Acoustics & Noise Control

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Module - 24
Lecture - 29
Electro Mechanical Analogies – examples

In the last class we looked at a few examples of discrete spring mass damper systems. And we looked at how to construct the electrical circuit analogous to that mechanical system. We also looked at the transfer matrix method. Today we will continue with some more examples hopefully a little more complicated one.

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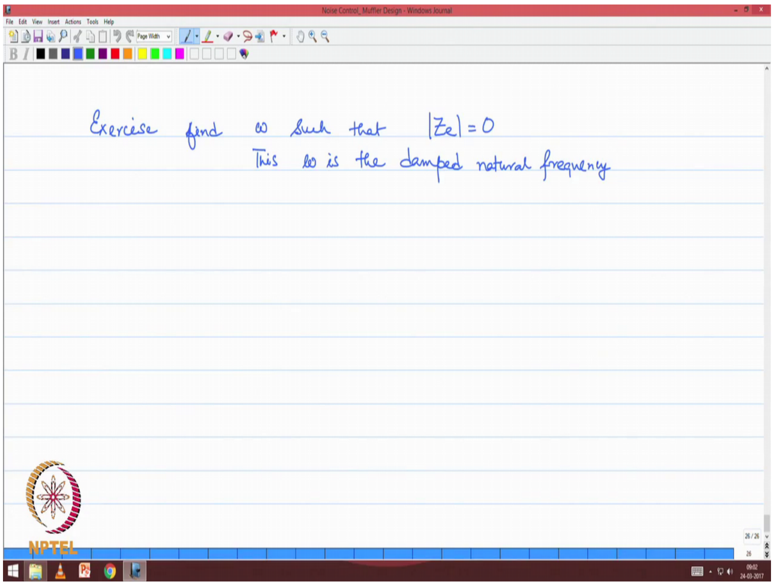

In the last call we had looked at different examples for mechanical systems and their I mean I would more specifically lump mechanical systems spring mass damper systems, their assemblies and then how to construct the electrical circuit analogue and then as well. We looked at the transfer matrix method. We will continue with a few more examples in that direction.

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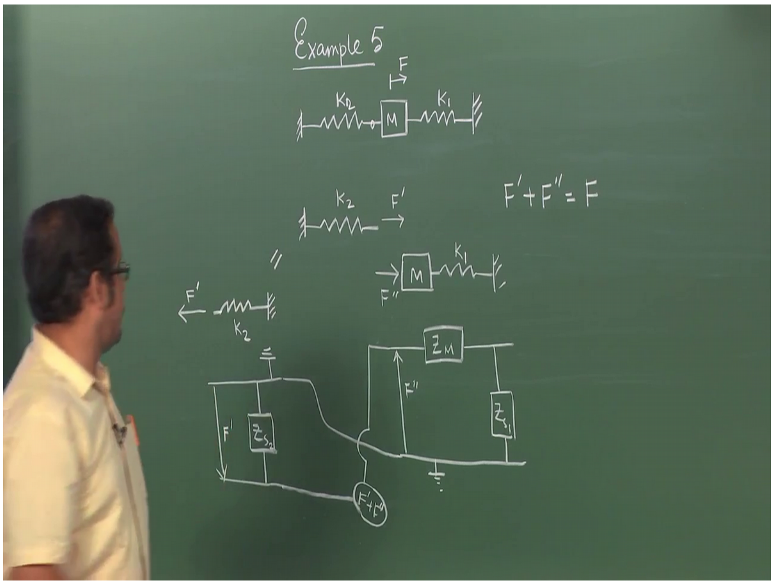

So, let us look at another example this would be example 5. So, here till now we were looking at a system which is ending with a mass or a spring, but which was having the force at the upstream end right.

The first the leftmost most was where having the force applied, but in this system we wish to put the force somewhere here. So, this is the force right. So, we wish to analyze this system using the electrical analogy as well as the transfer matrix method. So, let us see what happens. So, the force being applied to this mass can be resolved into 2 parts, one part is that associated with this spring which is sorry, I will call this K 1 and this K 2 and the other part is that of a mass and spring on the left hand and this forces F double prime. So, what we have done is that we have since this force is being applied at what we could assume is that this force is applied at this point ok.

