Acoustics & Noise Control
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Module - 11
Lecture - 16
Near Field Acoustic Waves

In the last class, we talked about fractional waves or bending waves that were generated in, that is possible to be generated in a beam.

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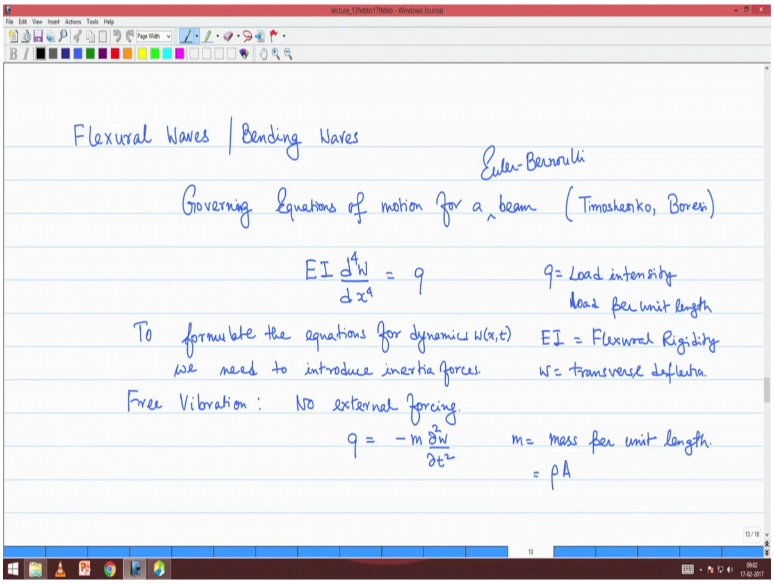

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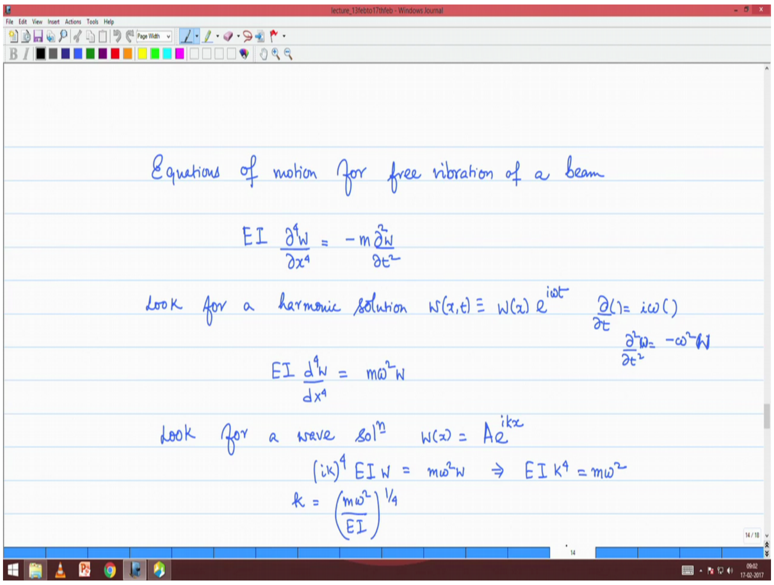

So, we started with the governing equations of beam and found out that at any frequency, it is possible that the fractional waves will be induced in a beam type of structure.

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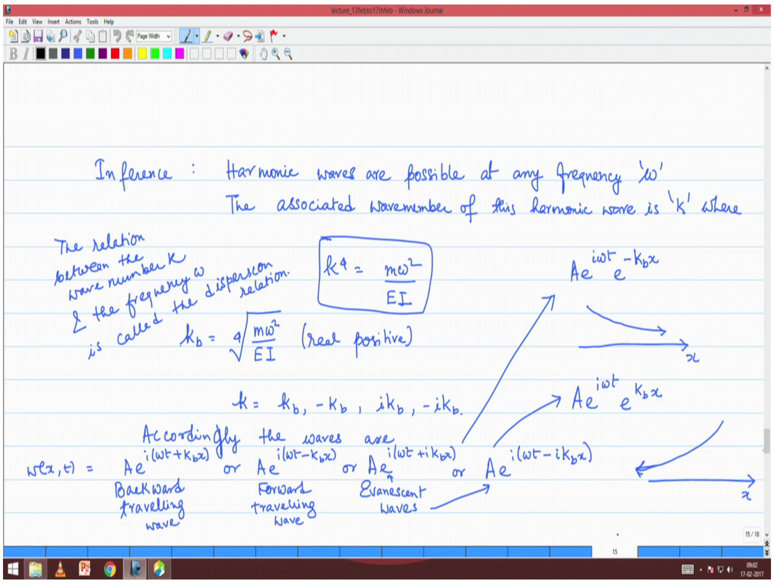

We found a very important result that in contrast to the acoustic waves or one-dimension acoustic waves, we have rather two different types of fractional waves; one is a travelling wave and the other is the evanescent wave. Evanescent wave is generally associated with the imaginary wave numbers, whereas the real wave numbers are associated with the travelling components, right.

