Acoustics & Noise Control
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Module - 14
Lecture - 19
Decibel Scale

Last class we talked about the Acoustic Intensity, the concept of acoustic intensity was introduced. So, let us have a quick recapitulation of that concept.

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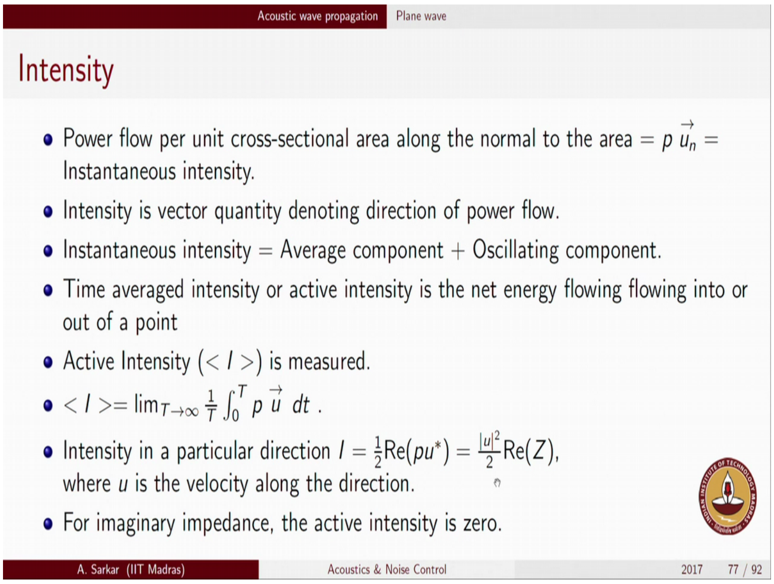

In mostly acoustic intensity or more precisely the instantaneous acoustic intensity is defined- as the power flow per unit cross sectional area along the normal to the area. So, we are contemplating a hypothetical area and we are trying to understand; what is the power that is flowing normal to the direction of this area. So, as was discussed in the last class the instantaneous expression is just force times velocity, and since you are talking about unit cross sectional area the force is basically boiling down to pressure term which is the acoustic pressure.

So, acoustic pressure times the normal velocity, velocity which is normal to the direction of this I mean which is aligned to the normal to this area is u n and p times u n gives the instantaneous intensity. Now you can understand intensity at a point we will therefore be a vector quantity, because it depends upon the area and in the direction of the unit normal of the area under consideration. So therefore, intensity is a vector quantity denoting the direction of power flow.

And what we understood in great details was that this instantaneous intensity could be decomposed into two parts: one part of which is the average component and the other is the oscillating component. The oscillating component will denote a fluctuating power contribution. So, in half the cycle it will contribute to a positive power which is the power that is expanded from one side of the medium to the other. And in the other half the cycle it will just revert to a negative number which essentially denotes the power flow is in the opposite direction.

So, virtually this oscillating component is not of that great significance so we will rather concentrate on this average component and which is called the active intensity. So, time averaged intensity or the active intensity is the net energy or possibly power flowing into or out of that point. So, average component has more physical relevance and more physical implications than this oscillating component. The oscillating component also is is of some significance in a different context which is beyond the scope of this lecture, but more often than not we are going to work with the active intensity rather than the instantaneous intensity.

And it so turns out and we will do that we will touch upon this that it is this active intensity quantity which can actually beyond the measured and the instrument to measure this is called an intensity pro. We will see as to how these measurements can be taken, and what are the instrumentation so hardwares associated with this sort of measurements.

So, it turns out that this average component or the active intensity component can actually be measured and just to denote how this averaging is done. So, it is just a time average if you may so call it. So, we integrate this instantaneous intensity over a time period which is pretty large and divided by 1 by T. So, that is basically the average of this instantaneous intensity.

So, that we will turn out to be as was shown in the last class for a single harmonic contribution we have shown; that we have firstly shown for a special case of a travelling harmonic wave and later on we took a general form of impedance. And we have shown that the active intensity is half of real part of p times u star; where u is the velocity along the direction of interest. And the star here denotes the complex conjugate. Remembered in our phasor representation we have allowed for complex magnitudes and as a result you could have u star which means that you now take the co conjugate; complex conjugate of that complex amplitude.

And if you recall p by u is z so that would mean that p could be written as u times z and u into u star would separate out as magnitude of u square; magnitude of u square being obviously real positive number so it can be extracted out of this real part function. So, what matters is the real part of impedance. As far as the intensity or rather the active intensity calculation goes it is only the real part of impedance that contributes to the active intensity. The imaginary part has got no role to play in this calculation of active intensity.

You will recall that for a travelling wave. The impedance was rho c, the characteristic impedance I should qualify it as saying characteristic impedance; the characteristic impedance is given by rho naught c which is a perfectly real number. So therefore, associated with a travelling wave you had a situation of the active intensity given by u square into rho naughty c divided by 2 which is what we derived in the last class also in terms of p, just p and u relations are inverted here. But, today we shall see certain counterintuitive relations where you will see intensity can be 0 in active intensity I mean can be 0 in certain situation.