So, this force we have separated into 2 parts F prime and F double. Prime F prime acts on the portion which is left of this point, and F double prime is acting to the portion which is on the right. Of this point, but; obviously, the 2 must add up to the applied force F. So, we have a condition that F prime plus F double prime must be equals to F right. It is trivial to a trivial at this stage I would say to construct the electric circuit for this system right. This system would have an electric circuit which is Z m impedance in line, and Z s 1 impedance in the shunt position. And then since the other end is fixed it is an open circuit right. And this is the ground line and the applied force this time is F double prime which means the voltage in the line is F double prime. This is this part should be simple this is already what we have done right.

Now, considered this system the system, on the left this system on the left is also the same as this system, I am just reflecting this system and that would result in to this situation right. Just a mere reflection this system and this system is no different in mechanical terms it is just of flipped version bottle, but the reason I am flipping it is that I wanted the convention in which we introduce the electro mechanical analogies there in, we thought that the it was convenient to make the rightmost ended fixed and the force to be applied at the leftmost end. So, this seems to work as per that convention. The only issue here is that this F prime force seems to have gone the other way right. As per our convention we said that at the upstream point the force by our convention the positive direction of force was chosen, from left to right. This is going the other way right, round from right to left.

So, this is the only change we need to incorporate. So, again as usual this part of the circuit would be an inla and impedance corresponding to this spring element, which is placed in shunt or in parallel position. But now the voltage is not positive, but negative, because the direction of F has reversed in this case. So, here I will choose this as the ground line and this voltage I will denote as F prime right. Now observe this is the electric circuit corresponding to the left portion, this is the electric circuit corresponding to the right. Portion these 2 lines are actually at the same level because both are ground lines. So, I might as well join these 2 lines right. And if I choose to join these 2 points then, I have to account for a potential difference of how much? F double prime above the ground F prime below the ground right.

So that means, a total potential difference would be F prime plus F double prime right. So, if I wish to join these 2 points, I could do that. But I have to account for a source which is giving F prime plus F double prime. So, that would be the equivalent circuit I could draw it a little more neatly.

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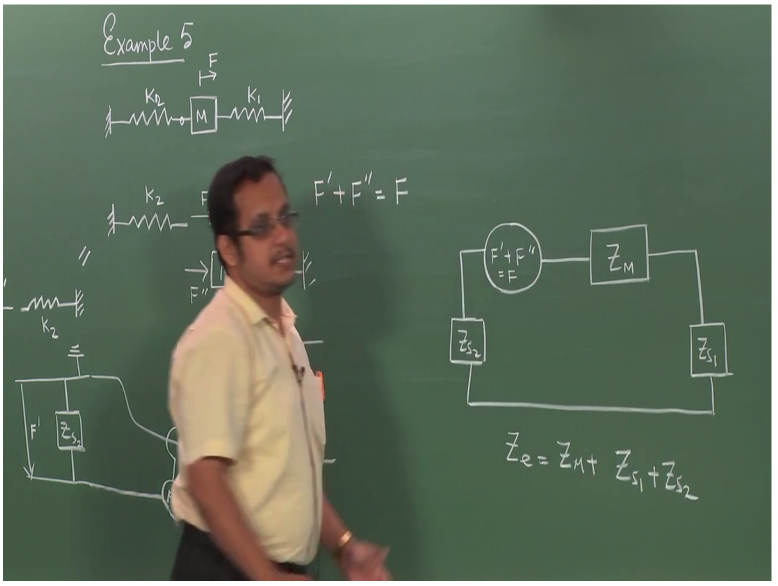

So, the equivalent circuit this time would mean that, you have a source of magnitude F prime plus F double prime. You have a mechanical impedance given by the mass Z m. You have Z s 1 on this side and you have Z s 2 on the other side, that is it right because Z s 2 is basically ending in an open end right. The Z s 2 is this is the only impedance that is there it is ending with a fixed mechanical support which incidentally implies that the circuit has to be left open.

So, this shunt position ends with a open circuit right. So, as a result this is the equivalent electrical circuit and from this equivalent electrical circuit it is easy to understand that the effective impedance is Z m plus Z s 1 plus Z s 2. In other words the spring stiffness of the 2 strings I just adding up right. As it should be because these 2 springs are arranged in this way if you appeal to your Newtonian mechanics again you should see that the effect is spring stiffness is just the addition of the 2 stiffness. And their the electrical circuit seems to have re enforced this understanding in any also. This is the next example 6, which is that of a dynamic absorber.

