Acoustics & Noise Control
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Module – 20
Lecture – 25
Muffler Analysis

In the last class we saw the principle behind the measurement of sound intensity. So, today we will just look at some best practices involved in intensity measurement methods.

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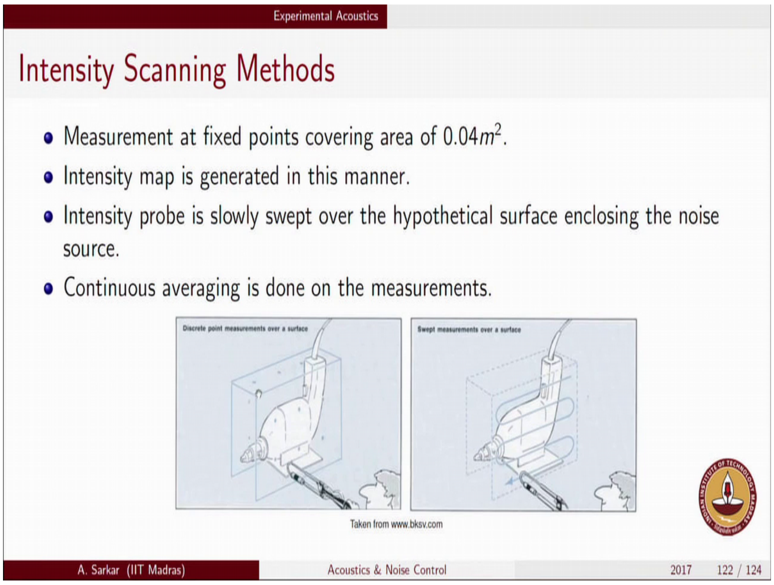

So, as we said that intensity will have to be measured to get the sound power from a machine such as this one which is a (Refer Time: 00:38), what you have to do is you have to take the intensity across various points over an enclosing area so obviously, measurements will be done at number of points. So, there are different ways in which you can scan for these different points these are some prevalent practices as is given in the document downloaded from bruel and kjaer website.

So, they recommend that they can you can take measurements at fixed points covering area of 0.04 meter square and then you can generate an intensity map which is like a control map and, but sorry this is not exactly a control map it will be a vector 3D vector field map because intensity is a vector. So, using the vector field map you can find out the hotspots and the exact components from where the sound power is having the maximal radiation. So, between these points you have to move in a very slow manner and you have to take measurements and in case you have repeated measurements then you have to take some averaging also which we have discussed.

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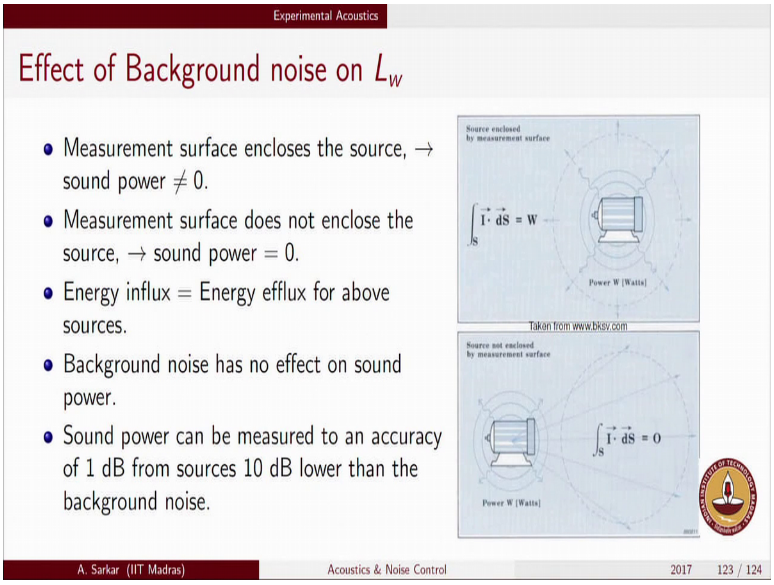

One very important reason why we should go for intensity measurement is that intensity measurement is unaffected by background noise and whereas, sound pressure level measurement is definitely going to be affected by background noise. So, let us see why this happens.