So, before doing that let us take a few illustrations. As I said for imaginary impedance the active intensity will be 0. So, we did come across a few instances by now where the impedances are actually not real, but they are purely imaginary. So, for purely imaginary impedance the formula tells us that the active intensity is going to be 0.

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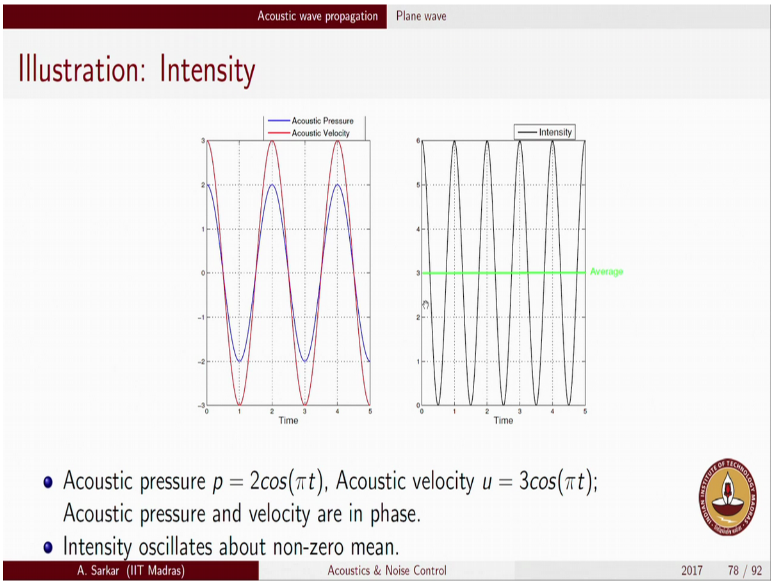

So, let us just take a few illustrative cases to understand what is happening. So, here I have plotted in blue the acoustic pressure and the acoustic velocity. Here you can see that the pressure and velocity are completely in phase does not matter about the magnitude I have just taken to arbitrary magnitudes 3 and 2, just for the purpose of illustration. But I wanted to show you the fact that the pressure and velocity in this illustration has been chosen to be in phase. So, in particular I have used the cosine function 2 cos pi t for the pressure and the velocity has been chosen to be 3 cos pi t, so both of them r in phase.

Accordingly the product of these two pressure and velocity is positive in the first half, again positive in the second half, again positive in this half. So, always the pressure and velocity is product is positive, but it repeats at half the time period. And therefore you will see that, whereas the period of oscillation for the pressure signal was given in a given here by a scale of 2. So, by within this time period of 2 you can now see that the intensity what has been plotted in black is the real in a sorry the instantaneous intensity; instantaneous in intensity has much more rapid fluctuation in fact double the fluctuations as was happening, if you compare it with the red and the blue signals .

So, here we see that the intensity does fluctuate but it remains positive, it is not going to a negative value because whenever the pressure is negative the velocity is also negative which makes the product always positive. And as a result though it fluctuates, in fact it fluctuates at the double the frequency of the pressure or the velocity signal, but the fluctuations will have an average which is positive and over and above this average there will be a fluctuation. So, when we refer to active intensity we will refer to this green line as the active intensity denoting that on the whole there is a positive pressure flow at the point under consideration.

Now we will take. So, intensity basically oscillates about a non-zero mean as is shown by this illustration.

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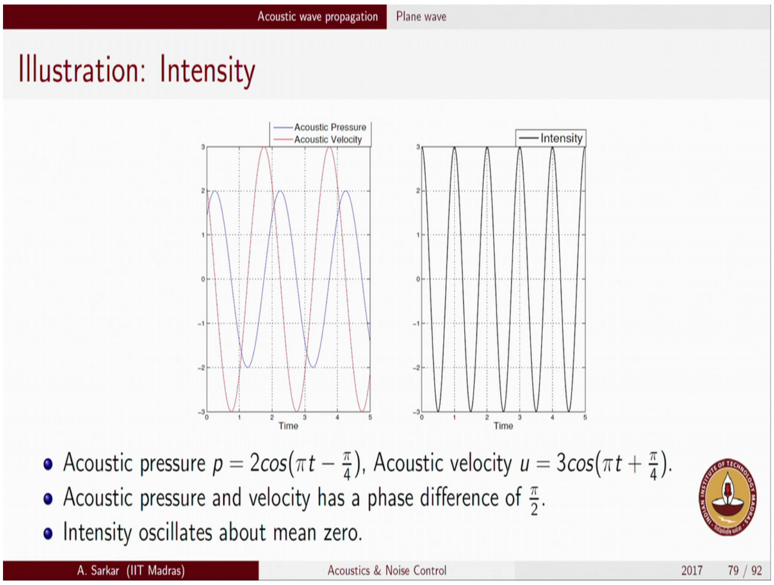

Now, we will take the opposite picture. So, here we have taken the pressure and the velocity signals which are given by these expressions. So, here you can see that there is a phase difference between the acoustic pressure and the acoustic velocity. In fact, the phase difference has been chosen to be pi by 2. So, cos pi by 2 minus pi by 4 is; what is the signal that has been chosen for pressure, for velocity it is pi t plus pi pi by 4 that makes a phase difference in totality between p and u to be pi by 2.