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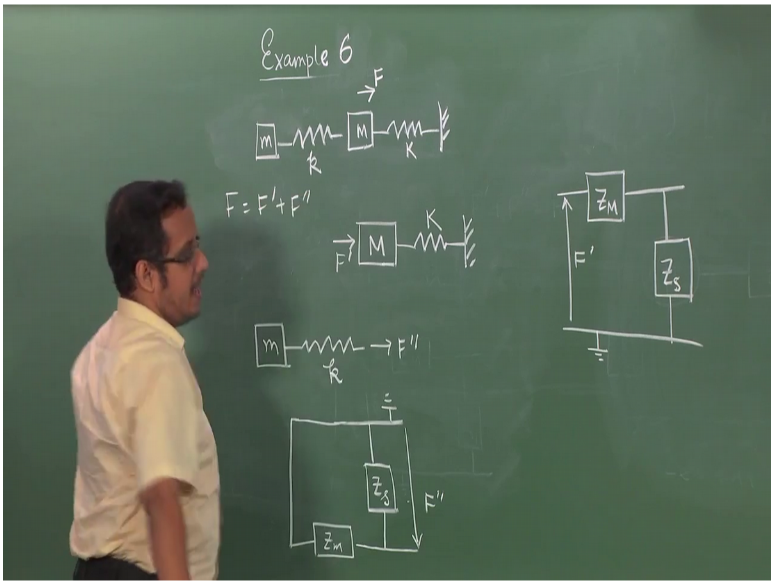

So, do you know why? So, there is a primary spring mass system which is capital M capital K, and there is a secondary spring mass system which is small k and a small m . There is a certain force acting on the primary mass, because of which the primary mass is suppose to oscillate, but then if you find that the vibration of this mass is beyond the certain level, then what you have to do is that you have to choose another secondary spring and mass system, such that the natural frequency of this spring mass system happens to coincide with the excitation frequency. And then if you work out the details from elementary vibration theory you will find, that the displacement of this primary mass is going to drastically come down.

So, that is the usual derivation which will be given in a vibration course, but here we will appeal to the electromechanical analogy to explain the situation. So, the situation is just the same we will have to resolve this into 2 parts, this is one system, where in we have the force F prime. And this is the other system, where in we have the force F double prime. This is capital M capital K and this is small m small k. So, we need to construct the electrical circuit for this situation. So, what will be the electric circuit for this right half that is pretty easy. So, you will have an impedance associated with the mass capital M which is in line, you will have the impedance associated with the stiffness capital K, which will be in the shunt position Z capital S and it will be left open, because there is the velocity here has got to be 0 which means, that there cannot be any current in this part of the segment. Which is why it will be left open and this is my grounding line and the voltage here would be F prime right.

Here exactly the same thing will happen, but we have to realize that F double prime has to be taken below the ground right because the direction is flipped right. As we did it in the previous example, if the we could treat this situation as a reflected system and in the reflected system we will see that F is going from right to left, right. So, that sort of defies our convention, remember in our convention the forces which are to the left of the element should go from left to right. And the forces which are on the right of the system should go from right to left, but this one seems to be going the other way round it is going from left to right. This force is to the right of the system, but it is going from right to left right which is why we have to take a negative sign associated with this force. So, other than that it is just the same. So, here as far as the circuitry is going to be first you will have a shunt associated with this stiffness.

So, I will call this as Z small s, because this is associated with the small k value. And then there will be an inline impedance associated with the mass small m right. What will happen here? How will the circuit be closed? How will be the circuit closed in this example? Should it be open or should not be closed? This is the this is one line, this is the other line, how should the circuit be closed? Should it be left open or should it get closed? Why should it be closed? It is not open? So, it has to be closed there is a velocity, but if it is getting closed in electrical terminology your shorting this 2 lines right; that means, there is no voltage drop between these 2 lines, what is voltage in mechanical force? There is no force at this mass. So, because there is no force on the mass the voltage has to be 0, which means these lines have got to be now interconnected or they have to be shot it.

