Acoustics & Noise Control
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Module – 02
Lecture – 03
Governing Equation 1

Friends welcome to the third lecture on Acoustics and Noise Control. So, we have been taxing for a while we have been giving you motivational staff us to why the topic of acoustics and noise control is a very important topic in today's engineering community, we will deeper today.

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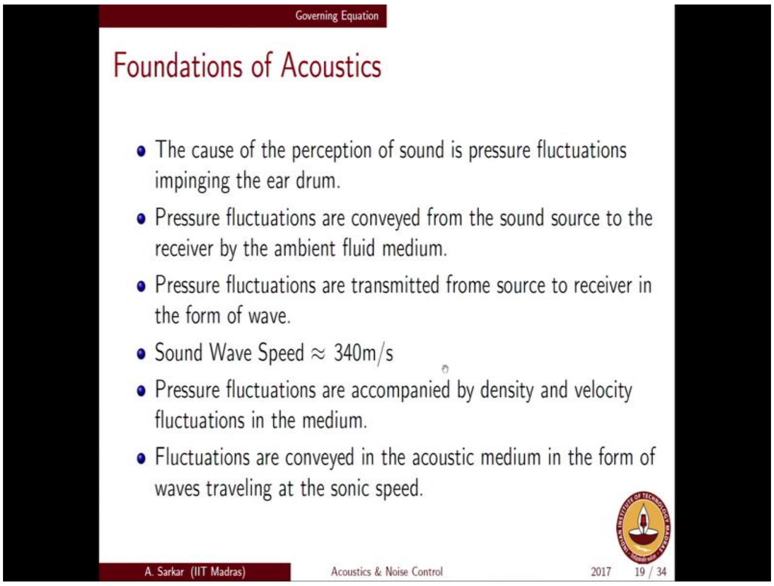

So, just to recall we understood that the cause the perception of sound is the pressure fluctuations which are impinging on our ear drum. And the pressure fluctuations are conveyed from the sound source to the receiver within the ambient fluid medium which in this case as we are talking is the air.

So, in certain Navil applications obviously, this fluid medium could be water. So, the question of transmission is off the sound is essentially boiling down to the question of how the pressure fluctuations are getting transmitted from the source to the receiver and it so happens to be in the form of wave. What is a wave and how do we arrive at this conclusion we will see that right in today's class; and just a few common things the it is well known the sound wave speed in air is about 340 meters per second it does a change a bit depending upon temperature atmospheric pressure and so on and so forth.

So, we will see to it that how the sound wave speed is derive. So, this is something that we at this point take it as an elementary fact that we just believe in, but we will derive it soon and half. So, the pressure fluctuations are not just an isolated phenomena, so the pressure fluctuations are accompanied by density and velocity fluctuations in the medium. So, there are density fluctuations which essentially mean that the fluid now essentially is not an incompressible fluid, but a compressible fluid because you have to accommodate for the fact that the density is going to change as the acoustic waves are going to travel. So, pressure fluctuations are accompanied by density fluctuations for sure and velocity fluctuations in the media. Fluctuations themselves are being conveyed in the form of the wave within the acoustic medium, and their travelling at the sonic speed which basically means the sound waves speed numerically the value is near about 340 meter per second.

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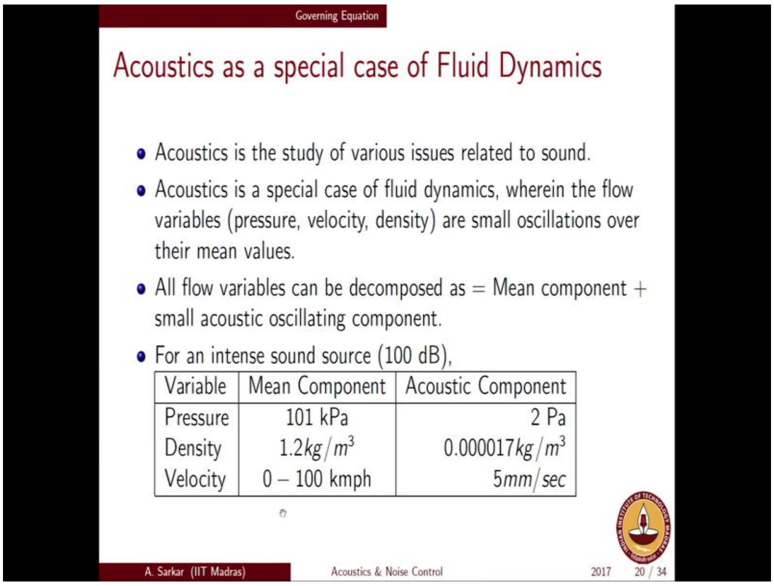

So, therefore, the prospective that we will look at is that the sound transmission within the fluid medium in particular air is just a special case of fluid dynamic; it is a fluid dynamic phenomena because of which the sound is travelling from one point to another at least within the air. We are not talking about structure bond sound at least for now, but we are talking about the transmission of sound from the source to the receiver; this is basically the transmission of pressure fluctuations accompanied by density fluctuations from the source point to the fluid to the receiver point. Accordingly we will treat acoustics or sound transmission more specifically sound transmission to be a special case of the fluid dynamic equations and that is how we will derive the governing equations of acoustic in the present class.