So, here we have a source again this drawing has been document downloaded from the primer in the bruel and kjaer website. So, what we know for sure is that if the measurement surface encloses the acoustic source then the sound power is that is passing through the surface is going to be a non 0 quantity. But if the measurement power does not, a measurement surface does not include the source where which is what is done in this second schematic drawing you can see whatever is the power that is coming in is actually that is the power that is going out right.

So, as I said that it is sound power is going to be constant over any hypothetical surface which encloses the sound source of interest. So, that is a very important point. So, if you were to measure any sound power across a hypothetical surface within which there are no sources of sound then that should give a sound power nearly 0 except for instrumental errors.

So, when sound power is 0 that essentially means the efflux of energy within this the neat efflux I should say is 0 which means the amount of influx should be equals to the amount of efflux something that you have also seen in fluid mechanics (Refer Time: 03:32). So, what it means is that if there is a background noise if there are let say 2 sources one is the primary source and over and above there are some other sources which are generating background noise. So, the sound power which is estimated or which is due to the background sources because the background sources are lying outside this hypothetical surface which encloses the primary source the sound power contributed by all those sources is going to be just dead 0.

So, what you are going to rid of is only the sound power which is due to the source of your concerned. So, this is a very important aspect in sound power measurement and sound power estimation which makes it very handy for us to in the noise control domain. To actually estimate in the presence of various realistic background noises which are present you can imagine in a factory setting it is not possible to say that you will take a measurement where all other machines have to be shut down and only the machine of your interest will be measured. So, SPL values obviously, will get corrupted because of the ambient noise whereas, sound power level will not be corrupted because whatever is the power that is entering this hypothetical surface due to a background source will also leave this hypothetical surface (Refer Time: 04:55). Sound power will be contributed by only those sources which are completely contained within this or completely enclosed within this hypothetical surface.

So, therefore, sound power can be measured to an accuracy of one db from sources ten db lower than the background noise right. So, this is pretty much effective this is not true for sound pressure for sound pressure things are far more respective. So, that is for the experimental acoustics as I said. So, with this I would like to sort of close our discussion on experimental acoustics at least for now may be if time permits we will do some other measurements of impedance and absorption, but this sort of gives you a brief over view as to how the important measurements of sound pressure and sound intensity would be done and what is the principle behind this measurement; obviously, by no means this is this information is exhaustive we will find much more information. Especially these manufacturing these manufacturers of these transducers will have very informative manuals and other literature which you should be able to read and we able to understand the integrities of the instrument ok.

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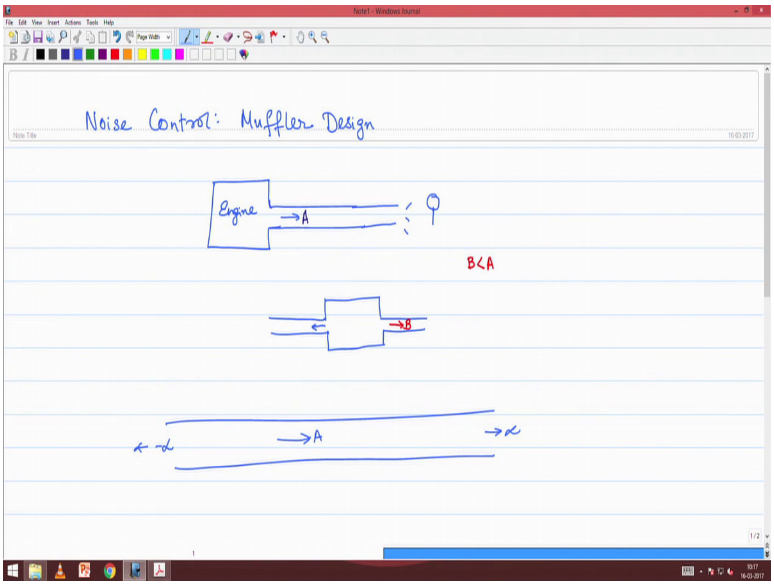

So, now we will take up a next topic which is going to be Muffler's. So, we will look at one very important application of noise control which is Muffler design. The most important application for Muffler design possibly arises in the case of exhaust noise control. So, as you know there is an engine the automotive engine because of the combustion process it has got to make certain noise a unwanted sound and that along with the exhaust we will be travelling through the exhaust pipe and then the microphone in the pass by noise will be able to pick up this noise right. As we have seen in one of the earlier talks that at certain cases exhaust noise could be the most predominant noise appearing in the vehicle. So, you need some ways and mains to control it by control here we generally mean reducing there is no reason why you would like to amplify it, but more often they are not I mean most prevalently you would like to reduce this noise.