So, today we shall see certain aspects of how these fractional waves once they are induced in the structure, then that leads to acoustic radiation. So, let us see what we do today.

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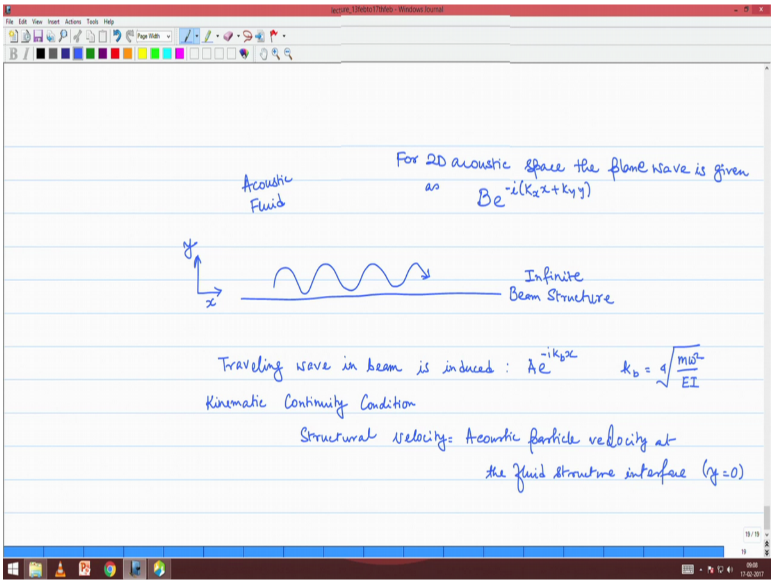

So, here is our beam structure and again at present we are taking this to be infinite, right. So, we have an infinite beam structure and we are assuming that there is a travelling wave which is somehow induced in this beam structure because something is happening upstream and that leads to a travelling wave. So, we will say that this travelling waves, so there is a travelling wave in beam is induced and just to keep things specific, we can call that a travelling wave of an amplitude a, but now since it is a travelling wave and it is also travelling in the positive direction, the travelling wave has to be of this form, where K b is fourth root of m omega square divided by E l, right.

So, there has got as per this hypothesis of this problem, we are saying that the structure is bearing a harmonic travelling one-dimensional wave right, but now as opposed to usual codes in structure dynamics where we assume that the structure is vibrating in vacuum here, we have the structure on one side of the structure, we have the acoustic fluid, right. So, these vibrations are supposed to be conveyed into the acoustic fluid and those oscillations of this structure should induce oscillations in the acoustic, neighboring acoustic fluid particle which should get communicated all the way up to our ear drums leading to the perception of sound, right.

So, therefore, what we are looking for is that because of these harmonic fractional waves which are induced in the beam, what is the acoustic pressure filled in the fluid. This is the very fundamental problem because we know that the vibration of the structure definitely leads to perception of sound, but the question is how is the vibration of the structure getting conveyed into the fluid and leading to the perception of sound. That is the question that we will try to answer through this simple exercise. So, accordingly what we do firstly is that we will mark off our coordinate axis as x and y. We had already said that the travelling wave in a beam is A e to the power minus i K b x.

So, the point is at we will again appeal to our usual continuity conditions. So, the kinematic continuity conditions here would be stated in the following manner that this structural velocity must be equal to the acoustic particle velocity at the fluid structure interface. The fluid structure interface is exactly this y equals to zero condition. So, at y equals to 0, you must have the structural particle velocities matching with the fluid particle velocities.

The displacements of the structural particle velocities should match with the displacement of the fluid particle velocities because the structural particles can neither penetrate into the fluid particles nor can it lead to some kind of a vacuous opening between the fluid and the structure. So, this is called what kinematic continuity condition is. It is a very obvious condition and it will keep coming back in various applications of both vibrio acoustics as well as in a general problem of fluid structure interaction. So, this is an important condition.