So, acoustics we know is a study of various issues related to sound, but we also understand from this argument that it is a special case of fluid dynamics where in the flow variables are small oscillations over the mean value. Remember that is the perspective we had right from the first class that over and above the atmospheric pressure there are some small fluctuations which is basically hitting your eardrum that is how you are hearing my voice. So, these small fluctuations are the acoustic quantities of interest, the fluctuations will be in the form of pressure, will be in the form of velocity will be in the form of density. And we will since there is a density fluctuation we are which we are going to trace out, so this density fluctuation can be accommodated only if we have a model of the fluid dynamic equations as that of a compressible fluid flow rather than an incompressible fluid flow. An incompressible fluid dynamic equation will not be able to cater for fluctuations in density.

So, therefore, the perspective that we will adopt from here on is that the acoustics is a special case of compressible fluid dynamics or compressible fluid flow, but unlike a true gas dynamics course where the these density fluctuations can be of any erotic fashion, what we have short of laid down is that these density fluctuations are necessarily small; what I mean by small I will come to it in a moment, but at least in conical times it is small in comparison to the mean value and I have some numbers to show you as to what I mean by that.

So, essentially what I mean is that all the flow variables namely pressure velocity and density can be decomposed into two parts, one which is the mean component and the other which is the small acoustic oscillating or fluctuating component right. So, that is how we will treat all the variables that arises in acoustics, over and above the mean value there is a small acoustic oscillating component and it is the small acoustic oscillating component the magnitude of this oscillating component that we will try to track we will try to figure out what are the implications of these small fluctuating components are and that is the acoustic quantity of interest.

So, that is the agenda for the day, just to give you a feel as to what the order of magnitude of different variables are. Considering a very intense sound source of 100 decibels which is like very close to a let us say an aircraft engine you will hear this kind of sound, it is something which is really heard and thankfully. So, that it is very rare that you will hear sound of the order of 100 dB so in fact, you should not be exposed to 100 db sound in normal day to day life, but a very intense sound of 100 decibels will have a mean component for this sound if we now try to decompose the fluid dynamic variables into two parts the mean component and acoustic component the breakup goes in this fashion for the fluid pressure there is a mean pressure which is 101 kilopascal effectively the atmospheric pressure right; the atmospheric pressure is 101 kilopascal so the mean component of the pressure is 101 kilopascal in this case.

The acoustic component is just 2 Pascal right. So, which means it is 2 divided by 100 into 10 to the power minus 3, that is the ratio between the acoustic component and mean component it is very very small. Similarly if you look at the density the density of air roughly is 1.2 kilo gram per meter cube in standard conditions, but then even due to a 100 decibel intense sound, the acoustic component is that much and it is up fifth decimal place that you get to see a significant digit. So, it is pretty small this acoustic component is pretty small in comparison to mean component right. Similarly if you look at the velocities if velocity is in air even if it is a severe cyclonic storm it can go at the most of the order of 100 kilometer per hour, but the acoustic component can really go more than 5 millimeter per second. So, 5 millimeter per second is all that you have in terms of the velocity of the fluid particles as it is oscillating.

But yes if you really considered a quiet room of the sort of this recording studio, then the velocity is can be assumed to be nearly 0 that there is no mean flow in this room. So, the mean component of the medium that we are talking it could range from 0 in a quiet room to a very large value which is happening in the cyclonic storm sort of a case, but the acoustic component done the same remains small. So, suffice to say that acoustic pressure acoustic density and acoustic velocity is are all small quantities in comparison to the mean quantities, its only under a quiet room assumption that you get to see the mean component of velocity to be also 0 in which case you just do not worry about the mean component, but in some error acoustic cases you are also interested to know what happens due to a bulk fluid flow, and those bulk fluid flows are definitely much larger than 5 millimeters per second.

So, this will give us the queue as to how we will go about tackling this problem of acoustic wave propagation from the source to the receiver. So, we will essentially breakup all our variables of interest in two parts a mean part and an oscillating part. The oscillating part we will do a book keeping and we will have a way to show that these oscillating party remains small, and we will track out the equations that we get off this small oscillating part. We are not interested in the mean part that is what is done in a course in fluid mechanics when they will talk about how the equations of mean flow evolve, but here we are not worried about the equation of mean flow here we are worried about the equation of the oscillating or the fluctuating part right that is the acoustic case for our interest.

