Acoustics & Noise Control
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Module – 19
Lecture – 24
Acoustic Measurements

In the last class we talked about condensor microphone, and the sensitivity analysis was done in a sort of an approximate fashion. And you will recall this is the graph that we would obtain which basically is the mechanical transmissibility plot.

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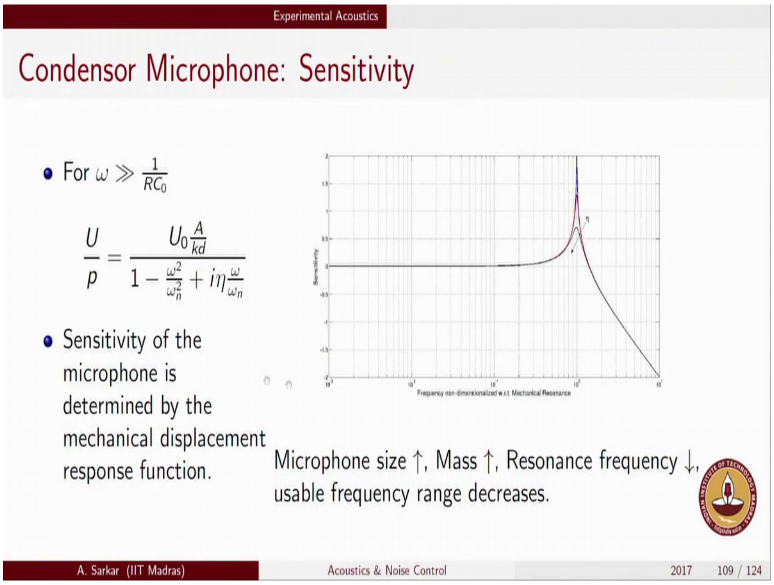

And that clearly shows that the sensitivity remains flat, but provided that the frequency range under consideration is like 0.3 maybe 0.4 times of it is of the mechanical resonance.

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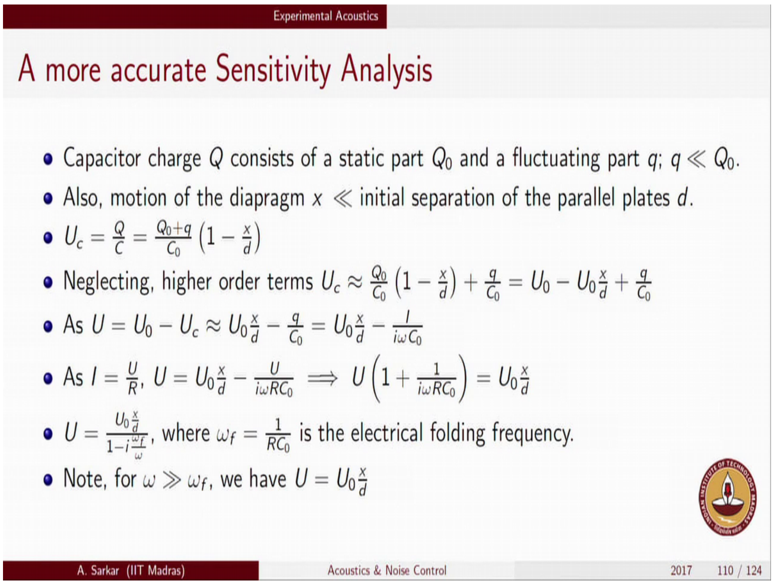

So, we will repeat the analysis today, but in a slightly more accurate fashion. So, in the last analysis you will recall that we will we had the I mean, simply because of the fact that we use the assumption that the resistance in the circuit was high. We said that the charge was basically frozen there is no amount of charge which is flowing from the parallel plate capacitor. But now we will repeat that analysis without invoking that assumption of high resistance, and essential we will account for the fact that the charge will have 2 part. One which is static and the other which is fluctuating. So, we will basically account for the fact that there is a possibility that there is a flow of charge despite the resistance being high, it is definitely not infinity, which results in some amount of fluctuation of this charge and hence the current in that is a part of the circuit.

So, accordingly this total charge will be broken down into a static part and a fluctuating part. Much like our pressures itself the ambient pressure as we understood, is comprising of a static atmospheric pressure over and above which the acoustic pressures are the fluctuating components. But again similar to our acoustic analogy, this fluctuating part of the pressure is much smaller sorry, frustrating part of the charge is much smaller than the static charge. That is the inherent assumption because the resistance is high. So, if the resistance is high therefore, you do not expect too much of a current to flow through the circuit, and as a result the fluctuating part of the charge is going to remain small. So, over and above the charge which is there in the initial condition due to the polarization over and above that there is a small fluctuating component of the charge which is there because of the dynamics of the condensor or the capacitor.

And we will continue with the previous framework, wherein we had said that the motion of the diaphragm is much smaller than the initial separation of the parallel plates. It is that this was something that we used in the last analysis also. In the last analysis we said that the diaphragm we ignored the factor x by d at places. And here also we will do the same thing we will basically ignore certain products of these 2 small components which are q and x. So, you will recall that U c which is the voltage across if I recall the circuit.

The U c is the voltage across the parallel plate capacitor. So, U c will be given by Q by C and this q is the total charge that resides in the parallel plate capacitors inclusive of the static component as well as the fluctuating components. And similarly c without any subscript denotes the capacitance and the when the when there is an arbitrary motion of x with a diaphragm right. And we already know that q is Q 0 plus small q and c is c 0 divided by 1 minus x by d. So, these parts of the derivations were already done in the last class. So, no point in repeating them.

Now, what happens is this, that because of our assumption that q and small x are small we essentially ignore this product what happens is that, this product of these 2 small terms q and x is going to be neglected, all other terms are going to be accounted right. So, we are going to account this Q 0 by c 0 1 minus x by d for sure. We are to account for Q by C 0 for sure, but we are going to neglect q times x, because both q and x I have been assumed to be small in comparison to their. So, we are neglecting the higher order terms which basically means that this q times x is thrown off, and that is why it is an approximate expression, but this approximation is hopefully better than the previous approximation, where we completely neglected that the fact that the charge is actually dynamic. It is not going to be entirely comprising of the static part.