So, one way of thinking about it is to reduce the noise at it source so that means, do something such that the engine makes less noise right. But that would mean, but what is the reason for because of which this noise arises the reason is the combustion process right. So, as a noise control engine air if you suggest that you rework the combustion process as that it is less noisy that possibly is not a good suggestion in the industrial context because that combustion process has been sort of frozen keeping very many different prospective in mind the most important of which being fuel efficiency talk and power delivery and things like that right. So, these are the primary driving factors which drive the engine design prospective definitely noise does not drive the engine design prospective at least till the present times. Hopefully it should, but the fact is it does not right.

So, the point here is as noise control engine as we would still like to do some refinement in this at least at the vehicle levels such that the least amount of noise escapes the vehicle and sort of intrudes in the pass by noise measurement something has to be done. So, if we realize carefully the source generates a certain sound and this is a ducked something that that we have been studying all through in our analysis of plane waves. So, hopefully we can say that within this narrow ducked that is the exhaust you should expect plane waves to be going out right. So, there are some plane waves which will be set up because there is a certain sound source it could be either acoustic, it could be some combustion driven whatever it is there is a certain source because of which acoustic pressure waves who are set in and they are traveling within the ducked right. And finally, at the end they will escape into the atmosphere.

Your job is to control the amount of sound which escapes into the atmosphere. So, now, that we understand it is that it is very difficult to implement a design solution where in you say that the let the engine characteristic switch in such that noise the audible noise that is generated in the first place is much lesser that is a very difficult solution to implement because that would make the engine design very complicated usually the engine designers would not like to take that into they are at least the first design iteration. So, the alternative root is that you put something in this exhaust pipe which colloquially we call it as silencer, but more technically it is called a Muffler. The job of the Muffler is therefore, to create a reflection whatever was this wave which was traveling nicely on interrupted within this exhaust pipe because of the silencer as we will show it will see a impedance mismatch and as a result a part of that incident wave will get reflected which means that now what is going to get transmitted beyond the Muffler should be lesser.

So, the objective is this that between these 2 pictures one which is unmuffled and the other which is muffled. So, we will call this B, we should have the B wave to be lesser than the A wave right. If that happens then we can hopefully think the of the situation that the Muffler is doing his job in the sense that what is getting transmitted downstream from the Muffler is actually getting reduced right and we know what causes reflection of plane waves we have seen we have made our pretty much detailed study where we had taken you know 2 different mediums and said that because of impedance mismatch because of the 2 mediums there is a reflection that is created or in other words reflection ratio was turning out directly to be a function of this impedance mismatch which was shown there in those examples. But those examples where mode like in academic nature because in this application problem we actually do not face any different medium the medium is anyway going to remain the same right.

So, but yet we will see that in this frame work there is an impedance mismatch which is getting introduced and let us see how do I analyze this problem. So, what we will do I think we can do that in the next page, we are going to analyze a simple expansion chamber Muffler.

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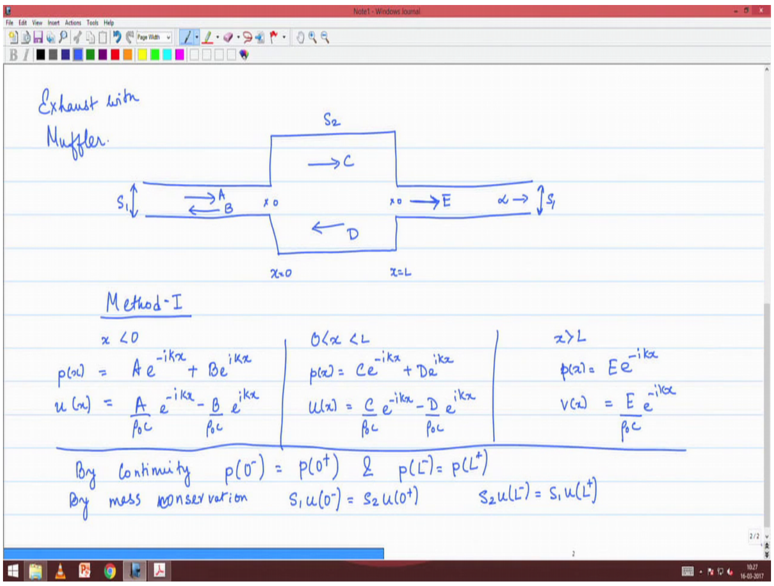