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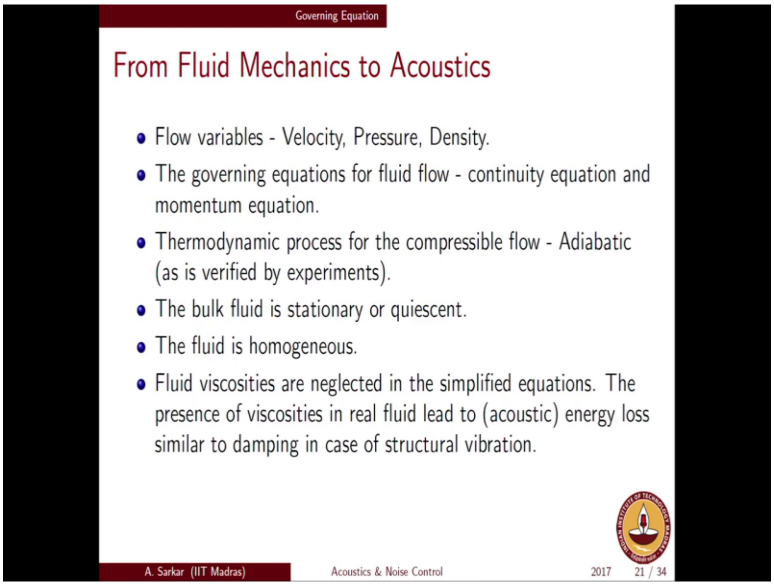

So the process is this way that there are flow variables which is velocity pressure and density and the governing equations for this fluid flow variables are continuity equations and momentum equation right; something which you have done in your undergraduate classes in case you need to refresh please look at any blocked in fluid mechanics to know what are the continuity equations and momentum equation you should you could possibly look at its derivation also.

But what we will do is we will appeal to these equations without really going to the derivation of continuity and momentum equation we will appeal through this equations together with a thermodynamic process, for the compressible flow because as you know the compressible flow essentially means that there is a change in pressure as well as density. So, if you assume a suitable model for the thermodynamic process which in this case will be assumed to be as adiabatic process and if you make that assumption then you will get one more equation which is the thermodynamic process equation if you can further we will make an assumption today that the bulk fluid is stationary or quite it is not having any mean flow.

So, we are typically interested as of now in an ambient fluid medium where there is no mean flow right, we are not talking about severe cyclonic storms why there are large mean flow created. We will assume that the fluid is homogenous that is the properties of the fluid is uniform throughout the region of interest, there is no change in density or there is no change in temp temperature of the mean components there can be change in the oscillating part, when we say the fluid is homogenous we will essentially mean that the mean density of the fluid at the mean pressure of the fluid is constant over the entire region of the fluid, it does change when the sound flows through, but that we will separately track through our accounting equations. Another very crucial assumption we will do to for now is that the fluid viscosity are neglected in the development that we will do today which is the acoustic wave equation.

So, in this development the reason why fluid viscosity is will be neglected is because the effect of fluid viscosity can be shown to be equivalent to an energy loss that is actually pretty easy to understand because viscosity is an energy dissipation mechanism. So, the inclusion of fluid viscosity which is truly there is going to induce some additional losses in to the acoustic wave propagation, because of which the wave will sort of decay as it travels further distances, but the presence of this decaying aspect will be ignored. We will simply assume today that there are no energy dissipation mechanisms in the fluid and we will try to figure out that what is the equations governing the transmission of waves on transmission of sound as I should say from the point of generation to the point of reception.

So, using these equations acoustic wave equation will be derived. So, let me show you the derivation.

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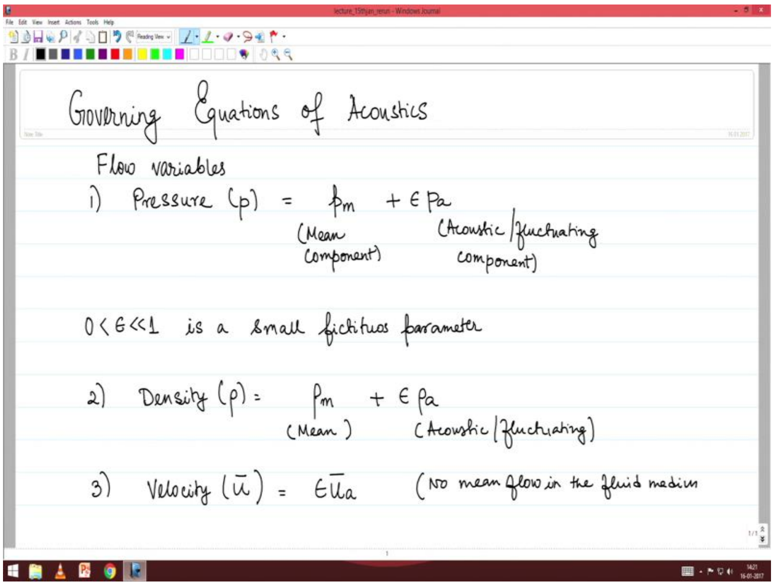