So, carrying ahead what we realize is Q 0 by c 0 is U 0 which is the initial voltage the DC voltage that has been applied across the parallel plate capacitors and Q 0 by c 0 include x by d that gives us U 0 x by d and Q by C 0. And as per the voltage relation that we had which is the voltage across the terminals was U 0 minus U c. And U c is given by this expression. So, therefore, U 0 minus U c can be taken as U 0 x by d minus Q by C 0 from these 2 relation.

And in the next step what we have done is remember q is the fluctuating component of the charge, which means q is a dynamic quantity whereas, Q 0 is a static component it is not going it is a constant it is does not change with time. So, again if we take a harmonic assumption which means that let us try to understand what is the voltage that is generated; across the all electrical network when this microphone is being impeached upon by a single frequency plane wave right.

So, if that if the acoustic excitation or to this electromechanical system is a single frequency then all the electrical as well as the mechanical quantities are carrying that same frequency. So, q will also necessarily be a harmonic quantity, not just a dynamic quantity dynamic by dynamic quantity we mean any quantity which changes with time. But when we say it is harmonic quantity we essentially mean the manner in which it change is of the form of a e to the power i omega t or harmonic simple harmonic expression.

So therefore, both q and the fluctuation I mean the dynamic part of the charge q is a harmonic quantity, and we know that the current across the circuit I is going to be dq by dt right. So that means, i omega times q is equals to the current I. So, this is I please do not mistake this for 1, this is how we will how the font 1 is this is I right. So, I is the expression for dq by dt, and dq by dt is i, I mean the small i here refers to square root of minus 1. So, small i omega q is equals to capital I which is the current.

So, in other words small q is capital I divided by small i omega right. That is how the simplification goes forward, but then the current in can also be understood the current in this part of the circuit is; obviously, U by R right.

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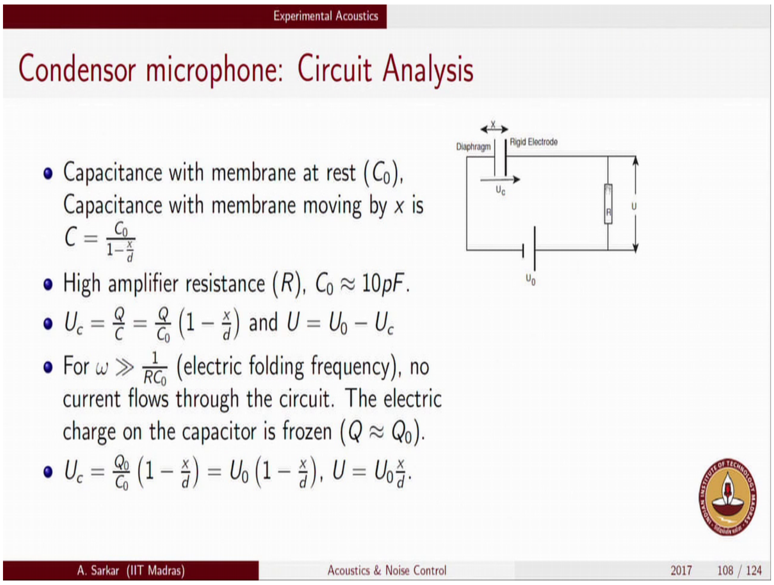

So, I can make that simplification, and when you do that when you substitute that part here I is equals to U by R in the previous expression this is what you will get right. And therefore, if you bring this back on the other side you will get this right. Please note at this stage the change of the expression between u and U 0. So, in the last analysis which we did which was supposed to be little more simplistic we get we had got U equals to U 0 x by d.

Now, we are getting an additional group of terms which basically depends upon the electrical folding frequency. So, 1 by R c 0 is the electrical folding frequency right. So, omega f is the electrical folding frequency, which is R 1 by R c 0. So, if this electrical folding frequency is very small, then what happens is, or in other words if omega f is very small omega by omega f will be very large. And then this term will not contribute right. And therefore, this is effectively 1. So, that is that was essentially the analysis that we did in the last lecture where we ignored the presence of this term.

But here we are not ignoring the presence of this term. If you remember in the last class we said that R was taken to be very high. So, if R is taken to be very high; obviously, this term is not active and it is effectively saying U equals to U 0 x by d which is what we derived in the last class. But here we are carrying this effect of the resistance and this is what we are getting as a result. And then the remaining part of the calculation is just the same we can relate this x which is the motion of the diaphragm to the acoustic pressure excitation which heats the diaphragm in actually identically the same fashion.

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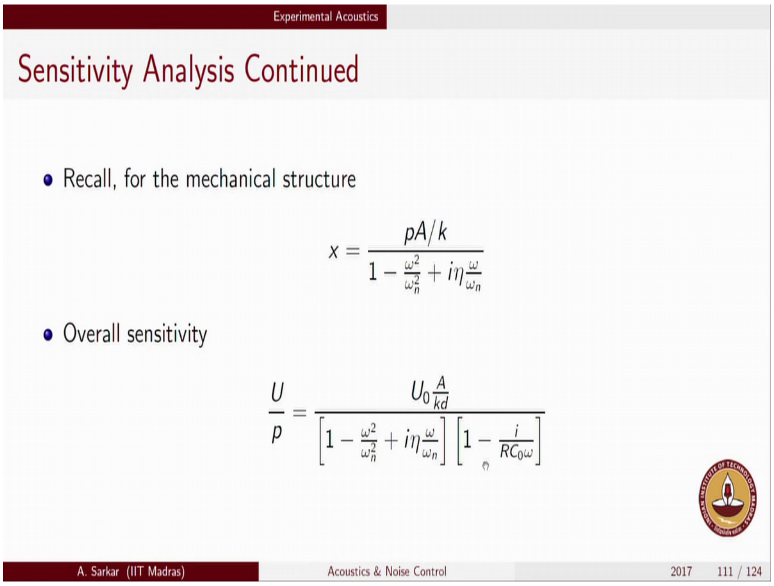

So, that would give us the expression of this form, where P into A that is the acoustic pressure times the area of the diaphragm is the total force divided by stiffness is the static deflection, and the quantity and the denominator is the magnification factor the dynamic magnification factor. So, this is how the pressure and the motion of the diaphragm is related, and the relation between the voltage and the motion of the diaphragm is given by this relation. So, combining the 2 we can get the sensitivity in this form.

