means whatever muffler you use. In fact, that you do not have to use any muffler because of a 0 pressure condition at the end everything should get reflected back towards the muffler nothing should get transmitted outside the tailpipe right.

So, this is something which we will revisit and we will find tune this analysis such that this contradiction is sort of mitigated and that we will bring us to the notion of radiation impedance. So, the concept of P equals to 0 was introduced in the earlier classes as the boundary condition of the opponent that is only approximate, that is not the correct incorporation of this boundary condition. A better incorporation of this boundary condition will be through the radiation impedance condition and when we will open out this radiation impedance condition, we will be able to show you that under certain special cases the radiation impedance condition will actually be that P equals to 0 condition also, but there is a difference between a radiation impedance in general and P equals to 0 condition. The problem here lies in the fact that if P equals to 0 then; obviously, nothing should go out.

So, if you put a microphone here or if you put a put your human ear outside just outside the tailpipe, which is where typically you are interested no 1 is interested to hear what happens inside the tailpipe you are interested to hear what happens just outside the tailpipe. So, if the analysis proceeds with the boundary condition that P equals to 0 at the tail pipe, then the travel is there is virtually no sound transmission beyond the tailpipe, which means that you basically do not need to do anything just by itself the sound that will come out of the exhaust should get kill, but that does not happen.

So, we will correct this situation that simplistic assumption that we have made thus part will be sort of corrected and we will talk about this more sophisticated condition of radiation impedance followed by the source impedance. So, similarly on the other end we will have the source and we will have to account for the source impedance. So, with that we should be able to close our discussion with mufflers.