So, here we will derive the governing equations for acoustics we will start by specifying all the flow variables of interest. So, flow variables of interest are one pressure, so that we will denote by the symbol p and we will break this up into two parts, the mean component pm and the acoustic component are the fluctuating part as we have said. So, this is the mean component and this is the acoustic or fluctuating component. So far so good now to make sure that we understand that the acoustic component is much lesser than the mean component what we will do is we will introduce a bookkeeping parameter epsilon. So, epsilon is a small book keeping fictitious parameter and the reason for its introduction is just to keep track of the orders of magnitude of different variables.

So, it does not have any physical relevance, but we will see it will be very useful in the derivation process if we introduce this epsilon notation here. So, similarly the density can be broken down into two components; one which we will do denote has rho subscript n meaning again the mean component plus rho a, and just as usual this is the acoustic or the fluctuating part of density right and again since we know that the acoustic or the fluctuating part of density is small in comparison to the mean density, we will precede this rho a with an epsilon term. So, wherever we see this epsilon level we will understand that it is small in comparison with other terms which does not have this epsilon component right and lastly for the velocity which is a vector we will denoted as u with a vector symbol, and we will ignore the presence of any mean component as we said that we are going to think that the fluid is not having any mean component it is having only a fluctuating components.

So, accordingly this will be denoted as epsilon u a right. So, there is no mean flow in the fluid medium as per our assumption. So, with this setup let us now look at each of the above each of the fluid dynamical equation we will start with the continuity equation.

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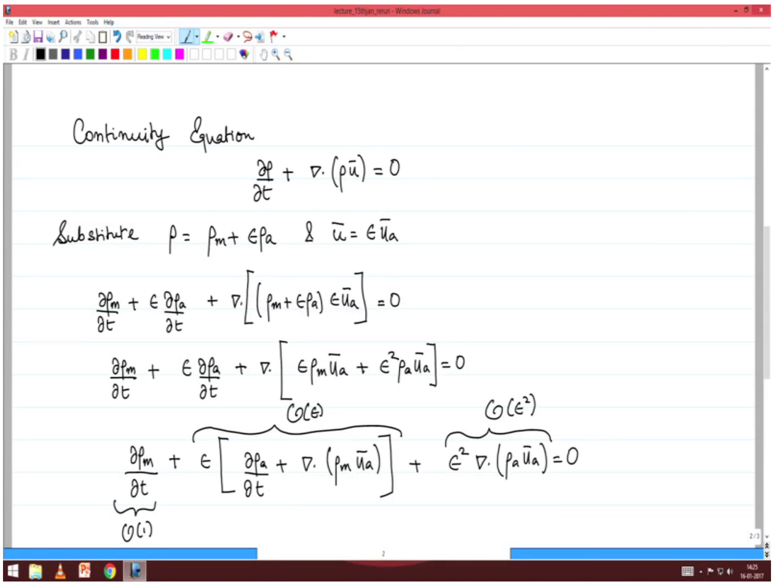

So, the continuity equation you will recall from any fluid dynamics undergraduate level text book is given by del rho del t plus del dot rho u is equals to 0, this is the continuity equation. So, if we substitute rho is equals to rho m plus epsilon rho a and u is equals to epsilon u a in this equation what we get is the following, del rho m del t plus epsilon times del rho a del t is what we get from the first term, plus class del dot rho m plus epsilon rho a into epsilon u a u vector is being replaced by u a because there is no mean component as per our assumption, so that must be equal to 0.

So, we will open this up and what we get is the following del rho m plus del t. So, derivative with of the mean density with respect to time, plus epsilon times del rho a del t. So, derivative of the acoustic density with respect to time plus divergence of the following quantity epsilon rho m u a, plus epsilon square rho a u a is equals to 0. So, this in turn would mean del rho m del t plus epsilon times del rho a del t, plus del dot rho m u a plus epsilon square del dot rho a u a is equals to 0 right? Now we note the following we understand that this is an order one term, where as this term which is the square bracketed term multiplied by epsilon since it is getting multiplied by epsilon, we call this term as order epsilon term and similarly this term is an order epsilon square term right? By our basic assumption epsilon is a small number which means that though this equation says that on the left hand side you have 3 terms and some of these 3 term should be 0, but we realize that there is a difference in orders of magnitude between the 3 terms.