So, this is the exhaust pipe and within the exhaust pipe what you have done is that you have created and expansion chamber which is sudden expansion area of cross section here is let say S 1 and the area of cross section here is also S 1 and suddenly the cross section is made to change Y 2 S 2. So, suddenly there is a contraction and similarly at this section suddenly there is an expansion in the upstream and suddenly there is a contraction in the downstream. So, there is a sudden expansion and there is a sudden contraction that happens.

So, as usual we are saying that there is an incident wave a there will be a reflected wave B because we are assuming that I will prove to you that there will be a reflection, but for now just for a moment taking me on trust that there will be a reflection which means there will be a B wave. Similarly within this zone we expect 2 waves and here we will say that this is going to infinity. So, there is an E wave. So, this is our Muffler configuration we will contrast this, so this is exhaust with Muffler and for our reference we will compare this picture with exhaust without Muffler.

So, we will pretend for now that the exhaust pipe is an infinite pipe, we will later come down to finite exhaust finite exhaust pipes also and there is only a incident wave which is going, within this pipe there is no change in medium there is no change in cross section nothing changes, so there is nothing which can create or reflection. So, whatever is the incident wave which will it will keep on travelling right.

But now what we are seeing here is that the wave which finally, travels out right is the E wave. So, we actually need to compare this amplitude of the transmitted wave with the transmitted of the incident wave A. This incident wave A will remain as it is if there is no Muffler it will just keep on travelling without any effect on its amplitude right which is the case of un muffled noise right. So, we will basically need to compare the amplitudes of E and A and hopefully if we can bring down the amplitude of E in comparison to A we should feel happy that yes the transmitted noise has reduced. So, what is going out from the Muffler is less then what is what it should have gone out of the exhaust pipe had there been no Muffler, had there been no Muffler then it would have gone as an a wave, but now that you have a Muffler you are introducing reflections and these reflections are creating the effect that the transmitted wave amplitude hopefully should be lesser.

So, how do we analyze this problem? The analysis is actually pretty simple. So, we will just take this as x equals to 0 x equals to L as our coordinates. So, I will do this analysis in very many different ways. The first analysis that we will do will sort of replicate the analysis that we have done for plane wave analysis right, but then we will soon develop this that analysis a little further such that we get into the integrities of these Muffler design issues. So, the first method which is typically not a very useful method in the sense that there is it is a little more combers up, that is why this method will not be used, but none the same I want to give you this method just for you to sort of connect with whatever we have studied in our plane wave analysis right.

So, with this as the problem in the region x less than 0 we know that the pressure wave will be given as A e to the power minus i k x plus B e to the power plus i k x right because in this region we have marked that there are two travelling waves A and B. So, the total pressure has got to be the superposition of these two that is it. In the region x between 0 to L the pressure is going to be a superposition of the C wave and the D wave right and finally, in the region x greater than L the pressure is going to be given by only the traveling E wave nothing else right. So, if pressure is given by this then the particle velocity u is going to be given by A by rho naught C e to the power minus i k x minus B by rho naught C e to the power i k x this is something that we had seen. That if pressure is given by a travelling wave then that pressure amplitude if divided by the appropriate impedance and the sign taken correctly in the direction of travel of the wave then you can easily recover the particle velocities from the pressure wave.

Similarly in this second segment we will have u of x to be equals to C by rho naught C e to the power minus i k x minus D by rho naught C e to the power i k x and lastly in the third segment we will have v as a function of x to be given by e by rho naught C e to the power minus i k x. This is exactly what we had done is just that we have to do it in three parts possibly the earlier analysis was all limited to two part analysis this is just easy extension of that. But remember what is our objective given A we should be able to find E right that is we need to find this transmission ratio. Given an a unit amplitude incident wave by how much does the transmitted wave reduce hopefully it reduces it does not increase right.