So, when we plot the intensity this is what it looks like. So, here we see that at some time in the first quarter cycle as I illustrated last class through a hand drawing. Here you see in this first quarter of this time cycle both pressure and velocity are positive which makes the intensity positive. But what happens in the next quarter from here to hear the pressure is positive, but the velocity is negative which makes the intensity negative. And again this keeps continuing. So, here what we see in black is the instantaneous intensity plotted against time; the instantaneous in intensity fluctuates at double the frequency, but this time it fluctuates over and average of 0.

As a result in this case there is no time everest or active intensity that is there at this point of consideration, so this active intensity will return as a value of 0. So, the active intensity in this case is 0, because it is oscillating about the mean 0. But the time plot of the instantaneous intensity is not 0, instantaneous intensity definitely has to fluctuate, but it is just there if it fluctuates about 0 on the whole there is no energy that is getting transmitted at this point for the direction under consideration.

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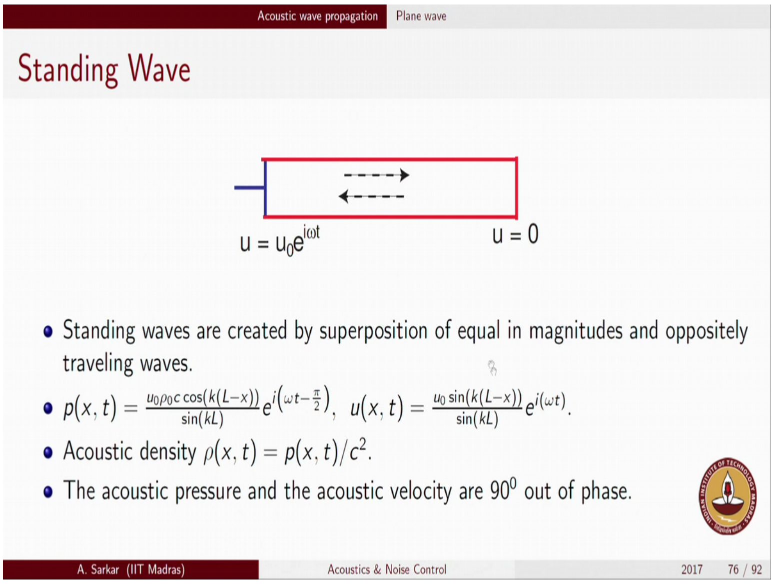

So, you will not recall that we had done examples of standing waves for this duct problem in a few classes earlier we talked about this rigid duct that problem which was excited by rigid oscillating piston at one side and that was undergoing a harmonic excitation. So, we did derive the complete expressions of the acoustic pressure and the acoustic velocity at each and every point.

So, if you recall that derivation you will find that the pressure expression was given in this form, whereas the velocity express is given in this form. Now from this it is pretty clear that the pressure and velocity expression has a pi by 2 phase difference and that is disastrous as far as active intensity is concerns. So, this again tells us that the active intensity associated with this problem is going to be 0.

Another physical way in which you could understand why the active intensity or the time average intensity will be 0 is in the following fashion. You will recall that a standing wave is made up of two travelling waves each of them are travelling in the opposite direction. And if you recall this derivation we had the relation by which we could relate the amplitudes of a and b. It turned out that the amplitudes of a and b are exactly the same. Just that one was given by e to the power i 2 k L times the other. So, a turned out to be e to the power i 2 k L times b, but then e to the power i 2 k L has a magnitude of unity, so therefore the magnitude of a is same as the magnitude of b. And these also make sense because this is rigid termination whatever is the amplitude of the incident wave has to be the amplitude of the reflected wave, except possibly for a phase factor which is e to the power i 2 k L. That also made sense is physically.

So therefore, the fact that you have a standing wave which basically in disguise is two travelling waves of equal amplitude and each of these travelling waves must carry the same amount of energy or active intensity associated with each of these travelling waves has got to be exactly the same. So, whatever is the active intensity conveyed by the light travelling wave is the same as the active intensity that is conveyed by the left travelling wave. As a result in totality you do not have any active intensity in this situation.