There is a the first term which is of order one, there is a second term which is after which is order epsilon that is it is necessarily smaller compared to the first term and if you note the third term the third term is even smaller because it gets pre multiplied by epsilon square if epsilon is small epsilon square is even smaller. So, therefore, what we have here is sum of 3 terms on the left hand side equated to 0, but then each of these 3 terms are having different orders of magnitude. So, using the fact that these 3 terms of different orders of magnitude there is no possibility that these 3 terms can mutually cancel each other because they are at different orders of magnitude, one is very large the other is small the third one is even smaller.

So, therefore, the only way in which the sum of these 3 terms can go to 0 is by the argument that each of these 3 terms has to be 0 this is what we call as an asymptotic argument or an order of magnitude study.

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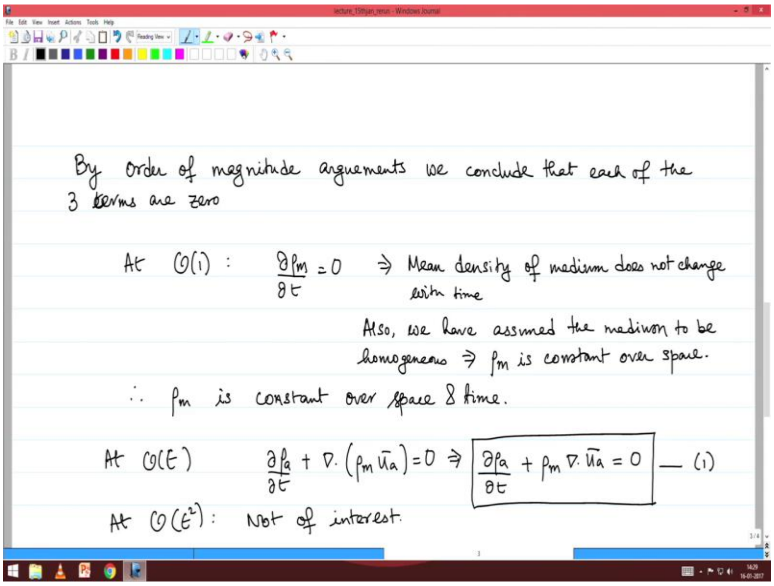

So, by order of magnitude arguments we conclude that each of the terms each of the 3 terms are zero right which means at order one at order one we have del rho m del t equals to 0, which implies that the mean density of the medium does not change with time also we have assumed the medium to be homogenous which implies that mean density rho m is constant over space as well. So, we have as a result rho m is constant over space and time right. So, this is our inference at order one, going back to order epsilon what we get is the following at order epsilon if we look at the equation we will get the following that del rho a del t, plus divergence of rho m u a has got to be 0. So, del rho a del t plus del dot rho m u a has got to be 0, but rho m as we have seen is constant over both space and time which means you can pull rho m out of the differentiation process within this divergence operation.

So, that essentially means del rho a del t plus rho m del dot u a has got to be 0. This is an important equation which we mark it as equation one; at order epsilon square you get another equation, but this is not of our interest. So, continuity equation basically gives us two very important inferences one is that the mean density is actually constant overtime it is also assume to be constant over space because my assumption we are now dealing with a homogenous medium, at order epsilon we get this equation one as our result.

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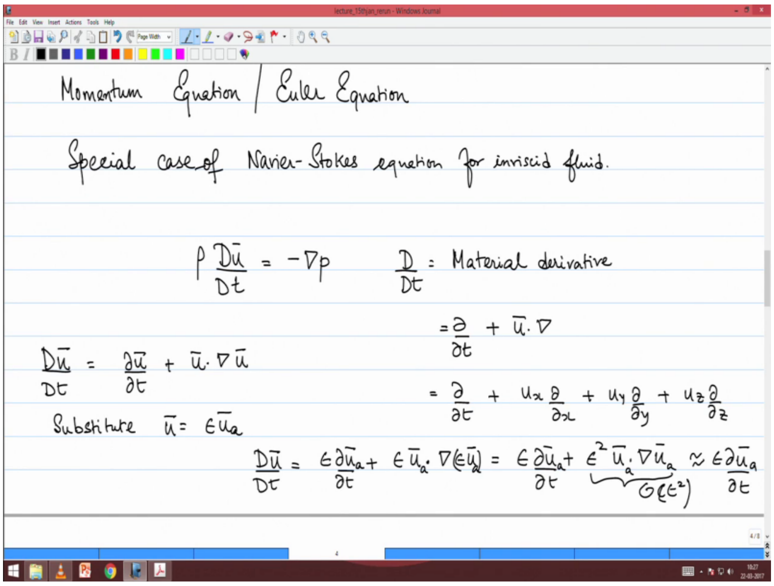

Now coming to the momentum equation or the Euler equation; Euler equation is a special case of the Navier-Stokes equation for inviscid fluid recall by our assumption we are dealing with inviscid fluid for now. This equation is stated is the following rho times D u D t and u is a vector is equals to minus gradient of p, and D u D t, D D t rather is called the material derivative and it has two parts one is it is simply partial derivative with respect to time plus u dot del.