Now, in this problem we are expecting this is the two-dimensional problem because the structure is 1D. The acoustic fluid is in a two-dimensional space. So, we already know that in a two-dimensional space, we are expecting solutions in the form of, so for 2D acoustic space, the plane wave is given as I will call this b e to the power minus i K x x plus K y y, right. So, if K x and K y are both positive, then the wave will look in this fashion. If K x and K y are both negative, then the wave will look in this fashion, right. If one of them if let us say K x is positive, but K y is negative, then how will it look like? If K x is positive, but K y is negative, then it will look like this, right. The wave will look like this.

So, let us do this carefully. So, again let us draw the picture. This is the beam structure.

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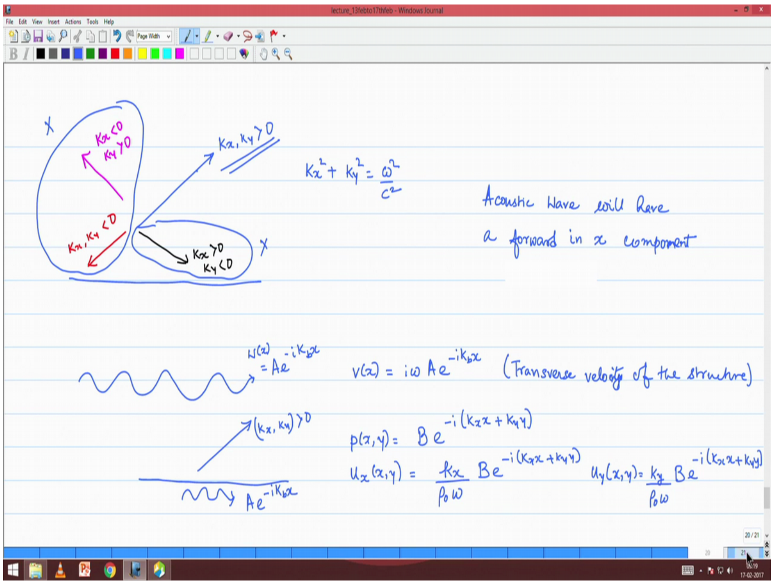

So, if K x and K y are both positive, then the acoustic wave, plane wave that is going out that is being set out in the fluid is going to be looking in this fashion in red. We can mark the opposite situation, where both K x and K y are negative. If K x is positive, but K y is negative, then it will have this situation. So, this is the situation when K x is positive and K y is negative and finally, we will have this situation where you will have K x as negative, but K y as positive.

So, these are the four possible plane waves which could go out in the acoustic space, but in all these situations, you must have K x square plus K y square to be equal to omega square by c square, but let us think carefully that out of these four waves if we can rule out certain possibilities, right the bending wave that was going in the structure. So, the bending wave in the structure is having the form a e to the power i minus i K b x as we wrote in the previous page. So, the bending wave has this form and K b is positive, but then if this is the pattern of the bending displacement, then what is the profile for the bending velocity.

The bending velocity I can call them as, call it as b x is given by i omega a e to the power minus i K b x, right. So, the bending wave in terms of its velocities rather that transverse velocities of the particles on the beam will have a functional dependence on x which is given as e to the power minus i K b x, right.

Now, as I said i kinematic continuity condition you have got to match the conditions at the interface. So, the velocity of the acoustic fluid at the interface which is y equals to 0 plus right from the positive side if you look at the acoustic particle velocity along the x direction, that must also bear a functional dependence of exactly the same form which is e to the power minus i K b x. In other words, the wave number associated with the x component should be same as I mean should have the same feature as the wave number associated with the bending wave, right because you have to match the continuity condition at the interface.

So, therefore, what we see is that the acoustic particle velocity, the acoustic wave will have a forward in x component which means I am taking the 2D acoustic plane wave as b e to the power minus i K x x plus K y y, right. So, with K x and K y both 0, I am going to get this forward sign here, the wave to be travelling in the forward x direction as well as the forward y direction, right.

Now, this is not yet ruled out, but these two possibilities, therefore are ruled out which will have a negative travelling component in the x direction. It has to have a positive travelling component in the x directions. So, these two possibilities are ruled out, right. So, K x must, it must travel in the positive direction is what we are getting right now between these two components.