Till now we had seen examples where the lines were left open, because it was ending with a rigid termination the mechanical system was ending with a rigid termination. Here we are saying that the mechanical system is ending with the free termination. Free termination means, 0 force, 0 force means, 0 voltage in electrical analogy. So, that is why, this has to be done, and remember the grounding in this case will be in the opposite direction. So, this is the ground line. So, what happens here is that the voltage is exactly the same as the ground, because this is exactly the ground like there is no difference that you have got in this ground like. And this time the potential is in the opposite direction F double prime right.

So, this is the equivalent circuit for the left half of the system this is the equivalent circuit for the right half of the system. How do you join it you realize that these 2 points can be anyway join they are at the same ground. Between these 2 points what is the potential difference this point is at F prime above ground this point is at F double prime below ground. So, the total potential difference is F prime plus F double prime right. Which is actually F. So, F was F prime plus F double prime and we can combine these 2 pictures and put our electric circuit in this fashion, Z M Z S both capital.

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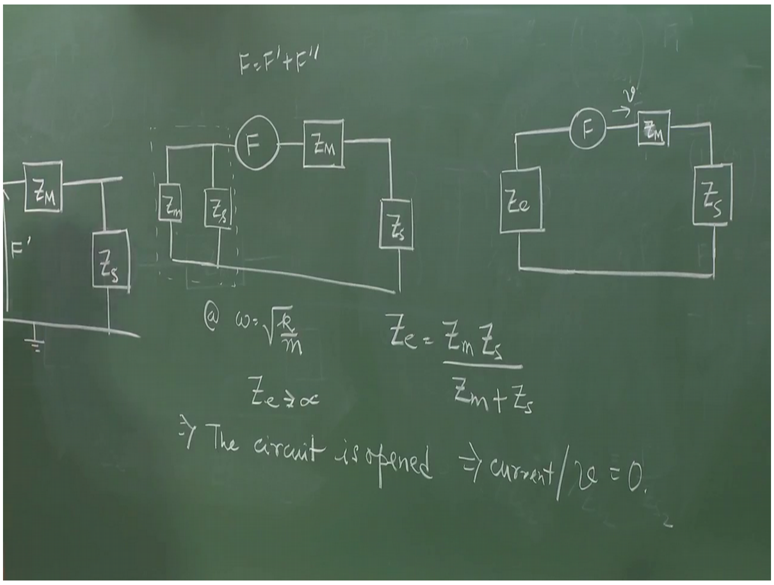

And then there is a voltage source which is capital F and capital F is equals to F prime plus F double prime which is the voltages across the 2 individual segments of the circuit. And it gets completed through shunt which is Z small s and also through Z m ok.

So, this is Z m Z small m and that one is Z capital M. So, this seems to be the equivalent circuit of the dynamic absorber right. Now let us try to explain the fact that, what happens at excitation frequency which is equals to square root K by small m that is at the resonance of this secondary spring mass system. So, let us towards that end let us try to find the equivalent impedance of this portion of the segment. So, we would like to find out and equivalent impedance Z e, which is due to the secondary mass and the secondary spring. All other aspects of this circuit remains just the same, all other aspects of the circuit remains just the same right.

So, now what is Z e? In terms of Z small m, and Z small s these are 2 impedances in parallel right. So, therefore, the effective impedance has got to be Z small m Z small s divided by Z m plus Z s. That is the equivalent impedance of 2 impedances in parallel. So, what will happen at this frequency, Z m plus Z s will be how much? 0 we did that in the la previous class that at the resonance of a spring mass system it is equivalent impedance turns 0. So, at omega equals to square root K by m this Z e we will have a 0 in the denominator which means it will tend to infinity right. So, if Z e tends to infinity what is happening basically? This circuit is getting opened up not shorted it is getting opened up, there is infinite impedance. And if this circuit is left open what will happen to the current? In the circuit nothing will flow right.

So, at this frequency one of these impedance which is Z e, which basically is the effective impedance of this block. The secondary mass and the secondary spring that goes to a very high value which means that the impedance turns infinite. And this implies the circuit is opened right. Which means that the current which is V is equals to 0. So, therefore, minimal current flows through the circuit, and that is why finally, what is the current flowing through the circuit? Is same as the velocity of this mass. So, the velocity of this mass is going to 0 and that is why you are system is saved.