So, in totality there is no energy flow that is happening in this situation wherein you have standing wave which basically comprises of two equal and opposite travelling; which is made up of two waves: one travelling in the rightward direction and the other traveling the leftward direction and with the two waves having the same amplitude. At the two waves been of different amplitude then you can expect that there will be some intensity left between corresponding to whichever is the more dominant of these two waves. But it just so happens that since you have a complete reflection condition at the end.

The complete reflection condition induced a reflecting wave which is having the same amplitude. And therefore whatever is the incident active intensity is going to get nullified with the reflective and active intensity. And as a result the total active intensity at any point within this that is going to be 0. This might appear and little counterintuitive for the beginners, but this is how live phase is.

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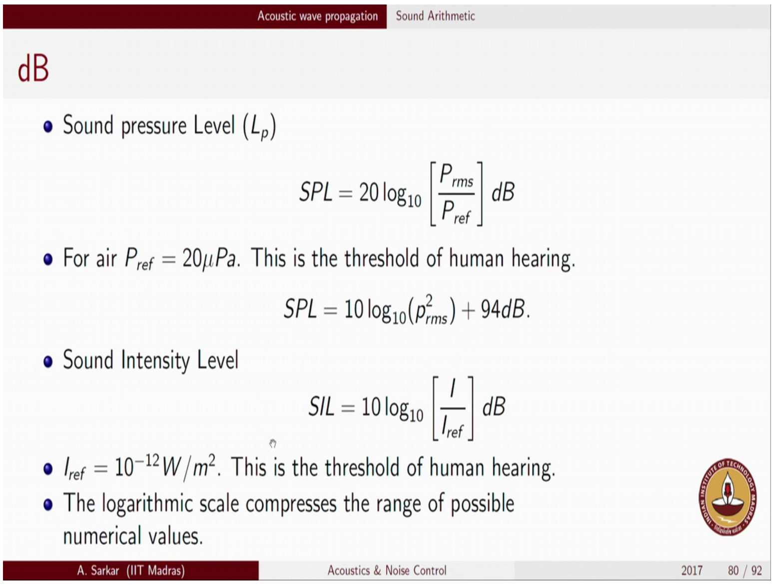

So, with this brief recapitulation of intensity let us now enter into a new topic which is that of the decibel levels. Till now we had been looking at sounds as in the form of acoustic pressure, and accordingly the units associated with acoustic pressure was given as Pascals. But now we will make a more realistic change of variables that which is associated with the human hearing aspects.

So, I will tell you the story as we go along, but just for a opening- the definition of sound pressure level which is at times abbreviated as SPL or at times as L p is given by 20 log 10 of the factor which you see in brackets which is the rms value of the pressure and divided by the reference value of the acoustic pressure. The reference value of the acoustic pressure happens to be 20 micropascal. So, when this it is arrived at by exhaustive experimentation on different human subjects that this happens to be the threshold of human hearing. When different human subjects have been exposed to different kinds of acoustic pressure of different amplitudes it has turned out that we no human could possibly here a sound which is lesser than 20 micropascal.

It is the rms amplitude of a reference sound at which you just tend to here particular sound. There is a slight difference between the rms value and the amplitude value which is what I will talk about in the next slide; I will discuss in great details how to calculate the rms value. So, by now I will just revert my explanation once more.

So, the reference value of 20 micropascal is the rms value of the sound which is just audible to a healthy human being. So, that happens to be 20 micropascal. Now, if you do the usual calculations associated with the logarithm function. So, instead of carrying this 20 as a factor you could make this 10 into 2 and this 2 you could conveyed, you could put it back within this bracket. So, that makes it 10 log 10 P rms square divided by P ref square. And when you do this calculation for P ref by putting this value of 20 micropascal and that means 20 into 10 to the power minus 6 you will get 94. So, what the formula of SPL tells us is 20 log of P rms square plus 94 dB is what we are going to use.

So, similarly sound intensity level is given by a similar formula. Again here intensity refers to active intensity it is not instantaneous intensity. So, we will take the intensity amplitude and divided by the reference intensity amplitude which is again associate that, which is associated with the threshold of human hearing. And this now happens to be 10 to the power minus 12 watt per meter square. You can verify that associated with a reference rms pressure of 20 micropascal you will actually get a reference intensity of 10 to the power minus 12.

In other words the intensity value that we formula that we calculated in the last class was given by P square by 2 rho naught c; rho naught is the value of the ambient here density c is the value of the sound speed in air which is taken as 340 meters per second. So, as we will see the rms value or rather the rms square is basically P square by 2. In our calculation we had shown that if P is the amplitude of the acoustic pressure then P square by 2 rho naught c is what we get for active intensity. So, this factor P square by 2 turns out to be exactly P rms square. So, P rms is given by; with the value of P rms which is 20 micropascals you can recover that I reference is going to be 10 to the power minus 12. So, the reference threshold of human hearing is not a new quantity it is that which is associated with these 20 micropascals itself.