Again if you look carefully what is happening is that here it is the black wave is actually coming from infinity towards the structure whereas, the blue wave, the one with K x and K y both greater than 0 is going from the structure towards infinity. So, this is actually an outgoing wave. Outgoing in the sense that it is going outward from the structure into the region of the fluid, whereas the wave which is shown in black is coming from infinity and hitting the structure; it is like an incident wave on to the structure here. We are taking this perspective that we are trying to evaluate the wave which is radiated due to the vibration of the sound.

So, the physical cause of the acoustic wave is the vibration of the beam structure, right. So, therefore, the physically plausible condition would be this wave number because that will lead to an outward wave whereas, the other one will lead to an inward wave. That inward wave we have seen time and again, we are ruling out the possibility of inward waves in an infinite medium because inward waves if at all it happens, it will happen because of reflection, but we are at present assuming that the acoustic fluid is infinite in its extent. So, there is no possibility of reflection. So, that rules out this combination also. So, therefore, the only possibility that we will look at is that this is the structure and this is the acoustic wave with components K x and K y greater than 0.

So, again to repeat myself the bending wave is denoted by as a e to the power minus i K b x and the acoustic plane waves p x, y can now be written as b e to the power minus i K x x plus K y y, right because this is the only component which seems to satisfy both the condition that is it is able to lead to a to a condition, where the kinematic continuity conditions can be matched at y equals to 0 and it is also leading to an outward wave rather than an inward wave.

The red and the black wave that is shown in this diagram is an inward wave whereas, the magenta and the blue are the outward wave, but the magenta wave will not satisfy the condition of the interface which demands that you at the interface, you must have a wave which goes in the positive x direction and not in the negative x direction. The component of the magenta wave is travelling in the negative x direction. So, that also is ruled out.

So, therefore, the only possibility that is left is p x, y must be this situation. So, now again u x which is the part acoustic particles velocity can be determined in a similar fashion as we derived in the last class, K x divided by rho 0 b e to the power minus i K x x plus K y y, right. So, that is the particle velocity associated with this acoustic wave, but this particle velocity is in the x direction. Particle velocity is a vector and it has its direction. We are looking at x direction. So, what we must have is that the particle velocity in the y direction which we we call is u y x, y that will also be given as K y rho 0 omega b e to the power minus i K x x plus K y y, right.

Now, y equals to 0 and the normal velocities should match, right because the tangential velocities would not match because we are assuming an inviscid acoustic fluid, but the normal velocity should match. What is the normal velocity at y equals to 0? What is the direction of it? It is perpendicular to the direction of our beam structure which is in the direction of y.

So, by kinematic continuity condition we must have have u y for all axis, but y to be taken as 0 plus, just it is at 0.

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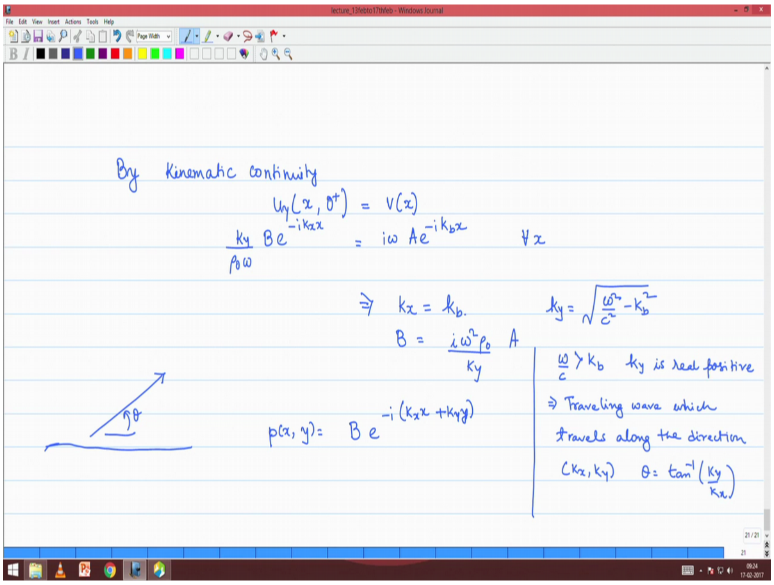

On the positive side of 0 to know to sort of demarcate that it is the point corresponding to the acoustic fluid and that must equal the structural velocity which is v x. Remember v a w x is the transverse displacement of the structure. It is the displacement which is occurring normal to the axis of the beam. So, it is a transverse displacement v x. Similarly it is a transverse velocity of the structure. So, we are basically equating y directional velocity of both this structure as well as the fluid; so the y directional velocity as a function of x. So, v as a function of x is this and this is the structural velocity exactly at the plane y equals to 0 which corresponds to the structure, right and that must match the plane which is just neighboring to the structural plane, but that plane comprises of the acoustic fluid particles. So, that is what we are balancing.