So, this is U by P you will recall this is exactly the same expression except for this second group of term appearing in the denominator. So, in that sense this is a little more accurate expression a little more I mean, accurate in the sense that we do not have the assumption that the effect of the electrical resistance is ignored by saying that the resistance is high. Whatever is the resistance we are able to see for ourselves the effect of it. But then the point is this analysis tells you that you should actually choose a very high resistance in the circuit as was shown. Because if you now start plotting this diagram then you will see, That the previous expression that we did in the last class is plotted in red.

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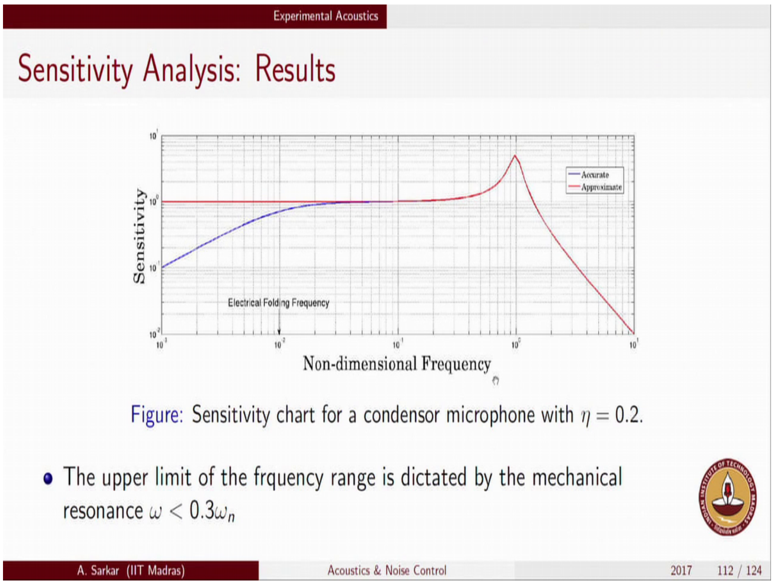

And the expression that we derived here is plotted in blue ok.

You will see just around the electrical folding frequency which is 1 by R c 0 again the sensitivity drops right. So, basically the usable range of this usable frequency range of this condensor microphone is going to be in this zone, which is higher than the electrical folding frequency, but lower than the mechanical resonance frequency. These 2 parts of the electro mechanical analysis is very important, that there is a mechanical resonance there is an electrical folding frequency and the usable frequency range of the sensor happens to be lying in this intermediate zone. Because once you approach either of these resonances you are going to deviate from a flat sensitivity graph which is not a desirable characteristics of the transducer.

And here I have just taken I think 10 to the power my. So, in the electrical folding frequency that I have chosen for this plot is 0.01 times the mechanical resonance. So, this is the non dimensional frequency which is non dimensionalized with respect to the mechanical resonance. So, 1 denotes, 1 in this skill denotes the mechanical resonance. And the electrical folding frequency has been chosen as 0.01 it is 10 to the power minus 2. So, in that case we see that the usable range is like in this zone which is around 10 to the power minus 1. So, it is more than electrical field in frequency, but less than the mechanical resonance frequency.

So, this is what I wanted to at least the sensitivity aspects of a condenser microphone what are the limitations. I hope you get a fair idea as to how the sensitivity analysis is done. And the crucial factor as I said to remember is that it is dictated by 2 factors, mechanical resonance on the upper side and electrical holding frequency on the lower side.

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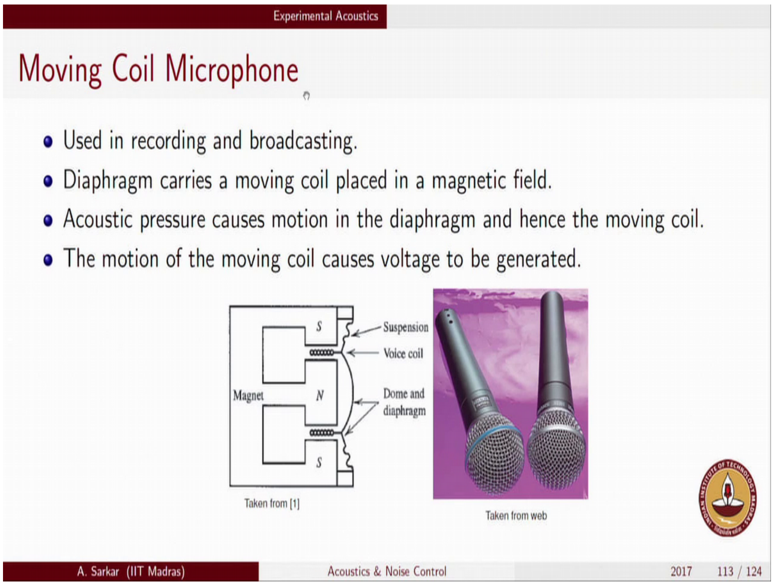

Now, we will just quickly look through just another kind of microphone, the moving coil microphone. And this is very oftenly used in recording and broadcasting, if you see public speeches or singing a singer performing not with the collar mic. But at least the public address system the mikes have this kind, this is a wireless mic.

So, most of these mikes would be a moving coil microphone. The principle of operation of this moving coil microphone is quite different from that of the condenser microphone that we have studied in details. So, I will just catch up on the principle of operation we are not going to deal in any further details how this any analysis of this moving coil microphone will be exempted from this discussion. So, what happens here is that, we again have a diaphragm, but this time it is like a dome shaped and this diaphragm is connected with a coil which and therefore, as the diaphragm moves the coil is moving right. And this coil is embedded within a magnetic field.