So, towards that end we must understand now that there are five variables as we see A B C D and E, but we need to find B C I mean what we are mostly interested is to find E in terms of A right. When I say in terms of it essentially means A is known we could as well assume that A is of unit amplitude and find what is the resulting E. So, that gives us the ratio of E by A right. So, there are essentially four ratios, there are essentially four ratios which are unknown or there are essentially four amplitudes which are unknown in terms of A right; B C D E are unknown we need to find B C D E in terms of A right . So, somehow to find this four equations we need four; sorry four unknown we need four equations.

What do you think are the four equations? See we are doing it in segments we are saying that in this segment x less than 0 you have a certain form of pressure and velocity in the second segment between x equals to 0 to L you have some other form of pressure and velocity and thirdly in the last segment you have some other form of pressure and velocity right. But if you do it in segments is there anything that you have to match or.

Student: (Refer Time: 20:38).

You have to match continuity definitely of pressures. So, the conditions that are required by continuity argument would be given in the following manner. So, by continuity we should demand that p at 0 minus should be equals to p at 0 plus. So, if you take two neighboring points here just on either sides of that expansion chamber then you should expect that the pressure to be in continuous there is no reason that the pressure to jump drastically. Similarly if you take 2 points on either side of the contraction segment you should expect that p L minus should be equals to p L plus right, that should be nice and simple should we expect the same relation for velocities also; should we expect the similar relation for velocities also?

Student: Yes.

Yes, emphatic yes, emphatic yes.

Student: (Refer Time: 21:55).

Area changes right. So, area times velocity is mass. So, it is the mass which should be conserved not the velocity right. So, this is a little change from what we had done in our previous analysis. See in the previous analysis we never said that we are looking at a duck which is suddenly changing in its cross section right we either said that we are looking at a segment which is completely infinite in its width or we said that the duck is of uniformly then that is why we sort of some times said that let the material property change right. But here we realize that the velocity should not be fixed rather it is the mass which should be fixed which means that by mass conservation we should have the densities anyway same for the fluid throughout.

So, essentially the mass flow is given by density times area times velocity right, but the density part is anyway same. So, no need to account for that. So, we will anyway account for only, the volume velocity volume velocity is area times the particle velocity, but the area is changing. So, that is what, that is why I said the area change will create a new problem for us this is what it is showing up. So, what we require is S 1 multiplied by u at 0 minus should be equal to S 2 multiplied by u at 0 plus and similarly we should have S 2 at u compute is to S to multiplied by u computed at L minus should be equals to S 1 multiplied by u computed at L plus. This is what is demanded by mass conservation.

So, essentially you have two variables sorry four variables and four equations which you now see. So, hopefully there is some hope to be able to solve this problem in this fashion. So, let us keep looking at taking this derivation a bit longer.

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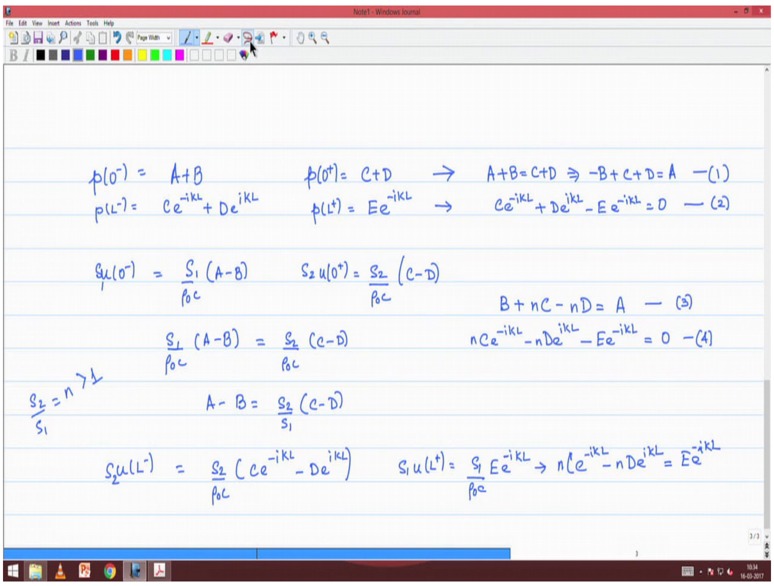

So, p 0 minus will be derived out of this first segment. So, this is this expression what happens for x lesser than 0. So, p 0 minus definitely is A plus B right and what is p 0 plus.