Now, if u y x 0 plus if we now put in this formula, if we bring y equals to 0, what we will get is the following K y by rho 0 omega b e to the power minus i K x x, right. That is all that remains because y has been put as 0, right and that must be equal to i omega A e to the power minus i K b x. This is what we demand and this must be true for all x. The only way to make this happen is to choose K x equals to K b. There is no other possibility can this equality be satisfied for all axis. It can be satisfied for some axis, but it cannot be satisfied for all axis unless you have the condition that K x must be equal to K b.

So, therefore, K x must be equal to K b is one condition and the amplitude can also be determined as i omega square rho 0 by K y into a i omega square rho 0 divided by K y is the associated amplitude, but then if K x equals to K b, then what is K y? K y is square root omega square by c square minus K b square, right. So, therefore associated with this, the wave which we started off with is b e to the power minus i K x x plus K y y where K x is equal to K b and K y is as it gives. So, all this undetermined constants b K x K y have been determined at this stage, but the physical interpretation of this equation is what I want to emphasize on.

There are two conditions that we should look at one condition is when omega by c or the acoustic wave number if it is greater than K b, then no issues. K y is real positive. We will rule out the negative K y's because the negative K y's will be associated with an incoming acoustic wave that is physically in plausible. So, that does not bother us.

Omega by c greater than K b under this condition, K y is real and we will choose the positive sign out of it which simply means that a under this circumstances we are going to get a travelling wave which travels along the direction given by the wave number vector. So, K x, K y basically reveals the direction of the wave or in other words, theta is equal to tan inwards K y by K x will give us the direction of this wave. So, this is our structure, this is the acoustic wave and this is theta which is tan inwards K x by K y by K x. This is all nice and simple, but what happens if we have the other condition that is omega by c is less than K b. So, that condition is equally interesting.

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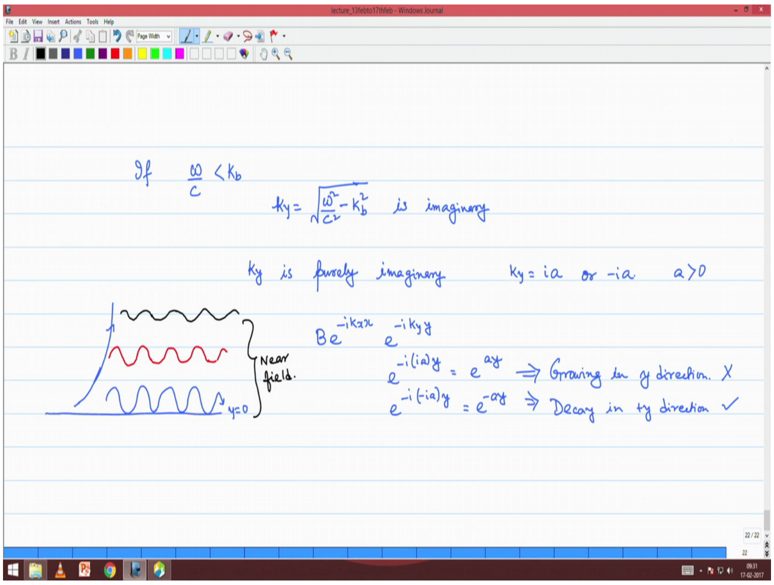

So, if we will have omega by c to be less than K b, remember K b is the bending wave number and as I said the bending wave is generated by a process which is set up in the structure. The structure has some excitation on it. Because of those structural excitation, there is a bending wave which generated within the structure. So, that bending wave number is completely independent of what is happening in the acoustic space, right.

So, we could have a situation where the bending wave number of the structure can be lesser than the value omega by c which incidentally is the acoustic plane wave, wave number associated with the acoustic plane wave or it could be the other way round when K b is less than omega y c. If K b is less than omega by c, the associated K y is real positive and we have a plane wave condition all that is verify, but if then K y in this condition if K b is greater than omega by c, then we will have omega by c omega square by c square minus K b square.