So, this is the magnetic I mean, basically the base structure is the structure which is immobile is a magnet is a north pole this is a south pole this. So, this coil which is embedded together with the moving diaphragm. When the moving diaphragm moves the coil moves and when the coil moves the because it is moving in a magnetic field it generates some electrical field and that is picked up as a voltage right. So, that is how the motion of the diaphragm is basically converted into a voltage signal. And a motion of the diaphragm in turn is linearly related to the excitation which is the acoustic pressure.

So, please remember that most of the transducers in that is no matter how diverse be the application. Essentially the system is this way the acoustic excitation will cause mechanical response the mechanical response in turn should lead to a transduction which means that it should be converted in some way or the other to a voltage signal. You could do it either with the use of capacitor or you could do it with a moving coil microphone. So, there are reasons why moving coil microphone has not found. So, much place in as far as acoustic measurements are concerned. But I do not wish to dwell on those aspects. So, as I said we are in at least acoustic laboratory measurements what is (Refer Time: 15:33) used is that of a moving condensor microphones. Coming to another crucial aspect.

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We need to calibrate our microphones every time we take a measurement. And there is this instrument which is called as acoustic calibrator, and it looks somewhat like this.

It is like a box shaped and you put your microphone into this hole, and there is a switch or a button on one side. So, this is how it looks like that the microphone is inserted within this hole, and there will be some sort of button here and when you press that button and then in your front end of the data acquisition system you have to say that I am calibrating. This is supposed to be a source which emit is sound at a prescribed decibel level at a single frequency or multiple frequencies usually single frequencies.

So, you need to see to it that your measurement is able to read of exactly this calibrated value. Usually it would not and process of calibration is essentially to adjust the game in your circuitry says that it is now able to read the calibrated value right. Whatever is the prescribed value associated with the calibrator it should be able to read it off. So, accordingly you will adjust the gain value in your system such that you are able to read of the right. Value and this is where your question comes that can we not use the transducer which is having a sensitivity which is not flat right.

So, in other words then you have to adjust for the games of each and every frequency right. But here since you know you at the most you can have that say 3 calibrations done per in an experiment 3 maximum 5 right. So, then you can you cannot talk intermediate points, you are sure that these 3 points are going to read off the right. Values, but what about the intermediate values if you have a crazy sensitivity graph which goes up and down then essentially you need to calibrate all the frequency values. That is not possible, because then you will be the whole shelf of calibrators and also not only that it will be really tedious to do the experiment because every time you have to calibrate, maybe 10 20 times of these frequencies right.

But once you are assured that the sensitivity is flat, if you calibrate for any frequency then you are sure that as long as the sensitivity factor is flat, things will not be different across the different frequencies right. So, this is also called a piston phone because basically what happens is that this here since it is here what were actually talking of is the river this is not a sensor rather this is a source of sound. So, just like in the condenser microphone what happened is that the acoustic pressure cause the motion of the diaphragm, the acoustic pressure was causing the motion of the diaphragm. Here it is the opposite the voltage of the circuit causes a motion of the diaphragm, and the diaphragm motion in turn causes sound right.

So, essentially any speaker, in a loud speaker works on this principle any source of sound will have a certain mechanical component which vibrates. And the mechanical component is made to vibrate again using either a moving electrodynamic principle as in a moving coil microphone as you have seen. Or it can be made to vibrate based on that principle of condensor microphones, that is you have 2 charged plates and then if you apply a voltage essentially the force between the 2 charged plates changes and once you have a change in force one of those charged plates if it is allowed to move it will move the other charged plate is obviously held fixed.

So, once this motion of the diaphragm is set up then the diaphragm is supposed to emit acoustic radiation and that is what we hear as sound. So, this is the principle of sound source and that is why it is called a piston phone. Because it is like a pistol which is moving up and down and this is precisely what you have done in your problems of one d duct which is excited by a piston which is harmonically moving right.

The only point of discrepancy here is that this duct is or duct line is much smaller, but still it will possibly go for a few wavelengths (Refer Time: 20:09), but this is a sort of situation that we had worked out in our earlier examples right. The sound is generated into the coupler cavity. So, this is basically the coupler cavity that wherein the sound is generated and that is why the microphone is inserted into this cavity. And you are supposed to record the sound in this cavity. You have to take care while calibrating the ambient acoustic pressure temperature and humidity affect the calibrator performance.

And therefore, if you think that the atmospheric pressure temperature and humidity is not as per the prescribed values which is given into the in the manual of this instrument then appropriate correction factors, which are also quoted in the manuals should be applied. And all these values are can be done settings can be done when you use your front end. Usually these instrumentation companies by the way these pictures have been taken from the bruel and kjaer website which is a leading manufacturer for acoustic and vibration instrumentation. They have their frontend and in their front end you can; obviously, do these settings get these settings done for correction and calibration factors and so on.

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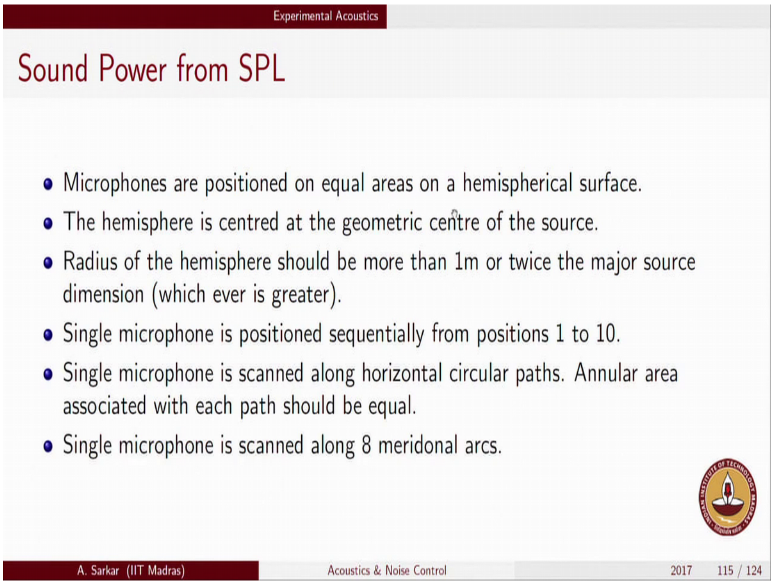

Now let us look at sound power. So, till now we have been looking for measurements of sound pressure, we will quickly look at how to calculate sound power from the sound pressure levels. In fact, you will remember that we have done this, when we did the dB calculation also, but I just wish to reiterate how this will be done in an experimental settings. So, towards that end I just have a few slides. So, microphones will be positioned on equal areas on a hemispherical surface, It something like this.