U dot del you may open it up in the following fashion del del t plus u x del del x, plus u y del del y plus u z del del z if you want to. So, this is this statement of the Euler equation which is available in any fluid mechanics book if you want you can check it. So, now, we will proceed from here on in our derivation for the acoustic governing equation. So, this is given as follows so firstly, we will try to make a simplification for the material derivative. So, the material derivative Du Dt will be written as del u del t, plus u dot del of u itself right the vector u itself.

So, therefore, now substitute the fact that the velocity vector is actually comprising of the small acoustic component and nothing else. So, we have already define the velocity vector to be u is equals to epsilon u a, u is equals to epsilon u a was define. So, using this definition we will simply make the appropriate substitution in the material derivative that we see here. So, again you see that D u D t therefore, is epsilon del u del t plus epsilon u dot gradient of epsilon u. So, that again means that the first term is order epsilon and the second term is epsilon square u dot del u.

So, therefore, as you as one can understand from these derivations this is a higher order term or order epsilon square term. So, I will put in a here. So, del u del t the formula for del u del t will now read as the following will have epsilon del y a del t, plus epsilon u a dot del gradient of epsilon u a. So, u a is a vector quantities, I appropriately put a vector sign here and then in the next step if I simplify the second term you will see that what I have done is I have taken the two of epsilons out and this is epsilons square. So, the second term is epsilon square u a dot of gradient of u a. Now please note that we have already said that epsilon is a small fictitious quantities, epsilon square is even smaller.

So, as a result this second term is order epsilon square, where as the first term is of order epsilon. So, definitely the first term is larger than the second term because it has only order epsilon sitting in front of it, where as the second term is having an order epsilon square effect and as a result is small. So, we might as well choose to ignore the second term and write this as epsilon del u a del t from here. So, therefore, a material derivative of u in the Euler equation will from here on the simplified to epsilon del u a del t, del del t is just the partial derivative with respect to the time.

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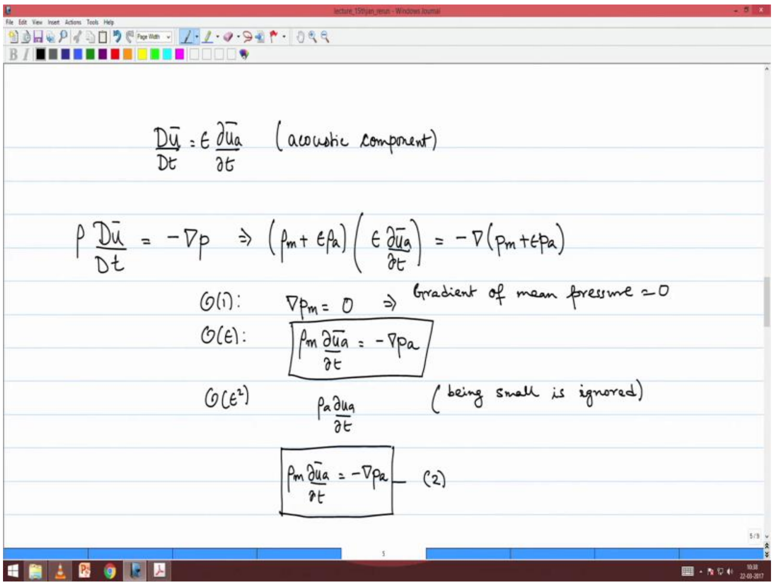

So, what we have derived is that the material derivative Du D t is equal to del u a del t, is equal to del u a del t for the acoustic component, that is what we have derive in the previous page of the notes. So, I will pick it up from here that del u del t is effectively epsilon times del u a del t. So, next we appeal to the momentum equation which we had written it here. So, this momentum equation I am bringing it again here, and now what I will do is I will keep substituting the form of density in its mean plus the acoustic or the fluctuating form d u d t is already known to be epsilon del u a del t and finally, on the right hand side we have gradient of mean pressure plus epsilon times acoustic pressure.

Now, what we do is that we again from this equation look at what happens at each order. So, at order one, we include those terms which do not have any effect of epsilon that gives us gradient of P m to be 0. So, this means that the gradient of the mean pressure is 0, that is actually no surprise because you will recall we have neglected the effect of any mean flow since the effect of mean flow is neglected it is, but natural that the mean pressure gradient has to be 0.