So, this is an explanation which comes out of the analogous electrical circuit associated with this mechanical system right. So, similar things can be tried out; obviously, this explanation is just an alternative explanation you could explain the effect of dynamic absorber purely from a mechanistic perspective, without going into this electrical circuit theory which is typically done in a theory of vibration or a first course in vibration theory, introduction to vibrations or something like that, but here I present an alternative explanation exploiting the equivalent circuit theory ok.

The next question is slightly different. So, therefore, can we say that for all systems we will have an equivalent circuit. We have seen quite a few examples 6 of them to be particular and. In fact, we had constructed springs and masses in series and we said that even for those cases, where in springs and masses are in series we are going to get some equivalent circuit.

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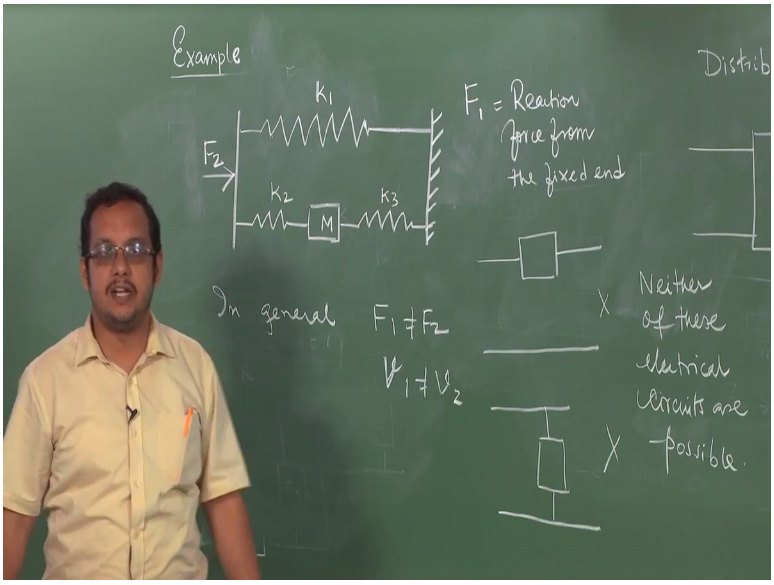

But then the question remains that, does the equivalent circuit idea always work? Can you always replace a mechanical system with an equivalent circuit system? Try this one. Try this one. Firstly, try to answer this question, is there an equivalent circuit possible for this system? This is a 2 port system, is nothing wrong in the mechanic sense, if we have this system right. There is an input port there is an output port. Will there be an equivalent circuit I mean in the sense that something is inline something is in shunt in that fashion?

Please recall that our entire argument of putting electrical equivalents registered on the fact that electrical elements were either placed in line or in shunt. If it was placed in line then the forces between the input and the output port is possibly different. But the velocities are same right. And if it is in shunt position the velocity is a possibly different, but the forces are same. What does that argument lead you to for this structure? Can we say anything about the input and the output ports in terms of equality of the forces and the velocities? Can you say for sure that F 1 will be equal to F 2 or V 1 will be equal to V 2? You cannot right. So, that their lies the problem. So, here our idea of lumped electric lumped system modeling we will fail, though this system I have constructed it out of lump systems, but it is entirety it is not as an assemblage, it is not lumped system it is now becoming a distributed system. Because the you cannot say that. So, we will call this port as 1 we will call this port as 2.

So, in general F 1 is not equals to F 2 and m 1 sorry, V 1 is not equals to V 2 V 1 is not equals to V 2, is obvious because these are 2 springs and the spring at it is 2 ends can have different displacements right. There is nothing constraining the 2 ends of the spring to move identically. What about this point? Between the 2 ends of the spring the forces getting identically transmitted, between these 2 ends it is getting identically transmitted, but since the force has to move a mass. Between the 2 ends of the mass the force need not be the same right which means that you cannot say that the force also is same. Or in other words whenever you have a difference in force that difference in force will cause the oscillation of this mass, which is fine right.

So, neither of these 2 conditions is satisfied which means you cannot have an electrical circuit either of this kind or of this kind. Neither of these electrical circuit is are possible. You cannot have an electrical circuit neither of these electrical circuit is are possible, but then the situation is not that hopeless.