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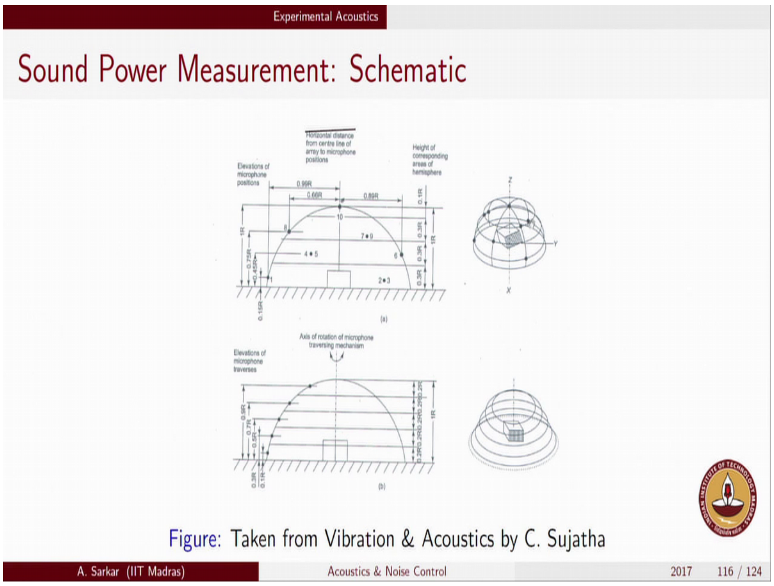

So, you will have a source of this kind and this is some like ISO standards, I think it is the standard number I can give you, but it as it is as per an ISO standard that you have to enclose this sound source in a hemispherical in a hypothetical hemispherical surface. And take readings at all these points which has been marked up. So, the exact location of this points can always be found from that ISO handbook. In fact, from this Sujatha's book has the details given if you wish you can refer to it, but I just wish to touch upon the principle of measurement rather than the nitty gritties of it.

So, you simply have to record the sound pressure level at all these location that is the first step. So, the microphones are positioned on equal areas on a hemispherical surface the hemisphere is centered the geometric centre of the source. The radius of the hemisphere there is a guideline that it should be more than 1 meter or twice the major source dimension whichever is greater. So, this is the standardization guideline. And then a single microphone is positioned sequentially from positions 1 to 10. If you have 10 microphones would you place all of them together, why should you move a single microphone from position 1 to 10? If you have if you are rich enough and hopefully if at least your company is rich enough, if not you should have 10 microphones or at least you can sort of wine to your boss that give me 10 microphones. Maybe he is a good man he will give it to you if you have 10 microphones it will cause more scatter right.

So, to make sure that the scattering effect is minimum, you should rather use a single microphone and sequentially take it take readings in all the 10 positions. And for some reason as is the guideline suggest that you should make the reading sequentially along the horizontal surfaces. So, you should move in this fashion, in the horizontal fashion. I really do not know why, but that is what is given in the guideline. And then it should be taken along 8 meridonal arcs. So, there are 8 of these arcs, 8 arcs and the so many horizontal annular arcs.

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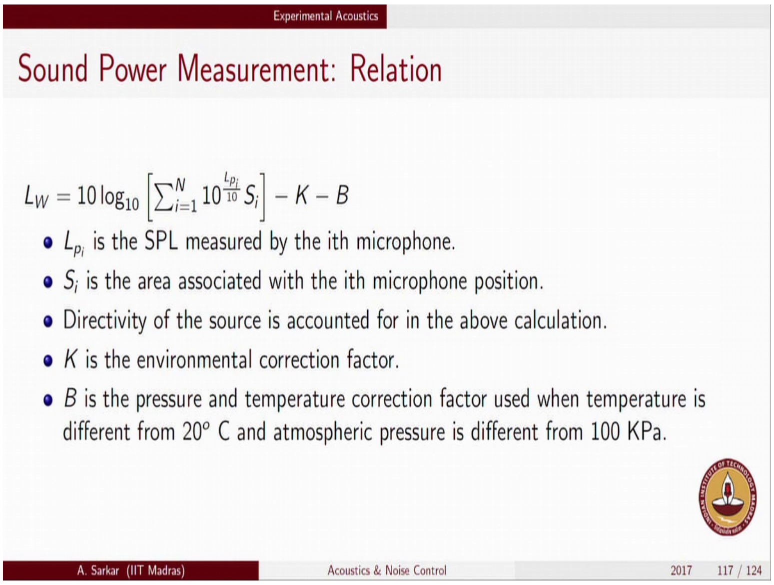

So, in this fashion these records of the sound pressure level will be taken in each of these 10 microphones, and then you would have seen this formula, and this formula is applied this remember this is the formula, without the K and the B part; obviously, this formula basically suggest that the, how we can convert the pressure to the sound power level formula.

So, S i denotes the area associated with each microphone. So, is the area associated with each of the of the ith microphone position this is the S i. So, applying this formula you can get the l w, and K and B are the environmental correction factor and pressure and temperature correction factor. Because as you know this microphones are designed for a specific temperature and pressure conditions, if you do not have those conditions then you have to apply appropriate correction factor.

Here unlike the derivation that we have done, we had done where we took an omni directional source and got our calculation that lw and L P will differ by 8 dB if it is actually an omni directional source here the question of directionality is taken care of by actually taking multiple readings, and multiplying the associated area right. So, if it is a source which is not omni directional in and it has directivity, then each of these L P 1, L P 2 will differ right. Whereas, in the analysis that we had done few classes back we had said all these L p's are actually same, because we took an omni directional source right.