Because remember if there is a mean pressure gradient there has to be a mean flow accompanying that part, but now since we are ignoring the effect of mean flow we are treating that the medium is essentially quite there is no mean flow so therefore, mean pressure will not bother us so the gradient of mean pressure will be 0. So, will usually not worry about this order one equation at the acoustic level, this is what will give us the mean flow effects if at all it is there; here we choose to ignore them in for effect so therefore, it is not to our interest from here up.

Next if we collect the terms of order epsilon we get to see rho m del u a del t on the left hand side, and that must be equals to minus gradient of p a this is what happens at order epsilon. There is also one more term at order epsilon square which would read as rho a del u a del t, but we understand that order epsilon square being small we will choose to ignore this affect, we are only going to look at equations at order epsilon. So, the equation at order epsilon as we have written is this equation. So, this equation is basically the momentum equation for our acoustic case. Please understand that this is a simplification to the Euler equations are the general Navier stokes equation, now what we are reading as the momentum equation from here on in the acoustic case will be this and this is what we will use correspondingly in the subsequent derivation also, but for now this is the final form of the momentum equation for the acoustic fluid.

So, there are two equations that we have derived, a third equation will come from the thermodynamic process.

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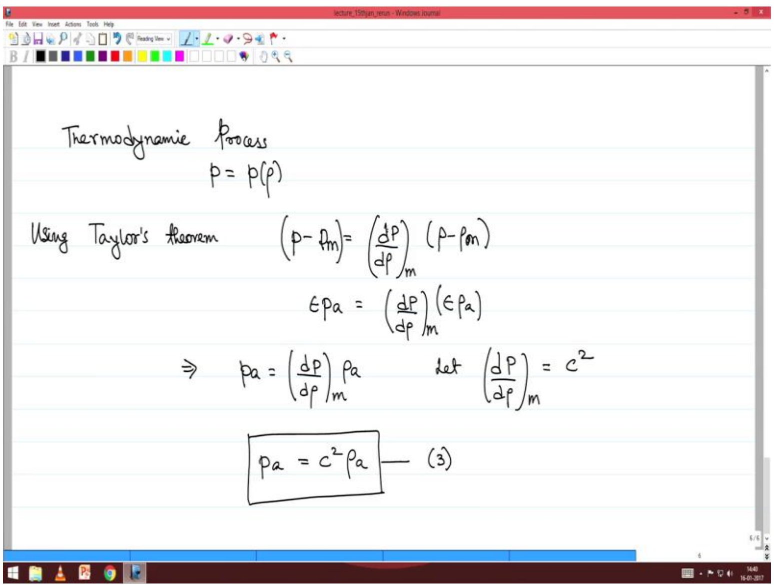

So, the thermodynamic process states that the pressure and the densities have got to be related. So, the thermodynamic laws essentially relates the pressure of the fluid and density of the fluid as the compressible flow process is taking place. Now using Taylors theorem and treating this as any obvious function nice and smooth and continuous, we could write this as P minus P 0 is equals to d P d rho evaluated at the mean quantity into rho minus rho m, m is the subscript for the mean quantity some times in some books it is just called 0, but we will use the subscript m for the mean quantity.

So, the value of the pressure about the mean point can be obtained in terms of the value of the density about the mean point, and the function relationship between the two is just related by the gradient of the pressure with respect to the density, but then P minus P m exactly the acoustic pressure. So, P minus P m if you recall is epsilon P a. So, that must be equals to d P d rho into epsilon rho a. So, similarly rho minus rho m by our definition is epsilon times rho a. So, what we have got as a result of this thermodynamic analysis is that the acoustic pressure is related to the acoustic density through this relation right.

We will call this gradient d P d rho evaluated at the mean condition as c square, we will see that c eventually will turn out to be the wave speed, but that discussion will have it probably in the next lecture, but at present we are just assuming this quantity to be c square and let us see where that takes us. So, now, what we have is P a is equals to c square rho a right. So, the acoustic pressure is related to the acoustic density in the above manner. So, we have got all the 3 ingredients in place continuity equation, momentum equation and the thermodynamic process what remains is just do a few simplification.

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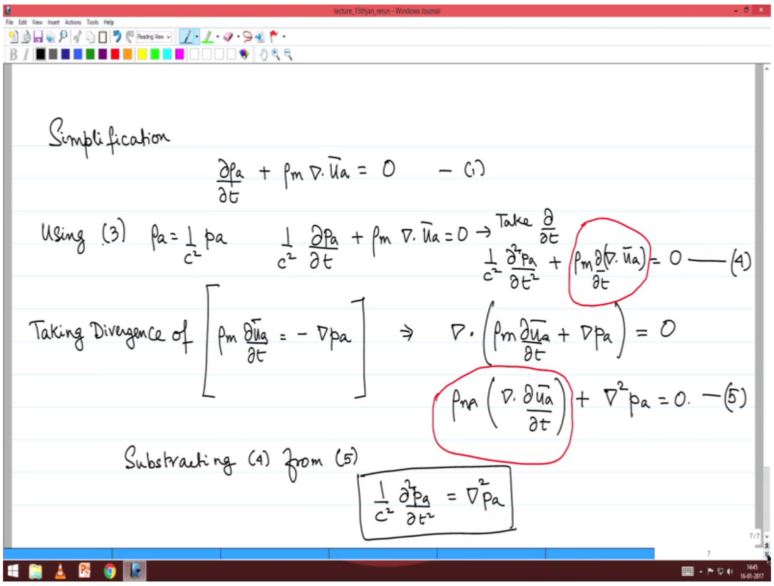