So, the reason why we use this logarithmic scale must be cleared to you. It is to compress this large scale that we use a logarithmic scale and that is not just true in acoustics it is there in: for example, your bode plots you will recall is a logarithmic plot and the reason why we do that is we wish to compress a very large scale, because as was probably discussed earlier also that associated with different sounds that we here in our day to day life as well as in industrial applications you get very very different orders of magnitude of these acoustic pressures. If you had to report it in linear scale you have to cover a large order, a large range.

Therefore, to compress this range of numbers we are using a logarithmic plot, logarithmic function to convert this nicely into a compress scale. And this factor 20 sort of makes us feel good; in the sense that we have numbers which looks like our numbers that we would typically report for our final examination. So, typically the numbers would come out as 60, 70, 80, something which we are pretty accustomed with right from our school level.

So, that factor 20 has sort of been adjusted so that we get to see numbers which are in the range of 60, 70, 80, 90 rarely we will it cross 100; similar to our examination marks it cannot cross 100, but yes it does cross 100 in certain very sort of specialized applications in acoustics which are not definitely a date today operands. But the factor of this 20 is just a factor which has been adjusted so that you get numbers which sort of look athletically more appealing. But it has this is the convention that we have taken for defining the SPL and we have to stick by this convention.

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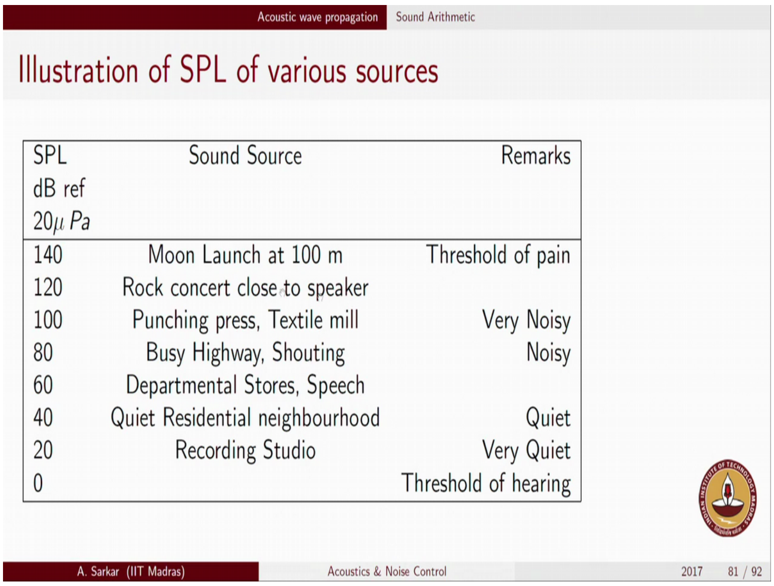

So, before going further as I said there is a range of values that happens for different sound sources. So, here is a typical value of different sound sources in decibels. As I said the threshold of human hearing is 0. So, that is because if you put a P rms to be equals to P ref this quantity within bracket we will give you unity, a log of unity is 0; so that means, 0 disable is the threshold of human hearing. This is also very comforting to note that you will not hear sound with the negative disable, it is not possible just like we would not like to get in our examination some negative marks as our final total. This is sort of way we have defined this formula we are quite assured that we will not get to this threshold of human hearing.

But if you have done and audiometric test ever for yourself or for your relatives if you have accompanied then you will see that most of us who reportedly are healthy human beings do not reach anywhere near 0 we are hopefully 20 maybe 30 decibel is our starting point. And also these values do differ, that is typically what is done in an audiometric test. In audiometric test there will subject you to different frequency is of sound and you will have to sort of indicate whether you are able to hear what is the threshold of your human hearing.

So, it turns out at least for me that at certain frequencies I was able to here only my threshold was 30 decibels. But thankfully the doctor reported that I am a perfectly healthy human being I do not need to worry. So, this 0 decibels I guess is possibly not true for it is the Indian urban conditions, maybe for some arctic population, but typically you do not find any noise probably then here is a custom to this people noise. But one thing is for sure that 0 decibel is a very difficult limit to breach for any human.

But anyway for the recording studio I do not know whether this recording studio would report 20 decimal values, but maybe for a recording studio that is used in let us say our Bollywood probably you will get about 20 decibels. So, this like extremely fable sound.

40 decibels is probably an IIT Madras neighborhood, I am not quite. Departmental stores 60 decibels, now you are going higher busy highway we are going to have about 80 decibels. And now when you enter different industrial factories you are going to hit that 100 decibels. Rock concert is supposed to be 120 decibels. And as I said if you are just about 100 meter away from launch vehicle take off you will possibly here 140 decibels.

But, typically no human is subjected to that sort of a high decibel levels, you are supposed to wear precautionary earplugs if you are subjected to that high levels of noise. So, typically I would say crossing 100 decibel there are some physiological and psychological disorders that are reported, but this is the range of values that we are supposed to get. And as I said that the compression scale that we have used in terms of the logarithmic formula that essentially make sure that we are going to have sound between realistically speaking let say 30 dB to about 110 dB max 120 dB.