So, the fact that the directionality is different will be incorporated in this formula by the fact that you are actually accounting for the individual measurements to be different, and then you are also multiplying it with the area corresponding to the territory of each of these microphone, and when you do that hopefully you should get the right answer for sound power. The question is that I will come back to another important question that why, I think sound power as I said sound power is possibly more important when you wish to characterize the source, rather than the sound pressure level.

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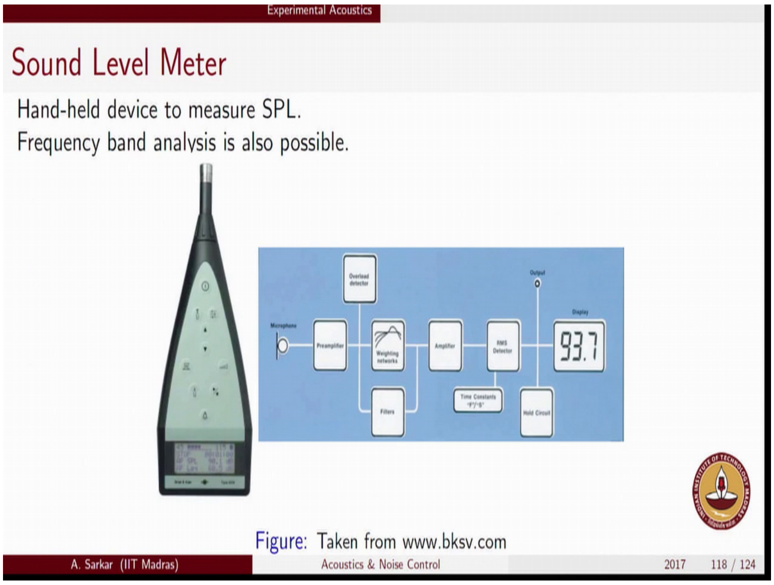

So, I will come back to that question in bit of a time, but before that let us talk about sound level meter. So, in comparison to microphone which is like a real the sophisticated instruments in order to diagnose our (Refer Time: 26:56) related problems the acoustic related problems, sound level meter is a handy device. In fact, our noise police man will have only this sound level meter.

If you just wish to check whether the particular machine is working as per the prescribed condition all you need is a sound level meter. This is not usually meant for any diagnostic purposes, for diagnostic and for really understanding the frequency in a narrow band it is a microphone together with a digital acquisition system which has to be used. But if you are just interested to understand the total sound pressure level that is emanated or maybe sound pressure level at different one third octave bands then, you have this handheld device. The advantage of this device as compared to a microphone is that it is very, very portable.

In fact, as consultants if we have to go to site and take measurements probably the first cut measurement should be taken. Only with the sound level meter only for a detailed analysis you have to carry your laptop with the front end with the acquisition system and other hardwares together with the microphone. But that is only when a detailed analysis is called for if you just have a baseline analysis for example, even in a pass by noise if you want to make sure that whether your vehicle is passing or failing the pass by noise criteria all you need is a sound level meter. So, this is how our sound level meter looks like and then this is been taken from bruel and kjaer website, which who is one of the leading manufacturers of different acoustic instrumentation.

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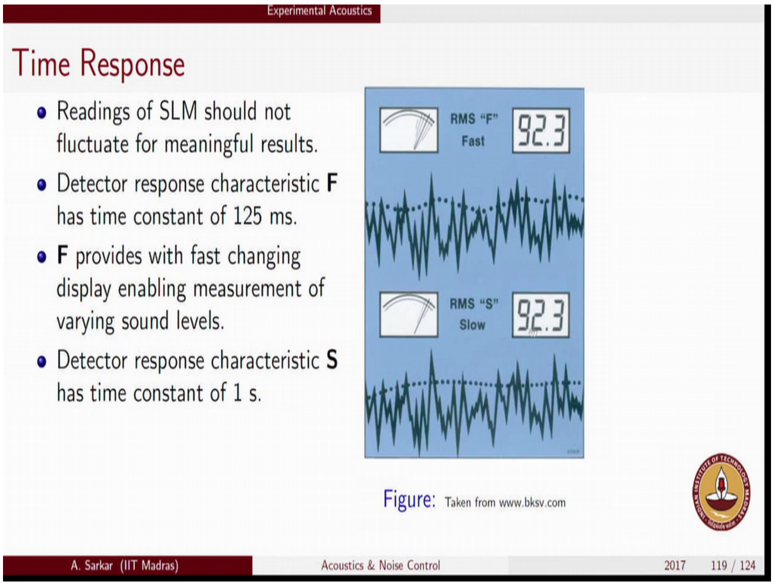

So, just to understand how the important factors associated with this sound level meter instrument. So, one aspect is that the band analysis is possible, but narrowband analysis is not possible usually it is octave band and one third octave band maybe one tenth octave band is also coming, but at least for the one that I have in the lab is octave band and one third octave band. So, that is why we say that it is not supposed to give you a very detailed diagnostic results. The other factor here is that we do use this kind of a machine for internet and noise measurement. So, as we said that if it is a steady state noise if the process that generates the noise is not changing with time then; obviously, the sound levels will not change, but at times we are looking for product let us say community noise. So, in this if you take this road in a busy street and you wish to find out what is the level of noise that is coming in the busy street the question is, you should look at what time window of noise. So, the time window is taken with the help of a particular setting which is there within the circuitry of this instrument, and that basically defines the time response of the system.

So, there are different settings the what bruel and kjaer calls it as the fast setting and the slow setting. In a slow setting the time constant of this instrument is larger which means that you have a larger time window over which the RMS calculation is done. Remember for the RMS calculation you essentially have to take the integral of the mean over a certain time window. So, when you have a slow setting; that means, you take the RMS over a larger time window, when you go for the fast setting the time constant of the machine is smaller the time window associated is small.

So, the display in your sound level meter will essentially change fast, for the fast response and display in your sound level meter will change slowly, if it is under slow response. So, slow response will sort of tend to average out over large periods of time whereas, here this display will change very fast. And at times you may not like it if it changes very fast you may not be able to observe the maximum value and the minimum value that efficiently because it is changing very fast. There are few other settings also associated with impulsive noise and things like that, but those things I am skipping. Another important aspect of sound as we know is sound intensity right.