So, the simplifications will be in the following manner, if you recall the equation one that we wrote down for continuity equation stated the following that this is the equation one. So, equation one I will just write it once again del rho a del t, plus rho m del dot u a is equal to 0, and similarly this is basically equation one.

So, now using 3 what we have is rho a is equals to 1 by c square P a right. So, therefore, we have del P a del t, 1 by c square plus rho m del dot u a as our equation of interest; and similarly if we now look at equation two what we get was this. So, this equation two I will rewrite again with gives me as rho m del u a del t is equals to minus del p a. So, this equation I will take divergence of this equation. So, taking divergence of this equation which means I will just do a del dot operation on this equation or maybe I will do that in the next. So, step taking divergence of this equation. So, this equation can be rewritten as rho m del u a del t, plus del P a the left hand side of it. So, now, I want to take divergence of it. So, I will give del dot and that must also be equal to 0.

So, therefore, what we have is rho m del dot del u a del t remember rho m is a constant in space and time. So, the divergence operational got nothing to do with rho m, it just comes out and del dot del p a is del dot del is the laplacian. So, we will get del square p a that must be equals to 0. Now in this equation we take time derivative once again we take del del t of this equation once again. So, when we take del del t of this equation what we get is the following 1 by c square, del 2 P a del t 2, plus rho m del dot u a and this guy has got to take a time derivative of right.

Now look at these two equation. So, this I will call it my equation 4, and this I will call it my equation 5. Look at these two equations you have the same term sitting here, these two terms this one and this one adjust the same. The order of the differentiation process can be inter changed the first you can take the time derivative and then to the divergence or first you can take the divergence and then do the time derivative both will give you the same answer as long as you are dealing with nice and continuous function.

So, these two equations have sometimes mistake in the miss the sign for the vector please be here with me on that, but now that I have corrected it you can understand these two terms are same. So, therefore, if we take a subtraction between these two equation, so subtracting 4 from 5 we can get to this equation 1 by c square del p a del t taken twice is equals to del square P a this is the acoustic wave equation.

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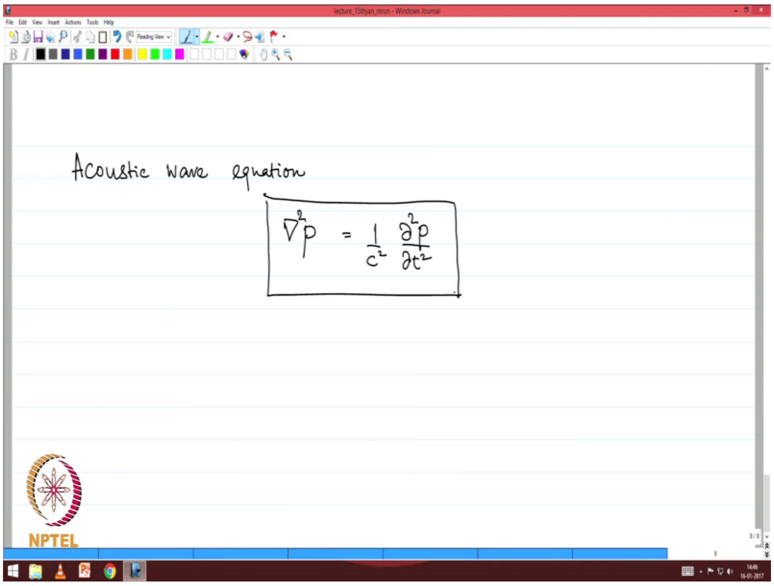

So, acoustic wave equation is the equation dell square P a which is on the acoustic pressure is equals to 1 by c square, del 2 P a del t 2. Since from here on we will be always dealing with acoustic pressure we would not be dealing with mean pressure. So, without loss of any clarity we may as well omit this subscript a, the understanding would be that when we write P without any subscript essentially means that it is the acoustic pressure because we are not interested in the mean pressure in this course at least so therefore, the acoustic wave equation will be given in this form. In the next class we will elaborate more about the properties of this acoustic wave equation, but that is it for now.

Thank you.