So, this is the range of our numbers that we will going to work with in any applications of acoustic. So, that is where the formula, the tweaking in the formula instead of reporting the acoustic pressure in Pascal's. The fact that we are reporting it in decibels it helps us to work with a much lesser range of numbers, but the fact that we have tweak this pressure scale to a disable scale and that two with a logarithmic function that introduces a certain settled changes in the way we perceive our regular arithmetic operations and that is precisely what I wish to talk about in this lecture.

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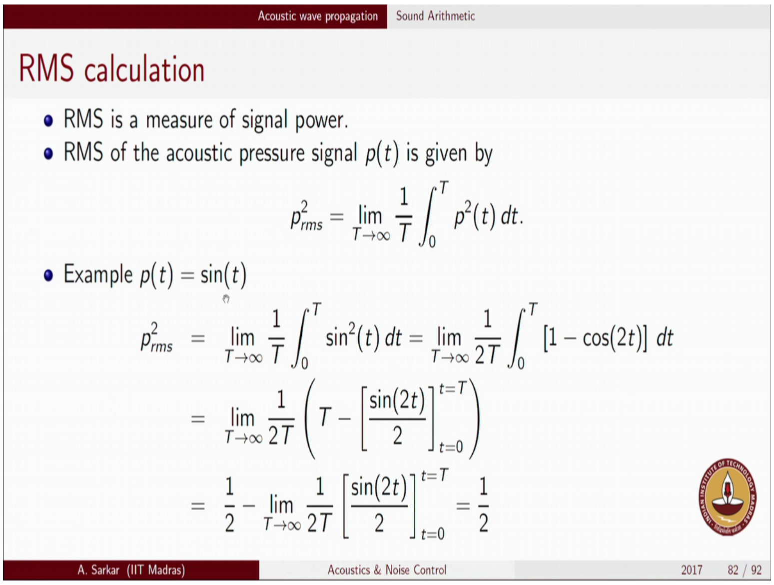

Now let us look at how is this RMS quantity calculated. Those of you who are aware of different signal processing techniques you will know that RMS is actually the measure of signal power. And the way to calculate it is as follows: RMS abbreviation stands for root of mean of square, but when you calculate it is the other way around: it is the square of mean of root.

So, first we square the signal, then we calculate its mean; how do we calculate its mean? We on the squared signal we integrate its value over a certain time duration and then divided by the time duration or the time interval that we have chosen. And we choose this time duration to be sufficiently large when we do that you get to the mean square value. Finally, to arrive at the root mean square you have to take a square root of this and that gives you P rms.

So, I would rather give you the formula for P rms square, because I did not have I think adequate space in this slight to put another square root on top of it. But you can understand the calculation, the easy way to remember the calculation process for root mean square is just do the calculation the in the opposite fashion: first square it, then minute, and finally take the square root of it. That leaves you with the rms value of a signal.

So, this is the formula for calculation of the rms value of any signal, but here we are interested to calculate the rms value of acoustic pressure. But the same formula works whether it is a voltage signal whether it is a current signal velocity signal anything. So, let us take our first example: let us take p t to be sign of t, so just a single frequency calculation to keep life easy.

So, we will do first we will square it then we will integrate it and we will choose the time period of integration to be as large as possible; That is this steps that are there for the rms calculation. And remember this is the formula for P rms square not P rms, the rms itself will be the under root of this quantity whatever we calculate. And then sin square t is given by 1 minus cos 2 t divided by 2, so that part is nice and simple. And then when we integrate this for the first term we get a t, and for the second term we get sin of 2 t divided by 2 and the limits are from 0 to capital T.

The first part of it the limit is pretty simple, because capital T and on the numerator and capital T on the denominator get cancelled out and we get half. And the second part however we looks little bit complicated, but what is your guess about this part? What is the value of this limit? The point is you have to realize this number is going to be bounded right sin of 2 capital T, whatever be the value of capital T, whether it is pi whether it is pi by 2 see if you choose pi by 2 this number is going to be 0; if you choose capital T to be pi by 4 this number is going to be 1.

So, the point is whatever be the choice of capital T this square bracket at quantity is going to be bound it is not going to be 0. I am never said that it is going to be 0. The bracketed quantities going to be bounded, it is a finite number it cannot go out of bounds, whereas the denominator will go un bounded. So, denominator will go large by this limiting process the limiting process will take the denominator to a large value, but the numerator will keep itself bounded which means that the numerator divided by denominator is 0. And that is why you get a half.

So, in other words if you have an amplitude of a harmonic term to be 1 they mean square; not the root mean square the mean square value is half and the root mean square is 1 by square root 2 or 0.707. So, that is precisely the point that if you have at least now you should be clear; that if you have a single harmonic quantity as your signal of interest then the root mean square is given by 1 by square root 2 of its amplitude. So, that is how the rms value and the amplitude is related. The rms value is not the amplitude as I said few moments back, but it is 1 by square root 2 factor of the aptitude value.