This is imaginary, right. You cannot have a real number for it because omega square by c square is less than K b square. You will not be able to get a positive number sitting inside the under root sign. There has to be a negative number and therefore, there is no possibility that the associated K y will now be a a real positive sign which is associated with the travelling wave, right.

So, here you will get K y is purely imaginary because it is square root of a negative number, right. Now, what is the sign that we are going to choose for this imaginary number? Is it positive imaginary or is it negative imaginary? To look at that let us look at it carefully. B e to the power minus i K x x is not a problem. The next part of it is minus i K y y, right. So, K y by now we know has got to be either i times some number a or minus i times the same number a and a is positive, right.

So, the point is which number should I choose. What happens if I make the first possibility? If I choose the first possibility, it is minus i times i a into y which gives us as e to the power a y minus i square is plus 1. So, that goes e to the power a y. So, what is the characteristics associated with this? This is growing in the y direction. So, this implies growing in y direction. What is associated with the other possibility is, e to the power minus i into minus i a y. This would lead to minus and minus will be plus and then, i square will be minus. This is minus a y and a is positive.

So, this will lead to decay in plus y direction, right. Now, again we will have to appeal to our physical reasoning to choose between these two signs. This is physically in plausible because it is leading to a growing acoustic pressure wave as you go further and further away from the structure. Remember the structure is at y equals to 0, right. So, it is very physically infusible to contemplate the situation that you will have a very high acoustic pressure as you go further from the source, right.

So, this is associated with the growing wave solution, but that is not physically plausible. The other solution therefore has to be taken in which is a decaying wave solution. What is a decaying wave solution? The decaying wave solution implies that this is the structure which is y equals to zero plane and along the direction of y, the acoustic pressure amplitude keeps on decreasing, but along the x direction, it remains draw better diagram. So, at x at y equals to 0 along x, this is how the plot would look like, but if I travel a little up to a positive value of y, then I would get something like this.

So, I hope you realize that the amplitude of the red wave is lesser than the amplitude of the blue wave right, but the wavelength I mean peak to peak distance in the red and the peak to peak distance in the blue is same because the K x value did not change across the different layers of y, but the K y value seems it is leading to a decay that has caused actually a reduced amplitude of this sinusoid as you travel upwards in the direction of y, right. So, at the end of the day what is it that you will get it? So, by the time it has gone quite a few, quite some distance, what will happen is that these amplitudes will become extremely low to be of any consequence, right. It is extremely low and therefore, what we will say is that this sort of a response will be present only in the near field. So, this is any significant acoustic pressure is there only in the near field. Once you go far away and by far away I mean exactly this condition. So, if a y, right what is a? A is the factor which is leading to the exponential decay in the y direction.

So, if a y is taken as let us say 10, if a y value is 10 or you choose a value of y such that a y value becomes 10 which means y is 10 by a, in that case you are going to get a response which is e to the power minus 10 times that which is happening at this structure and e to the power 10 is pretty large number, right. I mean it is e to the power minus 10, I am sorry is a very small number.

So, therefore the response would have gone down by a huge factor, right from this structure to a distance let us say y which is given by 10 by a is a huge reduction in the acoustic pressure response and that is happening naturally. You are not doing anything about it, but it is just that the acoustic fluid is not being able to convey this structural disturbance within its domain.

The acoustic disturbances are concentrated within a very narrow region or what we call as near field region, away from the near field region, the responses are too feeble to even start accounting for it or do analysis for it. So, therefore we say that these are very near field waves and this is exactly the evanescent character that you are now seeing even in the acoustic domain, right.

In the last class, we talked about an evanescent wave characteristics from the perspective of the fractional wave. There we saw that the fractional wave can lead to a travelling wave component. It can also lead to a decaying wave component. So, the decaying wave component was also alternatively qualified as a evanescent wave characteristics. Here we have an evanescent wave, acoustic wave characteristic which is coming out in the y direction and not in the x direction because it is coming out in the y direction, we are calling it as a near field wave because all the activities seems to be concentrated in the values of y which are near the structure.

The structure is at y equals to 0. For small values of y, you will get an appreciable acoustic pressure response, but soon enough as you travel to distances y which are greater and you are getting into a condition, where a y is becoming large, you are getting a sharp fall in the acoustic pressure by itself and this is what is needing to a near field acoustic wave or evanescent acoustic wave in the y direction.