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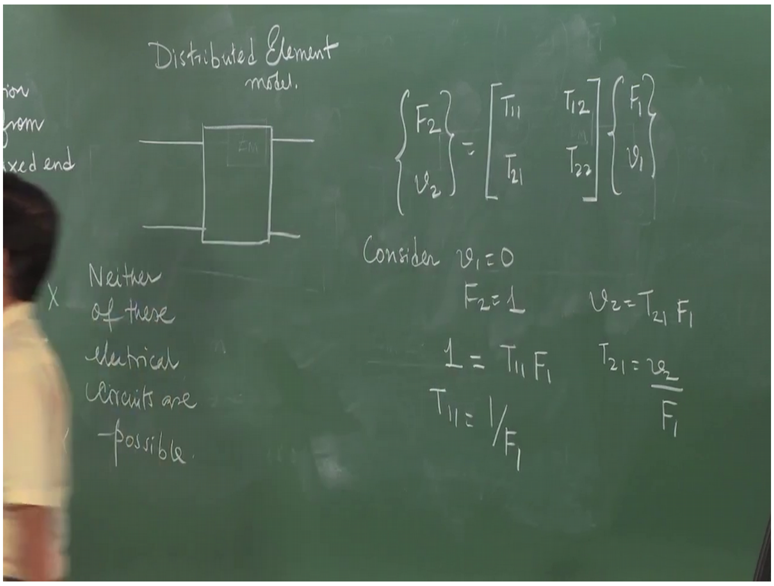

What we will say is that the electrical circuit is going to be represented in this form this is remembered just a representation. Because we realize that it cannot be shunt it cannot be in line. So, therefore, it is a combination possibly of both inline and the shunt, and this is what is called distributed element modelling.

So physically it may not be conceivable to arrive at this circuit using imp inductances and capacitances and resistances, but none the same at least in terms of analysis, we could work this out and all that, we will do to represent this block is the transfer matrix. We will see that the transfer matrix works beautifully that does not have any restriction. So, let us quickly understand what do we need to do for the transfer matrix evaluation transfer matrix should be pretty simple, that is we need to evaluate F 2 V 2 we need to understand how F 2 V 2 has to be related to F 1 V 1. And the individual entries will be the transfer matrix right. So, this will be the transformer matrix the question is, how do we determine this transfer matrix values? The determination also is actually pretty easy, we could contemplate different situations of this mechanical system.

So, consider one simple case where in V 1 is 0 and F 2 is equals to 1. So, that basically implies that this edge is fixed, and here you will have a force F 2 going in there is a perfect mechanical system we could write the equations of this system without much of a fast is just from first principle we could write it. So, if we solve this system and put this back in our transfer matrix what we get is one is equals to T 1 1 F 1 right. And what is the F 1? F 1 is the reaction force that will come from the fixed end of this system right. So, F 1 can be obtained. So F 1, I will write it here F 1 equals to the reaction force from the fixed end. That can be obtained, not a big deal right. So, once you obtain that reaction force T 1 1 is going to be just the reciprocal of that reaction force ok.

So, the way to obtain T 1 1 would be to first solved this problem purely as a mechanical problem appealing to Newton's laws of motion appealing to equilibrium conditions. You can find out what is F 1, and the which is just the reaction force from the fixed end the reciprocal of the reaction force from the fixed end for this problem, is going to give you T 1 1 associated with this transfer matrix right. That is what the theory tells us. Next if we look at the next equation V 2 will be T 2 1 times F 1 right. So, in other words T 2 1 would be V 2 divided by F 1. And V 2 is precisely the displacement of this end. So, again you can compute the displacement at this end divided by the reaction force the resulting value is going to be T 2 1 right.

So, in this fashion we have computed one column, the other 2 columns can also be contemporary calculated in exactly the same fashion. But this time we can fix this end and apply a force on the other end we will get the other 2 entries. And that is way we can complete the entries of each and every element of this transfer matrix. This is calculation we will do in details in the next class because this will help you to do the transfer matrix method. And now you will see that the transfer matrix method can be done for any system you need not always think in terms of a lumped impedance or an lumped inline impedance or a lumped shunt impedance. But you could think in terms of transfer matrix.

So, this element whatever this electrical element is I do not know what it is in realistic terms, but at least in terms of my what the purpose of the analysis this electrical element is that, which corresponds to this transfer matrix. And we can implement it exactly in that same way through simu link or other system simulation software. So, hopefully with these 2 examples we can complete the electromechanical analogy part, then we can shift to the acoustic anlogies also.

Thank you.