Student: (Refer Time: 34:35).

That is C plus D. So, as a result from the first continuity condition what you will get is A plus B is equals to C plus D which also can be written as minus B plus C plus D is equals to A right. So, this is my equation one. Now let us look at p at L minus that will give us C e to the power. So, I need to replace x as L right. So, C e to the power minus i k L plus D e to the power i k L is p at L minus and what is p at L plus.

That is in terms of E e to the power minus i k L right. So, continuity argument demands that these 2 quantities must be same which means C e to the power minus i k L plus D e to the power i k L minus E e to the power minus i k L is equals to 0 that is the second equation. Let us look at the third equation u at 0 minus how much is that A by rho naught C minus B by rho naught C I can as well take the rho naught C outside right, let me write it in a neat way 1 by rho naught C into A minus B right and that should get multiplied with the area which is S 1. So, S 1 into u 0 minus reads as this condition right. Similarly S 2 at u 0 plus would read as S 2 divided by rho naught C, C and D, C minus D right.

So, therefore, this equation would read in the following fashion - S 1 by rho naught C, A minus B is equals to S 2 by rho naught C, C minus D right. So, doing a little more simplification we get A minus B is equals to S 2 by S 1 C minus D. So, we will call S 2 by S 1 as the expansion ratio n and this is greater than 1 by the nature of construction. So, this is S 2 this is S 1 S 2 is more than S 1. So, we have a minus B as S 2 by S 1 into C minus D which means the third equation would could now be written as B plus nC minus n D should be equals to A this is the third equation right.

Finally we need u at L minus which is going to be S 2 by rho naught C, C e to the power minus i k L minus D e to the power i k L that is u at L minus right this is S 2 into u at L minus - u at L minus is C e to the power minus k L minus D to the power i k L divide by rho naught C that is get multiplied at with S 2 and finally, S 1 into u at L plus reads as S 1 divided by rho naught C E e to the power i k L right. So, therefore, what you have equating them is the following. So, you will get n C E to the power minus i k L minus n D e to the power i k L is equals to E e to the power i k L I hope I have done the algebra correctly E to the power i k L n C e to the S 2 by S 1 is n right looks right.

Student: E (Refer Time: 29:46) minus (Refer Time: 29:47).

This one is minus, I was almost sniping some trouble D e to the power E e to the power minus i k L right because this is the forward wave. So, then what we will get out of this equation is the following n C e to the power minus i k L minus n D e to the power i k L minus E e to the power minus i k L equals to 0 this is the forth equation. So, we had four variables we have got the four equations.

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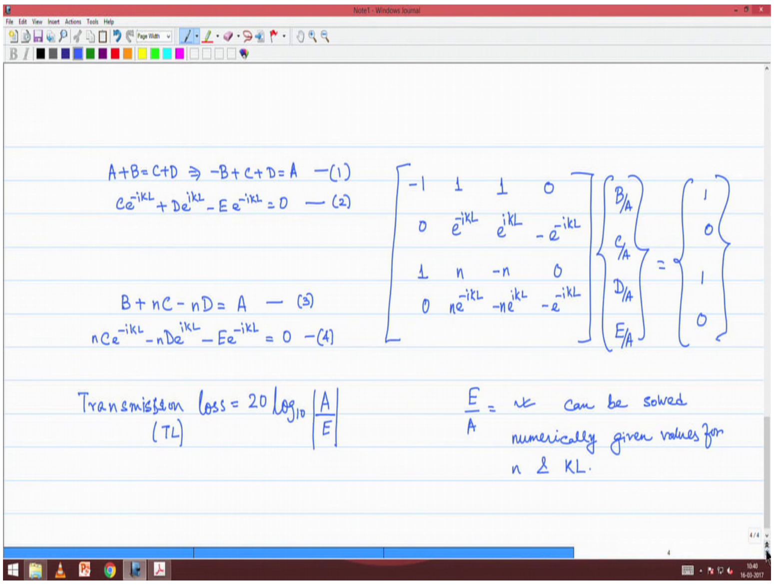

So, in matrix form this equations could be written in the following fashion. So, the arguments the unknowns are B C D and E. So, each equation will now write it the first equation has minus 1 as the coefficient of B plus 1 as the coefficient of C plus 1 as the coefficient of D, but 0 as the coefficient of E right and this on the right hand side leads us to A right.