Now let us see what happens if there are more than one frequency. So, now, I have taken two sound sources.

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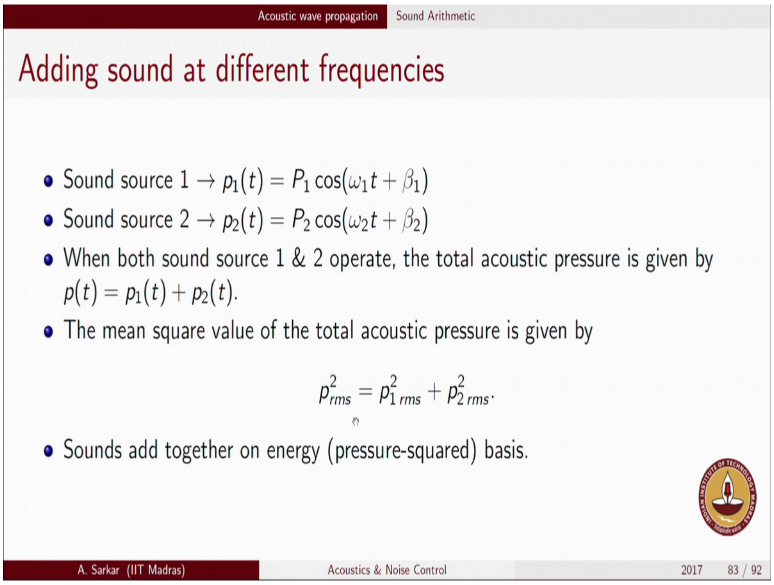

Let us say there are two speakers and each of these two speakers are putting out two different sounds at two different frequencies: one at omega 1 and the other at omega 2. And just to keep things general I have put associated a phase factor of beta 1 and beta 2 associated with this. Now let us see what happens to the root mean square of this combination.

We understand that as for as acoustic pressure goes the two acoustic pressures will add up, because everything is linear that we are studying thus for. So, if one source is put up putting up a acoustic pressure given by p 1 t and the other one is giving acoustic pressure of p 2 t then in totality the sound that will be audible will be p 1 t and plus p 2 t; this is just because of linearity that is there in the very basic differential equation that we are working with.

So, the total acoustic pressure is just given by the sum of it. And now we need to calculate. So, in other words the total acoustic pressure which is hitting our ear drums is just the sum total of this term, but to report our results in decibel level we do not need p of t rather we need the rms value of p of t. Remember p of t obviously depends up with time, but the rms value of the signal is a need constant number associated with that signal.

So, we now need to report the rms value of this plus this. So, towards that in we just simply follow the same process and we will be able to show that it is adding in the in the mean squared series. So, I will do the derivation here.

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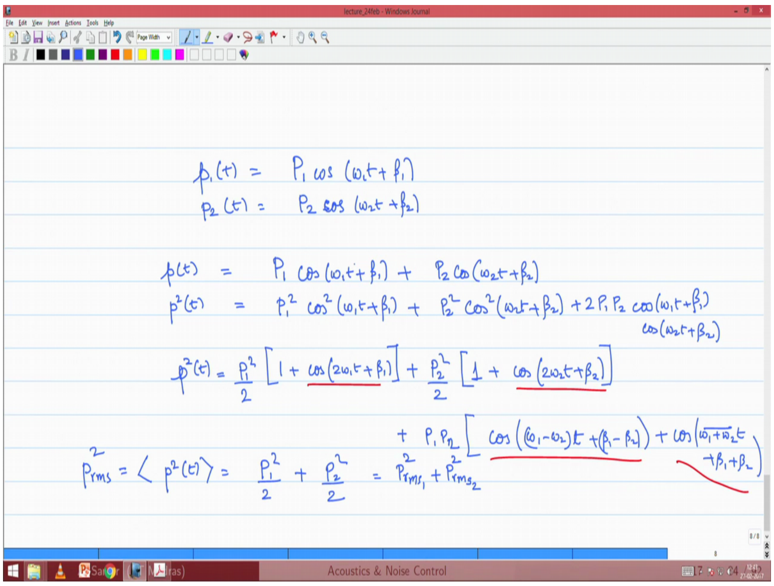

So, we have p 1 of t as capital P 1 sin omega 1 t plus beta 1, and we have p 2 t as capital P 2 sin omega 2 t plus beta 2. I think calculations would be little simplifier if I choose cos, because I like the cos formula instead of the sin formula. Does not matter I can always change between sin and cos by appropriately changing this beta effect. So, p of t is P 1 cos omega 1 t plus beta 1 plus P 2 cos omega 2 t plus beta 2. What is the step? First we have to square it which means P 1 square cos square omega 1 t plus beta 1 plus P 2 square cos square omega 2 t plus beta 2 plus 2 P 1 P 2 cos omega 1 t plus beta 1 cos omega 2 t plus beta 2.