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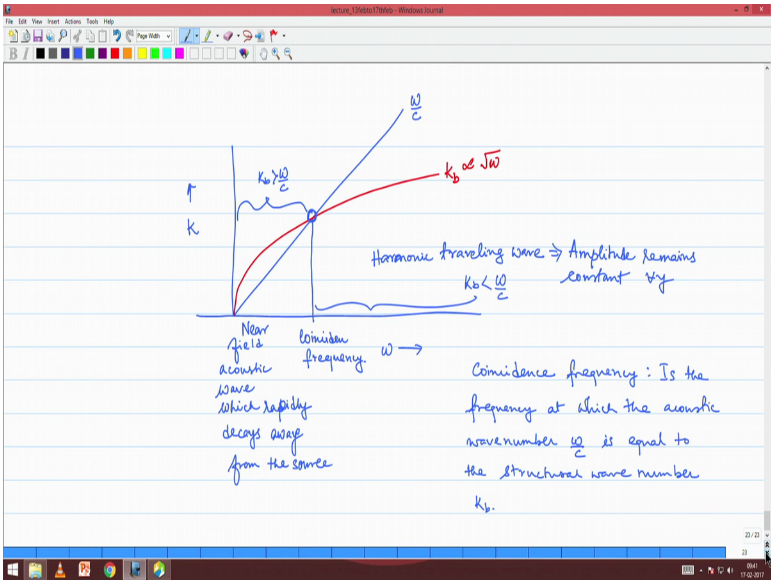

So, therefore to summarize what we have done is that we have found two conditions. So, I will again draw this graph. This is the graph of omega by c and omega by c is the acoustic wave number, right. So, I am drawing the dispersion curves omega by c is associated with the plane acoustic wave, right and the other graph is associated with K b which is the bending wave and it is proportional to square root omega, right. So, what we have seen is that for situations where K b is less than omega, this situation we are going to have a radiation region, right.

The condition of radiation is met that is you are going to get a plane wave and this plane wave is going to have the same amplitude throughout. There is no decay associated with plane wave, with this plane wave as it travels. So, this is nice and fine. So, the radiation condition will be met in this region which is more, I mean sorry here the the region were you will have K b to be less than omega by c, right where as in the other region where K b is greater than omega by c, there you are not going to have a travelling wave component, but you are only going to have a near field acoustic wave, right.

So, in this region you are going to have a near field acoustic wave which rapidly decays away from the source. By source I mean the structural vibration or the structural bending wave which is source, right. The source of the sound is the structural bending wave, but then this rapidly decays, right. So, that is a unimportant observation where as in the region where you have K b to be less than omega by c, you will have harmonic traveling wave and here you will have the amplitude remains constant for all y.

There is no drop in amplitude, there is no growth either, but the amplitude for all points within the acoustic fluid is got to remain same. Even if you are sitting in infinity, you are going to get an appreciable amount of sound, right. So, this is what is resulting in this characteristics and this frequency where things are just merging the acoustic wave number is matching with the bending wave number is called the coincidence frequency. Coincidence frequency is the frequency where the acoustic wave number when c is the frequency at which the acoustic wave number omega by c is equal to the structural wave number K b.

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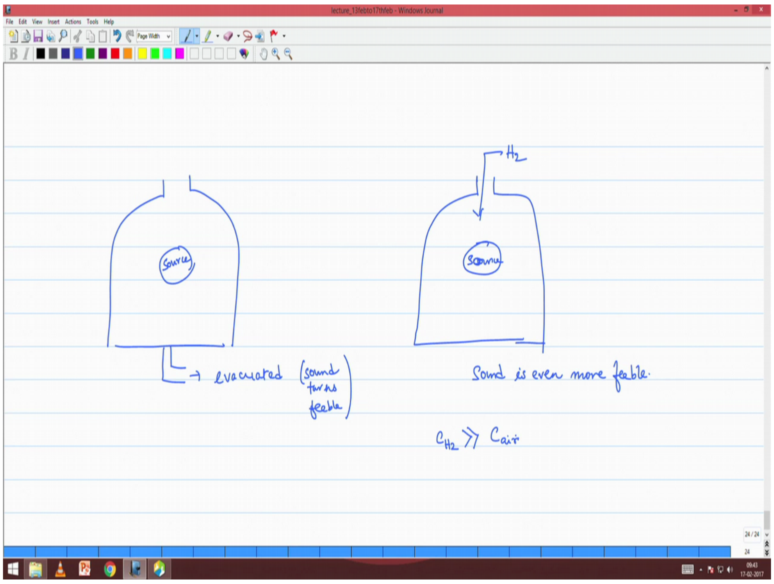

Now, let us look try to interpret that modified bell jar experiment as I told you. So, as I said that in the bell jar experiment, you had a sound source which is inside a certain bell jar. So, this is the source of sound, right. In the first step you evacuated this. So, when you are evacuated, then the sound turns feeble. In the second step of this experiment which is now the modified bell jar experiment, you put back hydrogen in the same bell jar. So, now you can no longer say that the sound is turning feeble because there is no material, right.