So, that is the first equation the second equation reads as there is no B here, but C will come with a coefficient of E to the power minus i k L, D will come with a coefficient of E to the power i k L and E will come with a coefficient of minus 1 right and on the right hand side we will have the 0. Similarly third equation will read as 1 n minus n 0 and on the right hand side you have A, right.

Student: (Refer Time: 31:41).

I missed a minus is it.

Student: (Refer Time: 31:44).

Yeah.

Student: E power (Refer Time: 31:54).

E power minus i k L right, E to the power minus i k L with A minus sign right, so then n 0 n E to the power minus i k L minus n E to the power i k L minus E to the power minus i k L right and on the right hand side we have a 0. So, this matrix equation hopefully can be solved and do not ask me to solve it, but you should also not try to solve it yourself in pen and paper, but this can we solved in a symbolic computation package within MATLAB you have a symbolic MATLAB tool box you can solve it or even numerically you can solve it for different values of k L the only factor that you need is give me n give me k L you should be able to find numerically this matrix invert it and get the answer of B in terms of A or B by a right. You could change over from this notation now by putting 1 and 1 here and dividing it throughout by A.

So, essentially you are dividing by a throughout right. So, we do not have much need to know what is B by A and C by A and S by A our primary interest lies in.

Student: E by A.

E by A right. So, E by A can be known, can be solved numerically given values for n and k L or it can be solved even symbolically in software such as mathematica and even in the symbolic MATLAB toolbox. But all that you can do is probably take different values of n and solve it and see a graph of this quantity or maybe you should plot directly the transmission laws. I will just rephrase that part.

After having obtain E by A we define a quantity which is called transmission laws. So, the transmission expected when there is no Muffler is A right, the transmission after you put the Muffler is E right, but then this ratio you should not just look at it in a linear fashion, but it make sense to look at it in a logarithmic fashion right because we understand that our sound perception in terms of decimals the even perception is in terms of logarithmic scale rather than in terms of a linear scale. Just like we have rebuilt the units we converted from Pascal units of pressure to a decimal units using the logarithmic scale, so what we need is a logarithmic reduction.

If you simply have E to be just half of a that possibly will not lead to much perceptual difference right what you need a logarithmic reduction. So, to have this logarithm idea built in, so we will not only look for a by a by E which in general could be complex. So, we will take the absolute value of this ratio and then we will take the logarithm of it and then multiply it with 20 and this is called the transmission laws. So, what I want you to do in an assignment which is almost trivial is that plot this as a function of n and plot this as a function of k L for different values of n.

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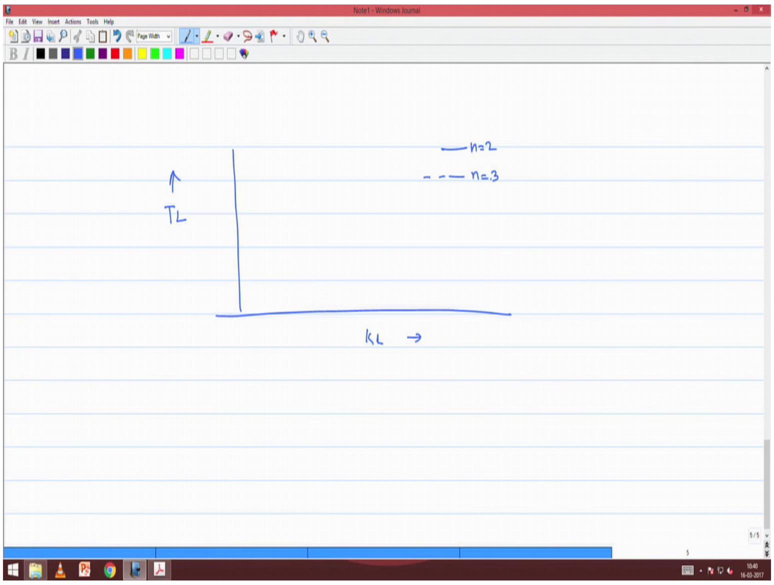

So, what I want you to do now is K L against T L should be plotted for let say n equals to 2, n equals to 3 and so on.