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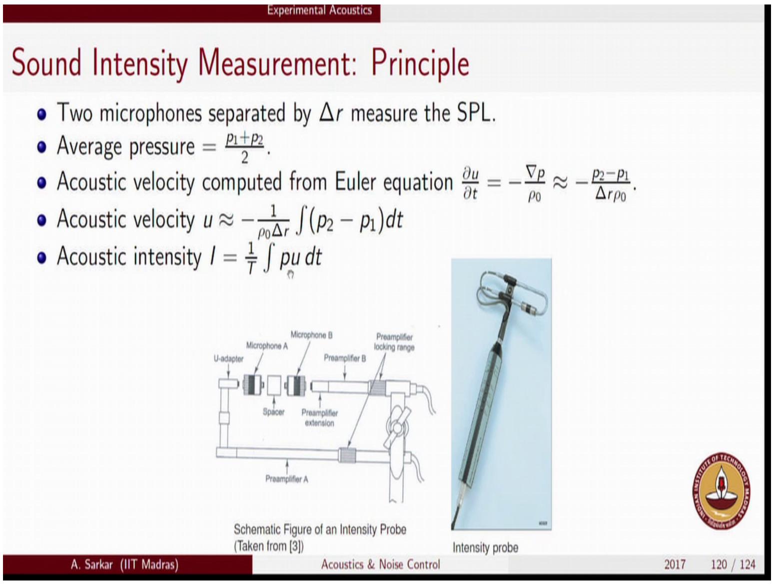

So, sound pressure is getting as we said is usually measured using microphones. For sound intensity we need 2 microphones. And there is a good reason why we need 2 microphones?

So, let us come to the principle of this measurement. So, you will recall that the Euler equation which relates the finally, what is intensity is a quadratic in pressure and velocity right. So, you cannot get away only with a sound pressure measuring instruments in, if you are really interested to measure intensity you have to measure both pressure and velocity. There is no way in which you can get out from this right. So, probably if you have another measurement which another instrument which measures the acoustic particle velocity at this point of interest, then it will work fine.

But then I usually though presently there are few instruments which does measure acoustic particle velocity, but traditional speaking that and also in terms of economics it is probably easier to have measurement of the acoustic velocity done in a little round about fashion. So, instead of directly measuring velocity what we will do is we will measure pressure at 2 different points. So, here we have 2 microphones microphone A and microphone B. Which are separated by a certain distance, and that distance is controlled by this spacer. So, there is a spacer which is a mechanical sort of adaptor which fixes the distance between the 2 microphones right.

And then if we get back to our theory which says from the Euler equation that rho 0 del u del t has got to be minus gradient of P right. And so, therefore gradient of P if we take something like a finite difference approximation, gradient of P this if we now take the component form of this vector relation gradient means, essentially derivative in a particular direction. So, the derivative along these this direction the direction between from microphone A to microphone B the spacing between this microphones is denoted by delta r. So, the gradient could be approximated using a finite difference formula as P 2 minus P 1 by delta r.

Obviously it is an approximate formula therefore, this is an approximation sign. So, P 2 minus P 1 divided by delta R is an approximate estimate of the gradient. And there is a division by rho 0. So, del u del t can be easily found from 2 of these microphone measurements you know what is delta R you know what is rho 0. So, del u del t has been found out what remains is that you could integrate this and as you know, the electrical circuit is do have integrating circuit is So, you could put this factor in an integrator circuit and as the result of this integration will essentially give you the velocity at the point.

So, the velocity at the point will not be at the position of A or at the B, but we will say it is it some intermediate position right. And similarly you need to find the velocity sorry the acoustic pressure at the same point. So, we will say that if microphone a measures P 1 and microphone B measures P 2 then, the velocity at this midpoint is going to be P 1 plus P 2. So, for the point of interest, we are actually not going to take any measurement at the point of interest we are going to take measurements of P 1 and P 2 in 2 neighbouring points which are separated by a distance of delta R. And we are going to assume that the pressure at the point of interest is going to be just the arithmetic mean of these 2 measuring points.

Obviously this is true only if the wavelength is large compared to delta R, if wavelength is small compared to delta R this will not be true. So, therefore, you actually have different adaptors which control the spacing between these 2 microphones, because depending upon the wavelength which in turn depends upon the frequency of interest you have to control this spacing right. And similarly the average velocity is computed as per this relation as I Have said.

But then this is possibly not a good way to measure at least if the you know that the sound source is steady. If it is study you can apply the principle of sorry before that once the velocity is measure intensity can be simply measured by the integral formula which is this.

So, all these computations are basically done in time domain. So, this will work whether the sound is steady or whether the sound is unsteady it is stationery non stationary both will go through using this idea.

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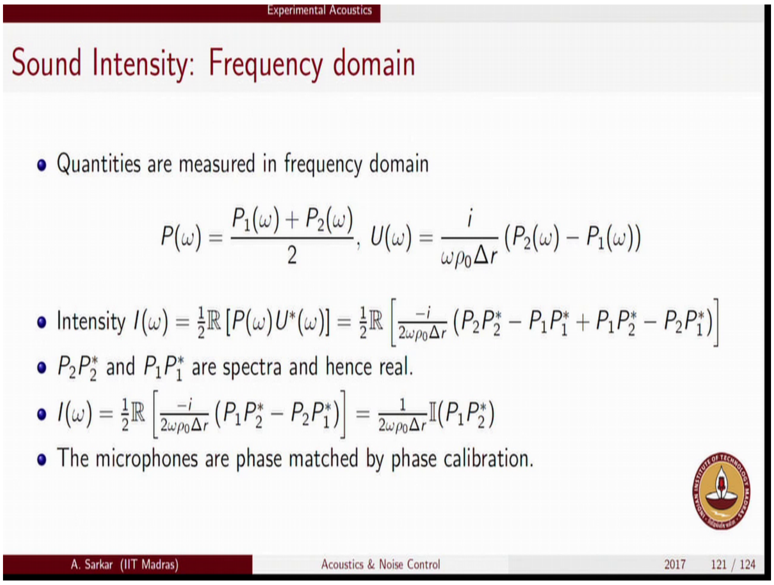

But if we know for sure that the sound intensity the sound source that we are dealing with is a steady state source, and it is stationary the characteristics do not change with time. We will appeal to this frequency domain techniques. And the various frequency domain technique will be win over is this that the estimation techniques associated with those integration in and other things will be much more robust right.