Definitely sound requires material for its propagation and the bell jar. The classical bell jar experiment demonstrates that a material is required for the propagation of the sound, but then if I replace this evacuated chamber with the source and everything is same, but I pump in hydrogen, right and now I rest out the atmospheric pressure within this chamber. So, it is the medium there, but just that the medium has been replaced by hydrogen instead of air. Here it turns out that the sound is even more feeble.

Let us understand why this is happening? It turns out that the sound speed associated with hydrogen is much greater about five times than with the sound speed with air, right. So, what is happening let us draw these dispersion characteristic graphs once more.

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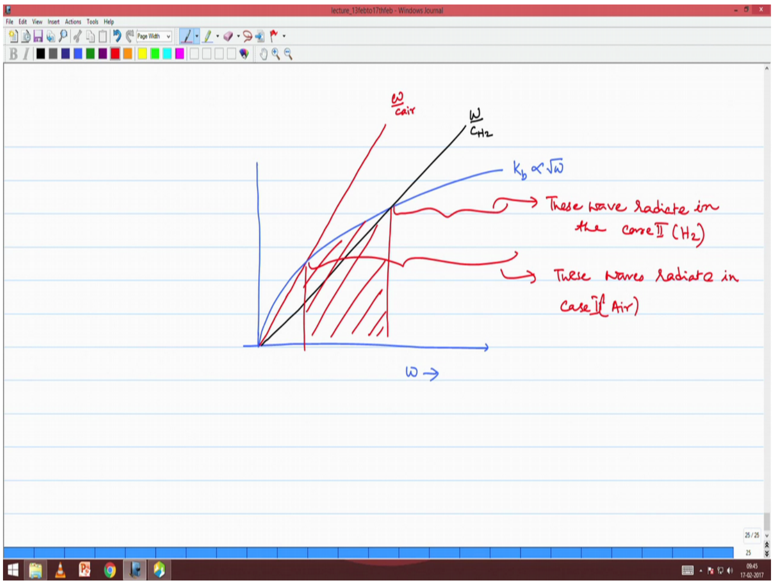

So, the structural wave will assume remains the same. There is no change in the structural characteristics. Accordingly whatever waves are there in the structure that is not affected whether you put hydrogen, whether you put air, whether you evacuate the chamber does not matter. So, this K b graph which is proportional to square root omega remains the same, right. When you have air, you had an omega by c which looks like this. So, this is omega by c air right, but now you know that c of hydrogen is more than c of air which means the omega by c of hydrogen plot will be always lower than the omega by c of air plot because c of hydrogen is more the denominator being, more the factor omega by c will be lower, right.

So, let us plot in graph what that would look like. So, omega by c of hydrogen would look like this, right. So, in the first case we had how many waves? I mean what would be the waves which would radiate the waves? All these waves would satisfy the coincidence condition in the first case whereas, in the second case only these waves are going to satisfy the coincidence condition. The radiation condition will be met only by these waves. So, these waves radiate in the case 2 whereas, here these waves radiate in case 1 which is air and case 2 is hydrogen, right.

Therefore, now you realize that quite a few waves are not going to contribute these regions. All this bending waves are rendered ineffective when you are refilling the chamber with hydrogen because these waves which are shown in these halves portion are no longer able to meet the radiation condition and though they are leading to the same vibration of the sound source, that vibration is not producing any acoustic radiation far away from the source.

By far away you mean one means further away from the near field of the acoustics and it so happens that the near field conditions I mean our ear it happens to be in the far field and not in the near field because finally you are outside the bell jar and the conditions will be met, such that you are not in the near field, but in the far field. So, there is a rapid decay associated with these waves. These waves will rapidly decay out as it travels away from the source, right and your ear is away from the source and that is why the sound is turning more feeble than it was with air right and this is very important observation in vibrio acoustics.

This will end the class here for the day.