This could be written as using our good old trigonometric formulas: 1 plus cosine of 2 omega 1 t plus beta 1. The second term can be similarly written as 1 plus cosine of 2 omega 2 t plus beta 2 plus P 1 P 2 cosine of omega 1 minus omega 2 t plus beta 1 minus beta 2 plus cosine of a plus b which is omega 1 plus omega 2 t plus beta 1 plus beta 2. This is the P square t value, now we have to find the average of this quantity. So, what do you think will be the average? Please note I have broken it down into trigonometric terms and constant terms precisely for this reason that the average calculation should be made easier.

So, the average of this P square t will be just those associated with the constant terms, the ones associated with the trigonometric terms will not contribute to the average. These terms underlined in red this term, this term, this term, and this term. None of them are going to contribute to the average value they are all fluctuating values. So therefore, we could simply write this mean square term as P 1 square by 2 plus P 2 square by 2. And this by the way is the definition of P rms square. And what is P 1 square by 2? P 1 square is P rms of the first guy square or the mean square value of the first signal right it was already shown that if the amplitude is one then the mean square value is 1 by 2. So, the amplitude is p 1 then the mean square value will be P 1 square by 2.

So therefore, this is the mean square value of the first signal plus mean square value of the second signal is not rms, but rms square. So, what we are seeing is a very important effect which is not just through for acoustics, but any general signals in at least in the case of linear signals in linear systems that you have a mean square addition. If you have a collection of more than one frequency in your signal then it is the mean square values which will get added up. And that is precisely the form of energy conservation that we have in or power conservation if you mean so call it.

Another way to interpret it is through Parseval's theorem. Parseval's theorem is exactly going to give you the same result. But in whichever way you interpret this and here I have just taken a very fundamental approach without relying anything on signal processing. But this result as I said is a fairly general reserve which is true for not just acoustic pressure signals, but a lot of other signals where in your assumption of linearity holds.

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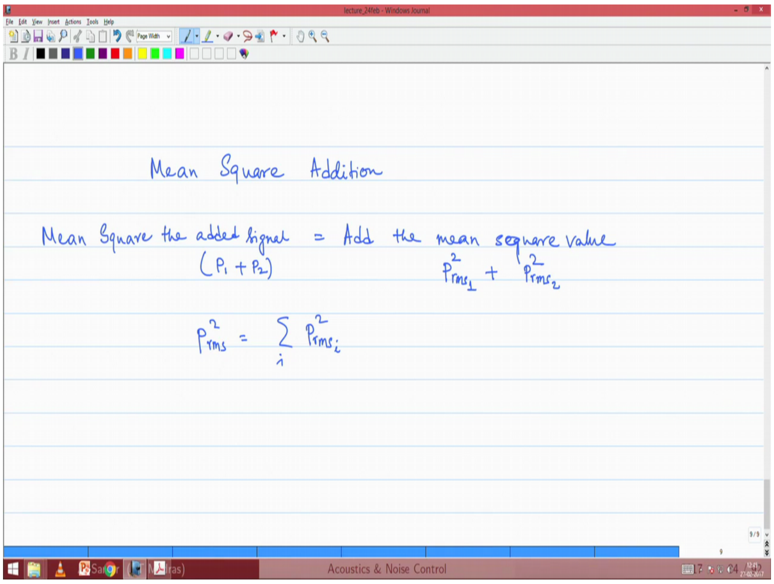

So, for all such cases you will get to see the phenomena of mean square addition, which means you add the mean square values; you first find the mean square values and then add it or you mean square the added signal. So, these two operations of mean squaring and adding can be commuted; mean square the added signal. The added signal is P 1 plus P 2. Then we calculate the mean square of this added signal and that turns out to be same as P rms 1 square plus P rms 2 square.

So, the important thing is that you have mean square addition, we have shown this for two frequencies, but you can generalize this situation and carry this over to multiple frequency is also. If you have three frequencies what will happen is that there will be few more crossed terms and there will be a P 3 squared terms coming exactly the same derivation will go through.

So, effectively you have P rms square is P rms i square summation over i. So, this is how the mean square value of a signal can be computed wherein the signal has already been composed into its frequency component. That if you have multiple frequencies then for each of those frequencies you calculate its mean square values then at them up and this added mean square values is actually the mean square value of the total signal which is heating your ear drum.

So, when we say 140 dB is the sound of that space vehicle launch at 100 meter from the space vehicle; obviously we are not saying the space vehicle is emitting 1 frequency it has a emitted so many different frequencies, but using this operation we are now able to calculate the rms value of the total signal. The rms value of this added signal is equals to the addition of the mean square values of each of these frequency components. And that is the mean square value which is used in the dB calculation formula.

That is it for today.