But at this stage it is really difficult to derive from first principles what this plot will be looking like because it is a mammoth fourth order equation sorry 4 by 4 equation and it is actually pretty difficult to solve it in pen and paper. So, I will not try to attempt doing the solution of a 4 by 4 system, but none the same I wanted to emphasize to you that even at this stage without knowing anything additional in terms of you know the subject area of Muffler acoustics you are actually able to find the transmission laws of an expansion chamber Muffler. Just that this is probably a little more combustion because as you see you are getting a 4 by 4 system which you have to solve.

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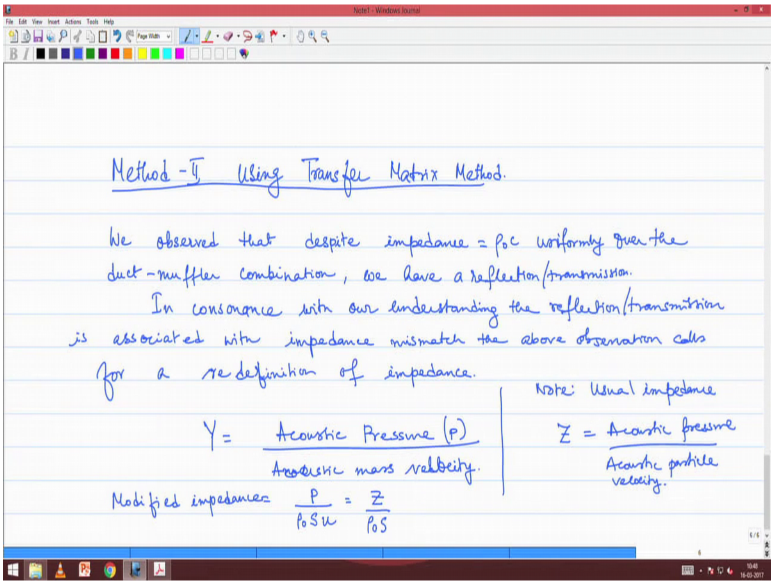

So, now we will redo this problem in the manner in which the duct acoustics community does recommend and that is call the transfer matrix method. So, we will redo this problem now in a little more elevate fashion using transfer matrix method. So, as you will see this method is much more easy to implement and you can derive a lot of useful information without jumping on to the computer and trying to plot a graph or something like that because the systems are much smaller here you will get only 2 by 2 systems for all your duct acoustic needs the systems will remain 2 by 2 which means that they will be pretty much easy to solve for you do not need to necessarily go to a computed to solve it. So, this is what we will now redo it.

So, therefore, we would for this class of problems we will try to redefine the impedance in a slightly different manner as we understand that it is not the velocity which is important in this class of problems, it is rather that the mass velocity or the volume velocity which is important right. So, we will redefine as we go along now we will redefine the impedance not in terms of particle velocity, but in terms of mass velocity. So, we observed that despite impedance equals to rho naught C uniformly over the duct Muffler combination we have reflection transmission. So, this in consonance with our understanding that reflection transmission is associated with reflection transmission is association ted with impedance mismatch. The above observation calls for redefinition of impedance.

So, this redefine impedance we will call it as Y and this will be acoustic pressure p divided by acoustic mass velocities right instead of acoustic particle velocity. So, I will just put a contrast here note usual impedance is defined as acoustic pressure divided by acoustic particle velocity, but now that we see that with this definition we are getting a sort of unsettling observation that impedance mismatch is not there you get reflection is happening. So, therefore, we are tweaking the or rather reinventing a new sort of impedance because we realize that if here it is not the velocity which is supposed to be continue across the segment, but it is the mass velocity which is supposed to be continuous.

And obviously, this is pressure by mass velocity is density times cross sectional area times particle velocity right. So, p by u is the usual definition of impedance which we can call it as Z. So, this is Z. So, Z by rho naught C rho naught S is this new impedance. So, this is what we can call it as modified impedance and we will use at least for duct acoustics we will use this form of impedance rather than the previous one. In the next class we will take it from here that will be all (Refer Time: 42:32).