So, let us see how it goes through. So, as I said the average pressure has to be computed as this P 1 plus P 2 by 2, but instead of computing in time domain since this is a completely linear relation we might as well do it in the frequency domain. So, instead of taking P 1 and P 2 as functions in time we could as well choose P 1 and P 2 to be functions in frequency. Similarly this relation with u, and now one advantage that comes is that del u del t can be written as i omega u right. So, i omega and we can bring this omega in the denominator and there is a minus sign here. So, you have a minus 1 divided by I which is why you get an I in the numerator right. So, i omega rho 0 by delta R delta R is this and P 2 by minus P 1. So, this is the expression for u and this is the expression for p. So this obviously looks much simpler.

But it is even better if you do the next part of the calculation, intensity in frequency domain we have seen this formula is a real part of the complex pressure amplitude, into the complex velocity amplitude with a conjugate right. Real part of P times u star, u star is the complex conjugate of u right. So, now, u has been taken in this form. So, if you open this if you put u star, u star will be minus I P 2 star into minus P 1 star right. You have to put a minus associated with this factor I and P 2 and P 1 will both be complex conjugated.

And then if you do the multiplication I am sure you will be able to get at this step which is just a product of all the cross terms. So, P 1 and P 1 star will pick you will get a product term with a minus sign because a minus comes here and P 2 and P 2 star is going to be plus, and then there are the cross terms. So, you will be able to follow up with this calculations I am sure. So, you will have to take real part of this quantity, but please understand P 2 into P 2 start is magnitude of P 2 right. P 1 into P 1 star is magnitude of P 1 both of these terms are actually going to be real right. And you see a multiplication with i following, I mean there is a product of these 2 terms with the i and then you have to take the real part of it.

So; obviously, the real part of this 2 terms is going to be 0 because there is a I sitting on top right. Whereas, this term which is the cross term and this is exactly called cross spectra. This is called the auto spectra one into one star is called the auto spectrum of P 1 and P 2 into P 2 star is called the auto spectrum of P 2. And similarly terms of this kind P 1 P 2 star and P 2 P 1 star similarly is called the cross spectra right. So, the cross spectrum is in general complex right.

So, therefore, the auto spectrum does not contribute. It is only the cross spectrum which contributes. And also please remember P 1 P 2 star and P to P 1 star will have the same as I mean there are just complex conjugates of each other right. So, therefore, when you have a complex number and subtract it from is complex conjugate what will happen you will get twice it is imaginary value right. So, there will be a 2 coming here with when you do this substraction, that will get nullified with the 2 in the denominator, and there is a I sitting on it because this is basically the imaginary part of the cross spectra. And this imaginary part will multiply with this I sitting here and you will get a real quantity.

So, it is the imaginary part of the cross spectra, which is going to be important in deciding the intensity. So, here as you see the calculations do not require you to integrate. So, you are saving the integration I mean no matter how good is your electrical integrating circuit, it will have some error. So, using plain and simple mathematical formula you are getting away from those integration related issues you simply have to calculate the cross spectrum associated with 2 signals, and there are very, very robust and accurate techniques by which spectrum can be estimated, for a dual channel measurement you need 2 channels because P 1 and P 2 both has to be measured simultaneously, the phase difference between P 1 and P 2 has to be appropriately captured.

So, you need 2 microphones wherein the phase has to be matched. If there is a delay between these 2 channels if there is a phase difference that is inherent in these 2 microphones then the readings will be corrupted. You need phase match microphones using the phase match microphones all that you have to do is that you have to calculate the cross spectrum or rather estimate the cross spectra accurately using any of the signal processing tools which are well developed. And the final formula therefore, will read out as I elaborated, to be the imaginary part of the cross spectra. One of these tools will get cancelled and therefore, you will have the imaginary part of the cross spectra divided by i omega rho 0 delta R. And the phase matching is very important. And therefore, In fact, this phase matching should be calibrated every time you take the measurement and there is there are ways to do it.

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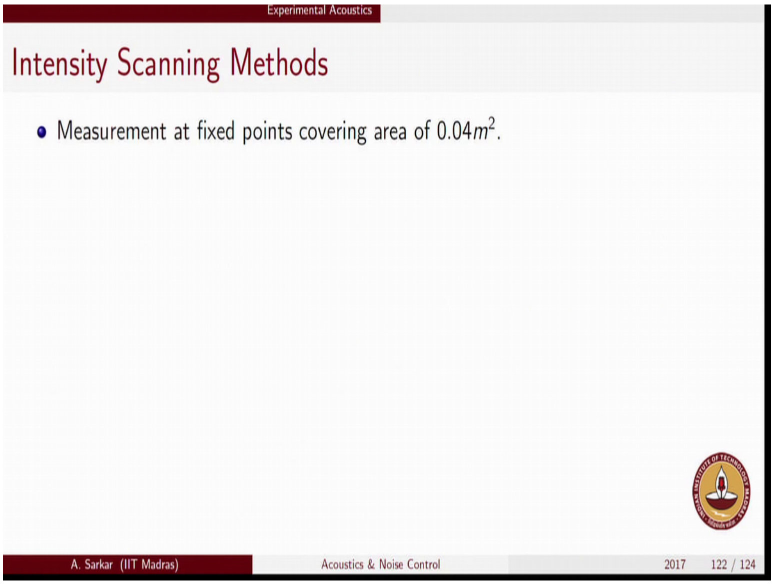

I think we will have to end here, but I hope you get the idea of sound intensity measurements with this. So, hopefully we will be able to complete the experimental acoustics by within a few minutes in the next class.